Ch8 Coherence Theory

Correlation: for example: A↑ B↑

Temporal Coherence: $\vec{E}(\vec{r},t)$ & $\vec{E}(\vec{r},t-\tau)$

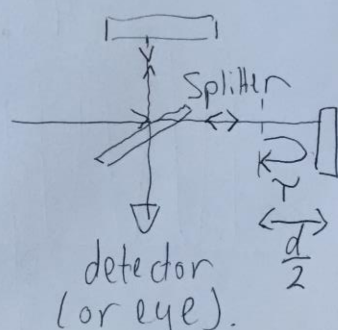
Spatial Coherence: $\vec{E}(\vec{r},t)$ & $\vec{E}(\vec{r}+\Delta\vec{r},t)$

High coherence → consistently constructive, or destructive

Low Coherence → time-averaged signal shows no interference.

8.1 Michelson Interferometer:

- path difference $\underline{\underline{d}}$ leads to time delay τ.



$$\tau = \frac{d}{c}$$

Temporal coherence

Assume input light is a plane wave (2)

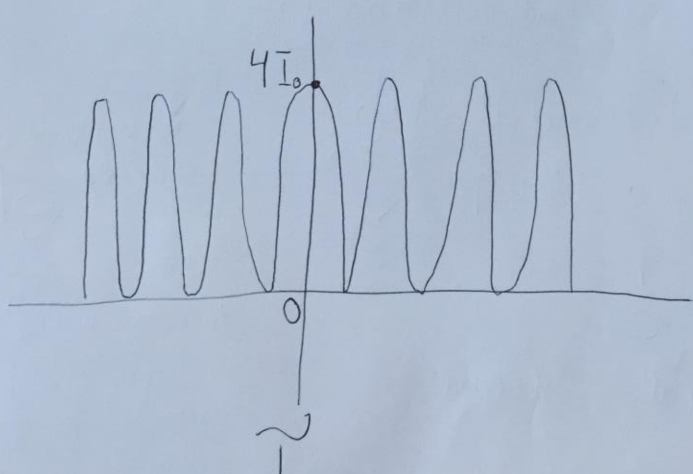
: We have two signals at detector

$$\vec{E}_0 e^{i(kz-wt)} \quad \text{and} \quad \vec{E}_0 e^{i(kz-w(t-\tilde{\tau}))}$$

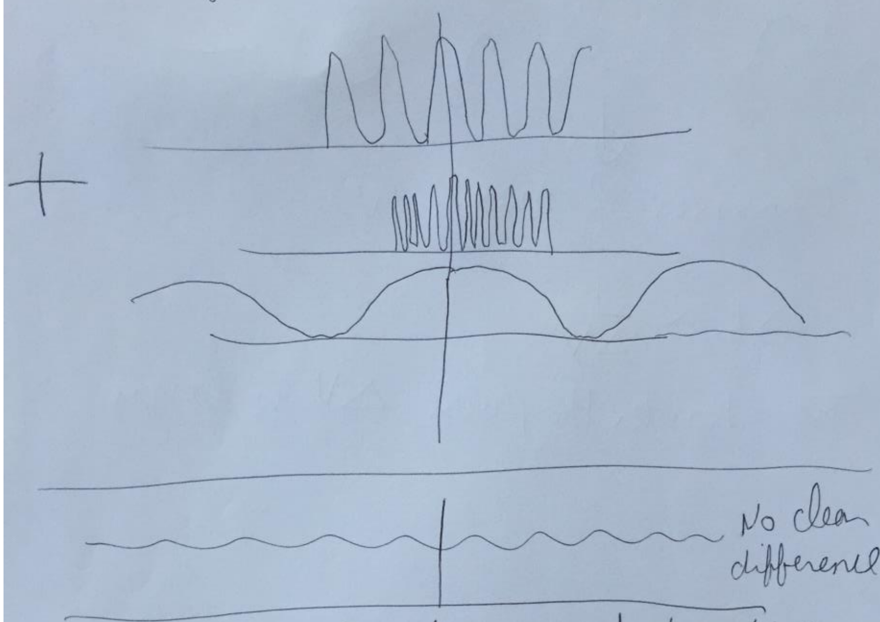
$$I_{tot}(\tau) = \frac{c\varepsilon_0}{2}\left[\vec{E}_{tot} \cdot \vec{E}^*_{tot}\right]$$

$$= \frac{c\varepsilon_0}{2}\left[\vec{E}_0 e^{i(kz-wt)} + \vec{E}_0 e^{i(kz-w(t-\tau))}\right]\left[\quad\right]^*$$

$$= 2 I_0 \left[1 + \cos(w\tau)\right] \quad ; \quad I_0 \equiv \frac{c\varepsilon_0}{2}\vec{E}_0 \cdot \vec{E}_0^*$$

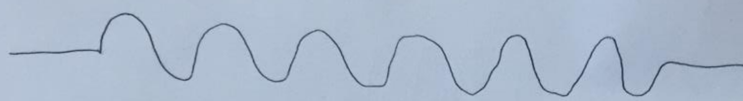


What if we have multiple frequencies? 3



No constructive or destructive interference seen at detector.

Wavetrain



Consists of many W's.

$$\Delta t \propto \frac{1}{\Delta V}$$

The shorter the pulse, ΔV is larger.

What happens if the light incident consists of a continuous band of frequencies? (3a)

Let $\vec{E}(t)$ be an arbitrary waveform, e.g. a pulse with many frequencies.

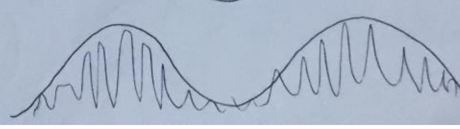
$$\vec{E}_{tot}(t,\tau) = \vec{E}(t) + \vec{E}(t-\tau)$$

$$I_{tot}(t,\tau) = \frac{n c \varepsilon_0}{2} \vec{E}_{tot}(t,\tau) \cdot \vec{E}^{*}_{tot}(t,\tau)$$

$$= \frac{c\varepsilon_0}{2}\Big[\vec{E}(t)\cdot\vec{E}^{*}(t) + \vec{E}(t)\cdot\vec{E}^{*}(t-\tau) + \vec{E}(t-\tau)\cdot\vec{E}^{*}(t) + \vec{E}(t-\tau)\cdot\vec{E}^{*}(t-\tau)\Big]$$

$$= I(t) + I(t-\tau) + \frac{c\varepsilon_0}{2}\,\mathrm{Re}\left\{\vec{E}(t)\cdot\vec{E}^{*}(t-\tau)\right\}$$

$\rightarrow$ rapid oscillations averaged out. We only keep the slowly changing envelope.



The detected signal is $Sig(\tau)$ (4)

$$Sig(\tau) \propto \int_{-\infty}^{\infty} I_{tot}(t,\tau)\,dt$$

$Sig(\tau)$ is time - integrated instensity

↳ called the "fluence"

Why ??

let $\mathcal{E} = \int_{-\infty}^{\infty} I(t)\,dt = \int_{-\infty}^{\infty} I(t-\tau)\,dt$

(8.9) $Sig(\tau) \propto 2\mathcal{E}\left[1 + \mathrm{Re}\{\gamma(\tau)\}\right]$

(derivation in book; not important).

$\gamma(\tau)$ = degree of coherence function

$$\equiv \frac{\int_{-\infty}^{\infty} I(\omega)e^{-i\omega\tau}\,d\omega}{\int_{-\infty}^{\infty} I(\omega)\,d\omega}$$

$$\mathcal{E} = \int_{-\infty}^{\infty} I(t)\,dt = \int_{-\infty}^{\infty} I(\omega)\,d\omega$$

Example 8.1

Compute output signal when a gaussian pulse with spectrum (7.25) is sent into a Michelson Interferometer.

given: $$I(\omega) = \frac{\varepsilon_0 c}{2} \vec{E}_0 \cdot \vec{E}_0^* \, T \, e^{-T^2(\omega-\omega_0)^2}$$

T = pulse duration

$$\int_{-\infty}^{\infty} I(\omega)\,d\omega = \frac{\varepsilon_0 c}{2} \vec{E}_0 \cdot \vec{E}_0^* \, T\sqrt{\pi} \,; \left(\begin{matrix}\text{Example} \\ 7.3\end{matrix}\right)$$

$$\gamma(\tau) = \frac{T}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-T^2(\omega-\omega_0)^2} e^{-i\omega\tau}\,d\omega$$

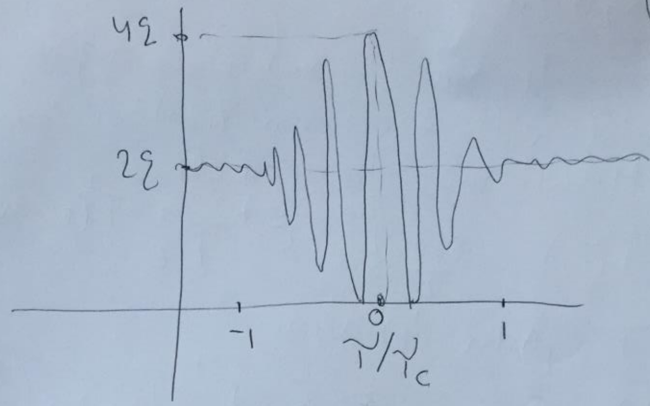
$$= e^{-\frac{\tau^2}{4T^2}} e^{-i\omega_0\tau}$$

$$Sig(\tau) \propto 2\mathcal{E}\left[1 + Re\{\gamma(\tau)\}\right]$$

$$= 2\mathcal{E}\left[1 + e^{-\frac{\tau^2}{4T^2}} \cos(\omega_0\tau)\right]$$

$\longrightarrow$

(6)



$$\boxed{\tilde{\tau}_c \equiv \int_{-\infty}^{\infty} |\gamma(\tilde{\tau})|^2 d\tilde{\tau}}$$

$\tilde{\tau}_c$ = coherence time

= delay necessary for $\gamma(\tilde{\tau})$ to become steady (no oscillation).

l_c = coherence length

$\equiv c\,\tilde{\tau}_c$

