

FACTOR MODELS

To compute the efficient set, you need estimates of expected return and the covariances between the securities in the available population. One way you can get these estimates is by sampling from past returns. For example, the expected monthly rate of return can be estimated from the mean of the series of past monthly returns. The covariance between two securities can be estimated on the basis of the sample covariance, as given in Equation (3.1), over the same series of monthly returns.

While this is the most straightforward approach to obtaining these estimates, it has some problems. Sample means of stock returns are unstable and unreliable as estimates of the true expected rate of return. In order to reduce sampling error, sample estimates also require a lengthy history of past returns. Unfortunately, the farther you go back in time, the more likely the series of stock returns doesn't reflect the contemporary character of the firm. For example, the firm may have recently merged or significantly changed its capital structure by adding more debt. Given this, the firm today has a significantly different character from its past. The return series of its future is likely to be significantly different from its past. Accordingly, sample estimates may prove to be unreliable estimates of the future.

To provide better estimates of covariances and expected returns, you can employ factor models. Risk factors are variables (the rate of inflation, the growth in industrial production) that induce stock prices (and rates of return to stocks) to go up and down from period to period. Different stocks respond to movements in the risk factors to different degrees. The differential responses ultimately determine the differential future covariances of return between different stocks. Expected return factors are firm characteristics (relative risk, firm size, liquidity, etc.), which may be useful in explaining (and ultimately predicting) why some firms systematically produce higher returns, on average, than others.

RISK FACTOR MODELS TO ESTIMATE VOLATILITY OF RETURN

THE SINGLE-FACTOR MODEL

Essentially, the single-factor model assumes security returns are correlated for only one reason. Each security is assumed to respond, in some cases more and in other cases less, to the pull of a single factor, which is usually taken to be the market portfolio. As

the market portfolio makes a significant movement upward, nearly all stocks go up with it. Some stocks rise in price more than others, but as we observe the movement of stock prices over time, it is assumed that variability in the market portfolio accounts for all of the comovement we see among the stocks. This is, in fact, the assumption of the single-factor model: The model assumes all the numbers in the covariance matrix can be accounted for by the fact that all the stocks are responding to the pull of this single, common force.

To state the assumption of the single-factor model more precisely, consider Figure 6.1, where we have related the returns on an arbitrarily selected stock to the returns on the market portfolio. The broken line running through the scatter is the line of best fit (minimizing the sum of the squared vertical deviations of each observation from the line), or an estimate of the stock's characteristic line. The intercept of the characteristic line is given by A , and the slope by the beta factor, β . As defined in Chapter 3, the vertical deviations from the characteristic line are called residuals or shock terms, ε .

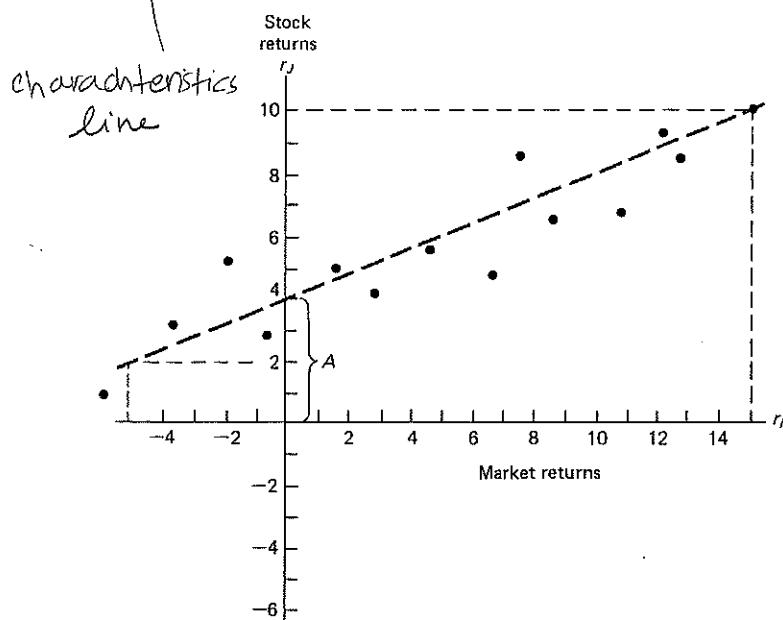
The rate of return for the stock in any one month can be written as

$$r_t = A + \beta r_{M,t} + \varepsilon_t$$

where r_t is the rate of return to a security or portfolio and $r_{M,t}$ is the rate of return to the market portfolio.

The single-factor model implicitly assumes that two types of events produce the period-to-period variability in a stock's rate of return. We refer to the first type of event as a macro event. Examples might include an unexpected change in the rate of inflation, a change in the Federal Reserve discount rate, or a change in the prime rate of interest. In any case, macro events are broad or sweeping in their impact. They affect nearly all firms to one degree or another, and they may have an effect on the general level of stock

FIGURE 6.1 RELATIONSHIP BETWEEN THE RETURNS ON AN INDIVIDUAL INVESTMENT AND THE RETURNS ON THE MARKET PORTFOLIO.



2 - events produce
variability in stock
returns
MACRO
EVENTS

prices.¹ They produce a change in the rate of return to the market portfolio, and through the pull of the market, they induce changes in the rates of return on individual securities. Thus, in Figure 6.1, if the return to the market portfolio in a given period were equal to -5 percent, we would expect the return to the stock to be 2 percent. If the market's return were 15 percent instead, we would expect the stock's return to be 10 percent. The difference in the stock's expected return can be attributed to the difference in the pull of the market from one period to the other.

MICRO
EVENTS

The second type of event that produces variability in a security's return in the single factor model is micro in nature. *Micro events* have an impact on individual firms but no generalized impact on other firms. Examples include the discovery of a new product or the sudden obsolescence of an old one. They might also include a local labor strike, a fire, or the resignation or death of a key person in the firm. These events affect the individual firm alone. They are assumed to have no effect on other firms, and they have no effect on the value of the market portfolio or its rate of return. Micro events do affect the rate of return on the individual security, however. They cause the stock to produce a rate of return that might be higher or lower than normal, given the rate of return produced by the market portfolio in the period. Micro events, therefore, are presumed to cause the appearance of residuals or deviations from the characteristic line.

Other types of events have been assumed away by the model. One might be referred to as an *industry event*, an event that has a generalized impact on many of the firms in a given industry but is not broad or important enough to have a significant impact on the general economy or the value of the market portfolio. Events of this nature also may, conceivably, cause the appearance of a residual, but the single-factor model assumes residuals are always caused by micro events.

The foregoing scenario is consistent with the assumption that the residuals or shock terms for different companies are uncorrelated with one another, as depicted in Figure 6.2. The residuals will be uncorrelated if they are caused by micro events that affect the individual firm alone but not other firms.

As noted, the single-factor model assumes all the numbers in the covariance matrix for the returns on securities can be accounted for by the fact that each of the stocks responds, to some degree, to the pull of a single common factor, the market. In fact, given the assumption of the single-factor model, we can write the covariance between any two securities, J and K , as

$$\text{Cov}(r_J, r_K) = \beta_J \beta_K \sigma^2(r_M)$$

The right-hand side of this equation is the product of three terms. The third, which is the variance of the rate of return to the market, specifies the magnitude of the market's movement or the strength of its pull; the first two, which are the beta factors for the two securities, specify the extent to which each of the two securities responds to the pull.

THE SINGLE-FACTOR MODEL'S SIMPLIFIED FORMULA FOR PORTFOLIO VARIANCE

On the basis of the foregoing assumptions and conditions, we can derive an alternative formula for the variance of a portfolio. We begin by noting that after passing a line of best fit through points representing pairs of returns between security or portfolio returns

¹ Alternatively, you may want to account for the macro factors individually. In this case you would use a multifactor model with factors such as changes in interest rates, inflation, and industrial production.

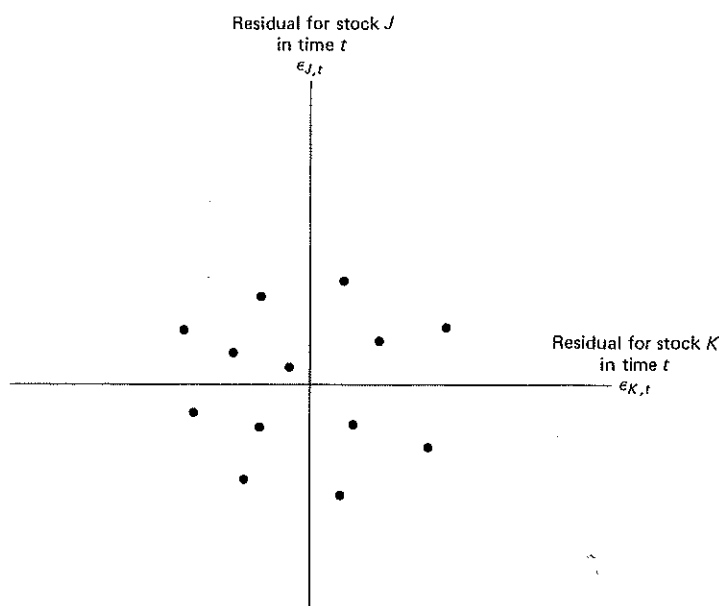


FIGURE 6.2 RELATIONSHIP BETWEEN RESIDUALS ON STOCKS J AND K.

and market returns, as in Figure 6.1, we can always split the variance of the return on a security or portfolio into two parts:

$$\sigma^2(r) = \beta^2 \sigma^2(r_M) + \sigma^2(\epsilon) \quad (6.1)$$

Total variance = Systematic risk + Residual variance

The first term on the right-hand side of Equation (6.1) is called the systematic risk of the investment. Under the assumptions of the single-factor model, it accounts for that part of the security's variance that cannot be diversified away. This part of the security's variance is contributed to the variance of even a well-diversified portfolio of many different stocks. The second term is called residual variance or unsystematic risk. It represents the part of a security's total variance that disappears as we diversify. Because of residual variance, the variance of a portfolio is less than the weighted average of the variances of the securities in the portfolio.

We can see from the equation that variability in return to a stock may account for two components. The systematic risk accounts for one part of the total variability. This part results when market movement pulls the security *along* its characteristic line. Note that systematic risk itself is the product of two terms. The first term involves the security's beta, which tells us the extent to which the security responds to the up and down pull of the market. The second term is the market's variance, which tells us the extent to which the market is pulling up and down. The remaining component of a security's variance is the residual variance. This accounts for the part of the variability that is due to deviations *from* the characteristic line. Thus, when we think of the total variability in a security's returns under the single-factor model, part of it is due to movement by the security along its characteristic line and part of it is due to deviations from the characteristic line.

Equation (6.1) holds for both an individual security and a portfolio. Rewriting equation for the case of a portfolio, we get

$$\sigma^2(r_P) = \beta_P^2 \sigma^2(r_M) + \sigma^2(\varepsilon_P) \quad (6.2)$$

At this point, we need equations for the beta factor and residual variance of a portfolio as functions of the characteristics of the securities we put in the portfolio. Or we have these equations, we can substitute them for portfolio beta and residual variance and obtain a more simple, alternative expression for portfolio variance to use in finding the minimum variance set.

The beta factor for a portfolio of M securities is a simple weighted average of betas of the stocks in the portfolio, where the weights are the relative amounts invested in each security.

$$\beta_P = \sum_{j=1}^M x_j \beta_j$$

Portfolio beta = Weighted average of security betas

Thus, if we have two stocks, one with a beta of 1.00 and the other with a beta of 0.0 and we invest 75 percent of our money in the stock with the larger beta and 25 percent in the other stock, the portfolio would have a beta of .75.

Now consider the formula for the residual variance of a portfolio. To determine what the residual variance is, we can use the same procedure we used to determine the variance of the portfolio's *returns* (as opposed to its *residuals*) in the Markowitz model. That is, we could employ the covariance matrix for the residuals on the various stocks. For the case of a three-security portfolio, the matrix would look like this:

	Security	x_A A	x_B B	x_C C
x_A	A	$\sigma^2(\varepsilon_A)$	$\text{Cov}(\varepsilon_B, \varepsilon_A)$	$\text{Cov}(\varepsilon_C, \varepsilon_A)$
x_B	B	$\text{Cov}(\varepsilon_A, \varepsilon_B)$	$\sigma^2(\varepsilon_B)$	$\text{Cov}(\varepsilon_C, \varepsilon_B)$
x_C	C	$\text{Cov}(\varepsilon_A, \varepsilon_C)$	$\text{Cov}(\varepsilon_B, \varepsilon_C)$	$\sigma^2(\varepsilon_C)$

Now the *assumption* of the single-factor model comes into play. The covariance between the residuals on any two securities is assumed to be equal to zero. Given this assumption, all the covariances in the matrix that are *off* the diagonal are assumed equal to zero. This means, to compute the residual variance of a portfolio, we need only go down the diagonal of the matrix, taking each security's residual variance and multiplying it by the portfolio weight at the top of the column and again by the portfolio weight at the left-hand side of the row. Because both these two weights are equal to the portfolio weight for the security itself, we have the following relationship:

$$\sigma^2(\varepsilon_P) = \sum_{j=1}^M x_j^2 \sigma^2(\varepsilon_j) \quad (6.3)$$

Portfolio residual variance = "Weighted average" of security residual variances where portfolio weights are squared

Thus, the residual variance of a portfolio is also a weighted average (of sorts) of the residual variances of the securities in the portfolio. However, this time, in taking the average, we square the portfolio weights.

Given the assumption of uncorrelated residuals among securities, the residual variance of a portfolio begins to disappear as the number of securities in the portfolio is increased. Consider the residual variance formula, and suppose we have a large number of securities, each with a residual variance equal to 10 percent. If we invest half our money equally in two of the securities, the residual variance of the two-security portfolio is 5 percent according to the foregoing formula.

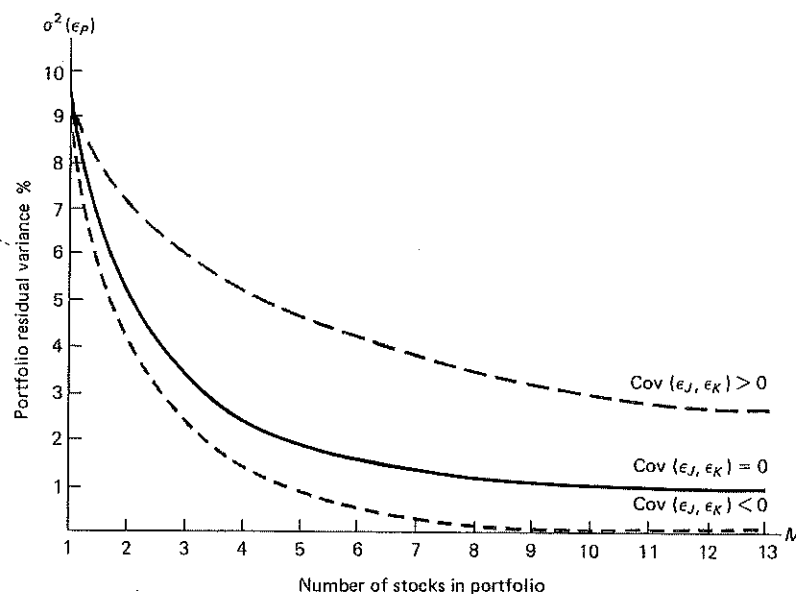
$$\sigma^2(\epsilon_P) = (.50^2 \times .10) + (.50^2 \times .10) = .05$$

In the same sense, if we invest a third of our money in each of three of the securities, the residual variance of the portfolio would be 3.33 percent and so on, as shown by the solid curve in Figure 6.3.

As we diversify, the residual variance of the equally weighted portfolio approaches, but never quite reaches, zero. This is because the residuals in the portfolio are presumed to be uncorrelated, and the good individual things that are happening to some of the securities are being offset by the bad things happening to others. Some are above their characteristic lines, but others are below; the residual of the portfolio, being the average of the residuals of the individual securities, can be expected to be quite small if the number of securities is large. In fact, when we are dealing with a portfolio that is weighted *equally* among the various securities, the residual variance of the portfolio is equal to the average residual variance of the stocks, divided by the number of securities in the portfolio. Of course, as the residual variance of the portfolio gets smaller and smaller, the correlation of the portfolio's returns with the market gets larger and larger, as shown in Figure 6.4.

These relationships depend crucially on our assumption that the residuals for different securities are uncorrelated. Suppose this is an invalid assumption. Suppose that industry-type events frequently occur, and the covariance between the residuals for

FIGURE 6.3 THE EFFECT OF DIVERSIFICATION ON THE RESIDUAL VARIANCE OF A PORTFOLIO.



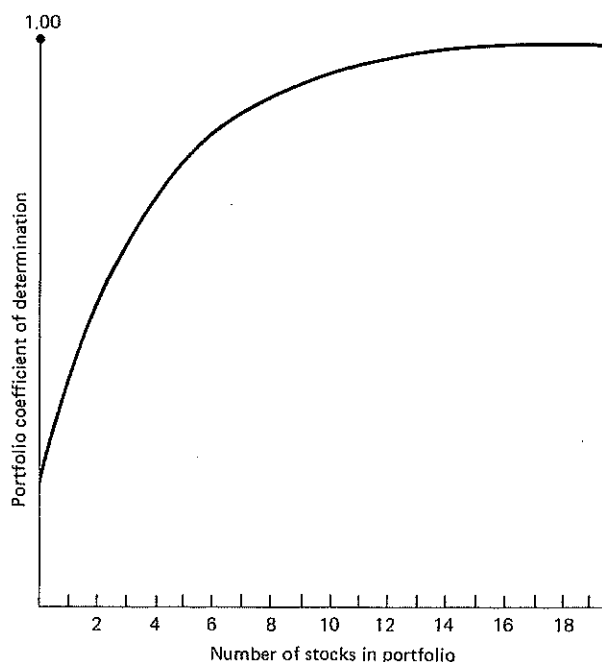


FIGURE 6.4 RELATIONSHIP BETWEEN THE COEFFICIENT OF DETERMINATION AND THE NUMBER OF STOCKS IN THE PORTFOLIO IN THE SINGLE-FACTOR MODEL.

different securities is typically *positive* and not zero. In this case, the off-diagonal elements of the matrix for residual covariances will be predominantly positive numbers. If we follow the single-factor model formula for portfolio residual variance and simply go down the diagonal of the matrix, we will underestimate the true residual variance of the portfolio. The actual residual variance will be larger than the single-factor model tells us it is, based on its assumption, because it is ignoring the positive elements in the sum that are off the diagonal. The relationship between the true residual variance and the number of securities in the portfolio may really look like the upper broken line of Figure 6.3.

Suppose, on the other hand, the covariances between the residuals for the securities in the population are typically *negative*. This might be the case for two stocks issued by companies that are very competitive. In this case, any event that is positive for one of the companies is negative for the other. If the numbers off the diagonal in the covariance matrix for the residuals are predominantly negative, the single-factor model gives an overestimate of the true residual variance of the portfolio. The actual residual variance, obtained by summing the products obtained for each element in the matrix, will be smaller than the sum obtained by simply going down the diagonal. The actual relationship between residual variance and the number of securities in the portfolio might then look like the lower broken line of Figure 6.3.

To summarize, the beta factor of a portfolio is equal to a weighted average of the betas of the securities in the portfolio, where the weights are equal to the fractions of our money we invest in each security. The residual variance under the single-factor model is assumed to be given by a similar weighted average, but this time, in taking the average, we square the portfolio weights.

We know that in the context of the single-factor model we can split the variance of any investment, including a portfolio, into two components, systematic risk and residual variance, as in Equation (6.2). Substituting the expressions we have derived for the

portfolio's beta and residual variance, we obtain the single-factor model's simplified formula for portfolio variance:

$$\sigma^2(r_P) = \left(\sum_{j=1}^M x_j \beta_j \right)^2 \sigma^2(r_M) + \sum_{j=1}^M x_j^2 \sigma^2(\varepsilon_j)$$

$$\begin{array}{ccccc} \text{Total portfolio} & = & \text{Portfolio systematic} & + & \text{Portfolio residual} \\ \text{variance} & & \text{risk} & & \text{variance} \end{array}$$

However, the reduction in the complexity of the model comes at a price. As we said before, the variance number obtained from the Markowitz formula is perfectly accurate, given the accuracy of the covariance estimates. The model makes no assumptions regarding the process generating security returns. The single-factor model, on the other hand, assumes the residuals, or deviations from the characteristic line are uncorrelated across different companies. The variance number obtained from the single-factor model, therefore, is only an approximation of the true variance. Even if the estimates of beta and residual variance that we feed into the model are perfectly accurate, the estimate of portfolio variance we obtain from the model is only as accurate as our assumption regarding the residuals.

It is obvious that the assumption isn't strictly accurate. After all, suppose something good happens to General Motors. This has an immediate impact not only on General Motors itself but also on the company's suppliers and competitors. Many companies would be affected simultaneously, some positively and others negatively. The residuals that appear for these firms would not be independent but rather would be generated by a common event. We know, therefore, that the residuals are correlated to some degree. We hope, however, that the degree of correlation is small enough that the inaccuracy of the single-factor model's portfolio variance equation doesn't transcend its relative efficiency.

AN EXAMPLE WHERE THE SINGLE-FACTOR MODEL WORKS

Consider two hypothetical stocks, Blue Steel and Black Rubber. In Table 6.1 are the rates of return for these companies, for the market portfolio, and for an equally weighted portfolio of the two stocks for five periods of time. The two-stock portfolio is assumed to be rebalanced to equal weights at the beginning of each period. Given this, the return for the portfolio is a simple average of the returns to the stocks in each period.

The returns for each stock and for the portfolio are plotted against the returns for the market in Figures 6.5, 6.6, and 6.7. Note that the beta factor for Blue Steel is

TABLE 6.1 Rates of Return to the Market, Two Stocks, and a Portfolio

Period	Market Portfolio r_M	Blue Steel r_S	Black Rubber r_R	Two-Stock Portfolio r_P
1	30%	30%	55%	42.5%
2	40	60	40	50
3	20	50	30	40
4	35	45	27.5	36.25
5	25	15	22.5	18.75

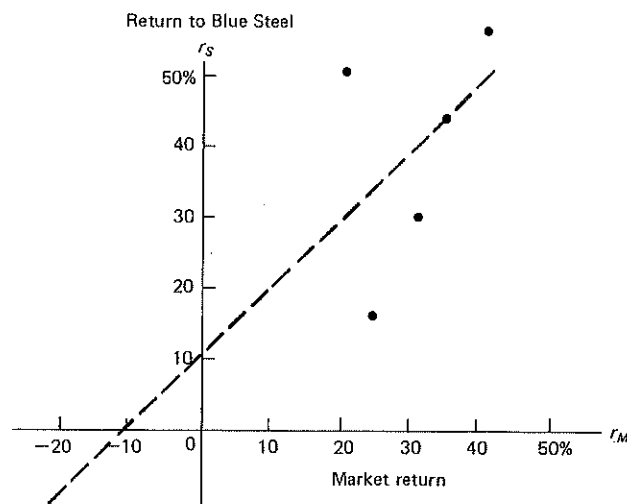


FIGURE 6.5 BLUE STEEL.

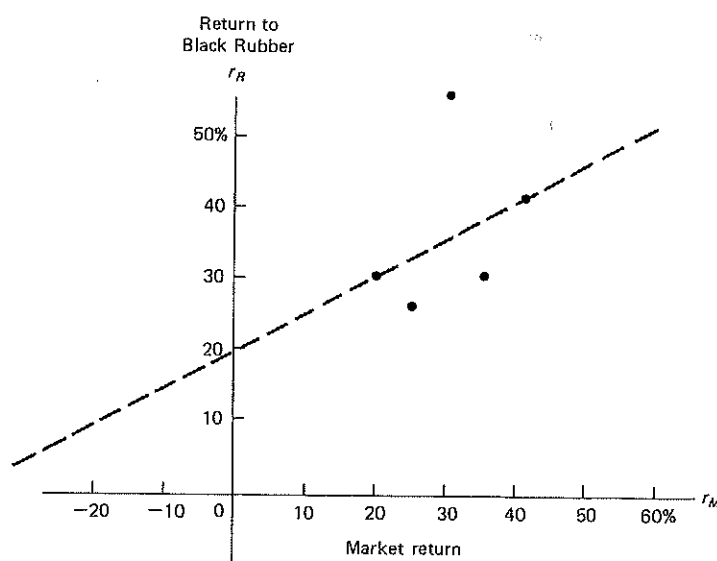


FIGURE 6.6 BLACK RUBBER.

equal to 1.00, the beta for Black Rubber is equal to .50, and the beta for the portfolio is the average of the two, or .75. The intercept of the portfolio (15 percent) is also the weighted average of the intercepts on Blue Steel (10 percent) and Black Rubber (20 percent).

Recall from Chapter 3 the general statistical procedure for computing residual variance: First compute the differences between the actual rates of return to the investment and the rates of return you expect the investment to produce, given its characteristic line and the market return for the period. The difference for any one period would be equal to

$$r_t - (A + \beta r_{M,t})$$

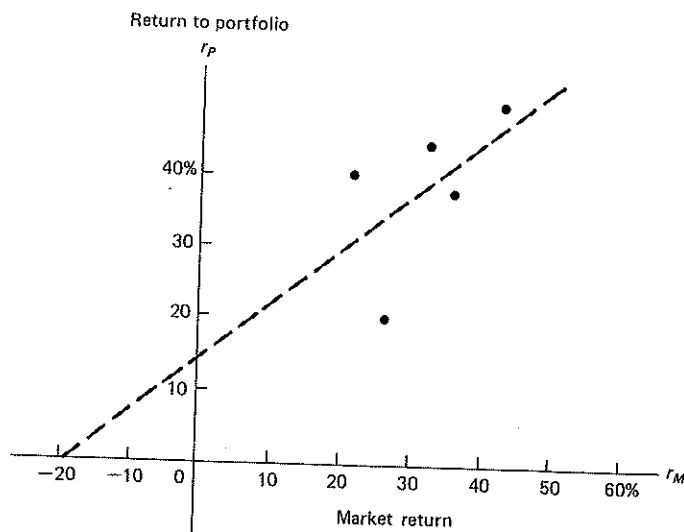


FIGURE 6.7 TWO-STOCK PORTFOLIO.

The differences for each period are then squared and the squared differences summed. The sum is divided by the number of periods observed, less 2. Therefore, the residual variance of Blue Steel can be computed as

$$\begin{aligned}
 & [.30 - (.10 + 1.00 \times .30)]^2 \\
 & + [.60 - (.10 + 1.00 \times .40)]^2 \\
 & + [.50 - (.10 + 1.00 \times .20)]^2 \\
 & + [.45 - (.10 + 1.00 \times .35)]^2 \\
 & + [.15 - (.10 + 1.00 \times .25)]^2 \\
 & \hline
 & .1000 \\
 & .1000 / (5 - 2) = .0333
 \end{aligned}$$

and the residual variance for Black Rubber as

$$\begin{aligned}
 & [.55 - (.20 + 0.50 \times .30)]^2 \\
 & + [.40 - (.20 + 0.50 \times .40)]^2 \\
 & + [.30 - (.20 + 0.50 \times .20)]^2 \\
 & + [.275 - (.20 + 0.50 \times .35)]^2 \\
 & + [.225 - (.20 + 0.50 \times .25)]^2 \\
 & \hline
 & .0600 \\
 & .0600 / (5 - 2) = .0200
 \end{aligned}$$

The residual variances for the portfolio are given by

$$\begin{aligned}
 & [.425 - (.15 + 0.75 \times .30)]^2 \\
 & + [.500 - (.15 + 0.75 \times .40)]^2 \\
 & + [.400 - (.15 + 0.75 \times .20)]^2 \\
 & + [.3625 - (.15 + 0.75 \times .35)]^2 \\
 & + [.1875 - (.15 + 0.75 \times .25)]^2 \\
 & \hline
 & .0399 \\
 & .0399 / (5 - 2) = .0133
 \end{aligned}$$

The portfolio's residual variance conforms to the value predicted by the single-factor model, a weighted average of the residual variances of each stock, where we square the portfolio weights.

$$.0133 = (.50)^2 \times .0333 + (.50)^2 \times .0200$$

That happens because the example was constructed so the correlation coefficient between the residuals was equal to zero.

AN EXAMPLE OF A POTENTIAL PROBLEM WITH THE SINGLE-FACTOR MODEL

To illustrate the potential problem with the single-factor model, consider the following example. Suppose we have two stocks, Unitech (U) and Birite (B). The stocks have the following characteristics:

	<i>Beta</i>	<i>Residual Variance</i>
Unitech	0.50	.0732
Birite	1.50	.0548
Market index variance	.0600	

Given this information, the variance of the two stocks can be written as the sum of their respective systematic risks and residual variances:

$$\sigma^2(r_j) = \beta_j^2 \sigma^2(r_M) + \sigma^2(\hat{\epsilon}_j)$$

$$\text{Unitech } .0882 = .50^2 \times .060 + .0732$$

$$\text{Birite } .1898 = 1.50^2 \times .060 + .0548$$

The covariance matrix for the *rates of return* to the two stocks is assumed to be given by

<i>Stocks</i>	<i>Unitech</i>	<i>Birite</i>
Unitech	.0882	.0594
Birite	.0594	.1898

Under the assumption of the single-factor model, the covariance between the *returns* on the two stocks is equal to the product of their betas and the variance of the market index.

$$\text{Cov}(r_U, r_B) = \beta_U \times \beta_B \times \sigma^2(r_M)$$

$$.0450 = .50 \times 1.50 \times .060$$

The actual covariance between the rates of return is greater than this number, which means the residuals for the two stocks are positively correlated. The covariance matrix for the *residuals* is in fact assumed to be given by

<i>Stocks</i>	<i>Unitech</i>	<i>Birite</i>
Unitech	.0732	.0144
Birite	.0144	.0548

Now suppose we form an equally weighted portfolio of the two stocks. The beta factor of the portfolio is given by

$$\beta_P = x_U \times \beta_U + x_B \times \beta_B$$

$$1.00 = .50 \times .50 + .50 \times 1.50$$

Under the assumption of the single-factor model, the residual variance can be estimated by going down the diagonal of the covariance matrix for the residuals.

$$\sigma^2(\epsilon_P) = x_U^2 \times \sigma^2(\epsilon_U) + x_B^2 \times \sigma^2(\epsilon_B)$$

$$.032 = .25 \times .0732 + .25 \times .0548$$

To compute the *true* residual variance of the portfolio, we would have to add to this number the two products from the two off-diagonal elements of the covariance matrix for the residuals.

$$\sigma^2(\epsilon_P) = .032 + 2 \times .50 \times .50 \times .0144 = .0392$$

Under the assumption of the single-factor model, we would estimate the variance of the equally weighted portfolio as

$$\sigma^2(r_P) = \beta_P^2 \times \sigma^2(r_M) + \sigma^2(\epsilon_P)$$

$$.092 = 1.00 \times .060 + .032$$

This is really an underestimate of the true portfolio variance. To find the true variance, we use the Markowitz technique, multiplying each element in the covariance matrix of returns by the portfolio weights for the two stocks.

$$\begin{aligned} &.50 \times .50 \times .0882 \\ &+ .50 \times .50 \times .0594 \\ &+ .50 \times .50 \times .0594 \\ &+ .50 \times .50 \times .1898 \\ &\hline &.0992 \end{aligned}$$

The difference between the actual portfolio variance and our estimate using the single-factor model (SFM) is equal to our underestimate of the residual variance.

$$\begin{array}{rcl} \text{Markowitz variance} & - & \text{SFM variance} \\ .0992 & - & .092 & = .0072 \\ \text{Actual residual variance} & - & \text{SFM residual variance} \\ .0392 & - & .032 & = .0072 \end{array}$$

Interestingly, most professional managers focus on optimizing tracking error in relation to a stock market index, such as the S&P 500 rather than volatility of return. Tracking error can be taken as the differences between your portfolio return and the return to your target market index. In optimizing, managers attempt to minimize tracking error given the expected return on their portfolios. As we shall learn in Chapter 10, the widespread use of this type of optimization brings lucrative opportunities to those who optimize on volatility of return.

MULTIFACTOR MODELS

In the single-factor model, we attribute the covariances between the returns on stocks to a single factor, usually market index. In a multifactor model, we attribute the covariances to two or more factors. Suppose, for example, we assume stocks tend to move up and down together because they are simultaneously responding to two factors. Those factors are movement in the rate of inflation and movement in the economy-wide growth rate in industrial production. The rate of return to any stock J in any period t is given by

$$r_{J,t} = A_J + \beta_{i,J}r_{i,t} + \beta_{g,J}g_t + \epsilon_{J,t} \quad (6.4)$$

where $\beta_{i,j}$ is the stock's inflation beta. It measures the response of the stock to unexpected changes in the rate of inflation. The term g_t is the unexpected growth rate in industrial production in any given period, and $\beta_{g,j}$ measures the stock's response to unexpected changes in the growth rate in industrial production. We say *unexpected* changes because the price of the stock is likely to be affected only by changes in industrial production not already anticipated by investors and discounted into the price of the stock. Just as, in the context of the single-factor model, the beta factor is estimated by relating the returns of the stock to the returns to the market index over a number of previous periods, so in a multifactor model the betas can be estimated by relating the stock's returns both to the unexpected change in inflation and to the unexpected growth rate in industrial production. One way of obtaining numbers for the latter series is to take the difference between the actual rates of inflation and growth in industrial production and the average rates forecasted by some group of professional economists.

In the context of a single-factor model, we slide a line of best fit through the data (stock returns versus market returns). Similarly, in the context of a double-factor model, like the foregoing one, we slide a plane of best fit through the data (stock returns versus unexpected inflation and changes in industrial production). If the plane is drawn to minimize the sum of the squared deviations from it, the residuals, or vertical deviations from the plane, must be uncorrelated with both the market returns and industrial production.

As with the single-factor model, portfolio variance can be separated into the sum of systematic risk and residual variance. Systematic risk, however, is now more complex. It is related to the variances and covariances of the sources of this risk—the factors—and the sensitivities of the portfolio to movements in the factors—the factor betas.

Similar to the procedure used in Chapter 4, with our two-factor model, the systematic risk of a portfolio of securities can be computed using the following covariance matrix:

$$\begin{array}{cc} & \begin{array}{cc} \beta_{P,i} & \beta_{P,g} \end{array} \\ \begin{array}{c} \beta_{P,i} \\ \beta_{P,g} \end{array} & \begin{array}{|cc|} \hline \sigma^2(i) & \text{Cov}(i, g) \\ \hline \text{Cov}(g, i) & \sigma^2(g) \\ \hline \end{array} \end{array}$$

The matrix shows the covariances between the *factors*. To obtain an estimate of the variance of return, we sum a series of products. Starting with the matrix of factor covariances, we take each covariance in the matrix and multiply it by the portfolio factor beta at the top of the column and by the portfolio factor beta at the left-hand side of the row. Thus, for the covariance between industrial production and inflation, you would obtain the following product:

$$\beta_{P,g} * \beta_{P,i} * \text{Cov}(i, g)$$

The portfolio's betas are, once again, weighted averages of the betas of the individual securities in the portfolio, where the weights are the fractions invested in each security.

With our two-factor model, you would compute four products. Add them up and you have an estimate of the amount of portfolio variance that's attributable to the portfolio's responses to movements in the two factors. With more factors, you simply have a larger matrix and more products.

By examining the matrix, we can see the advantage in using factor models to estimate the risk of a portfolio. Suppose, instead of a factor model, you simply use a past series of returns to compute the covariances of returns between stocks. On the one hand, the more months into the past you use, the lower is the sampling error for your estimate.

On the other hand, the further into the past you go in sampling returns, the more likely it is that the nature of the firms behind the stocks has significantly changed. Thus, you have a problem. Going further back in time reduces sampling error but increases the chance that the return observations are irrelevant to the current situation.²

However, if you employ a risk factor model, you can effectively address this problem. The nature of the underlying macroeconomy is probably less subject to dramatic change than is the character of an individual firm. If you believe this, you can estimate the inflation and industrial production betas (which are firm dependent) for the individual stocks in your portfolio over a relatively short period into the past and the variances and covariances between inflation and industrial production over a considerably longer period.³

To obtain an estimate of total portfolio variance, add, to your estimate of systematic risk, your estimate of residual variance. If you assume that you have accounted for all sources of the correlations of return with the various factors in your model, the correlations between the remaining residuals can be assumed to be zero, and portfolio residual variance can be calculated in accord with Equation (6.3).

To simplify matters, assume the residuals are uncorrelated and, in addition, that the rate of return to the market and the unexpected growth rate in industrial production are also uncorrelated with each other. Given this, we can write the variance of a portfolio of stocks as

$$\begin{aligned}\sigma^2(r_P) &= \beta_{i,P}^2 \sigma^2(i) + \beta_{g,P}^2 \sigma^2(g) + \sigma^2(\varepsilon_P) \\ \text{Total variance} &= \text{Systematic risk (inflation)} + \text{Systematic risk (industrial production)} + \text{Residual variance}\end{aligned}\quad (6.5)$$

The inflation beta for the portfolio is again a weighted average of the inflation betas of the stocks in the portfolio. The portfolio's beta with respect to unexpected changes in industrial production is also a weighted average.

If we now assume the residuals on any two stocks are also uncorrelated with each other, as with the single-factor model, we can write the residual variance of a portfolio as

$$\sigma^2(\varepsilon_P) = \sum_{j=1}^M x_j^2 \sigma^2(\varepsilon_j)$$

The final equation for residual variance is based on the presumption that we have now fully considered all the factors that account for the interrelationships among the returns on stocks. This being the case, the residuals for different companies will now be uncorrelated. If we should find, to our dismay, that the covariances between the residuals are still significantly different from zero, we haven't taken into account all the relevant factors. We need to move to a trifactor model or beyond. The search for such factors is now a matter of intense interest among practitioners in investments. The best evidence to date seems to indicate that the covariances among stock returns can be explained by as many as four or five factors.

² What about increasing the number of observations by shortening the return period to, say, a day rather than a month? The problem with this is that the returns may become nonsynchronous with each other. Most stocks don't trade continuously during the day. If your returns are measured from close to close, you may have a problem. The closing price on one stock may be from a trade at midday; the close on another from the last minute of the trading day. There may have been a significant change in the general level of stock prices between the final trade on the one stock and the other. Thus, the returns aren't really comparable, leading to an error in your estimate of the covariance between the two stocks.

³ It should be noted that if you estimate the betas and variances/covariances over the same period, you will find that you compute the same answer for portfolio variance as you would if you simply computed the variance directly from the returns to the proposed portfolio during the period.

OUT ON THE STREET

IN THE WEE SMALL HOURS

A bleary-eyed David Olson drains the last of his Coke from its can and then returns his attention to his computer terminal. It's 4:00 A.M., and David has been writing code for 17 hours without a significant break. As the hours pass, and his mind slowly turns to curly spaghetti, David fights to "straighten out the strands" so that they might continue to process logical thought.

David is director of systems development at Haugen Custom Financial Systems. His firm designs and builds customized financial software for financial consulting firms, brokerage houses, banks, and insurance companies. The software is used by these institutions for their own analysis or to retail to their clients as a service. David develops systems for asset allocation, performance measurement, and the risk management of equity portfolios. His bachelor's degree in computing science and his M.B.A. with emphasis on finance give him the combination of skills required to bring the complex tools and theories of modern investment theory into the hands of the professional investor. In addition to a thorough knowledge of the principles of finance, David's job requires the skills to put these principles to work writing user-friendly, interactive computer codes.

David is working at 4:00 A.M. because he's under a tight deadline. He is on the final stages of a particularly complex international asset allocation model.

The model is based on a factor model that divides the world into three *economic* regions: North America, Europe, and the Far East. Each region has a set of macroeconomic factors, such as aggregate consumption and industrial production. For each month, the value of any given factor is found by taking a weighted average of the factor across the countries of the region. The weights are based on the level of total consumption in each country. A given country is assigned to a particular region if the monthly returns to its capitalization-weighted common stock index are found to be best explained by changes in the region's macroeconomic factors.

The table shows the factor models for the North American economic region. The factors are listed in the left column. Most of them are self-explanatory. Unexpected changes are based on realized deviations from the predictions of a statistical model. For example, if the statistical model predicts that the price of oil will fall by 2 percent in a particular month, and oil ac-

tually increases in price by 1 percent for the month, the unexpected change is taken to be 3 percent. Crash is a dummy variable, which takes on a value of one for the month of October 1987 and zero otherwise. The factor model is estimated by regressing the factors on the monthly returns to the stock indices for each country. For some countries, a particular factor may not be an important determinant of the countries' returns. Therefore, the factor is not included in the country's factor model.

Canada, Malaysia, Mexico, Singapore, and the United States are all in the North American economic region. Apparently, Malaysia and Singapore are included because of their trade relationships with North American countries. Listed in the cells under each country are the signs of the relationships between each factor and the returns to the country's stock index. If the relationship is statistically significant, the sign is in boldface. An *L* indicates a lagged relationship. Thus, the returns to Canadian stocks in a given month are positively related to changes in the U.S. exchange rate relative to a weighted average of the other countries of the region in the same month and the previous month.

R^2 indicates the fraction of a country's stock returns that can be explained by movements in the region's factors. Thus, Canada shows the strongest relationship to the factors, and Mexico the weakest.

Interestingly, in the context of the model, Hong Kong and Australia are considered to be members of the European economic region, and Japan stands alone as the only member of the Far East economic region. The stock returns of all the other major countries of this *geographic* region are more closely related to the factors of other *economic* regions.

Once the factor models are identified, they can be used in the following ways:

1. The values for the factors for the forthcoming year can be estimated using time series models. Of course, time series models are not useful in *forecasting* unexpected changes in the factors or the returns to securities. Securities tend to move unpredictably, but their movements can be used to reliably predict variables like the price of oil and the rate of inflation. When time series forecasts of these variables

for the next year are imported to the factor model, expected returns to the various countries in the forthcoming year can be estimated on the basis of the sensitivities to the variables estimated in the factor model.

2. The institutions (brokerage houses, etc.) that may find the model useful have economists who specialize in forecasting by geographic regions of the world. Their forecasts of the future values for the factors can be brought to the model and translated into forecasts of future expected return by country.
3. Once expected returns are estimated by method 1 or 2, the model's mean-variance optimizer can be used to build portfolios of countries with maximum expected return given volatility exposure. Volatility of return is a major problem in international investing, because the volatilities of many countries are

high. (The volatility of Mexico, for example, is greater than 50 percent, compared to less than 20 percent in the United States.)

4. In other parts of the international asset allocation model, the user can run experiments, postulating different world economic scenarios (time paths for the factors). Then, through a technique called Monte Carlo simulation, the user can see the likely performance of alternative portfolios (constructed across countries). Likely portfolio performance is calculated on the basis of estimated portfolio sensitivity to movements in the factors under each proposed scenario.

David is about three hours away from completing a model that has filled the last four months of his working life. With a noon deadline that gives him the opportunity for five hours of sleep, he will take advantage of the couch in the next office.

Signs of Regression Coefficients for Country Equity Returns, January 1985–December 1994 (in U.S. \$)

North American Economic Region

<i>Factor</i>	<i>Can</i>	<i>Mal</i>	<i>Mex</i>	<i>Sgp</i>	<i>US</i>
Short-term bond return	—	—	—	—	—
Long-term bond return	+	+	—L3	—L1	+
% change in U.S. exchange rate to region	+L0, +L1	+	+	+	+L0, +L1
Inflation rate (CPI)			+	+	
Oil price (\$)	+L1	—L1			—
Unexpected change in oil price	—L1			—L1	—L0, —L1
Unexpected inflation (raw material)	+	+L1	+		—
% change industrial production				—	
Unexpected change industrial prod.	+	+	+		
% change industrial prod.					+
Unexpected change consumption (local currency)		+L1		+	—
Crash	—	—	—	—	—
<i>R</i> ²	.53	.30	.25	.30	.47
D.W.	2.03	2.16	2.08	2.30	2.20
<i>F</i>	15.3	8.2	5.2	7.3	12.1

Boldface indicates significance at better than 10 percent level.

L0 = Lag 0

ESTIMATING PORTFOLIO VARIANCE USING A MULTIFACTOR MODEL: AN EXAMPLE

Recall our earlier discussion of Unitech (U) and Birite (B) corporations. In the example, we estimated the variance of an equally weighted portfolio of the two stocks using the single-factor model. The single-factor model underestimated the actual variance of the portfolio because the residuals for the two stocks were positively correlated. A single factor was apparently inadequate for explaining the covariance between these two stocks. The actual covariance between the two stocks was .0594, greater than the covariance predicted by the single-factor model.

$$\begin{aligned}\text{Cov}(r_U, r_B) &> \beta_U \times \beta_B \times \sigma^2 r_M \\ .0594 &> .50 \times 1.50 \times .06\end{aligned}$$

Suppose the covariance between the residuals is caused by the presence of two factors, say, unanticipated changes in the rate of inflation and industrial production. The betas for the two stocks with respect to the inflation and industrial production and the true residual variances are assumed to be given by

	Industrial Production Beta	Inflation Beta	Residual Variance
Unitech	.50	1.20	.030
Birite	1.50	.40	.050

Assume that variance of unexpected changes in industrial production is 6 percent. The variance of the index of unanticipated inflation is assumed to be 3 percent.

The variance of each stock can be expressed as

Systematic risk

$$\begin{aligned}\text{Total variance} &= (\text{Industrial production}) + (\text{Inflation}) + \text{Residual variance} \\ \sigma^2(r) &= \beta_g^2 \sigma^2(r_M) + \beta_i^2 \sigma^2(i) + \sigma^2(\epsilon) \\ \text{Unitech: } .0882 &= .25 \times .06 + 1.44 \times .03 + .03 \\ \text{Birite: } .1898 &= 2.25 \times .06 + .16 \times .03 + .05\end{aligned}$$

Suppose we again form an equally weighted portfolio of these two stocks. The portfolio's betas with respect to the two indices is given by

$$\text{Industrial production beta: } 1.00 = .50 \times .50 + .50 \times 1.50$$

$$\text{Inflation beta: } .80 = .50 \times 1.20 + .50 \times .40$$

Given the assumption that the residuals are now truly uncorrelated, the residual variance of the portfolio can be computed as the weighted average of the true residual variances of the two stocks, where we square the portfolio weights.

$$\text{Portfolio residual variance} = .50^2 \times .03 + .50^2 \times .05 = .02$$

The total variance of the portfolio is estimated as the sum of the two systematic risk terms and the residual variance.

Systematic risk

$$\begin{aligned}\text{Total variance} &= (\text{Industrial production}) + (\text{Inflation}) + \text{Residual variance} \\ \sigma^2(r) &= \beta_g^2 \sigma^2(g) + \beta_i^2 \sigma^2(i) + \sigma^2(\epsilon) \\ .0992 &= 1.00^2 \times .06 + .80^2 \times .03 + .02\end{aligned}$$

If you recall, this is the answer we got when we computed the variance using the Markowitz technique. We get the correct answer this time because the example has been constructed assuming a double-factor framework.

FACTOR MODELS FOR ESTIMATING EXPECTED RETURN

Securities have different expected rates of return. Across *classes* of securities, such as bonds and stocks, the evidence is consistent with the notion that the primary determinants of these differentials are differentials in risk. Short-term bonds are far less risky than stocks, and because of this, they carry lower expected rates of return. For classes of securities, sample means of past rates of return are a good starting point for estimating future expected rates of return. Analysts usually start there, and then they make subjective adjustments to the estimates based on contemporary economic conditions that are now different from the past. For example, in making an estimate of the future rate of return on stocks, you might want to consider the fact that the stock market is less volatile now than it was in the 1930s. Lower volatility might induce investors to invest in stocks at lower expected rates of return. If this is the case, the prices of stocks may have risen such that expected equity returns are lower now than they were in the 1930s. Thus, you might want to adjust the long-term sample mean return on stocks downward to account for this. Similarly, since 1980 the volatilities of interest rates and the prices of long-term bonds have increased. Thus, the expected returns on long-term bonds may now be greater than their long-term averages.

While sample mean returns serve as a good starting point in estimating the expected returns of classes of securities, they are poor indicators of differentials in expected return between securities *within a class*, such as common stocks. The past returns to individual stocks have been affected by myriad idiosyncratic events that are unlikely to repeat themselves in the future. In addition, the contemporary character of an *individual firm* may differ considerably from its character in the distant past. Because of this, we must set aside sample means and look for a better approach.

Factor models can also be used to estimate expected returns. Here you try to estimate the tendencies of the market to produce differential returns (*payoffs*) to stocks with differential characteristics (*exposures*). You then project the future payoffs to the various characteristics and then relate them to a stock's individual exposures to the characteristics to produce an estimate of overall expected future return.

To illustrate the influence of one factor on expected return, over the long run the stocks of smaller firms have tended to produce greater rates of return. This may be due to the fact the investors want higher returns on small stocks because they consider them to be less liquid or more risky. In any case, the payoff to size has been negative over the long term. (The larger the company the lower its return.) Thus, if you are estimating the expected return to a stock with low exposure to size (it is relatively small), its expected rate of return may be increased if you are projecting a continuation of the negative payoff to size.

FIRM CHARACTERISTICS (FACTORS) THAT INDUCE DIFFERENTIALS IN EXPECTED RETURNS

One can profile a stock, and the company behind the stock, by many different characteristics—risk, stock liquidity, and so on. In a given period of time, say a month, the

OUT ON THE STREET

ARBITRAGE

The sound of the wind and the rain smashing against the large pane of glass 800 feet above downtown Los Angeles ramped up to the point of distraction. Dennis Bein looked up from his terminal to see several overlapping sheets of water cascading down the window across from his desk.

It was another Sunday in January. More work than usual this time because it was time to update the weights for ANALYTIC Investor's U.S. Long/Short Equity portfolio. At least he wouldn't be preoccupied by thoughts of dropping another 18-footer into the cup on his favorite seventh green. That cup was, undoubtedly, as flooded as the one he had left on the table this morning, outside the Oak Tree Cafe, just below the office.

He was in the process of constructing two complementary stock portfolios. Complementary in the sense that their monthly returns could be expected to be highly correlated with each other. Dennis was employing the Barra risk factor model to ensure that this would be the case. He adjusted the weights in each stock so that the "bets" on industries and sectors were very similar for each portfolio. At the same time, he was also making sure that the sensitivities (betas) with respect to the various Barra risk factors were not out of line between the one portfolio and the other. Dennis paid special attention to two of the factors in particular—the factor related to the monthly payoff to the relative market capitalization of the stocks in the portfolio and the factor related to the monthly payoff to the relative cheapness of the stocks. Dennis knew that, no matter how carefully he constructed the two portfolios, it was practically impossible to reduce the tracking error (volatility of the differences in their monthly returns) to less than 3 percent on an annualized basis. This was true because that much of the tracking error could be attributed to market forces that were unstable and unmanageable.

All of the stocks in both portfolios were among those included in the S&P 500 Stock Index. They were all fairly large companies. Nevertheless, the process Dennis was using to construct the portfolios took into consideration the likely market impact that his trades might have on the prices of the stocks he was buying and selling. Dennis would e-mail his trade list to Bear Sterns this evening. The trades would be executed early in the day on Monday. Execution was very important. You didn't want to temporarily bump the prices up or down because the "rebound" would work against your performance.

Although the two portfolios were matched in terms of their monthly correlation, they were diametrically opposed in terms of their expected rates of return. Dennis's optimizer contained estimates of the expected return (for the next month) for each stock in the S&P population. These expected returns had been estimated Saturday morning using a separate expected return factor model. The model employed more than 70 factors that profiled the characteristics of each stock. Each stock's profile was interfaced with the expected monthly payoff to each factor to obtain the overall monthly expected return for the stock. The two portfolios were constructed to achieve the highest possible difference in their expected returns while simultaneously maximizing correlation.

ANALYTIC would short-sell the stocks in the low expected return portfolio while buying the stocks in the high. The proceeds from short selling would be invested in cash. The relative positions in the two portfolios would be adjusted until their net beta factors against the market were zero. The combined portfolio would be market neutral. Movements in the returns to the S&P shouldn't affect the long/short portfolio's performance. The monthly return would always be equal to the difference between the return to the long and the short plus the return earned on the cash investment.

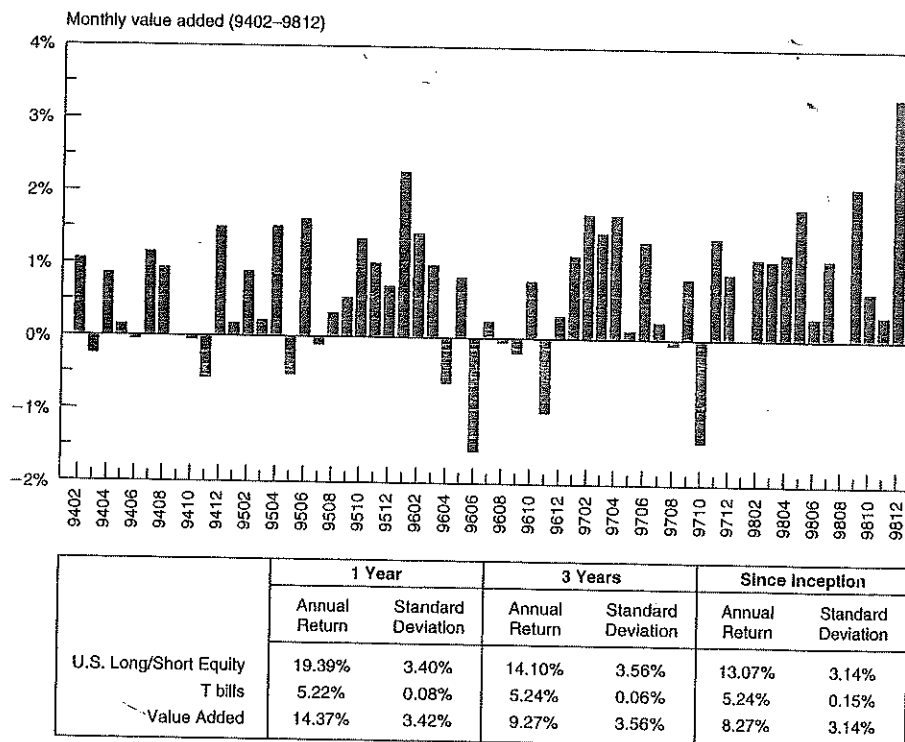
population of U.S. stocks will individually produce a wide array of returns. We will tend to find that, in the month, stocks of particular types will tend to produce larger or smaller rates of return. For example, especially in a month with strong market returns, stocks that are riskier might tend to have larger returns.

In using an expected return factor model, we will measure these tendencies, or *payoffs*, every month and develop a history of the monthly payoffs to the various elements of a stock's *profile*. From this history we will project the expected values of the various payoffs into the next period. Then, by interfacing the elements of the stock's

The simulated outperformance (relative to treasury-bill returns) of the portfolio is shown in Figure 6A. The strategy has produced an impressive annualized return of more than 13 percent with an annualized volatility of return of slightly more than 3 percent. Nearly all the volatility is attributable to the tracking error between the long and the short portfolios.

This should be an easy product to market. First, it has an expected return greater than that of stocks. Second, it has a risk level similar to that of short- to intermediate-term government bonds. Finally, it is "crash proof." If the stock market falls, although other investors will suffer a deterioration in the value of their holdings, the gains in the short positions for this fund should offset the losses in the longs.

FIGURE 6A LONG/SHORT EQUITY PROGRAM SIMULATED PERFORMANCE SUMMARY AS OF 12/31/98



current profile with the various expected payoffs, we can obtain an estimate of the stock's overall expected return for the coming period.⁴

⁴ In Part Three of this text we talk about theories of expected return. These theories restrict themselves to various measures of risk as the exclusive determinants of expected return. These theories are valid under their assumptions. In general, however, expected stock returns might be related to a wide variety of factors that collectively describe each stock's profile. Thus, the theories of Part Three might be best regarded as special cases of the expected return factor models discussed in this section, which can be expected to hold only their specific assumptions.

OUT ON THE STREET

PHASES AND FACTORS

Bob Marchesi's number-eight iron arched perfectly into the ball, sending it skyward with just the right amount of backspin. It hit just above the eighteenth hole, popped back, and rolled three feet in front of the cup. BINGO! What a day this had been. A warm sunny day—truly rare for Scotland—a light breeze, and with this sure bird, he'd be 10 over on St. Andrews, one of the toughest courses he'd ever played. In the days of his life, this will surely place in Bob's top 10.

Bob Marchesi is president of DeMarche Associates. Based in Kansas City, DeMarche is one of the major pension fund consulting firms. DeMarche advises hundreds of pension funds on the selection of investment managers and the measurement and analysis of their performance. He also advises on the optimal allocation of investments over broad classes of assets, such as domestic common stock, real estate, and government securities.

Much of the asset allocation analysis is based on the optimization techniques of Markowitz. Estimates of expected return and standard deviations of the asset classes are entered into the optimizer. So are estimates of the correlations between the monthly returns to the asset classes. An efficient set is produced, and an allocation is recommended on the basis of the plan's risk tolerance. More recently, the analysis had been extended to cover more detailed asset groupings. For example, DeMarche classifies the U.S. equity population into various indices based on the quality or risk of the stocks, size, and growth histories of earnings per share as well as other factors. In some cases, they use their optimizers to determine how much of a pension fund's equities should be invested in each type of stock.

DeMarche analyzes the asset allocation decision in terms of both short-run and long-run expectations for return. For the long-run picture, it bases expected returns to the asset classes first on the long-term track records of the asset classes in producing returns for their investors. It makes subjective modifications in these long-term realized returns when it is clear that changes have occurred over time in the overall investment climate that make the past an inaccurate guide to the future. DeMarche uses index or factor models to estimate short-run expected returns to the asset classes. The factors are usually macroeconomic variables, such as the rate of inflation. One of the models used by DeMarche includes the following set of factors:

1. The rate of return on a treasury bill (T bill).
2. The difference between the rate of return on a short-term and long-term government bond (term).
3. Unexpected changes in the rate of inflation in consumer prices (inflation).
4. Unexpected percentage changes in industrial production (ind. prod.).
5. The ratio of dividend to market price for the S&P 500 in the month preceding the return (yield).
6. The difference between the rate of return on a low- and high-quality bond (confidence).
7. Unexpected percentage changes in the price of oil (oil).

The estimated factor sensitivities of each asset class tell DeMarche the implications on expected return of a forecast of the economic climate (the factor values).

The various elements of a stock's profile might be grouped into the following families.

Risk *Differences in the risk of stocks* are important to investors. As discussed earlier, in forthcoming chapters we will learn about widely accepted theoretical models that define the nature of risk and predict the nature of the relationship between risk and expected future return. However, while it is important for you to understand these models because of their widespread use in the investment community, in building technology to provide the best forecasts of future return, it may be best to rely on a more comprehensive list of risk measures. These might include the sensitivity to market returns (market beta), the sensitivities to other macroeconomic variables (industrial production beta), simple volatility (standard deviation) of return, the relative amount of debt in a firm's capita

Bob was enjoying a well-deserved rest playing golf in Scotland. He'd been on the road throughout the United States promoting his firm's newest approach to asset allocation for pension funds. In this approach, DeMarche overlays the factor model with a market-phase model. DeMarche researchers have identified four distinctive phases of the market, which are based on the directional momentum in stock prices and earnings per share:

1. The initial phase of a bull market
2. The intermediate phase of a bull market

3. The final phase of a bull market
4. The bear market

Interestingly, for a given type of stock, the factor sensitivities can change dramatically as the market moves from one phase to the next. Thus, DeMarche estimates the factor sensitivities separately over each of the four market phases. The accompanying table lists the factor sensitivities for large versus small stocks in going from a bear market to the initial phase of a bull market.

Factor	Phase IV		Phase I	
	Small Stocks	Large Stocks	Small Stocks	Large Stocks
T bill	-6.45	-1.21	5.16	5.81
Term	.34	.45	.86	.92
Inflation	-3.82	-2.45	-3.23	-2.20
Ind. prod.	.54	.06	.00	.40
Yield	1.51	-.16	-.18	.00
Confidence	-.63	-.43	2.46	1.45
Oil	-.21	-.07	.26	.20

One way to approach the asset allocation decision is to identify the current market phase, calculate the factor values typically experienced in such a phase, and make modifications in these factor averages to reflect expectations for the forthcoming period (usually a year). Then calculate expected returns for the asset classes (such as large and small stocks) on the basis of the factor sensitivities in the phase. These expected returns can then be imported to an optimizer to determine the mix of in-

vestments that maximizes expected return given risk exposure for the forthcoming year.

So far, the market reaction to the new approach has been quite favorable. The factor/phase model is one of many quantitative innovations that have established DeMarche as a leader in the pension-consulting field.

Bob lines up his putt. Aha! Today turned out to be a 10 indeed!

structure, the variability of a firm's earnings per share, and so on. Collectively, you would expect the payoffs to these risk variables to be positive, with stocks having greater exposures to the variety of risk attributes having greater expected rates of return.

Liquidity Differences in the liquidity of stocks are also potentially important. In rebalancing their portfolios, traders must buy at the dealers' asked prices and they must sell at the dealers' bid prices. The bid-asked spread serves as part of the cost of trading. The expected impact of the trade on the price of the stock is also important. That is, if you want to buy a great deal of the stock, you may have to raise the price above the current asked price in order to attract the number of shares you want. Individual stocks are characterized by widely differing liquidity. To keep the expected rates of return, net of trading costs, commensurate across different stocks of differing liquidity, stocks must

have gross expected returns that reflect the cost of trading. Factors that determine the relative liquidity of a stock include its price per share, the volume of daily trading relative to total market capitalization (price per share times the total number of shares outstanding), the bid-asked spread as a percentage of price, the amount of institutional ownership, and the like.

In the United States, mutual funds have grown to the point where they exert considerable influence on the prices of stocks. These funds are subject to the cash flows coming into and out of the fund. As investors invest more money, they must buy more stock with the proceeds. As investors withdraw funds, they must be prepared to liquidate their investments to meet the cash demands. Mutual funds demand liquidity in their investments. They have a captive demand for the relatively liquid stocks. To the extent that the influence of mutual funds becomes greater, relative liquidity should play a greater role in determining the relative expected rates of return across different stocks. Overall, you should expect the payoffs to the various factors that represent differentials in liquidity to be negative, with stocks having the greater liquidity having lower expected rates of return.

Cheapness Factors related to cheapness in price indicate the relative magnitude of current market price in relation to the current cash flows available to the firm's stock investors. Factors representing cheapness in price include the relationship or the ratio of current market price to factors such as earnings per share, cash flow per share, dividends per share, book value per share, and so on. *Growth stocks* tend to have high ratios of price to current cash flows. This is because these cash flows are expected to grow fast to higher levels in the future. Thus, investors are willing to pay a relatively high price for these stocks today. *Value stocks* tend to have low values for these ratios. This is because the cash flows of value stocks are expected to grow relatively slowly in the future. Thus, investors aren't willing to pay a relatively high price for these stocks today.

Current research has shown that value stocks have earned much higher rates of return than growth stocks in recent decades. The source of these higher returns is the subject of much controversy. Some believe that value stocks are "fallen angels" and, therefore, are more risky. They believe the premium returns to these stocks are expected and required. Given that this is true, factors indicating cheapness in price actually belong in the risk category discussed earlier. Others, however, believe that the premium returns to value stocks are unexpected and systematically come as a surprise to investors. They believe that investors overreact to the past records of success and failure by firms. Proponents of overreactive markets believe that the forces of competition in a line of business tend to quickly drive profits to normal levels. Advocates of overreactive markets believe that by projecting relatively rapid rates of growth for long periods into the future, investors in growth stocks may drive prices too high. As the forces of competition come into play faster than these investors believed, they tend to be disappointed by the future reports of the profitability of growth stocks, and the future dividends and capital gains on these stocks tend to be smaller than expected (see Haugen 1999).

Irrespective of whether the payoffs to factors related to "value/growth" spring from risk or overreaction, you should expect these payoffs to be negative, with the stocks having the highest prices in relation to current cash flows having the lowest future expected rates of return.

Growth Potential Factors related to growth potential indicate the probability for faster than average future growth in a stock's cash flows. They include a firm's ratio of net earnings to book equity, the ratio of operating income to total assets, the ratio of

operating income to total sales, the ratio of total sales to total assets, and past rates of growth in the various measures of cash flow. *Given the magnitude of the factors that reflect cheapness in price*, the greater the growth potential for the cash flows, the greater the expected future rate of return. Thus, you would expect the payoffs to the growth potential factors to be collectively positive.

Technical Factors *Technical factors* describe the history of the rates of return to a stock. Recent research indicates the existence of at least three separate relationships between the nature of the past history of return and a stock's future expected return. First, there appears to be very short-term (1 month) reversal patterns in returns. If a stock went up significantly in price last month, there will be a tendency for the stock to come back down in price next month (Jegadeesh 1990). These short-term reversal patterns may be caused by price pressure induced by investors attempting to buy or sell large amounts of a particular stock quickly. An investor attempting to sell quickly may drive the price of the stock below its fair value. This being the case, the stock can be expected to recover and return to its fair value shortly thereafter. The opposite would be expected to happen to a stock driven above its fair value by a significant buyer. It is also possible that short-term negative serial correlation may be induced by a phenomenon called the "bid-asked bounce." Stocks fluctuate between bid and asked prices. Because of this, security returns measured over adjacent intervals may exhibit negative serial correlation (Roll 1984). Jegadeesh (1990), who discovered the short-term reversal pattern, argues that the bias due to bid-asked bounce is likely to be small. Moreover, he finds that trading strategies that attempt to exploit short-term reversals remain successful even when returns for the previous month do not reflect the last day of trading.

Second, there are intermediate-term (6 to 12 months) inertia patterns in stock returns, with stocks showing a tendency to repeat their performance over the previous 6 to 12 months in the next 6 months. This may be due to the market's tendency to underreact to *initial* reports of unusually high or low rates of profitability by firms. An initial good (bad) quarterly earnings report tends to be followed by one or two more. Failing to recognize this, the market underreacts to the first report and then completes its reaction as the next two are reported in the 6 months that follow (see Jegadeesh and Titman 1993). Finally, there are long-term (3 to 5 years) reversal patterns in stock returns (Jegadeesh and Titman 1993). This may be due to the fact that the market overreacts to a *chain* of positive (negative) reports of good (bad) earnings numbers. Believing that the chain will continue into the future for an extended period, investors drive the price up (down) to high (low). Consistent with our discussion here, as competitive forces come into play, the stocks that went up (down) in price in the past tend to reverse their performance in the future.

Some contend that these technical patterns aren't the product of market under- and overreaction. They believe, instead, that risk premiums on stocks become larger and smaller over time. Risk premiums in expected returns become larger and smaller as the risk of stocks becomes larger and smaller. Risk premiums may also change as investors' sensitivity to risk grows and declines. Both the levels of risk and risk aversion may change with the business cycle. As we move into a recession, the risk of common stocks may increase; we also become poorer, so our aversion to taking on risk may become stronger. Given this, the expected returns to stocks may be higher in recessions and lower in booms. To the extent that changes in prosperity occur in regular time patterns, the technical factors that we see in the history of stock returns may be induced by time-varying risk premiums. Irrespective of whether the serial patterns that have been observed in returns are caused by inefficient markets or time-varying risk premiums, you should

expect the payoffs to be (a) negative, (b) positive, and (c) negative, respectively, to a stock's performance in the past (a) 1 month (b) 6 to 12 months, and (c) 3 to 5 years.

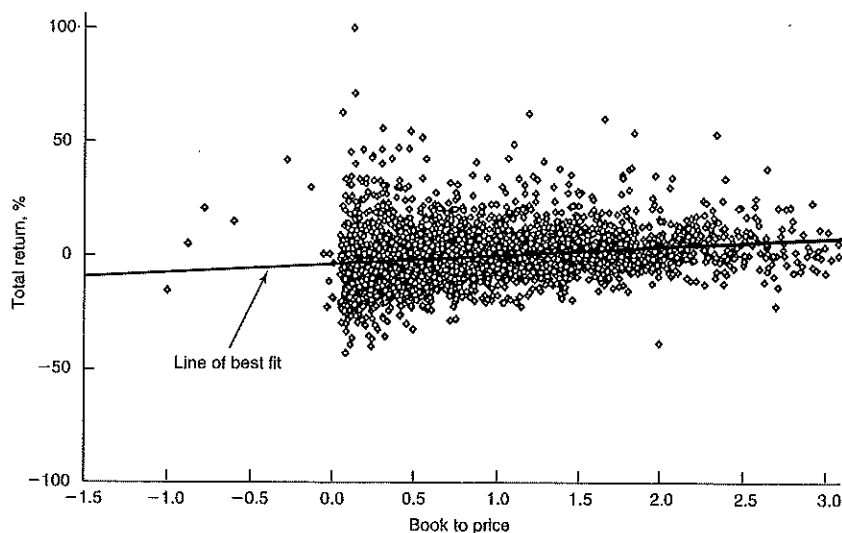
ESTIMATING AND PROJECTING FACTOR PAYOFFS

In building an expected return factor model, one needs to estimate the tendency for stocks with differing exposures to different factors, such as firm size, to systematically produce differing returns.

Suppose one wants to estimate the relationship between the magnitude of the last reported returns produced by different stocks in a given month, say, January 1988, and their book to price ratio as of the beginning of January. The relationship is shown in Figure 6.8. Each data point represents a particular firm, with a measure of a firm's relative book-price plotted horizontally and the rate of return in January 1988 plotted vertically. To interpret the horizontal axis, if a firm has a horizontal score of 1.5, it has a book-price ratio that is 1.5 standard deviations greater than an average firm, given the variability in the ratio across different firms. In this month firms with high book-price ratio tended to produce higher returns; *thus, the payoff to book-price in January 1988 is positive*. In this month the slope is actually equal to .04, indicating that, in this month, for each standard deviation increase in a stock's book-price ratio, its return in this month could be expected to go up by 4 percent.

Book-price isn't the only determinant of differences in the returns produced in a given month, so we will simultaneously estimate the payoffs to the variety of risk, liquidity, and so on, characteristics.⁵ The payoffs to the various firm characteristics are estimated (as the payoff to book-price was in Figure 6.8) over many months to obtain a history of the payoffs to the various factors. For example, the payoff to *size* is negative over most months in recent decades, with smaller firms tending to produce larger rates of return. In the United States this was true for much of the 1970s. In the 1980s, how-

FIGURE 6.8 RELATIONSHIP BETWEEN TOTAL RETURN AND BOOK TO PRICE RATIO, JANUARY 1988.



⁵ This can be done using multiple regression analysis.

ever, as pension funds and other institutional investors moved funds into portfolios that were designed to replicate a capitalization-weighted portfolio of largest U.S. stocks (usually the S&P 500 Index), the payoff to size tended to become positive, with the larger stocks producing higher returns under the price pressure of the pension funds.⁶

Thus, the payoffs to the various firm characteristics have interesting histories, and one can use the information in those histories to make projections of the magnitudes of the future payoffs in future periods. The projections might be based on simple moving averages of the magnitudes of the payoffs in trailing periods, or they might be based on more complex statistical time series models. The experiments reported in this chapter employ simple averages of the payoffs observed in the months prior to the month in which expected return is to be estimated.

For example, suppose one estimates the forthcoming payoff to *size* based on the simple average of estimated payoffs over the past 12 months. We want to estimate the expected return to a particular stock, and we begin by estimating the component of the total expected return that is attributable to the relative size of the company. Based on the cross-sectional variability across different firms, the company we are interested in is 1.00 standard deviation below the size of an average company in the market. Assume the estimate of payoff to size in the next month is -2.00. (For each standard deviation below the market average in size, expected return increases by 2 percent.) Given this, the component of the stock's expected return attributable to its relative size is:

$$\begin{array}{rclcl} \text{Factor exposure} & \times & \text{Projected payoff} & = & \text{Expected return component} \\ -1.00 \text{ S.D.} & \times & -2.00\% & = & 2.00\% \end{array}$$

Thus, based on its relative size alone, we expect this stock to produce a 2.00 percent greater rate of return than an average stock in the forthcoming period. We would now obtain similar expected return components for all the other factors in our model.

If you are employing an expected return factor model, with a wide variety of factors profiling the characteristics of the individual stocks, you might employ, for each stock, a spreadsheet like the table that follows for Green River Paper Company.

Spreadsheet for Expected Return (Green River Paper Company)

Factor	Green River exposure	*	Projected payoff	=	Return component
Size	-2 Standard deviation	*	-2.00%	=	4.0%
Volume	-.5 Standard deviation	*	-1.00%	=	.5%
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
% Debt	-1 Standard deviation	*	+1.50%	=	-1.5%
Total expected return:					2.60%

Only three factors are explicitly represented in the spreadsheet. The others are part of the computation of total expected return, but they are not explicitly represented. In looking at the first factor, we see that Green River is a relatively small company. Looking back at Figure 6.8, we see the cross section of stocks with respect to book-price in January 1988. At the beginning of the month for the spreadsheet, there is also a cross-sectional distribution for company size. Within that distribution, Green River happens

⁶ Chan and Lakonishok (1993) show that between 1977 and 1991 being a member of the S&P 500 contributed an average of 2.19 percent to a stock's rate of return.

to be 2 cross-sectional standard deviations below an average stock, in terms of its size. Based on the average of the estimated payoffs for the last 12 months, we project that the payoff for next month will be -2 percent per unit of standard deviation of size. Since Green River is 2 standard deviations *below* average, we increment up its expected return by 4 percent because (a) it is small, and (b) small stocks have tended to produce larger returns.

Green River is also less liquid than an average stock by .5 standard deviation in the cross section. Again, based on the last 12 months, we estimate the payoff to liquidity to be -1 percent per unit of standard deviation. So Green River's expected return is boosted .5 percent because (a) it is relatively *illiquid*, and (b) liquid stocks have tended to produce lower rates of return.

Finally, after considering the effects of other factors not explicitly shown in the spreadsheet, we *reduce* the expected return by 1.5 percent because Green River has a relatively small amount of debt in its capital structure, and more levered firms have tended to produce higher returns over the past 12 months.

After adding all the components (including those not explicitly considered), we conclude that given its overall profile and our projections of the individual payoffs to those components Green River has an expected return that is 2.6 percent greater than an average stock.⁷

A TEST OF THE ACCURACY OF EXPECTED RETURN FACTOR MODELS

Even though risk factor models of the type described earlier in this chapter are far more popular in the investment business, expected return factor models are actually relatively more accurate in their predictions. To see how accurate, we shall run a test using the 3,000 largest stocks in the U.S. population.

As we did with a single factor in Figure 6.8, we will simultaneously estimate the individual payoffs to an array of 70 factors for the 12 months of 1979.⁸ Going into the first month of 1980, the 12 payoffs of 1979 are averaged individually for each factor. For each stock, these individual projected factor payoffs are multiplied by the elements of the profile going into 1980, in accord with the spreadsheet discussed earlier. We now have an expected return for each stock for January 1980. The stocks are ranked by these expected returns and formed into deciles of approximately 300 stocks each. Decile 10 has the highest expected return and decile 1 the lowest.

We then observe how the deciles actually perform in this first month. Then we do the same thing for February. Dropping the payoffs for January 1979 and adding the payoffs for January 1980, we again take an average of the trailing 12 and multiply them by the new elements of each stock's profile for February 1980 to get expected returns for the stocks in the next month. The stocks are reranked and again formed into deciles.

How do the deciles perform?

The logarithm of the realized, cumulative returns to the deciles is provided in Figure 6.9. Note that the deciles correctly order themselves almost immediately after the first few months of the test. The annualized returns by decile are provided in Figure 6.10. A line of best fit is passed through the 10 points; the spread between the two ends of the line is actually 37 percent! Astonishingly, this spread is positive for each of the individual years of the test.

⁷ An average stock is one that is equal to the mean of the cross section for each element of its profile.

⁸ A multiple regression procedure is employed. See Haugen (1999a) for details.

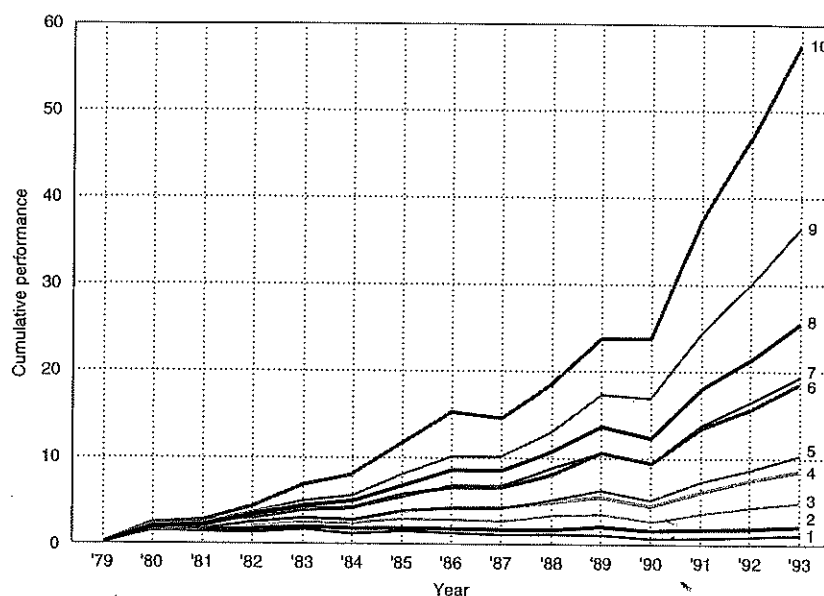


FIGURE 6.9 CUMULATIVE PERFORMANCE BY DECILE.

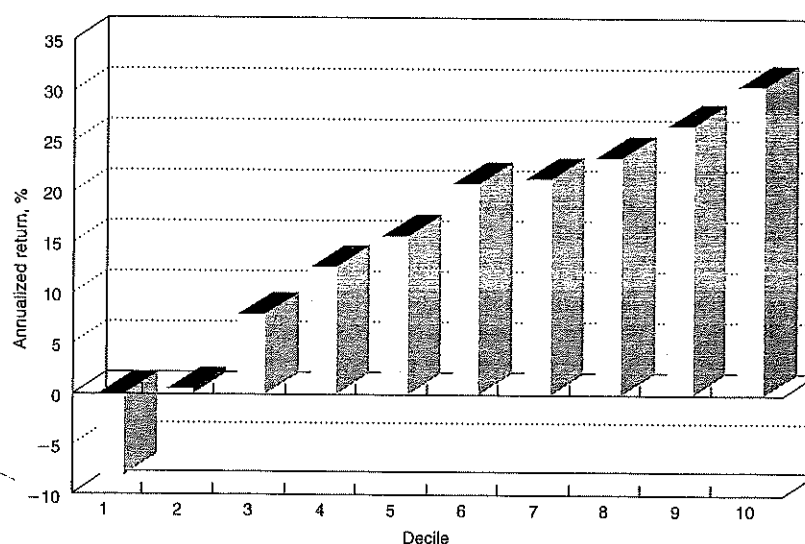


FIGURE 6.10 ANNUALIZED RETURNS BY DECILE, 1978-1993.

It must be said that the names of the stocks in the individual deciles are changing rapidly over time. Unless you can trade for free, the trading costs associated with this turnover would consume a significant amount of the return spreads across the deciles. However, in Haugen (1999a) it is shown that in the factor model, expected returns can effectively be employed in the context of portfolio optimization, where turnover is controlled and transaction costs are accounted for. In this context, portfolios can be constructed that have returns significantly higher than the S&P 500 Stock Index for similar levels of return variability.

USING FACTOR MODELS TO SIMULATE INVESTMENT PERFORMANCE

Factor models can be used to simulate possibilities for investment performance in past periods of time, even for securities or industries that didn't exist at the time.

Assume that the return to an asset class or a portfolio in period t is again linearly related, through time, to several factors ($I = 1$ through n) as in Equation (6.6).

$$r_t = \beta_1 I_{1,t} + \beta_2 I_{2,t} + \cdots + \beta_n I_{n,t} + \varepsilon_t \quad (6.6)$$

Here the betas represent the sensitivities of the returns to the periodic values for the factors and ε_t represents the component of the return that is unrelated to the factors.

In the examples that follow here and later in the book, we will assume that there are five factors:

1. The monthly percentage change in industrial production (I_1)
2. The monthly rate of inflation (I_2)
3. The difference in the monthly return to long- and short-term government bonds (I_3)
4. The difference in the monthly return to corporate and government bonds of the same maturity (I_4)
5. The monthly percentage change in the price of oil (I_5)

The third factor represents changes in the level of interest rates. In periods of rising rates, the returns to long-term bonds will be low relative to short-term bonds. The fourth factor represents changes in investor confidence. More confident investors will reduce their estimates of the probabilities of default on corporate bonds. This increase in investor confidence should increase the prices of corporate bonds and their returns relative to the returns on government bonds, which are not subject to default.

The first step is to estimate the five beta factor values for a particular investment by regressing the investment's monthly returns on the monthly values for the factors. The regression produces monthly values for the unexplained component of the returns, (ε) in Equation (6.6), as well as an estimate of its volatility over the period.

We will assume that unexplained components of return have a zero expected value and are normally distributed.

Now we estimate a possible sequence of future returns to an investment over some time period in the more distant past.

In this past period, we can observe the sequential history of the factor values. We multiply the five factor observations for the first month in the past period by the five factor betas. This calculation gives us the component of the first month's return associated with each factor.

We then "pull" an observation from the assumed, normal distribution of the unexplained components of return. By adding the six components, we obtain an estimate of the first month's return. The process is repeated for the second month, and so on, to obtain a sequence of possible returns for the past period.

To see how easily this might be done, you can go to the Web site www.TheNewFinance.com. Once there, go to the area labeled Modern Investment Theory. Install PManager into your computer. Then go to the area labeled Sessions. Copy the session called Industries into the directory in your computer called Optimize. You will find it on your C: drive.

Now go to the Programs section of Windows and run PManager. Go to Open under File, and select and open the file Industries. Bring up the window Expected Return and note that we have assumed a 10 percent base expected return for all industries. This is the assumed expected return, if all factor values in Equation (6.6) are taken to be zero.

Now open the window Select Period under Historical and Simulate. You will see a graph of the period February 1968 through June 1998 with recessions shaded in gray. Select Factor and highlight the 20-year treasury bond. You will see an index number (January 1982 = 100) for the yield-to-maturity on the bond. The period August 1979 through October 1981 has been selected on the screen. (Other periods can be selected by pointing the arrow to the broken lines and right-clicking to remove and left-clicking to restore. But, for now, let's work with the period already selected.)

Highlight Factor and then Zoom In to bring up the period selected. Then highlight Simulate under Historical and Simulate. You should see sequences of possible returns for the period (given the realized values for the factors) appear on the screen for the industry stock indexes—red for the banking industry and green for the health care industry. When the simulations are finished, three lines will appear for each industry. Ninety percent of the sequences fell within the two outer lines. The median sequence value is the middle line.

Now bring up the window Draw Graph under Back Test and Simulate. This shows the actual cumulative return to the two industries in the period. Highlight Tile under Window and the three graphs should be placed side by side. The actual result under Back Test may be viewed as one of the possible simulated results under Historical Simulation.

Note that the actual and expected possible results for the banking industry are much worse than for health care during this period. To see why, bring up the 20-year bond yield on the recession-shaded graph. Interest rates rose sharply during this period. This was bad for the interest-sensitive banking industry, and, at the same time, the health care stocks were largely unaffected by the recession.

To see the results for other industries, bring up the Select Portfolios window under Simulate. You can observe the results for only two industries at a time.

❖ SUMMARY

Factor models can be used to predict portfolio volatility and expected return. Volatility factor models are based on the presumption that the covariances between security returns are attributable to the fact that security prices respond to varying degrees to the pull of economic and financial variables like the return to the market index, inflation, industrial production, and so on. Volatility models have the advantage of being potentially more accurate in forecasting while at the same time being less computationally demanding.

Expected return factor models employ firm characteristics that can be used to predict the relative returns within stock populations. These factors can be classified into characteristics that describe the relative risk of a stock, its relative liquidity, the magnitude of its price in relation to current cash flows, the potential for growth in those cash flows, and the performance history of its rates of return. The components of expected return are the products of a stock's exposure to a particular factor (such as the size of the firm) and the projected payoff to the factor (e.g., to what extent will small firms produce greater returns than large firms in the forthcoming period).

❖ QUESTION SET 1

1. What assumption serves as the foundation of the single-factor model?
2. Given the following information and the assumption of the single-factor model, what is the covariance between stocks 1 and 2?

$$\beta_1 = .85$$

$$\beta_2 = 1.30$$

Variance of the market factor = .09

3. Assume the following:

<i>Residual Variance</i>		
Stock X	.02	$\text{Cov}(\varepsilon_X, \varepsilon_Y) = .01$
Stock Y	.06	

Also assume that a portfolio of X and Y is constructed, with a 2/3 weight for X and a 1/3 weight for Y.

- a. What is the residual variance of the portfolio if the single-factor model is assumed?
 - b. What is the residual variance of the portfolio without the single-factor model assumption?
4. Suppose you had estimated the following relationship for firm J's return as a function of the return on a market factor:

$$r_J = .03 + 1.3r_M + \varepsilon_J$$

- a. If the return on the market factor should fall by two percentage points, what is the expected change in firm J's return?
- b. What is the name given to the graphic representation of the preceding equation?
- c. What might account for J's *actual* return being different from that expected on the basis of the first two terms of the equation?

Refer to the following data for Questions 5 through 8.

<i>Security</i>	<i>Beta</i>	<i>Residual Variance</i>	σ_r^2
A	.5	.04	.0625
B	1.5	.08	.2825

Suppose an equally weighted portfolio of A and B is formed.

5. What is the beta coefficient for the portfolio?
6. Compute the residual variance of the portfolio assuming the single-factor model.
7. Compute the variance of the portfolio assuming the single-factor model.
8. Fill in the missing columns in the following table. Assume the variance of the market factor (M) to be .0016.

<i>Security i</i>	<i>Variance of i</i>	<i>Correlation of i with M</i>	<i>Beta</i>	<i>Systematic Risk</i>	<i>Unsystematic Risk</i>
$i = 1$	.006	.9			
$i = 2$	.006	.3			
$i = 3$	.006	0			

9. What is the meaning of unsystematic risk?

Refer to the following data for Questions 10 through 15.

Correlation coefficient between stocks A and $B = .50$

Standard deviation of the market factor (M) = .10

	Correlation of Stock with M	Standard Deviation
Stock A	0	.10
Stock B	0.5	.20

10. What are the beta values for A and B ?
11. What is the covariance between A and B , assuming the single-factor model?
12. What is the true covariance between A and B ?
13. Suppose a portfolio was constructed, with weights of .40 for A and .60 for B . What is the beta of this portfolio?
14. Compute the variance of the portfolio in Question 13, assuming the Markowitz model.
15. Compute the variance of the portfolio in Question 13, assuming the single-factor model.
16. What is a factor model (either a single-factor or multifactor) supposed to accomplish? What is the potential advantage of the multifactor approach, in comparison with the single-factor model?
17. Suppose you employed a two-factor model to estimate the following relationship for the percentage return on stock K

$$r_K = .5 + .8r_M + .2g + \varepsilon_K$$

where r_M represents the percentage return on the market index and g represents the unexpected growth rate of industrial production.

- a. If the market index's return is 5 percent and the unexpected growth of industrial production is 2 percent, what return would you expect for stock K ?
- b. What kind of *change* in stock K 's return would you expect if there were to be no change in g and a two-percentage-point decrease in r_M ?
18. Write the formula for the variance of a portfolio, assuming that a two-factor model has been used to explain returns and that the covariance between the factors is zero. Also, write the *general* expression for the portfolio's residual variance. If the two-factor model is really appropriate to account for the interrelationships among returns on individual stocks, what simplification occurs in the general expression for the portfolio's residual variance?
19. Compute the variance of stock X using the expression derived from the two-factor model and the following information. The two factors consist of the return on a market factor and a factor of unexpected growth in industrial production.

Stock X 's market beta = .75

Stock X 's growth beta = .40

Growth factor variance = .10

Market factor variance = .08

Stock X 's residual variance = .03

Refer to the following data for Questions 21 through 25. A two-factor model is being employed, one a market factor (M) and the other a factor of unexpected changes in the growth of industrial production (g).

	<i>Market Beta</i>	<i>Growth Beta</i>	<i>Residual Variance</i>
Stock 1	.6	.2	.05
Stock 2	.9	.1	.02

Variance of the market factor = .12

Variance of the growth factor = .10

Covariance between residuals of stocks 1 and 2 = .02

Covariance between M and g = 0

20. Compute the variance of stock 1.
21. Assume you had constructed an equally weighted portfolio of stocks 1 and 2. Compute the residual variance of this portfolio in two ways:
 - a. Making the simplifying assumption of the two-factor model about residual covariance.
 - b. Without making the simplifying assumption about residual covariance.
22. Compute the market beta and the growth beta for an equally weighted portfolio of stocks 1 and 2.
23. For an equally weighted portfolio of stocks 1 and 2, compute the variance of the portfolio in two ways:
 - a. Making the simplifying assumption of the two-factor model about residual covariance.
 - b. Without making the simplifying assumption about residual covariance.
24. Why would you want to compute portfolio variance by a single- or multifactor model rather than by the Markowitz model?
25. Discuss how we would arrive at an estimated beta value for a stock based on historical information. Further, try to speculate on potential difficulties in using historical information to estimate beta.
26. Why is the sample mean of stocks not a good estimate of future expected return?
27. Why are liquidity factors important in determining the expected return differentials?
28. Describe some of the liquidity measures.
29. What is the impact of mutual funds on the liquidity of stocks?
30. What is the cheapness factor? Describe some of the measures of the cheapness factor.
31. What is the difference between growth stocks and value stocks in terms of the cheapness factor?
32. What do the technical factors describe? State the empirical evidence on technical factors.
33. Suppose that stock A 's exposure to a risk factor is 1.0 standard deviation from mean, and the projected payoff to this risk factor is 1.5%. What is the contribution of this risk factor to A 's expected return?
34. What is the factor of growth potential? Describe several measures of the growth factor.
35. Explain why the leverage ratio (% debt exposure) has a positive payoff on the expected return of stocks.

❖ QUESTION SET 2

1. What is the difference between a covariance factor and an expected return factor?
2. What is the difference between a factor payoff and a factor exposure?

3. How are the Markowitz and factor models used in portfolio selection?
4. A new law in Brazil makes the construction and operation of steel factories a real bargain; however, the steel may only be used in the construction of South American automobiles, which rapidly become popular in the United States. If you were managing a portfolio that had three classes of securities—drugs, services, and machinery manufacturing—and used the single-factor model, would you expect the systematic or unsystematic variance to be affected by this event?
5. Under the single-factor model the relationship between returns to the market and returns to a security or a portfolio is expressed by the equation

$$r_i = A + \beta r_{M,i} + \varepsilon_i$$

- a. What is the name given this regression line?
 - b. Define each term in the equation.
 - c. What type of event is assumed to cause period-to-period movement along this line? What term in the equation accounts for this variability?
 - d. What type of event produces deviations from this line? Explain. What term in the equation accounts for this variability?
6. What is the single-factor model's key assumption, and what does it imply about the residual returns to securities in a portfolio?
 7. Suppose you are managing a portfolio consisting entirely of aerospace stocks. Is the single-factor model likely to accurately estimate the portfolio's residual variance? Explain.
 8. If, in a portfolio of stocks, those of a given industry respond in a similar manner to an industry-wide event, what is true of the SFM estimate of the portfolio's residual variance?
 9. Suppose the stock of two highly competitive companies is held in a portfolio. Would the SFM over- or underestimate this portfolio's residual variance?
 10. Current research has shown that value stocks have earned much higher rates of return than growth stocks in recent decades. What are some of the potential explanations of this phenomenon?
 11. Payoff to the size factor was negative in much of the 1970s in the United States. However, it tended to become positive in the 1980s. Explain why.
 12. When using factor models to run simulation on investment performance, we usually make some assumptions on the unexplained components of expected return. State one potential assumption and describe how it is applied in the simulation.

Refer to the following data for questions 13 and 14.

The expected returns of stocks *A* and *B* are affected by three factors. The factor exposure and the projected payoff are described in the following table:

Factor	Projected Payoff	Exposure to A (Std dev)	Exposure to B (Std dev)
Trading volume	-0.5%	2.0	-1.0
Price/book	-1%	-1.0	3.0
Size	-2%	1.5	-0.5

13. Compute the total expected returns of *A* and *B* and interpret your results.
14. What are the return components of the size factor to *A* and *B*?

❖ ANSWERS TO QUESTION SET 2

1. A covariance factor is typically an index of security prices, such as the S&P 500 Index, or a macroeconomic variable such as the rate of industrial production.

Covariance factors are sources of systematic risk. They account for the correlations that exist between the returns to different stocks.

Expected return factors relate to the characteristics of different firms, such as their relative size, their relative liquidity, or their relative sensitivities to the covariance factors. Expected return factors can be used to predict the relative returns to different portfolios in a forthcoming period.

2. Factor payoffs are the tendencies for stocks with different characteristics to produce different returns during a period of time. For example, in a bull market, stocks with high market betas will usually produce larger returns than stocks with small market betas.

Factor exposures describe the various characteristics of stocks that are useful in explaining their differential returns. For example, some stocks have large market betas, others small. Some stocks have large market capitalizations, others small.

3. In general, the Markowitz model is used for asset allocation between the classes of securities and factor models for portfolio optimization within a class of securities (usually common stocks). Factor models are used to select, within each class of assets, individual securities and determine the investment in each that will optimize the portfolio.
4. The systematic risk would be affected, since the developments would affect not only the automobile manufacturers but also their suppliers of parts and the steel industry.
5. a. The regression line expressed by the equation

$$r_i = A + \beta r_{M,i} + \varepsilon_i$$

is called the *characteristic line*.

- b. r_i = rate of return to a security (or portfolio) in period i .
 A = point at which the characteristic line intercepts dependent (r_i) axis. This point represents the expected rate of return to the security (or portfolio) if the market rate of return in the period is zero.
 β = beta is the slope of the characteristic line, measuring the extent that returns to a security (or portfolio) change in response to changes in rate of return to the market.
 $r_{M,i}$ = rate of return to the market in period i .
 ε_i = residual; the extent to which the actual return to the security (or portfolio) in period i differs from the expected rate of return.
- c. *Macro events*, which affect the rate of return to the market, are assumed to be the cause of movement along the characteristic line. The term $\beta r_{M,i}$ accounts for this variation.
- d. Vertical deviations from the characteristic line are caused by *micro events*. Company-specific factors cause the rates of return of individual securities to differ from the expected rate of return during a given period, i . The term ε_i accounts for this residual variance.
6. The single-factor model assumes that the returns to individual securities are correlated for one reason only: Each security is assumed to respond, in varying degrees, to the pull of the market. In the model, the degree of response is expressed as the security's beta factor. The implication of this assumption is that the residual returns of individual securities are uncorrelated.

$$\text{Cov}(\varepsilon_J, \varepsilon_K) = 0$$

That is to say, the cause of the covariance among individual securities is due solely to the common influence of macro events in the economy. Residual returns are the result of firm-specific microevents.

7. Industry events, such as an industry-wide rise in labor costs, could affect all the stocks in the portfolio but not have an appreciable effect on the market. Therefore, the covariance between the residuals of the stocks may be nonzero. The single-factor model, however, ignores the covariance of residuals among individual stocks and, consequently, will misestimate both the residual variance and total variance of the portfolio.
8. When returns to two stocks of a given industry change in the same direction in response to an event that affects the entire industry but not the general economy, the covariance between the residuals of firms in the industry is likely to be nonzero. Since the single-factor model ignores any covariance between the residuals for different stocks, it does not take this covariance into account and, consequently, it overestimates the portfolio residual variance and total variance.
9. When two companies are highly competitive, what is gained by one is usually lost to the other. Thus, the covariance between their returns is likely to be negative. The single-factor model ignores this negative covariance between stocks and, thus, overestimates the portfolio residual variance.
10. One explanation is that value stocks are "fallen angels" and therefore are more risky. As a result, the premium returns to these stocks are expected and required. Another interpretation is that the premium returns to value stocks are unexpected and systematically come as a surprise to investors. Investors overreact to the past records of success and failure by firms. One potential explanation to investor's overreaction to success is that forces of competition in a line of business tend to quickly drive profits to normal levels. By projecting relatively rapid rates of growth for long periods into the future, investors in growth stocks may drive prices too high.
11. In the 1980s, pension funds and other institutional investors moved funds into portfolios that were designed to replicate a capitalization-weighted portfolio of largest U.S. stocks (usually the S&P 500 Index). This drives up demand for large stocks. Because of the price pressure of the pension funds, the payoff to size tended to become positive.
12. Usually, the unexplained components of expected return are assumed to have a zero expected return and follow a certain distribution, such as a normal distribution. To apply this assumption in the simulation, we first calculate the return based on factor exposure and factor payoff, then we pull an observation from the assumed, normal distribution of the unexplained components of return. By adding this to the calculated return, we get an estimated return observation. Repeating this procedure many times, we are able to obtain a sequence of the returns.
13. Total expected return to $A = 2(-0.5\%) + (-1)(-1\%) + 1.5(-2\%) = -3\%$,
Similarly, total expected return to $B = -1.5\%$.
These are the extra expected returns relative to an average stock.
14. Size factor contributes -3% to A 's return and 1% to B 's return.

❖ PROBLEM SET

1. Given the following information and the assumption of the single-factor model, what is the beta factor of stock 1?

$$\beta_2 = 1.20$$

$$\sigma^2(r_M) = .3162$$

$$\text{Cov}(r_1, r_2) = .09$$

Refer to the following table for Problems 2 through 7.

<i>Stocks</i>	<i>Portfolio Weight</i>	<i>Beta</i>	<i>Expected Return</i>	$\sigma_2(r)$
A	.25	.50	.40	.07
B	.25	.50	.25	.05
C	.50	1.00	.21	.07

$$\sigma^2(r_M) = .06$$

- Given the assumption of the single-factor model, what is the residual variance of each of the foregoing stocks?
- What is the beta factor of the three-stock portfolio?
- What is the variance of the portfolio?
- What is the expected return on the portfolio?
- Given the actual (Markowitz) covariance between the stocks' returns, what is the actual portfolio variance?

$$\text{Cov}(r_A, r_B) = .020$$

$$\text{Cov}(r_A, r_C) = .035$$

$$\text{Cov}(r_B, r_C) = .035$$

- Why might the actual covariance differ from those found using the single-factor model formula?

Refer to the following data for questions 8 through 11.

Four factors are identified to contribute to the expected return of stock A, B and C. The following table lists the factor payoff and individual exposure to each factor:

<i>Factor</i>	<i>Projected Payoff</i>	<i>Exposures (Std dev)</i>		
		A	B	C
Size	-2%	-3.0	2.0	0.5
Trading volume	-1%	-2.0	1.0	1.0
P/E	-1.5%	2.0	-2.0	1.0
% debt	1%	1.0	0.5	-0.5

- Find the extra expected return above the average to stocks A, B, and C. Which one has the highest expected return?
- Why does P/E factor have a negative projected payoff?
- Is stock A most likely to be a small-growth stock, a small-value stock, a large-growth stock, or a large-value stock? What about B?
- Suppose you find an additional factor, a growth potential factor with a projected payoff of 2%. You estimate that stock C's exposure to this factor is 0.5 standard deviation above the mean. How will this change the expected return of stock C?

❖ ANSWERS TO PROBLEM SET

- Given the assumption of the single-factor model, we can write the covariance between any two stocks as

$$\text{Cov}(r_1, r_2) = \beta_1 \beta_2 \sigma^2(r_M)$$

By rearranging the terms, we can solve for

$$\begin{aligned}\beta_1 &= \frac{\text{Cov}(r_1, r_2)}{\beta_2 \sigma^2(r_M)} \\ &= \frac{.09}{1.20(.3162)}\end{aligned}$$

2. We know

$$\sigma^2(r) = \beta^2 \sigma^2(r_M) + \sigma^2(\varepsilon)$$

or

$$\sigma^2(\varepsilon) = \sigma^2(r) - \beta^2 \sigma^2(r_M)$$

Plugging in the known variables on the right-hand side of the equation, we find

$$\sigma^2(\varepsilon_A) = \sigma^2(r_A) - \beta_A^2 \sigma^2(r_M) = .07 - (.50)^2(.06) = .055$$

$$\sigma^2(\varepsilon_B) = \sigma^2(r_B) - \beta_B^2 \sigma^2(r_M) = .05 - (.50)^2(.06) = .035$$

$$\sigma^2(\varepsilon_C) = \sigma^2(r_C) - \beta_C^2 \sigma^2(r_M) = .07 - (1.0)^2(.06) = .010$$

3. The beta factor for the portfolio is simply the weighted average beta of the three stocks. From the text, we know

$$\beta_P = \sum_{j=1}^M x_j \beta_j$$

Therefore,

$$\begin{aligned}\beta_P &= x_A \beta_A + x_B \beta_B + x_C \beta_C \\ &= (.25)(.50) + (.25)(.50) + (.50)(1.00) = .75\end{aligned}$$

4. The variance of the portfolio can be split into two components, systematic risk and residual variance.

$$\sigma^2(r_P) = \beta_P^2 \sigma^2(r_M) + \sigma^2(\varepsilon_P)$$

From Problem 3, we know the beta of the stock portfolio is .75. Knowing this and the variance of the market, we can find the portfolio's systematic risk.

$$\text{Systematic risk} = \beta_P^2 \sigma^2(r_M) = (.75)^2(.06) = .0338$$

The portfolio residual variance under the single-factor model is the weighted sum of the elements on the diagonal in the covariance matrix.

$$\text{Residual variance} = \sigma^2(\varepsilon_P) = \sum_{j=1}^M x_j^2 \sigma^2(\varepsilon_j)$$

Note that the weights used are the square of the portfolio weights.

Using the residual variances computed in Problem 2, we can find the residual variance of the three-stock portfolio:

$$\begin{aligned}\sigma^2(\varepsilon_P) &= x_A^2 \sigma^2(\varepsilon_A) + x_B^2 \sigma^2(\varepsilon_B) + x_C^2 \sigma^2(\varepsilon_C) \\ &= (.25)^2(.055) + (.25)^2(.035) + (.50)^2(.010) \\ &= .0081\end{aligned}$$

With this information, we can now find the variance of the portfolio.

$$\sigma^2(r_P) = \beta_P^2 \sigma^2(r_M) + \sigma^2(\varepsilon_P) = .0338 + .0081 = .0419$$

5. The expected rate of return on the portfolio is a weighted average of the expected returns on each stock in the portfolio.

$$\begin{aligned} E(r_P) &= x_A E(r_A) + x_B E(r_B) + x_C E(r_C) \\ &= (.25)(.40) + (.25)(.25) + (.50)(.21) \\ &= .2675 \text{ or } 26.75\% \end{aligned}$$

6. The actual (Markowitz) portfolio variance includes the off-diagonal terms in the covariance matrix as well as the terms along the diagonal:

$$\begin{aligned} \sigma^2(r_P) &= \sum_{J,K=1}^{M^2} x_J x_K \text{Cov}(r_J, r_K) \\ &= x_A^2 \sigma^2(r_A) + x_B^2 \sigma^2(r_B) + x_C^2 \sigma^2(r_C) \\ &\quad + 2x_A x_B \text{Cov}(r_A, r_B) + 2x_A x_C \text{Cov}(r_A, r_C) \\ &\quad + 2x_B x_C \text{Cov}(r_B, r_C) \\ &= (.25)^2 (.07) + (.25)^2 (.05) + (.50)^2 (.07) \\ &\quad + 2(.25)(.25)(.020) + 2(.25)(.50)(.035) + 2(.25)(.50)(.035) \\ &= .0450 \end{aligned}$$

7. The actual (Markowitz) portfolio variance can differ from the portfolio variance found using the single-factor model if the single-factor model does not account for all the covariance among the portfolio's stocks.
8. Extra expected return to A = $(-3)(-2\%) + (-2)(-1\%) + 2(-1.5\%) + 1(1\%) = 6\%$. Similarly, extra expected return to B = -1.5% , extra expected return to C = -4% .
9. Price/earnings is a cheapness factor. Research has shown that stocks with high cheapness factors tend to produce lower returns. Therefore, the payoff on such factors is negative.
10. With its size much smaller than the average and high P/E, A is most likely to be a small-growth stock. B is more likely to be a large-value stock.
11. The extra expected return of C will be increased by 1% ($2\% \times 0.5$) from this additional factor.

❖ COMPUTER PROBLEM SET

1. The following table represents rates of return to four securities and an index of world wealth. Use PManager at www.TheNewFinance.com with the session called EfficientSets. Use the annual returns from 1974 to 1988 to estimate the model parameters. Print the table containing the estimated rate of return and standard deviation of the four securities as well as the tables for the 10 efficient portfolios. Draw a diagram of the efficient set in return/standard deviation space.

Annual Rates of Return (%)					
Year	Index	1	2	3	4
1	-5.52	1.31	-18.14	-12.23	-9.04
2	14.62	15.81	17.86	37.19	27.33
3	13.88	17.77	5.18	20.35	16.45

Year	Index	1	2	3	4
4	2.10	-16.92	-30.09	-28.47	-7.78
5	2.31	-26.80	-37.61	-32.71	-23.34
6	16.00	37.72	38.77	33.76	44.78
7	10.01	26.26	28.25	29.20	-7.56
8	11.35	-4.81	9.80	10.53	21.26
9	15.95	7.39	16.95	15.81	25.51
10	16.13	21.82	58.47	31.91	15.38
11	20.94	32.70	30.61	37.38	14.53
12	-3.18	-4.22	-6.10	-0.01	-8.93
13	11.96	20.72	4.93	21.87	5.74
14	7.66	23.00	28.27	22.17	22.84
15	5.17	6.88	-5.45	-9.31	1.67

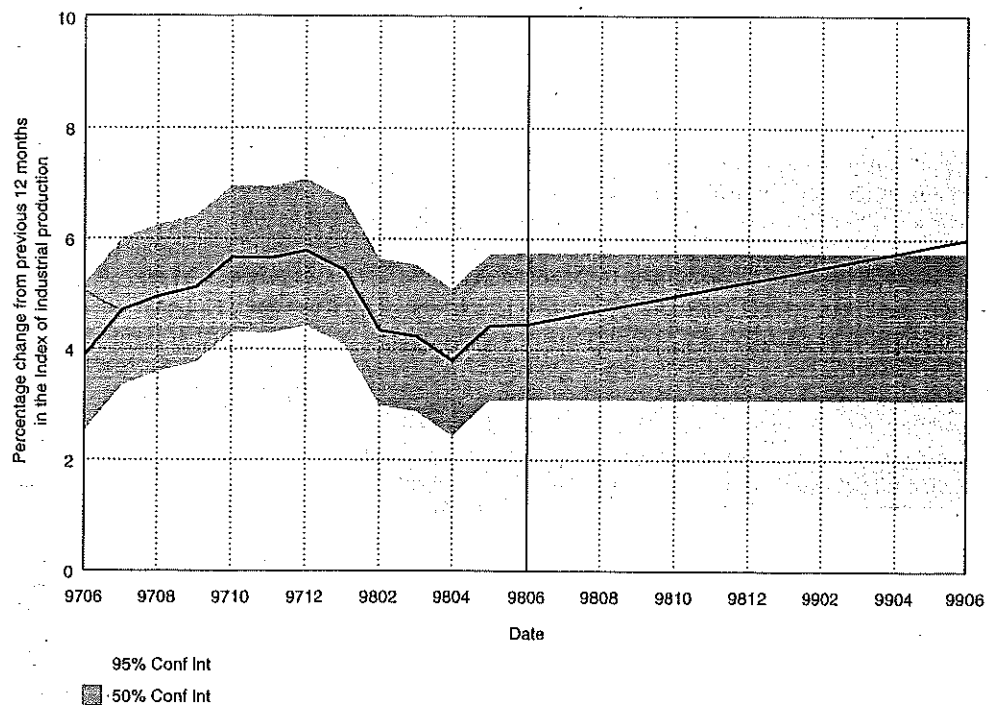
For problem 2, please refer to file *indust.ses*.

2. One type of mutual fund is called a specialized sector fund, which concentrates on a particular industry. Suppose you have investments in four such mutual funds A, B, C, and D, with the investment focus on the automobile, banking, construction, and drug industries, respectively.

The current time is June 1998; you would like to project the expected return of each fund for the period of July 1998 to June 1999. You have the following predictions of the economic factors for the next year:

Industrial Growth	30-Day T-Bill Yield	Oil Price
6%	6%	20

The numbers are end-of-period prediction, assuming a linear path from the current period. For example, the Industrial Growth window will look like



You employ a multifactor model with respect to these macroeconomic factors to project the fund performance next year.

- a. Suppose you use the information from 1993.07 to 1998.06 to estimate multifactor beta. What are the short-run expected returns for each fund over the next year? Plot the efficient frontier and identify the position of each fund.
- b. Now you expand the beta estimation to cover the period from 1983.07 to 1998.06. Repeat part a.
- c. What can you conclude from parts a and b?

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