

# DC Motors Suggested Problems

Dr. Muhammad Abu-Khaizaran

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Problems 8-1 to 8-12 refer to the following dc motor:

$$P_{\text{rated}} = 30 \text{ hp}$$

$$I_{L,\text{rated}} = 110 \text{ A}$$

$$V_T = 240 \text{ V}$$

$$N_F = 2700 \text{ turns per pole}$$

$$n_{\text{rated}} = 1800 \text{ r/min}$$

$$N_{SE} = 14 \text{ turns per pole}$$

$$R_A = 0.19 \Omega$$

$$R_F = 75 \Omega$$

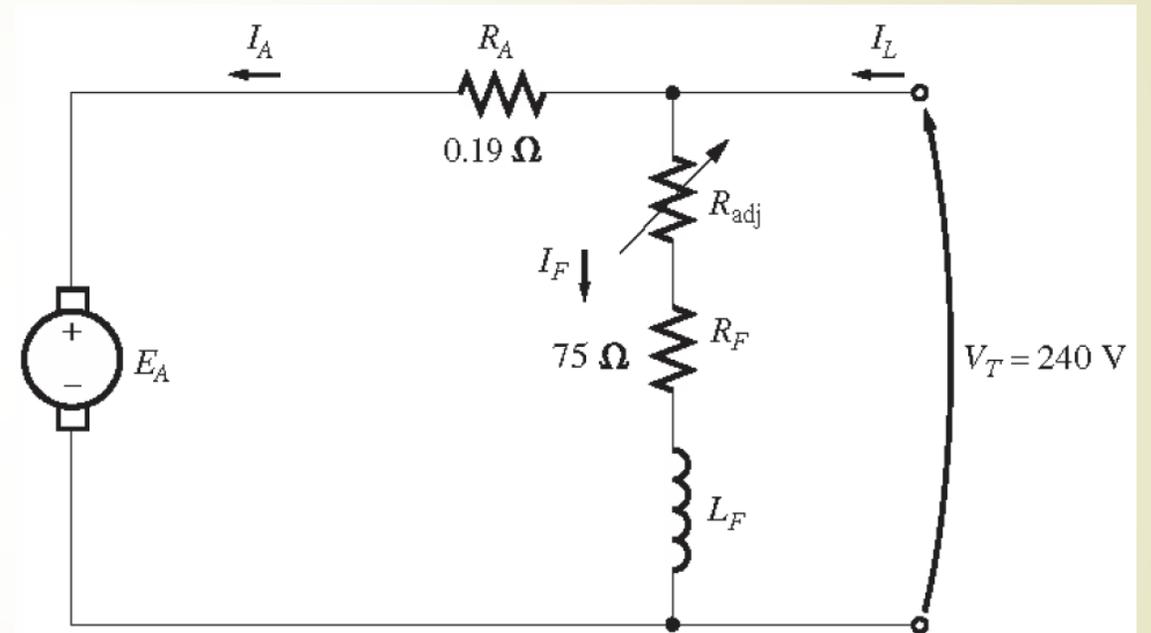
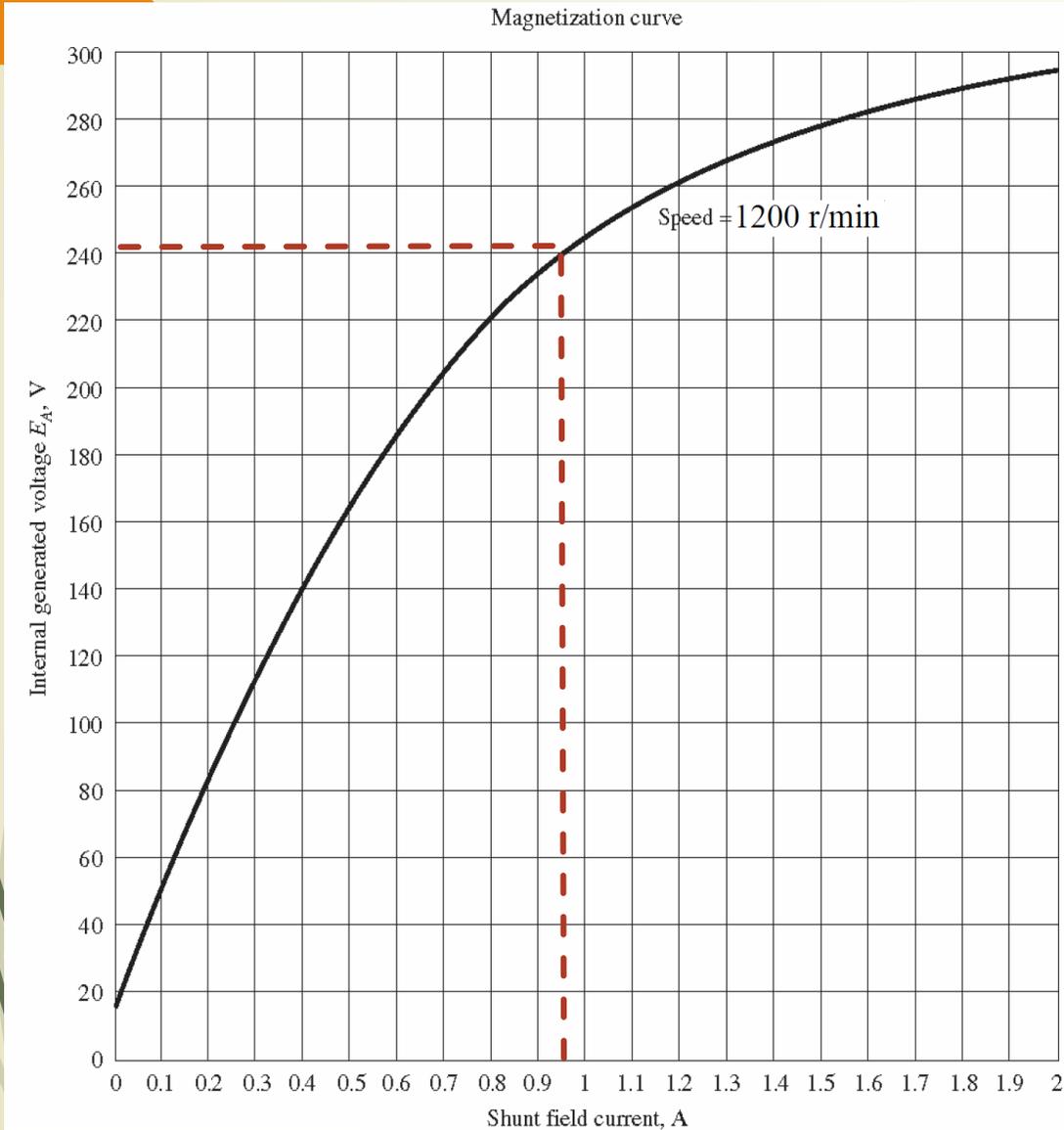
$$R_S = 0.02 \Omega$$

$$R_{\text{adj}} = 100 \text{ to } 400 \Omega$$

Rotational losses = 3550 W at full load

Magnetization curve as shown in Figure P8-1

In Problems 8-1 through 8-7, assume that the motor described above can be connected in shunt. The equivalent circuit of the shunt motor is shown in Figure P8-2.



**FIGURE P8-2**  
The equivalent circuit of the shunt motor in Problems 8-1 to 8-7.

**8-1.** If the resistor  $R_{\text{adj}}$  is adjusted to  $175 \Omega$  what is the rotational speed of the motor at no-load conditions?

SOLUTION At no-load conditions,  $E_A = V_T = 240 \text{ V}$ . The field current is given by

$$I_F = \frac{V_T}{R_{\text{adj}} + R_F} = \frac{240 \text{ V}}{175 \Omega + 75 \Omega} = \frac{240 \text{ V}}{250 \Omega} = 0.960 \text{ A}$$

From Figure P9-1, this field current would produce an internal generated voltage  $E_{Ao}$  of 241 V at a speed  $n_o$  of 1200 r/min. Therefore, the speed  $n$  with a voltage  $E_A$  of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$
$$n = \left( \frac{E_A}{E_{Ao}} \right) n_o = \left( \frac{240 \text{ V}}{241 \text{ V}} \right) (1200 \text{ r/min}) = 1195 \text{ r/min}$$

**8-2.** Assuming no armature reaction, what is the speed of the motor at full load? What is the speed regulation of the motor?

**SOLUTION** At full load, the armature current is

$$I_A = I_L - I_F = I_L - \frac{V_T}{R_{\text{adj}} + R_F} = 110 \text{ A} - \frac{240 \text{ V}}{250 \Omega} = 109 \text{ A}$$

The internal generated voltage  $E_A$  is

$$E_A = V_T - I_A R_A = 240 \text{ V} - (109 \text{ A})(0.19 \Omega) = 219.3 \text{ V}$$

The field current is the same as before, and there is no armature reaction, so  $E_{Ao}$  is still 241 V at a speed  $n_o$  of 1200 r/min. Therefore,

$$n = \left( \frac{E_A}{E_{Ao}} \right) n_o = \left( \frac{219.3 \text{ V}}{241 \text{ V}} \right) (1200 \text{ r/min}) = 1092 \text{ r/min}$$

The speed regulation is

$$\text{SR} = \frac{n_{\text{nl}} - n_{\text{fl}}}{n_{\text{fl}}} \times 100\% = \frac{1195 \text{ r/min} - 1092 \text{ r/min}}{1092 \text{ r/min}} \times 100\% = 9.4\%$$

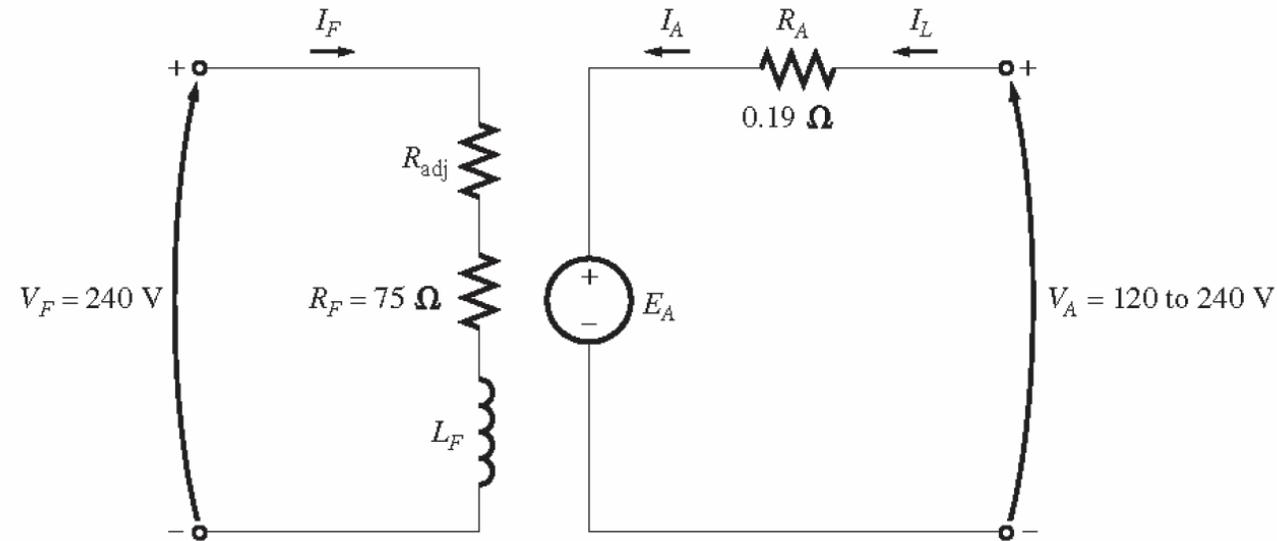
**8-6.** What is the starting current of this machine if it is started by connecting it directly to the power supply  $V_T$ ? How does this starting current compare to the full-load current of the motor?

**SOLUTION** The starting current of this machine (ignoring the small field current) is

$$I_{L,\text{start}} = \frac{V_T}{R_A} = \frac{240 \text{ V}}{0.19 \Omega} = 1260 \text{ A}$$

The rated current is 110 A, so the starting current is 11.5 times greater than the full-load current. This much current is extremely likely to damage the motor.

For Problems 8-8 and 8-9, the shunt dc motor is reconnected separately excited, as shown in Figure P8-3. It has a fixed field voltage  $V_F$  of 240 V and an armature voltage  $V_A$  that can be varied from 120 to 240 V.



**FIGURE P8-3**  
The equivalent circuit of the separately excited motor in Problems 8-8 and 8-9.

- 8-8.** What is the no-load speed of this separately excited motor when  $R_{\text{adj}} = 175 \Omega$  and (a)  $V_A = 120 \text{ V}$ , (b)  $V_A = 180 \text{ V}$ , (c)  $V_A = 240 \text{ V}$ ?

**SOLUTION** At no-load conditions,  $E_A = V_A$ . The field current is given by

$$I_F = \frac{V_F}{R_{\text{adj}} + R_F} = \frac{240 \text{ V}}{175 \Omega + 76 \Omega} = \frac{240 \text{ V}}{250 \Omega} = 0.96 \text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage  $E_{A0}$  of 241 V at a speed  $n_o$  of 1200 r/min. Therefore, the speed  $n$  with a voltage of 240 V would be

From Figure P8-1, this field current would produce an internal generated voltage  $E_{Ao}$  of 241 V at a speed  $n_o$  of 1200 r/min. Therefore, the speed  $n$  with a voltage of  $V_A$  would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$
$$n = \left( \frac{E_A}{E_{Ao}} \right) n_o$$

(a) If  $V_A = 120$  V, then  $E_A = 120$  V, and

$$n = \left( \frac{120 \text{ V}}{241 \text{ V}} \right) (1200 \text{ r/min}) = 598 \text{ r/min}$$

(b) If  $V_A = 180$  V, then  $E_A = 180$  V, and

$$n = \left( \frac{180 \text{ V}}{241 \text{ V}} \right) (1200 \text{ r/min}) = 986 \text{ r/min}$$

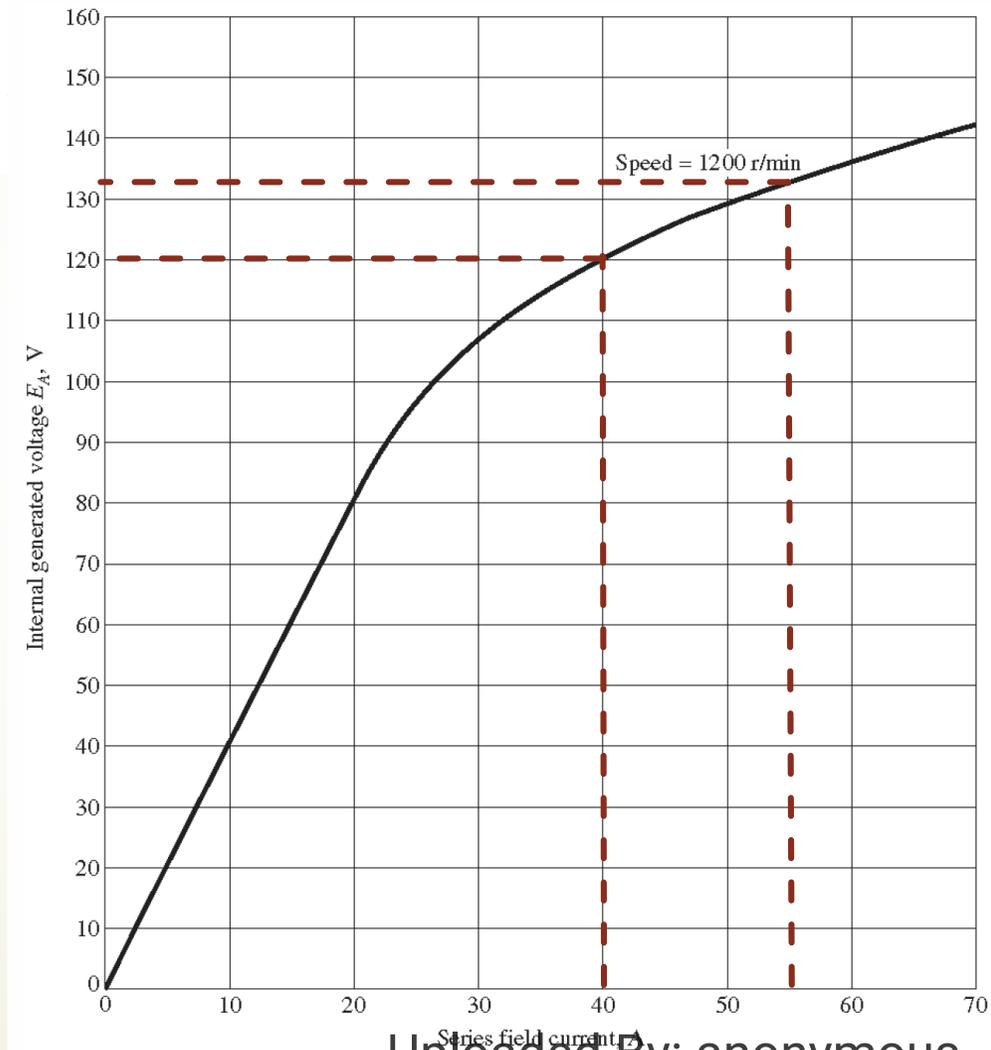
(c) If  $V_A = 240$  V, then  $E_A = 240$  V, and

$$n = \left( \frac{240 \text{ V}}{241 \text{ V}} \right) (1200 \text{ r/min}) = 1195 \text{ r/min}$$

**8-13.** A **7.5-hp** 120-V series dc motor has an armature resistance of  $0.1 \Omega$  and a series field resistance of  $0.08 \Omega$ . At full load, the current input is **56 A**, and the rated speed is 1050 r/min. Its magnetization curve is shown in Figure P8-5. The core losses are **220 W**, and the mechanical losses are **230 W** at full load. Assume that the mechanical losses vary as the cube of the speed of the motor and that the core losses are constant.

- (a) What is the efficiency of the motor at full load?
- (b) What are the speed and efficiency of the motor if it is operating at an armature current of **40**
- (c) Plot the torque-speed characteristic for this motor.

**FIGURE P8-5**



SOLUTION

(a) The output power of this motor at full load is

$$P_{\text{OUT}} = (7.5 \text{ hp})(746 \text{ W/hp}) = 5595 \text{ W}$$

The input power is

$$P_{\text{IN}} = V_T I_L = (120 \text{ V})(56 \text{ A}) = 6720 \text{ W}$$

Therefore the efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{5595 \text{ W}}{6720 \text{ W}} \times 100\% = 83.3\%$$

(b) If the armature current is 40 A, then the input power to the motor will be

$$P_{\text{IN}} = V_T I_L = (120 \text{ V})(40 \text{ A}) = 4800 \text{ W}$$

The internal generated voltage at this condition is

$$E_{A2} = V_T - I_A (R_A + R_S) = 120 \text{ V} - (40 \text{ A})(0.10 \Omega + 0.08 \Omega) = 112.8 \text{ V}$$

and the internal generated voltage at rated conditions is

$$E_{A1} = V_T - I_A (R_A + R_S) = 120 \text{ V} - (56 \text{ A})(0.10 \Omega + 0.08 \Omega) = 109.9 \text{ V}$$

The final speed is given by the equation

$$\frac{E_{A2}}{E_{A1}} = \frac{K \phi_2 \omega_2}{K \phi_1 \omega_1} = \frac{E_{A0,2} n_2}{E_{A0,1} n_1}$$

since the ratio  $E_{A0,2} / E_{A0,1}$  is the same as the ratio  $\phi_2 / \phi_1$ . Therefore, the final speed is

$$n_2 = \frac{E_{A2}}{E_{A1}} \frac{E_{A0,1}}{E_{A0,2}} n_1$$

; The actual output power since the input current and all power losses are given ( $P_{\text{stray}}$  neglected):

$$P_{\text{out}} = P_{\text{in}} - P_{\text{cu}} - P_{\text{core}} - P_{\text{mech}}$$

$$P_{\text{out}} = P_{\text{in}} - I_A^2 (R_A + R_S) - P_{\text{core}} - P_{\text{mech}}$$

$$P_{\text{out}} = 6720 - 56^2 (0.1 + 0.08) - 220 - 230$$

$$P_{\text{out}} = 6720 - 564.5 - 220 - 230$$

$$P_{\text{out}} = 6720 - 564.5 - 220 - 230$$

$$P_{\text{out}} = 5705.5 \text{ W}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} 100\% = \frac{5705.5}{6720} 100\% = 84.9\%$$

*$E_{A0,1}$ , and  $E_{A0,2}$  are the internally generated voltages at the two currents (56A and 40A, respectively), but at the same 1200rpm speed obtained from the magnetization curve!*

since the ratio  $E_{Ao,2}/E_{Ao,1}$  is the same as the ratio  $\phi_2/\phi_1$ . Therefore, the final speed is

$$n_2 = \frac{E_{A2}}{E_{A1}} \frac{E_{Ao,1}}{E_{Ao,2}} n_1$$

From Figure P8-5, the internal generated voltage  $E_{Ao,2}$  for a current of 40 A and a speed of  $n_o = 1200$  r/min is  $E_{Ao,2} = 120$  V, and the internal generated voltage  $E_{Ao,1}$  for a current of 56 A and a speed of  $n_o = 1200$  r/min is  $E_{Ao,1} = 133$  V.

$$n_2 = \frac{E_{A2}}{E_{A1}} \frac{E_{Ao,1}}{E_{Ao,2}} n_1 = \left( \frac{112.8 \text{ V}}{109.9 \text{ V}} \right) \left( \frac{133 \text{ V}}{120 \text{ V}} \right) (1050 \text{ r/min}) = 1195 \text{ r/min}$$

The power converted from electrical to mechanical form is

$$P_{\text{conv}} = \mathbf{E_{A2}I_{A2}} = (112.8 \text{ V})(40 \text{ A}) = 4512 \text{ W}$$

The core losses in the motor are 220 W, and the mechanical losses in the motor are 230 W at a speed of 1050 r/min. The mechanical losses in the motor scale proportionally to the cube of the rotational speed so the mechanical losses at 1195 r/min are

$$P_{\text{mech}} = \left( \frac{n_2}{n_1} \right)^3 (230 \text{ W}) = \left( \frac{1195 \text{ r/min}}{1050 \text{ r/min}} \right)^3 (230 \text{ W}) = 339 \text{ W}$$

Therefore, the output power is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} = 4512 \text{ W} - 339 \text{ W} - 220 \text{ W} = 3953 \text{ W}$$

and the efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{3953 \text{ W}}{4800 \text{ W}} \times 100\% = 82.4\%$$

(c) A MATLAB program to plot the torque-speed characteristic of this motor is shown below:

```
% M-file: prob9_13.m
% M-file to create a plot of the torque-speed curve of the
% the series dc motor in Problem 9-13.
% Get the magnetization curve. Note that this curve is
% defined for a speed of 1200 r/min.
load p85_mag.dat
if_values = p85_mag(:,1);
ea_values = p85_mag(:,2);
n_0 = 1200;

% First, initialize the values needed in this program.
v_t = 120;           % Terminal voltage (V)
r_a = 0.36;         % Armature + field resistance (ohms)
i_a = 9:1:58;      % Armature (line) currents (A)

% Calculate the internal generate voltage e_a.
e_a = v_t - i_a * r_a;

% Calculate the resulting internal generated voltage at
% 1200 r/min by interpolating the motor's magnetization
% curve. Note that the field current is the same as the
% armature current for this motor.
e_a0 = interp1(if_values,ea_values,i_a,'spline');

% Calculate the motor's speed, using the known fact that
% the motor runs at 1050 r/min at a current of 58 A. We
% know that
%
% 
$$\frac{E_{a2}}{E_{a1}} = \frac{K' \phi_2 n_2}{K' \phi_1 n_1} = \frac{E_{a02} n_2}{E_{a01} n_1}$$

%
```

```

%           Ea2   Ea01
%   ==> n2 = ----- ----- n1
%           Ea1   Ea02
%
% where Ea0 is the internal generated voltage at 1200 r/min
% for a given field current.
%
% Speed will be calculated by reference to full load speed
% and current.
n1 = 1050;           % 1050 r/min at full load
Ea01 = interp1(if_values,ea_values,58,'spline');
Ea1 = v_t - 58 * r_a;

% Get speed
Ea02 = interp1(if_values,ea_values,i_a,'spline');
n = (e_a./Ea1) .* (Ea01 ./ Ea02) * n1;

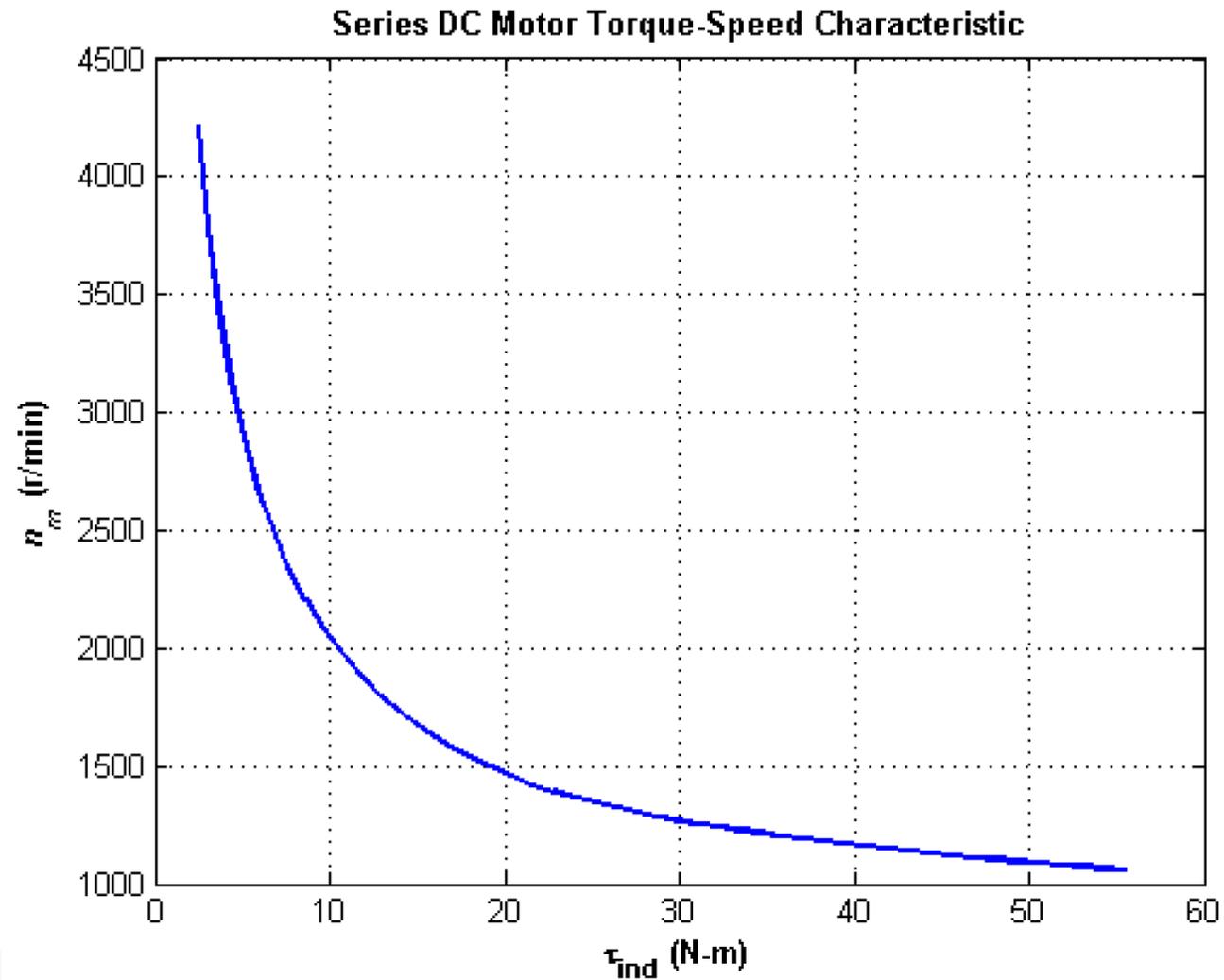
% Calculate the induced torque corresponding to each
% speed from Equations (8-55) and (8-56).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);

% Plot the torque-speed curve
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
hold on;
xlabel('\bf\tau_{ind} (N-m)');

```

```
ylabel('n_{\text{r/min}}');  
title('Series DC Motor Torque-Speed Characteristic');  
grid on;  
hold off;
```

The resulting torque-speed characteristic is shown below:





*Many Thanks  
for  
Your Attention!*



# Reference

- ▶ Instructor's Solutions Manual to accompany Electric Machinery Fundamentals by Stephen Chapman, 5<sup>th</sup> Ed., McGraw-Hill, Inc., 2012.