Calculus 2 summary
By : hanan alawawda

CLUB

Calculus (2)

Trigonometric integrals :

$u = \cos x$ $\quad \sin^2 x = 1 - \cos^2 x \Leftarrow$ اذا قوة الـ sin كانت odd

$\cos^2 x = 1 - \sin^2 x \Leftarrow$ اذا قوة الـ sin كانت even ، odd cos

$u = \sin x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$, $\cos^2 x = \frac{1 + \cos 2x}{2} \Leftarrow$ اذا قوة الـ sin و الـ cos كانو even

$$\sin mx \sin nx = \frac{1}{2}\left(\cos (m-n)x - \cos (m+n)x\right)$$

$$\sin mx \cos nx = \frac{1}{2}\left(\sin (m-n)x + \sin (m+n)x\right)$$

$$\cos mx \cos nx = \frac{1}{2}\left(\cos (m-n)x + \cos (m+n)x\right)$$

$$\sqrt{a^2 + x^2} = a\sec\theta \rightarrow x = a\tan\theta \rightarrow \theta = \tan^{-1}\frac{x}{a}, \quad \frac{-\pi}{2} < \theta < \frac{\pi}{2}$$

$$\sqrt{a^2 - x^2} = a\cos\theta \rightarrow x = a\sin\theta \rightarrow \theta = \sin^{-1}\frac{x}{a}, \quad \frac{-\pi}{2} < \theta < \frac{\pi}{2}$$

$$\sqrt{x^2 - a^2} = a\tan\theta \rightarrow x = a\sec\theta \rightarrow \theta = \sec^{-1}\frac{x}{a}$$

$$, \quad 0 \le \theta < \frac{\pi}{2} \text{ if } \frac{x}{a} \geq 1$$

$$, \quad \frac{\pi}{2} < \theta \le \pi \text{ if } \frac{x}{a} \le -1$$

Remember :

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

$\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

$\tanh^2 x = 1 - \operatorname{sech}^2 x$

$\coth^2 x = 1 + \operatorname{csch}^2 x$

$\cosh x = \sqrt{1 + \sinh^2 x}$

$\sin^2 x + \cos^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= 2\cos^2 x - 1$

$\sec^2 x = 1 + \tan^2 x$

$\csc^2 x = 1 + \cot^2 x$

$\csc x = \dfrac{1}{\sin x}$, $\sec x = \dfrac{1}{\cos x}$

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

	30	60	45	0	90	180
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	0	1	0
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	1	0	-1
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1	0	—	0

Remember :

$\sin x \; dx = \cos x$, $\cos x \; dx = -\sin x$

$\tan x \; dx = \sec^2 x$, $\cot x \; dx = -\csc^2 x$

$\sec x \; dx = \sec x \tan x$, $\csc \; dx = -\csc x \cot x$

$$\int \ln x = x \ln x - x$$

$$\int e^x = e^x \, dx$$

$\frac{1}{\infty} = \text{zero}$, $\frac{1}{0} = \infty$, $n^\infty = \infty$, $0^\infty = 0$,

$e^\infty = \infty$

$$a^x = e^{x \ln a}$$

$$a^x \, dx = a^x \ln a \; dx$$

$$\int a^u \, du = \frac{a^u}{\ln a}$$

$$\int \tan u \; du = \ln |\sec u|$$

$$\int \cot u \; du = \ln |\sin u|$$

$$\int \sec u \; du = \ln |\sec u + \tan u|$$

$$\int \csc u \; du = -\ln |\csc u + \cot u|$$

Remember:

$(\sinh u)' = \cosh u$

$(\cosh u)' = \sinh u$

$(\tanh u)' = \operatorname{sech}^2 u$

$(\coth u)' = -\operatorname{csch}^2 u$

$(\operatorname{sech} u)' = -\operatorname{sech} u \tanh u$

$(\operatorname{csch} u)' = \operatorname{csch} u \coth u$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$$

$\int \sinh u = \cosh u$ $\qquad$ $\int \cosh u = \sinh u$

$\int \operatorname{sech}^2 u = \tanh u$ $\qquad$ $\int \operatorname{csch}^2 u = -\coth u$

$\int \operatorname{sech} u \tanh u = -\operatorname{sech} u$ $\qquad$ $\int \operatorname{csch} u \coth u = -\operatorname{csch} u$

Remember :

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du \Rightarrow \sin^{-1}\left(\frac{u}{a}\right)$$

$$\int \frac{1}{a^2+u^2}\,du \Rightarrow \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$$

$$\int \frac{1}{u\sqrt{u^2-a^2}} \Rightarrow \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right|$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\cos^{-1}x + \cos^{-1}(-x) = \pi$$

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

$$\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$$

$$\cot^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \tan^{-1}(x^{-1})$$

$$(\sin^{-1}(u))' = \frac{u'}{\sqrt{1-u^2}}$$

$$\cos^{-1}(u) = \frac{-u'}{\sqrt{1-u^2}}$$

$$\tan^{-1}(u) = \frac{u'}{1+u^2}$$

$$\sec^{-1}(u) = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \cot^{-1}(u) = \frac{-u'}{\sqrt{1-u^2}}$$

$$\csc^{-1}(u) = \frac{-u'}{|u|\sqrt{u^2-1}}$$

$$\int_1^{\infty} \frac{dx}{x^p} \Rightarrow \begin{cases} \frac{1}{p-1} , & p > 1 \quad \text{converge} \\ \infty & p \leq 1 \quad \text{diverge} \end{cases}$$

$$\int_0^{1} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1} , & p < 1 \quad \text{converge} \\ \infty , & p \geq 1 \quad \text{diverge} \end{cases}$$

Improper integrals

$$\int_0^{\infty} f(x) \Rightarrow \lim_{b \to \infty} \int_0^{b} f(x)$$

$$\int_{-\infty}^{\infty} f(x) \rightarrow \int_{-\infty}^{0} f(x) + \int_0^{\infty} f(x) = \lim_{b \to \infty} \int_b^{0} f(x) + \lim_{b \to \infty} \int_0^{b} f(x)$$

$\int_0^{2} f(x)$ when $f(x)$ is discontinous at $2 \Rightarrow \lim_{b \to 2^-} \int_0^{b} f(x)$ (1)

If $f(x)$ is discontinous at $0 \Rightarrow \lim_{b \to 0^+} \int_b^{2} f(x)$ (2)

If $f(x)$ is discontinous in an inner point:-

$$\int_0^{\text{Discontinous Point}} f(x) + \int_{\text{Discontinous Point}}^{2} f(x)$$ then continue as (1) and (2)

1) $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

2) $\lim_{n\to\infty} \sqrt[n]{n} = 1$

3) $\lim_{n\to\infty} x^{\frac{1}{n}} = 1$, $x > 0$

4) $\lim_{n\to\infty} x^n = 0$ if $-1 < x < 1$

5) $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \Rightarrow \lim_{n\to\infty} \left(1 - \frac{x}{n}\right)^n = \frac{1}{e^x}$

6) $\lim_{n\to\infty} \frac{x^n}{n!} = 0$, for any x

- a sequence is bounded from above if I can get M where $M > a_n$
- a sequence is bounded from below if I can get m where $m < a_n$

a sequence is bounded if the previous two conditions are met

- a sequence a_n is non decreasing if $a_1 \leq a_2 \leq a_3$
- a sequence a_n is non increasing if $a_1 \geq a_2 \geq a_3$

The sequence is monotomic if it is either non decreasing or non increasing

- If a sequence is both bounded & monotomic then a_n is converges

Tests :-

1) P series $\sum_{n=1}^{\infty} \frac{1}{n^P}$, $\begin{cases} \text{conv} \rightarrow P > 1 \\ \text{div} \rightarrow P \leq 1 \end{cases}$

2) n^{th} Partial sum $\lim_{n\to\infty} \sum a_n = L \rightarrow$ conv

$\lim_{n\to\infty} \sum a_n = \infty \rightarrow$ div

3) n^{th} term test $\lim_{n\to\infty} a_n \neq 0 \rightarrow$ div

$\lim_{n\to\infty} a_n = 0 \rightarrow$ can't say

$\lim_{n\to\infty} a_n = \infty$ / DNE $\rightarrow$ div

4) geometric series $r > 1 \rightarrow$ div

$r < 1 \rightarrow$ conv $\Rightarrow \sum a_n = \frac{a_n}{1-r}$

5) Ratio test $\cancel{\Sigma}$ $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$, $\rho > 1 \rightarrow$ div

$\rho < 1 \rightarrow$ conv

$\rho = 1 \rightarrow$ fail

6) Root test $\lim_{n\to\infty} \sqrt[n]{a_n} = \rho$, $\rho > 1 \rightarrow$ div

$\rho < 1 \rightarrow$ conv

$\rho = 1 \rightarrow$ fail

7) limit comparasion test $\lim_{n \to \infty} \frac{a_n}{b_n} = c$, $c > 0$ both conv / div
, $c = 0$ and $\sum b_n$ conv $\Rightarrow a_n$ conv
, $c = \infty$ and $\sum b_n$ div $\Rightarrow a_n$ div

8) Direct comparasion test If $\sum b_n$ is div and $\sum a_n > \sum b_n$ then a_n is div too
If $\sum b_n$ is conv and $\sum a_n < \sum b_n$ then b_n is conv too

9) integral test $\int_1^\infty f(x)\,dx$ = conv $\Rightarrow \sum a_n$ conv
= div $\Rightarrow \sum a_n$ div

10) Alternating series test $\sum (-1)^{n+1} a_n$ conv if :-
1) $|u_n| > 0$
2) ~~a_n~~ u_n is decreasing for large n
3) $\lim_{n \to \infty}$ ~~a_n~~ $u_n = 0$ If not :

$\sum (-1)^{n+1}$ ~~a_n~~ u_n div by n^{th} term test

$\sum u_n$ is converge absolutly If $\sum |a_n|$ is conv

$\sum u_n$ is converge conditionaly If $\sum a_n$ is div

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^P} \begin{cases} P > 1, & \text{conv abs} \\ 0 \le P \le 1, & \text{conv condi} \\ P < 0, & \text{div} \end{cases}$$

$$\sum_{n=0}^{\infty} a_n (x-a)^n \Rightarrow \text{Power series}$$

to find R and IC ⇒ apply Ratio or root test

~~ty~~ taylor series generated by f at $x=a$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \dots$$

taylor series generated by f at $x=0$ ⇒ Maclaurim series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$