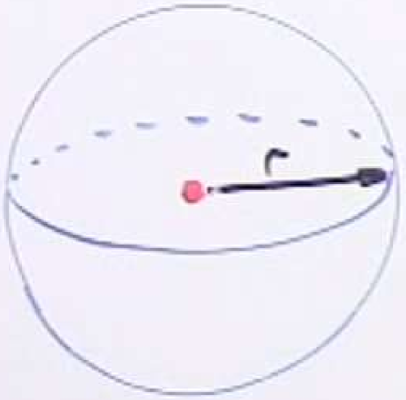
Spherical Mechanical Waves

① When a wave propagates from a source in all directions, the wave is three-dimensional. If the medium through which the wave moves is isotropic (constant density), such a wave is called a spherical wave.



As the wave moves outward, the energy it carries is spread out over a larger area.

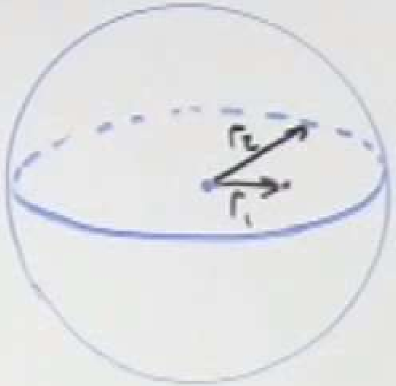
$$S = 4\pi r^2$$

Recall: $I = \dfrac{\bar{P}}{S} = \boxed{\dfrac{\bar{P}}{4\pi r^2}}$

② Suppose that we examine two different points at radius r_1 and r_2 from our source.

Ⓐ $I_1 = \dfrac{\bar{P}}{4\pi r_1^2}$ and Ⓑ $I_2 = \dfrac{\bar{P}}{4\pi r_2^2}$

$$\frac{I_2}{I_1} = \frac{\bar{P}/4\pi r_2^2}{\bar{P}/4\pi r_1^2} = \frac{r_1^2}{r_2^2} = \boxed{\left(\frac{r_1}{r_2}\right)^2}$$



So for example if $r_2 = 2r_1$, the the intensity decreases by a factor of 4!

$$\left(\frac{r_1}{2r_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \boxed{1/4} = \frac{I_2}{I_1}$$

③ Amplitude:

What is the relationship between A and r? Consider again two points at r_1 and r_2.

Recall: $\bar{P} = 2\pi^2 \rho S v f^2 A^2$

$\bar{P}_1 = \bar{P}_2 \Rightarrow 2\pi^2 \rho S_1 v f^2 A_1^2 = 2\pi \rho S_2 f^2 A_2$.

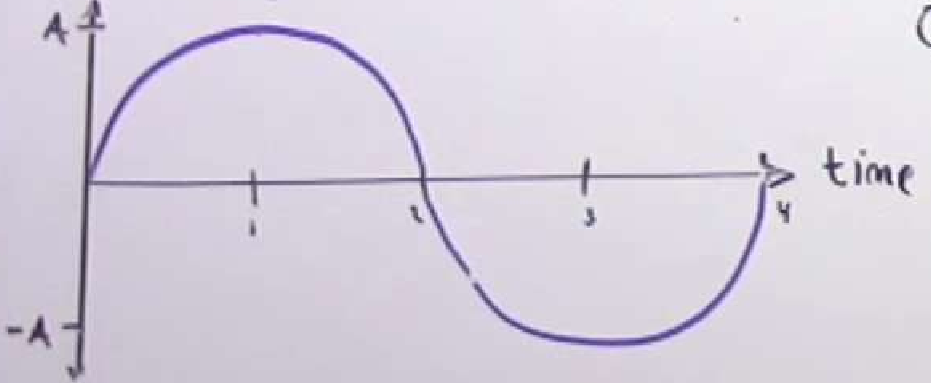
$\Rightarrow S_1 A_1^2 = S_2 A_2^2 \Rightarrow (4\pi r_1^2) A_1^2 = (4\pi r_2^2) A_2^2$

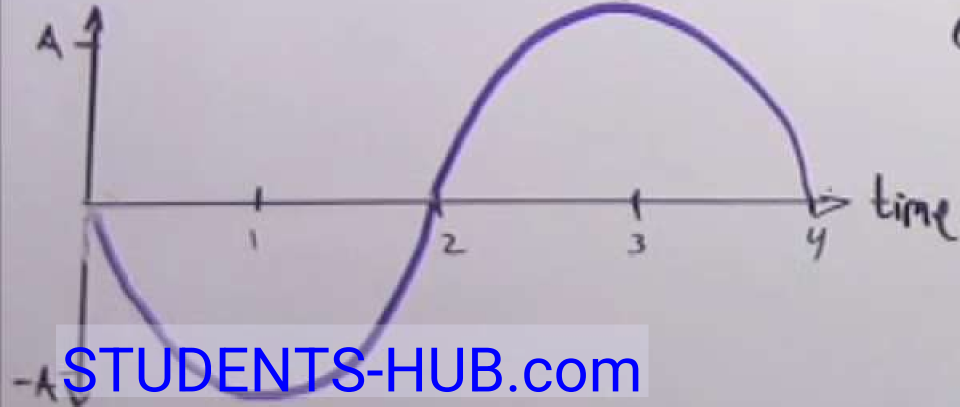
$\Rightarrow \dfrac{r_2^2}{r_1^2} = \dfrac{A_1^2}{A_2^2} \Rightarrow \boxed{\dfrac{r_2}{r_1} = \dfrac{A_1}{A_2}}$

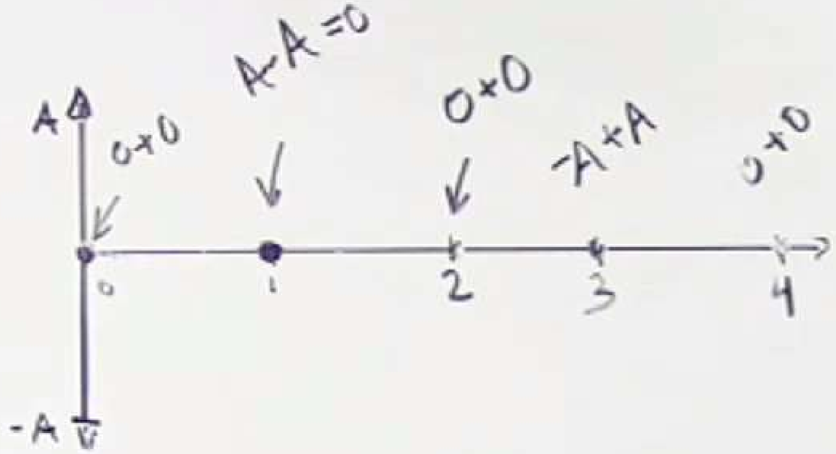
Principle of Superposition

Consider what happens when two or more waves travel in the same medium. When these waves pass through the same region of space at the same time, it is found that the actual displacement is the algebraic sum of the different displacements.

{**Note:** Valid for mechanical waves only if the displacement is not too large and restoring force obeys Hooke's Law.}







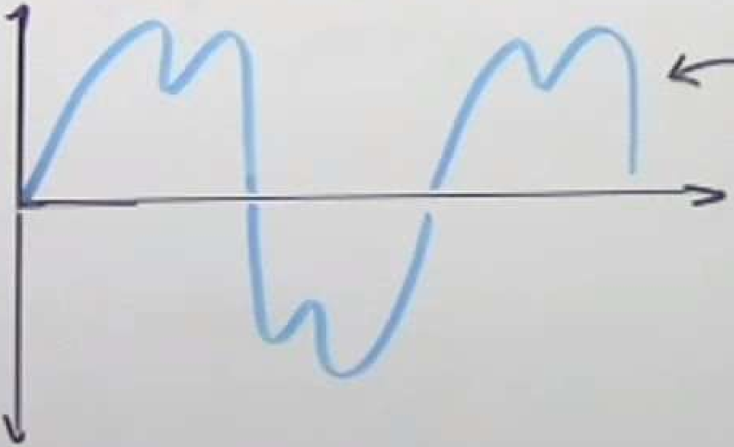
③ composite wave

the superposition makes the waves seem to disappear!

important result:

① If the two waves pass through the same region of space, those waves will continue to move independently of one another!

② Fourier's Theorem: Any composite wave can be considered to be composed of many sinusoidal waves.

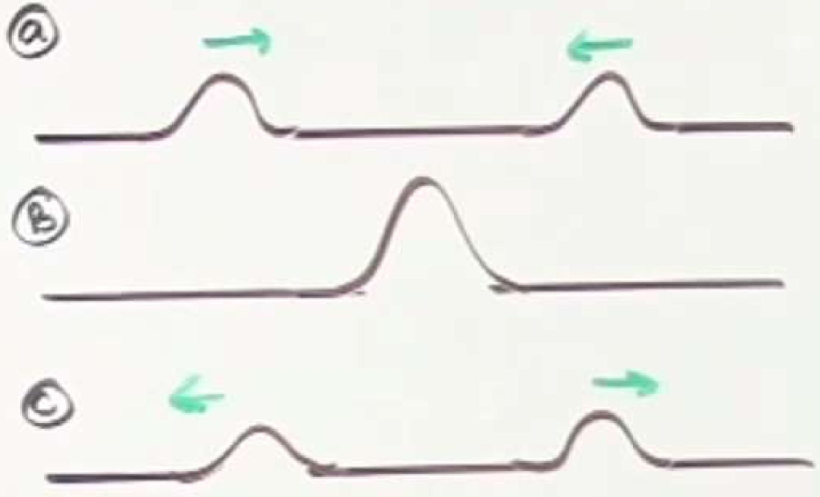


complex wave composed of many different sinusoidal waves!

Interference

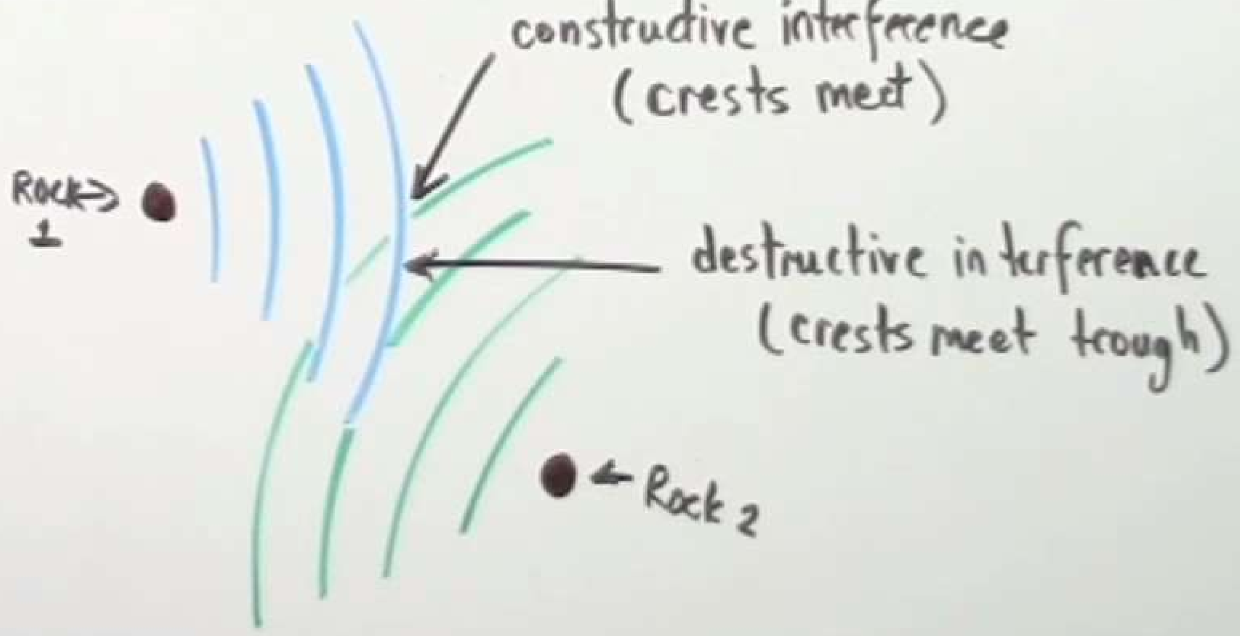
- When two or more waves pass through the same region of space at the same moment in time interference takes place.
- Principle of superposition provides us a way to combine the waves. The composite wave is obtained by taking the algebraic sum of displacements at every single point.

Constructive Interference



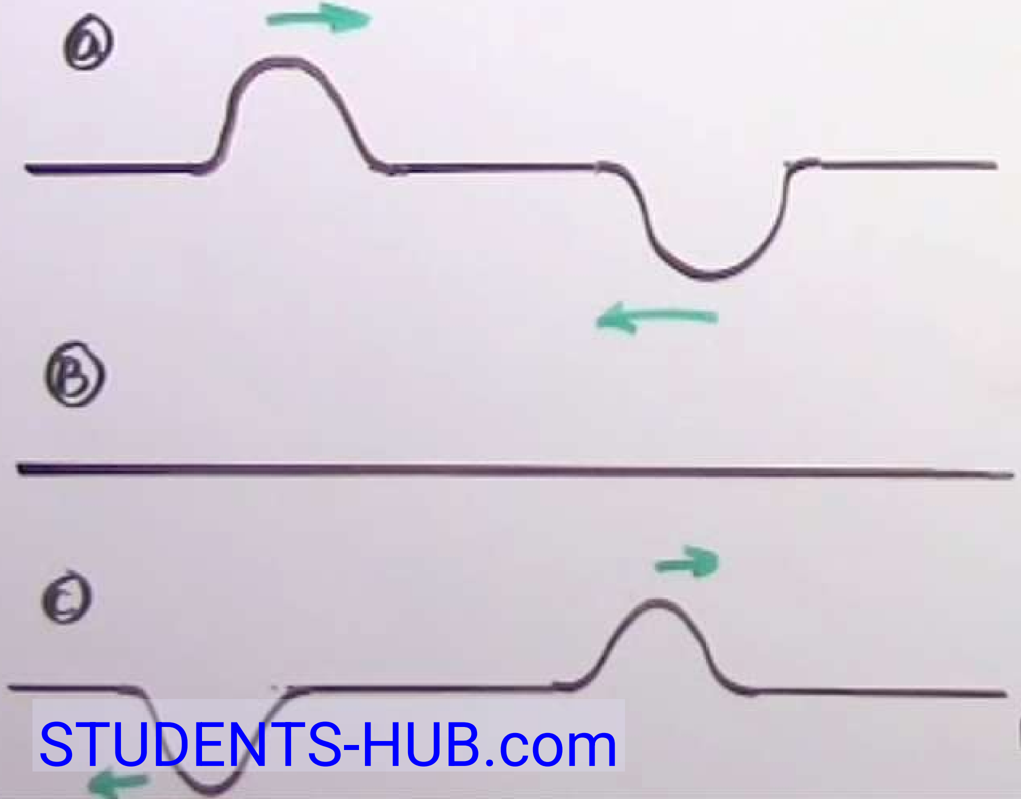
At the instant the two pulse waves overlap, they produce a resultant displacement that is greater than the displacement of either wave.

Spherical Waves



in-phase : constructive
out of phase: destructive

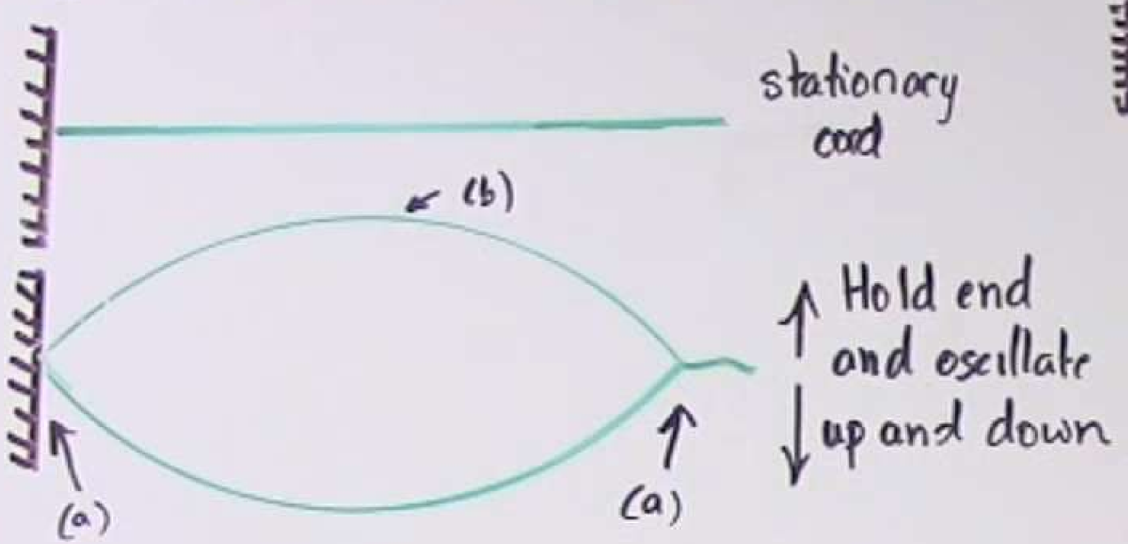
Destructive Interference



Two wave pulses, one right side up and the second inverted meet. If they have opposite displacements, destructive interference occurs

Standing Waves

① Thin cord tied at one end to a wall:

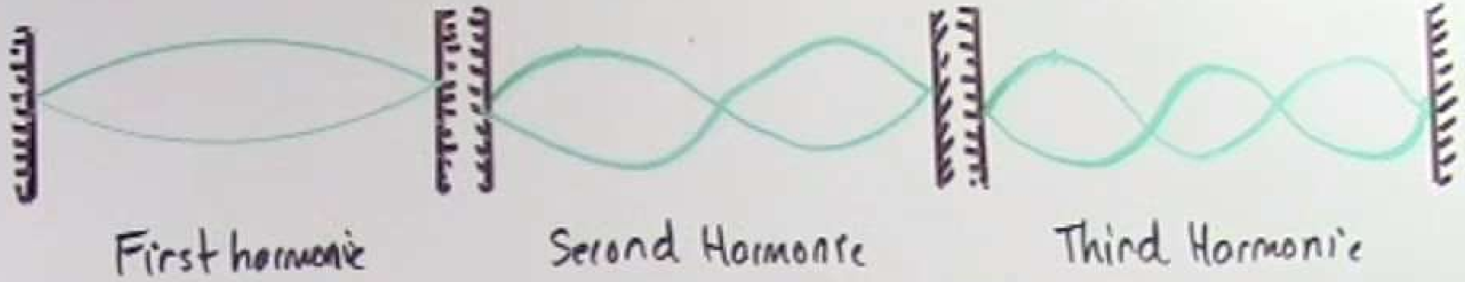


- As you begin to oscillate, the waves will travel to one end and back, creating wave interference. However, if you oscillate with just the right frequency, you will produce a standing wave, as shown.

- Destructive interference occurs at (a) ⇒ nodes
Constructive interference occurs at (b) ⇒ anti-nodes

Standing wave - large-amplitude waves that appear to be stationary, with nodes and antinodes.

Resonant frequency - natural frequency at which standing waves are produced.



- Lowest frequency at which a standing wave can be produced on a cord fixed at both end is called the fundamental frequency (or first harmonic).

$$\boxed{\ell = \frac{n\lambda_n}{2}}$$

$n :=$ # of harmonic ($n = 1, 2, 3, \dots$)
$\ell :=$ length of cord
$\lambda :=$ wavelength of standing wave.

- $$\boxed{v = \sqrt{\frac{F_T}{\mu}}}$$

velocity of standing wave
F_T = tension $\quad \mu$ = mass/length

Example: A piano cord is 1.0 m long and has a mass of 10.0 g. If the frequency of oscillation is 200 Hz, find the tension. Assume oscillation is at fundamental frequency.

$$\left.\begin{array}{l} ①\ v = \lambda f \\ ②\ 2\ell = \lambda \end{array}\right] \Rightarrow v = (2\ell)f = \sqrt{\frac{F_T}{\mu}} \Rightarrow (2\ell f)^2 \mu = \boxed{1600\ \text{N}}$$

If two successive harmonics of a vibrating cord are $\underbrace{240\ Hz}_{n}$ and $\underbrace{320\ Hz}_{n+1}$, find the frequency of the first harmonic.

Recall:

$v = \lambda f$

① • velocity remains constant at both frequencies!

$\begin{cases} \text{At harmonic } n, & f_n = 240\ Hz \\ \text{At harmonic } n+1, & f_{n+1} = 320\ Hz \end{cases}$

$$v_n = v_{n+1} \Rightarrow \lambda_n f_n = \lambda_{n+1} f_{n+1} \Rightarrow 240\lambda_n = 320\lambda_{n+1} \Rightarrow \frac{\lambda_n}{\lambda_{n+1}} = \frac{320}{240} = \boxed{\frac{4}{3}}$$

②

$$\boxed{l = \frac{n\lambda_n}{2}}$$

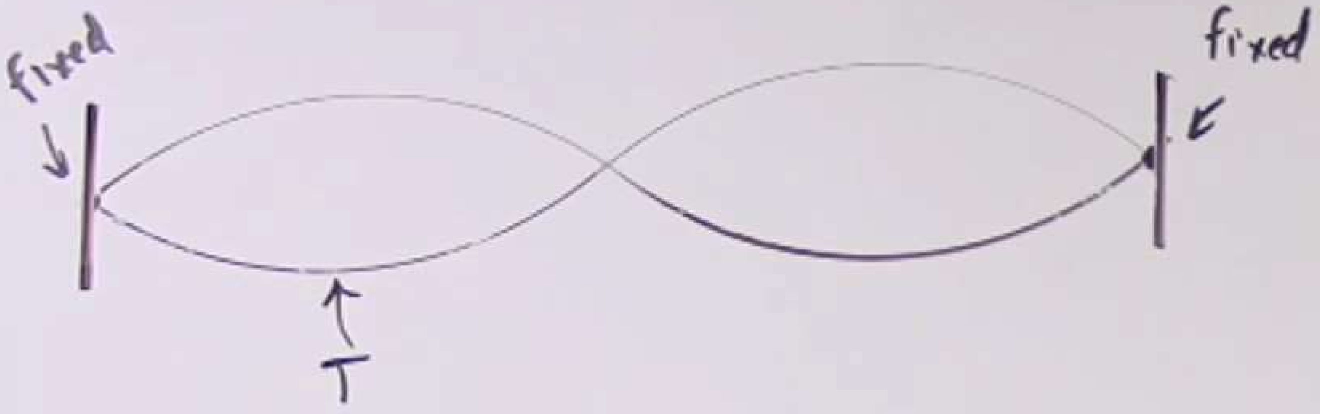
$$l_n = l_{n+1} \Rightarrow \frac{n\lambda_n}{2} = \frac{(n+1)\lambda_{n+1}}{2} \Rightarrow \frac{\lambda_n}{\lambda_{n+1}} = \frac{n+1}{n} = \frac{4}{3} \Rightarrow \boxed{n = 3}$$

③ What is f_{n-2}?

$$l_{n-2} = l_n \Rightarrow \frac{(n-2)\lambda_{n-2}}{2} = \frac{n\lambda_n}{2} \Rightarrow \lambda_{n-2} = 3\lambda_n \Rightarrow \frac{1}{3} = \frac{\lambda_n}{\lambda_{n-2}}$$

since $v_{n-2} = v_n \Rightarrow \lambda_{n-2} f_{n-2} = \lambda_n f_n \Rightarrow f_{n-2} = \frac{\lambda_n}{\lambda_{n-2}} f_n = \frac{1}{3} \cdot 240\ Hz = \boxed{80\ Hz}$

Suppose that a certain string vibrates at a frequency of 350 Hz. If we increase the tension in the string by 50 %, what is the new frequency?



Since $f_1 = 350\ \text{Hz}$ and $T_2 = 1.50\,T_1$

$$\frac{350}{f_2} = \sqrt{\frac{T_1}{1.50\,T_1}} = \sqrt{\frac{1}{1.5}}$$

$$f_2 = \frac{350}{\sqrt{1/1.5}} \approx \boxed{429\ \text{Hz}}$$

<u>Recall:</u> $v = \lambda f$ and $v = \sqrt{\frac{T}{\mu}}$

$$\lambda f = \sqrt{\frac{T}{\mu}} \implies f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$$

Suppose f_1 = frequency before increase

and f_2 = frequency after increase.

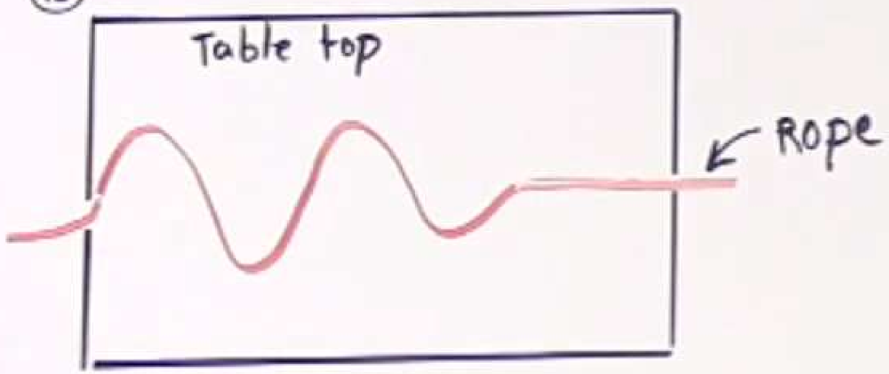
$$\frac{f_1}{f_2} = \frac{\frac{1}{\lambda}\sqrt{T_1/\mu}}{\frac{1}{\lambda}\sqrt{T_2/\mu}} = \sqrt{T_1/T_2}$$

Mechanical Waves

① Mechanical waves require the presence of matter to propagate.

(a)

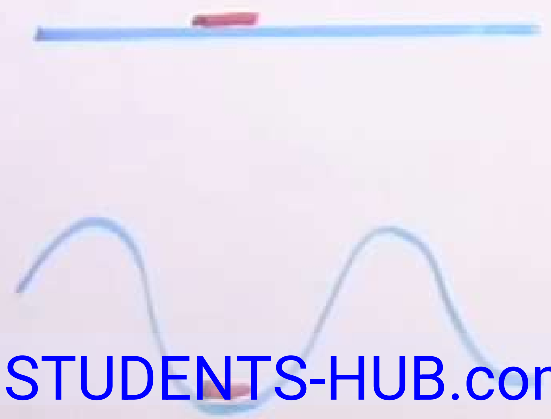
(B)



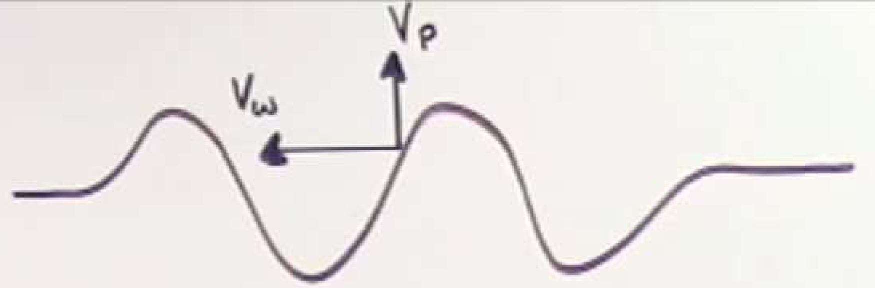
Rock thrown into water will create mechanical waves which will propagate outward in all directions

Mechanical waves can also travel through a cord, as shown above.

②



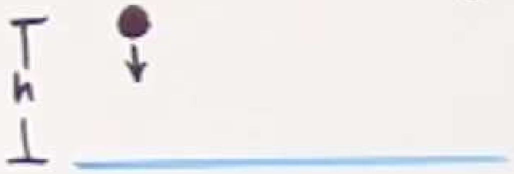
Waves can move over great distances but the medium oscillates in SHM about an equilibrium point.



V_p = velocity of particle medium particle

V_w = velocity of wave

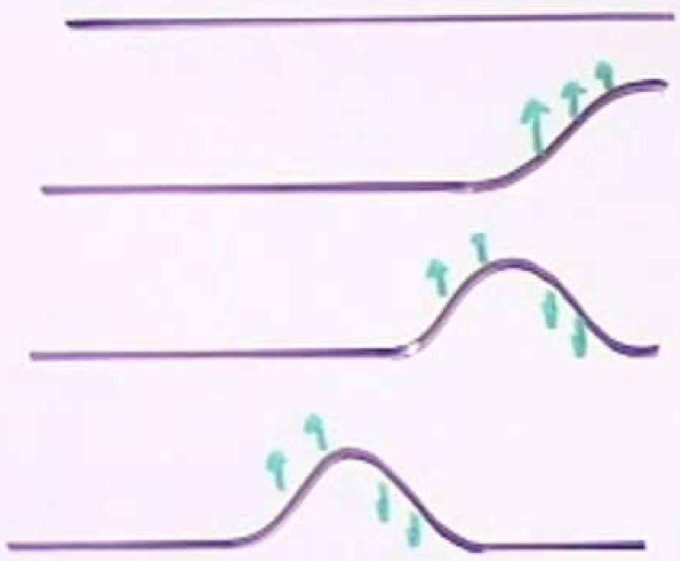
③ Waves carry energy



Gravitational potential energy is completely transformed into kinetic energy right before impact. During impact, the energy is carried by propagating mechanical waves.

*All traveling waves transport energy

Wave Motion



(i) Stationary cord

(ii) Hand pulls on one end of the cord

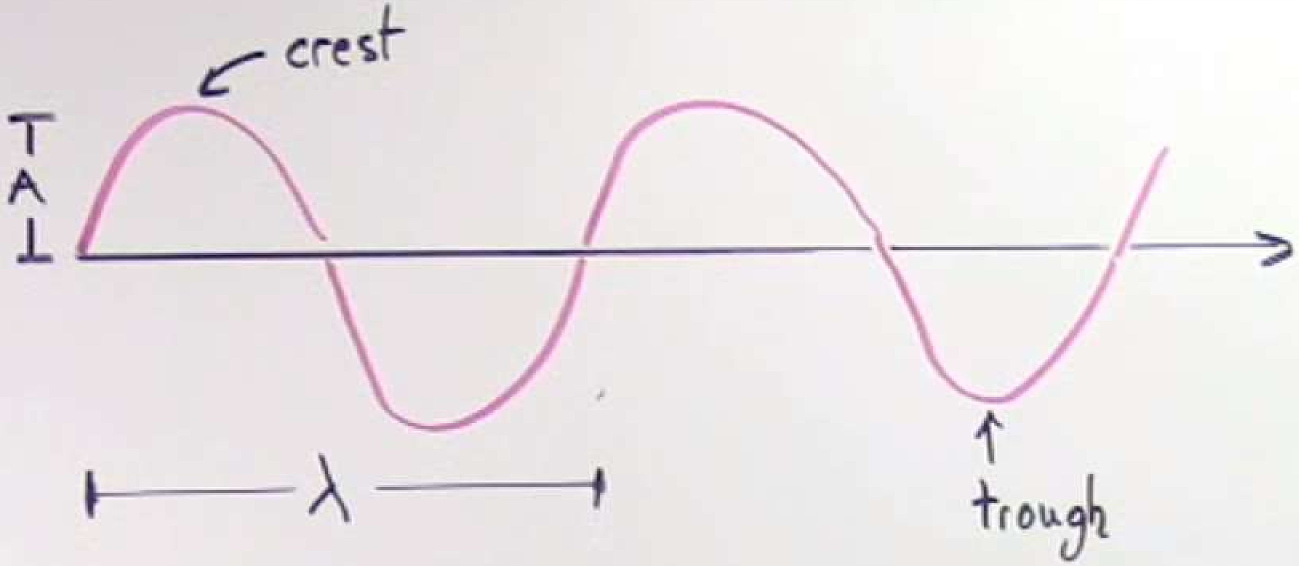
(iii) Adjacent sections of cord feel the same force.

(iv) As succeeding section moves, wave crest moves outward

* The quick up-and down motion of the hand creates a single wave bump called a <u>pulse</u>.

<u>Continuous / Periodic wave</u>: a wave that has a periodic source of disturbance known as an oscillation or <u>vibration</u>.

=> If the source (the vibration) obeys simple harmonic motion, the wave itself will also be sinusoidal (assuming medium is elastic).



A = maximum height of crest or maximum depth of trough relative to equilibrium point

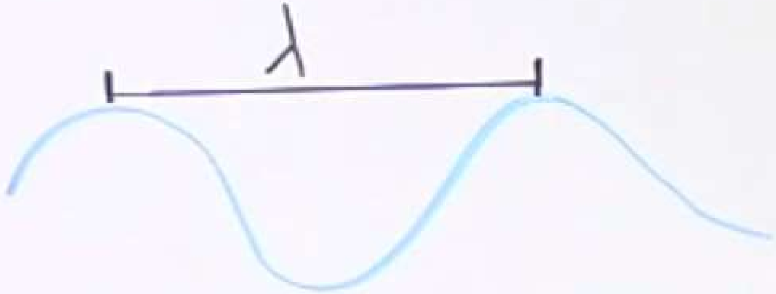
λ = distance from one position to a second at which the wave begins to repeat itself.

f = number of cycles that are completed over some given period of time (cycles/second)

v = velocity at which wave crests move forward

$$\boxed{v = \lambda f = \frac{\lambda}{T}}$$

① A fisherman notices that wave crests pass his boat every single 2.0 seconds. He measures the distance to be 6.0 meters between any two crests. How fast are the waves traveling?



$\lambda = 6.0 \text{ meters}$

$f = \frac{1 \text{ cycle}}{2 \text{ sec}} = 0.5 \, s^{-1}$

$v = \lambda f = (6.0 m)(0.5 s^{-1}) = \boxed{3 m/s}$

② A certain traveling mechanical wave has a frequency of 400 Hz and travels with a velocity of 333 m/s. Calculate the wavelength.

$v = \lambda f \Rightarrow \lambda = \frac{v}{f} = \frac{333 m/s}{400 s^{-1}} = \boxed{0.8325 \text{ m}}$

③ Certain radio waves have a frequency that range between 550 kHz and 1600 kHz. and travel with a velocity of 3×10^8 m/s. Calculate the range of wavelength of such waves.

$v = \lambda f \Rightarrow \lambda_{min} = \frac{v}{f_{max}} = \frac{(3 \times 10^8 m/s)}{(1600{,}000 Hz)} = \boxed{187.5 m}$ $\lambda_{max} = \frac{(3 \times 10^8 m/s)}{(550{,}000 Hz)} = \boxed{545.5 m}$

Types of Waves

Transverse Waves

When the particles composing the medium through which the wave travels oscillate perpendicularly to the direction of the wave, the wave is called a transverse wave.

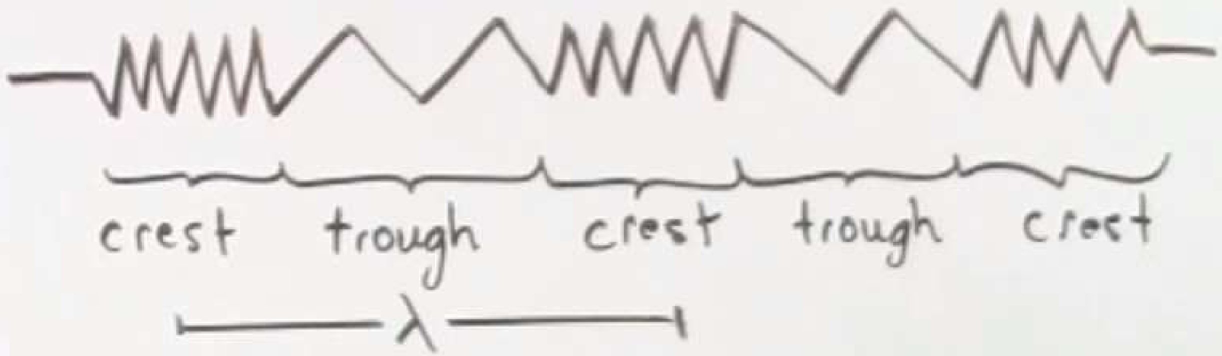
ex: wave in a cord

Longitudinal Waves

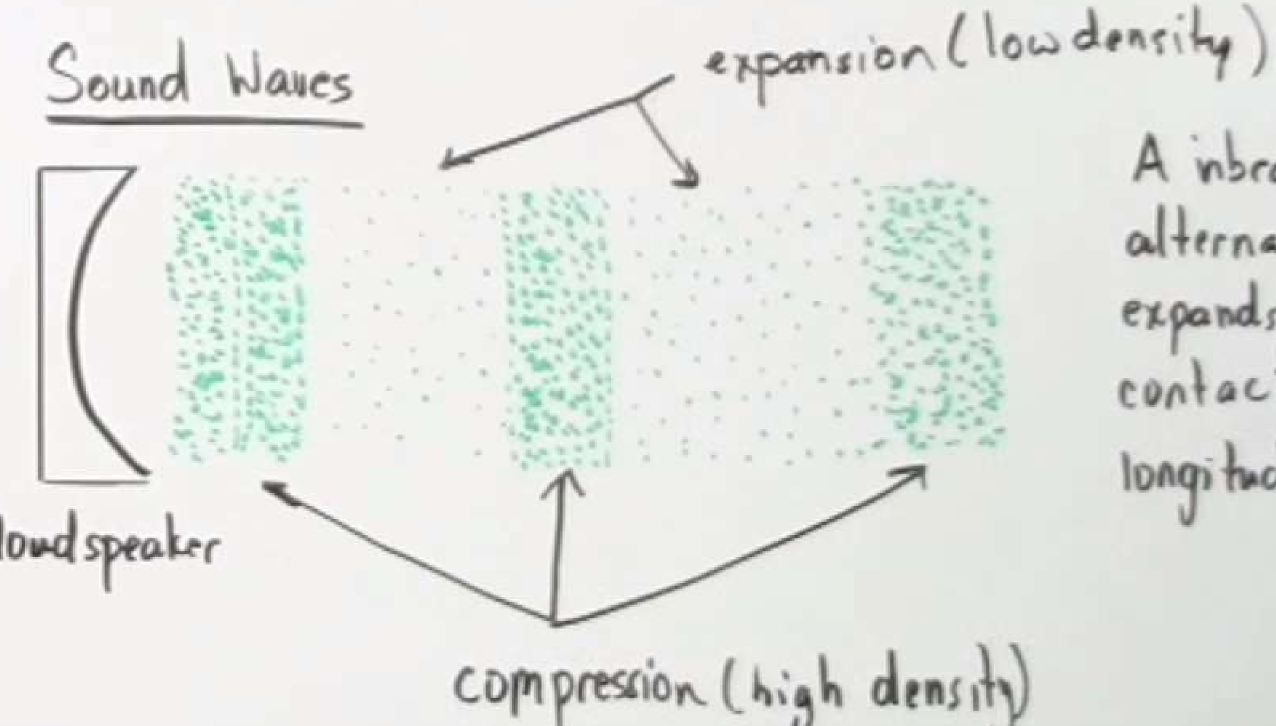
When the particles oscillate along the direction of motion of the wave, such a wave is known as a longitudinal wave.

In a longitudinal wave, a series of compressions and expansions propogate the wave along the spring

- Compressions: Regions where the coils are close together
- Expansions: Regions where the coils are far apart.



Sound Waves



A vibrating loudspeaker alternately compresses & expands the air molecules in contact producing a longitudinal wave.

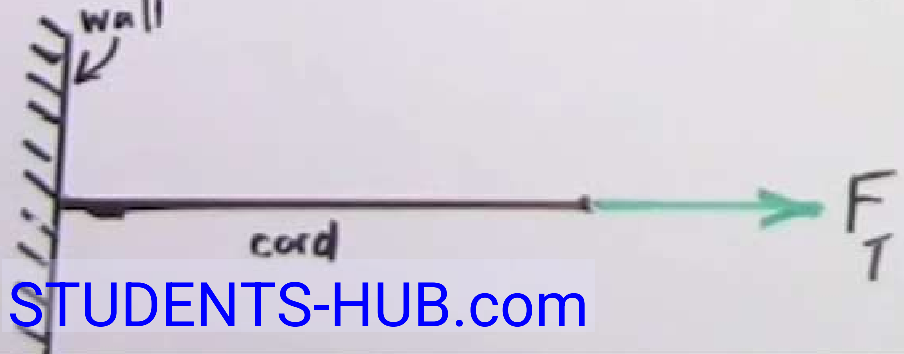
Velocity of Transverse Waves

The velocity of a wave depends on the medium through which it travels. A mechanical wave traveling in a cord has a velocity given by:

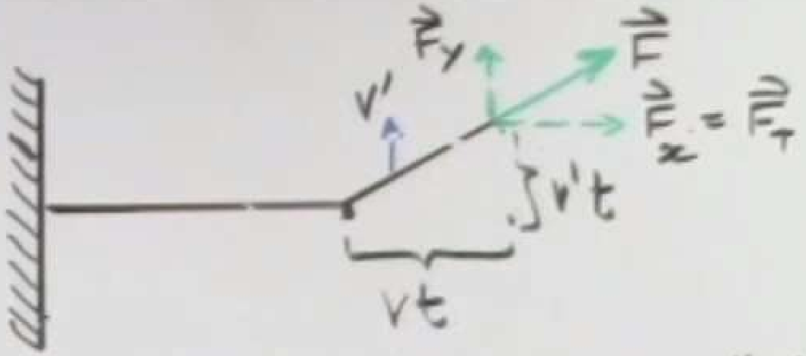
$$\boxed{v = \sqrt{\frac{F_T}{\mu}}}$$

$^*\mu = m/\ell$

- the greater the mass per unit length, the more inertia the cord has to resist change in motion.
- the greater the tension, the greater the velocity because each segment of cord is held tighter (higher force)



cord is under tension



Suppose now the cord is pulled upward by Force $\vec{F}_y$ with velocity v'. The velocity v is the propagation of wave.

vt = distance wave moves to left

$v't$ = distance end of cord moves up

$$\Rightarrow \left[\frac{F_T}{F_y} = \frac{vt}{v't} = \frac{v}{v'}\right]$$

Recall: Impulse is force multiplied by time.

$$\vec{F}_y \cdot t = \frac{F_T v'}{v} \cdot t = \Delta p = (\mu v t) v' \quad \left[\frac{kg}{m} \cdot \frac{m}{s} \cdot s \, \frac{m}{s}\right]$$

$$\frac{F_T v'}{v} t = \mu v t v' \Rightarrow \boxed{v = \sqrt{\frac{F_T}{\mu}}}$$

Velocity of Longitudinal Waves

Recall: velocity of transverse waves in cord is given by:

$$v = \sqrt{\frac{F_T}{\mu}}$$

F_T ← elastic factor

μ ← inertia factor

① **Longitudinal Wave in Long Solid Rod**

$$v = \sqrt{\frac{E}{\rho}}$$

$E :=$ Young's Modulus

$\rho :=$ density

② **Longitudinal Wave in Fluids**

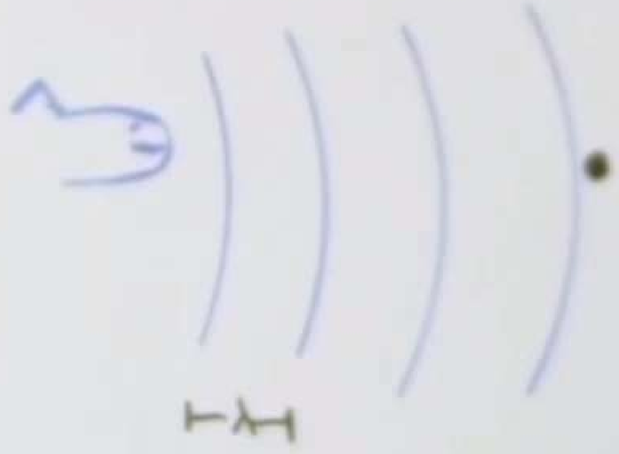
$$v = \sqrt{\frac{B}{\rho}}$$

$B :=$ Bulk modulus

$\rho :=$ density

Example: The process of echolocation is used by animals such as dolphins for sensory perception. The dolphin emits a pulse of sound (longitudinal) which reflects off of objects and returns to the dolphin. Suppose the frequency of such a wave is 100,000 Hz.

ⓐ Calculate the wavelength of such a wave.



$$v = \sqrt{\frac{2\times10^{9}\ N/m^2}{1.0\times10^{3}\ kg/m^3}}$$

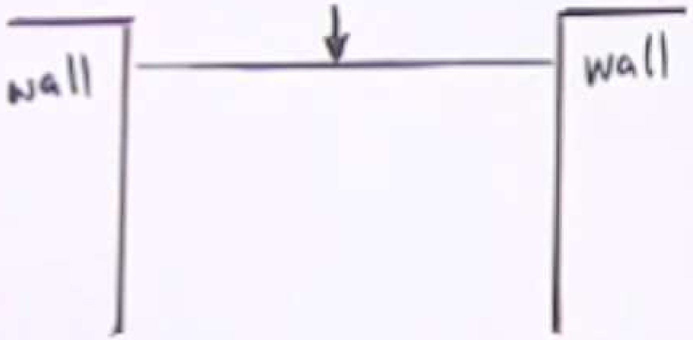
$$v = 1,400\ m/s$$

$$v = \lambda f \Rightarrow \lambda = \frac{v}{f} = \frac{1,400\ m/s}{100,000\ Hz} = \boxed{0.014\ m}$$

ⓑ Suppose there is another animal 120 m away from where the dolphin emits the pulse. How long does it take the wave to travel back to the dolphin?

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{2(120\ m)}{1400\ m/s} = \boxed{0.17\ s}$$

A 100.0 m long wire with a diameter of 2.0 mm is stretched between two walls. A bird lands at the center of wire, sending a pulse in both directions. The pulse travels to both ends, reflects and arrives at the initial position after 0.8 seconds. Determine the tension in the wire. (assume wire is copper).



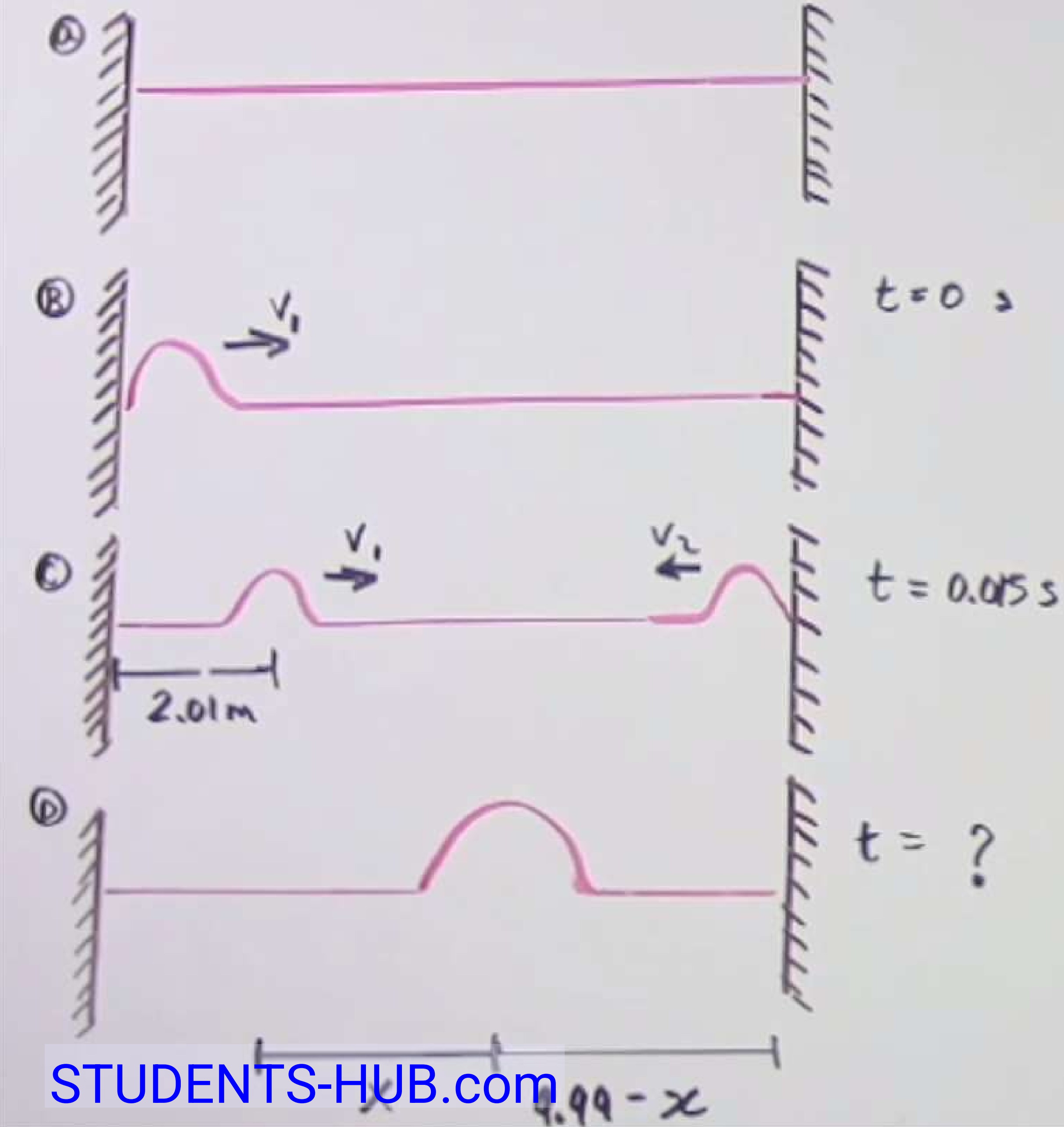
Each wave pulse travels a total distance of 100m and the time it takes it to travel that distance is 0.8 seconds.

$$\frac{\text{distance}}{\text{time}} = \frac{100\text{ m}}{0.8\text{ s}} = 125\text{ m/s}$$

$$v = \sqrt{\frac{F_T}{\mu}} \Rightarrow F_T = v^2\mu = \frac{v^2 m}{\ell} = \frac{v^2 \rho V}{\ell} = \frac{v^2 \rho A \ell}{\ell} = v^2 \rho A$$

$$F_T = (125\text{ m/s})^2 (8.9\times10^3\text{ kg/m}^3)(\pi(0.001\text{ m})^2) = \boxed{437\text{ N}}$$

Suppose a certain string has a mass of 200 grams and a length of 12.0 m. A wave pulse is created at one end at $t = 0$ seconds and 15 ms later a second pulse is generated on the other end. Assuming the tension in the cord is 300 N, calculate where the two pulses will meet.



① $V = \sqrt{\dfrac{F_T}{\mu}} = \sqrt{\dfrac{F_T}{M/\ell}} = \sqrt{\dfrac{300\,N}{\frac{0.2\,kg}{12.0\,m}}} = 134\ m/s$

② After $t = 0.015\,s$, wave 1 has traveled:

$v_1 t = vt = (134\ m/s)(0.015\ s) = 2.01\,m$

③ Since $V_1 = V_2 = 134\,m/s$,

$x = 9.99\,m - x \Rightarrow 2x = 9.99\,m$

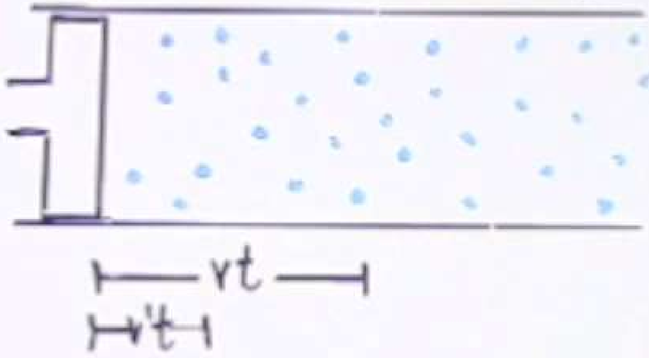
$x = 4.995\,m$

④ Wave 1 travels: $2.01\,m + 4.995\,m = \boxed{7.005\,m}$

Wave 2 travels: $\boxed{4.995\,m}$

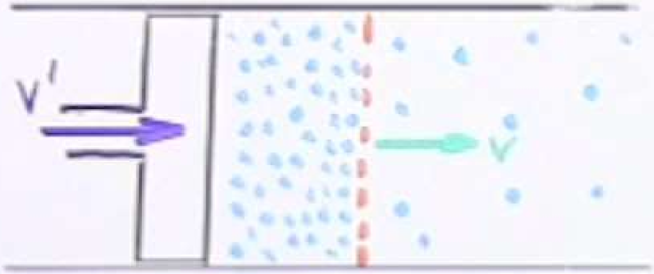
Derivation of Longitudinal Velocity in Fluids

ⓐ Piston is stationary



P_i = initial pressure

ρ = density



* dashed line leading edge of compressed fluid

① Piston moves right with velocity v', compressing the fluid right in front of it. If the piston moves in time period t, then:

$v't$ = distance piston moved

② Compressed fluid also moves at velocity v' but the front of the compressed region moves with another velocity, lets say v.

(assume $v \gg v'$)

the leading edge moves

Let $\underbrace{P_i + \Delta P}$ be the pressure in compressed fluid.

The piston must exert a force of $\underbrace{(P_i + \Delta P)A}$

$$\vec{F}_{net} = \underbrace{(P_i + \Delta P)A}_{\text{force on compressed fluid by piston}} - \underbrace{P_i A}_{\text{Force on compressed fluid by fluid in front}}$$

$$\vec{F}_{net} = P_i A + \Delta P A - P_i A = \boxed{\Delta P A} \quad (i)$$

Recall: $\text{impulse} = \underline{F_{ave} t = \Delta p = m \Delta v}$

$$\text{impulse on fluid} = F_{net} \cdot t = mv'$$

$$\Rightarrow \Delta P A t = \rho V v' = \rho (A\ell) v' = \rho A (vt) v'$$

$$\Rightarrow \boxed{\Delta P = \rho v v'} \quad (ii)$$

Recall: $B = -\dfrac{\Delta P}{\Delta V / V_i} \Rightarrow B = -\dfrac{\Delta P}{\dfrac{v'tA}{vtA}} = B = -\dfrac{\Delta P}{v'/v}$

$$\Rightarrow B = \frac{\rho v v'}{v'/v} \Rightarrow \boxed{v = \sqrt{\frac{B}{\rho}}}$$

Energy Carried by Waves

① How do mechanical waves carry energy?

molecules of the medium

②
The energy is transferred from one molecule to another as vibrational energy. If the particles move in SHM (as in a sinusoidal wave), the energy is given by:

$$E = \frac{1}{2} k A^2 \qquad \text{Recall: } f = \frac{1}{2\pi}\sqrt{k/m}$$

$$\boxed{E = 2\pi^2 m f A^2} \quad \text{Ⓐ} \qquad \Rightarrow 4\pi^2 f^2 m = k$$

↳ transverse/longitudinal 2D waves in medium

③
We can use equation Ⓐ to find energy of 3D waves traveling through medium

Since mass of medium is $\underbrace{m = \rho V}$ through which 3D wave moves

$$m = \rho V = \rho S L = \rho S v t \Rightarrow \boxed{E = 2\pi^2 \rho S v t f A^2} \quad \text{Ⓑ}$$

↑ transverse/longitudinal 3D waves in medium

conclusion:

Energy carried by wave is proportional to the frequency and the square of amplitude.

④ Power and Intensity

$$\bar{P} = \frac{\Delta E}{t} = 2\pi^2 \rho S v f A^2$$

} average power is the rate of change of energy

$$I = \frac{\bar{P}}{S} = 2\pi^2 \rho v f A^2$$

} intensity is the average power transferred per area ⊥ to energy flow

Example: Two waves of the same frequency travel through the same medium, but wave 1 carries 9 times as much energy as wave 2. Find the ratio of amplitudes of waves.

$$\frac{9\bar{P}_1}{\bar{P}_2} = \frac{9(2\pi^2\rho S v f A_1^2)}{2\pi^2 \rho S v f A_2^2} \Rightarrow \frac{A_1}{A_2} = \sqrt{9} = \boxed{3}$$

The intensity of a wave created by an earthquake (spherical wave) detected a distance of 80.0 km from point of origin is 50×10^4 W/m². Calculate the intensity of the wave a distance of 350 km away.

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow I_2 = I_1\left(\frac{r_1}{r_2}\right)^2$$

$$I_2 = \left(50 \times 10^4 \frac{W}{m^2}\right)\left(\frac{80{,}000\,m}{350{,}000\,m}\right)^2 = \boxed{2.61 \times 10^4\ W/m^2}$$

We are comparing the intensity at two different points.

The intensity of an earthquake passing through the ground is 5×10^6 W/m² at a distance of 52.0 km away from point of origin.

ⓐ Calculate what the waves intensity was when it was 2.0 km from origin.

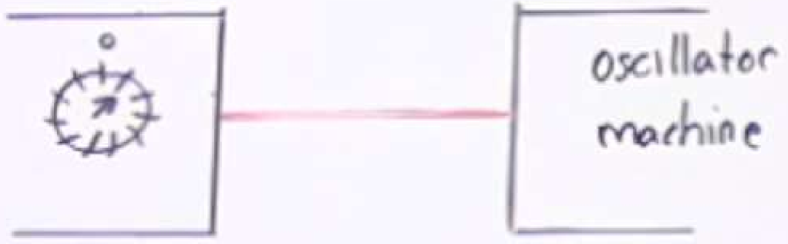
$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow I_1 = I_2\left(\frac{r_2}{r_1}\right)^2 = \left(5 \times 10^6 \frac{W}{m^2}\right)\left(\frac{52{,}000\ m}{2{,}000\ m}\right)^2 = \boxed{3.38 \times 10^9 \frac{W}{m^2}}$$

Ⓑ Find the rate at which energy passed through an area of 5.0 m² at 2.0 km from origin.

$$I_1 = \frac{\bar{P}_1}{S} \Rightarrow \bar{P} = I_1 S = \left(3.38 \times 10^9 \frac{W}{m^2}\right)\left(5.0\ m^2\right) = \boxed{1.69 \times 10^{10} \frac{J}{s}}$$

A copper wire of radius 5.0 mm is under 10.0 N of tension while connected to an oscillator. The frequency of oscillation is 80.0 Hz while the maximum displacement is 2.0 mm.

ⓐ What is the average power of the oscillator?



$$\bar{P} = 2\pi^2 f^2 \rho S v A^2$$

$$\rho = 8.9 \times 10^3 \ kg/m^3$$

$$S = \pi r^2 = \pi (0.005 m)^2 = 7.85 \times 10^{-5} \ m^2$$

$$f = 80 \ Hz$$

$$A = 0.002 m$$

$$v = \sqrt{\frac{F_T}{\mu^*}} = 3.78 \ m/s$$

$$\left[\mu^* = \frac{m}{\ell} = \frac{\rho V}{\ell} = \frac{\rho S \ell}{\ell} = \rho S\right]$$

$$\bar{P} = (2\pi^2)(80 \ s^{-1})^2 (8.9 \times 10^3 \ kg/m^3)(7.85 \times 10^{-5} \ m^2)(3.78 \ m/s)(0.002 m)^2$$

$$\boxed{\bar{P} = 1.33 \ W}$$

ⓑ What happens to the amplitude of oscillation if power out remains constant but frequency doubles?

$$\bar{P} = 2\pi^2 \rho S v \underbrace{f^2 A^2}$$ => amplitude decreases by factor of 2 (from 2.0 mm to 1.0 mm)