CH6 Three-phase Induction Motor

① The distinguishing feature of induction motor is that no field circuit current is required to run the machine — advantage

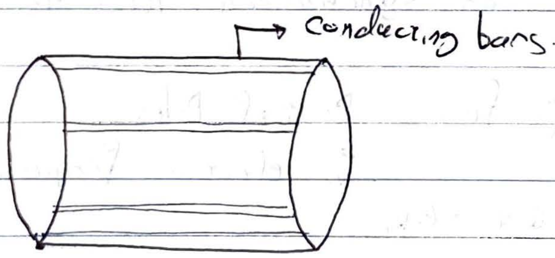
The construction of induction motor.

- it has the same stator construction of synchronous motor, with different construction Rotor

- The induction motor has two rotor types:
① ~~sq~~ squirrel cage rotor
② wound rotor.

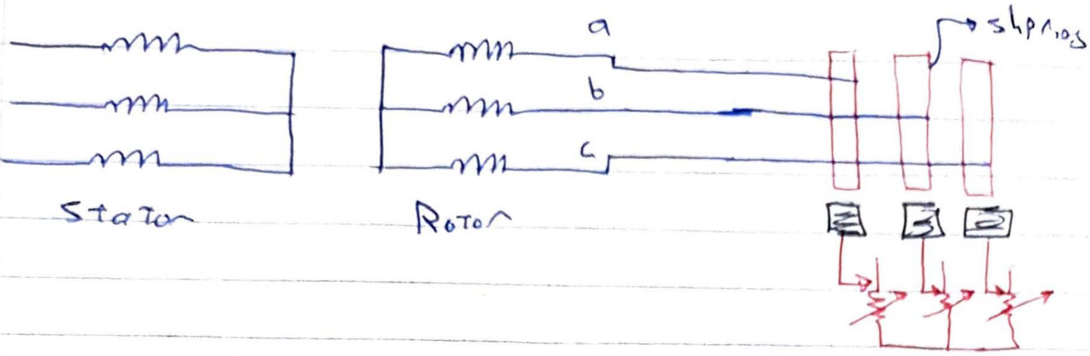
① Cage Rotor

it consists of series conducting bars laid into slots carved in the face of rotor and shorted at either by large short rings ~~are either by~~ 1.



② Wound Rotor:

It's consists of a set of 3ϕ windings that are mirror to the stator winding. The winding are usually Y connected. The terminal a,b,c are tied to the slip rings on the rotor's shaft. The terminal are shorted via brushes.



Notes: • In wound rotor type, an external resistor can be added to the rotor circuit to modify the torque-speed characteristic.

• The wound rotor is more expensive & it requires much more maintenance.

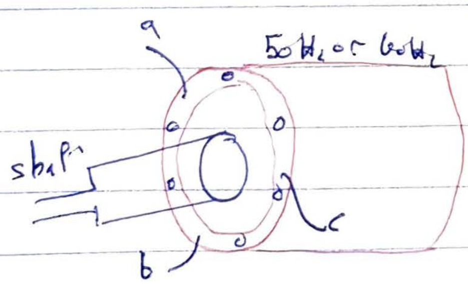
* Operating principle of induction motors.

* A set of three-ϕ voltage is applied to the ~~rotor~~ stator windings to cause a set of 3-ϕ current to flow in stator windings.

* the current will produce a rotating stator magnetic field, $\vec{B}_s$ which rotates at synchrounous speed n_s. where n_s is given by

$$n_s = \frac{120}{P} f_e$$

P: # of poles
fe: electric frequency



* The rotating magnetic field will pass over the conducting ~~rot~~ bars and induces voltage on them.

$e_{ind} = (V \times B_s) \cdot L$, where V is relative velocity of bars with respect ~~of~~ to the speed of $\vec{B}_s$

⊛ The induced voltage will produce a rotor current $\vec{I_r}^*$

⊛ $\vec{I_r}$ produces $\vec{B_r}$

⊛ $\vec{B_r}$ interact with $\vec{B_s}$ to produce the induced Torque in the machine

$T_{ind} = K \vec{B_r} \times \vec{B_s}$

Note: The Rotors can speed up to synchronous speed, but it can never Reach it [to keep $E_{ind} \neq 0$]

speed of $\vec{B_s}$: n_s ,

speed of $\vec{B_r}$: n_s , where $n_m < n_s$

speed of Rotor: n_m ,

⊛ Concept of slip:

slip speed n_a: its the difference between n_s & rotor speed

$n_a = n_s - n_m$ oR $\omega_a = \omega_s - \omega_m$

synchronous speed ↲ ↳ mechanical speed

slip (a): its the Ratio of slip speed and synchronous speed

$a = \frac{n_a}{n_s} \Rightarrow \boxed{n_a = a n_s}$

$a = \frac{\omega_a}{\omega_s} \Rightarrow \boxed{\omega_a = a \omega_s}$

$a = \frac{n_a}{n_s} = \frac{n_s - n_m}{n_s} = \frac{\omega_s - \omega_m}{\omega_s} \Rightarrow a\omega_s = \omega_s - \omega_m$

$$\boxed{\omega_m = \omega_s (1 - a)}$$

$n_s a = n_s - ?$

$n_m = n_s(1-a)$

Rotor frequency (f_r)

when the Rotor is locked ($n_m = 0$, $s = 1$) $\Rightarrow$ $f_r = f_e$

when the Rotor Turns at n_s ($s = 0$) $\Rightarrow$ $f_r = 0$

at any other speed than n_s, the rotor frequency is proportional to the slip

$$\boxed{f_r = s f_e}$$

$$n_s = \frac{120}{P} f_e \Rightarrow \boxed{f_e = \frac{P n_s}{120}} \quad , \quad f_r = (s)\left(\frac{P}{120}\right) n_s$$

$$\boxed{f_r = \frac{P}{120} n_{slip} = \frac{P}{120} (n_s - n_m)}$$

Ex A 208 V, 10 hp, four pole, 60 Hz, Y-conected induction motor has a full Load slip of 5%.

a) what is the synchronous speed of this motor?

b) what is the Rotor speed of this motor?

c) what is the Rotor frequency of this motor at rated speed?

d) what is the shaft Torque of this motor at rated Load?

Solution $\rightarrow$

a) $n_s = \frac{120}{P} f_e = 1800$ rpm

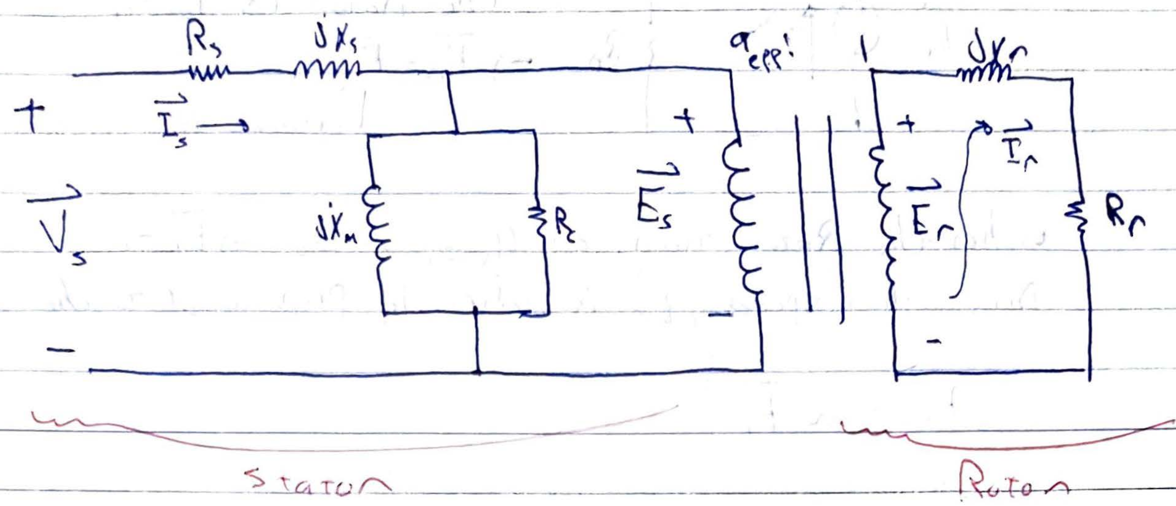
b) $n_m = (1-s) n_s = (1-0.05)(1800) = 1710$ rpm

c) $f_r = s f_e = (0.05)(60) = 3$ Hz

d) $T_m = \frac{P_r}{\omega_m} = \frac{(10)(746)}{(1710)(2\pi)/60} = 41.7$ N.m

Equivalent circuit of Induction Motors:

its very similar to the per-phase equivalent circuit of Transformer.



R_s : Resistance of stator winding.
R_r : Resistance of rotor bar or windings.
X_s : stator Leakage Reactance
X_r : rotor Leakage Reactance.
X_m : Magnetizing Reactance
R_c : Resistance of core accounting for eddy current & hysteresis.

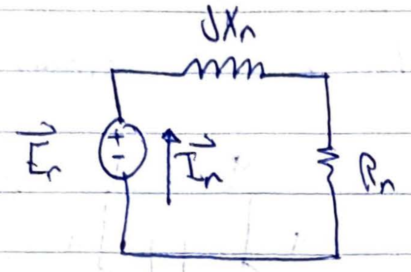
a_{eff} : its the effective turns ratio of the motor which couples between $\vec{E}_s$ & $\vec{E}_r$.

Note: we can easily find it (a_{eff}) in the case of wound Rotor

$\vec{E}_s$: Internal stator voltage.
$\vec{E}_r$: rotor induced voltage.
$\vec{I}_r$: rotor current.
$\vec{I}_s$: stator or source phase current.
$\vec{V}_s$: stator or source phase voltage.

⊛ مشكلة eq circuit - غير عملياً لا stator هو عدم توازن ال Frequency

Rotor Circuit Model



For $\vec{E_r}$:

The Rotor is locked
$\Rightarrow \vec{E_r} = \vec{E_{r_0}}$
↳ maximum Rotor Voltage

- when the Rotor turns at $u_s \Rightarrow a = 0 \Rightarrow \vec{E_r} = 0$

Any other speed, $\vec{E_r}$ is directly proportional to the slip

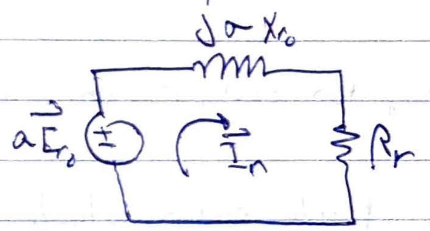
$$\boxed{\vec{E_r} = a \vec{E_{r_0}}}$$

For $\vec{X_r}$

$$X_r = W_r L_r = 2\pi f_r L_r = 2\pi (a f_e) L_r = a X_{r_0}$$

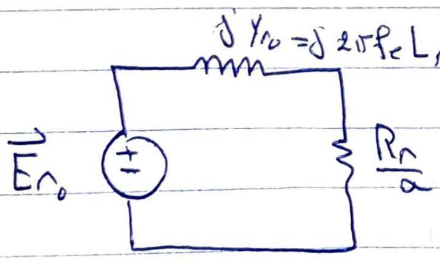
where $X_{r_0} = 2\pi f_e L_r$
↓
Maximum leakage rotor reactance.

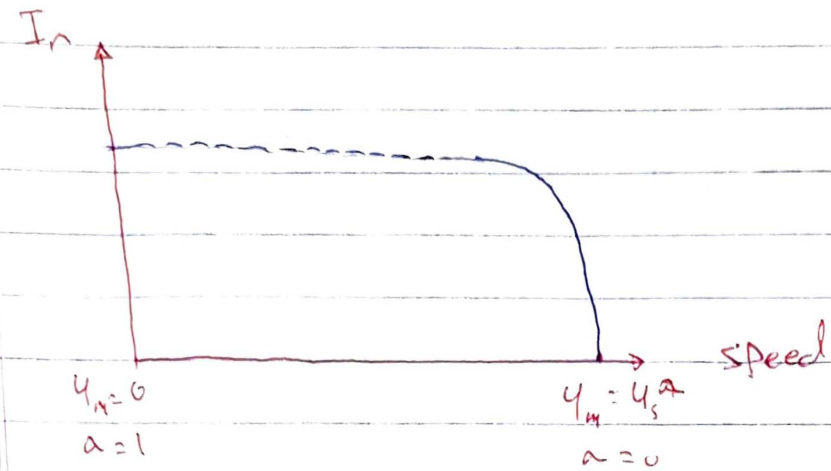
So



By applying KVL in the Rotor Circuit.

$$\vec{I_r} = \frac{a \vec{E_{r_0}}}{R_r + (jX_{r_0})a} \Rightarrow \vec{I_r} = \frac{\vec{E_{r_0}}}{\frac{R_r}{a} + jX_{r_0}}$$





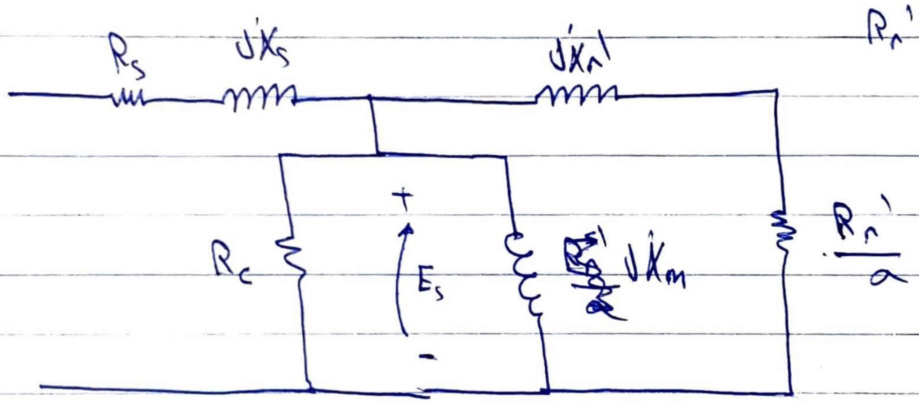
when a is small $I_r = \dfrac{\vec{E}_{ro}}{\frac{R_r}{a} + jX_{ro}} \cong \dfrac{\vec{E}_{ro}}{\frac{R_r}{a}} \approx \dfrac{a E_{ro}}{R_r}$.

Note: At starting the Rotor current is usually high compared with the motor nominal current.

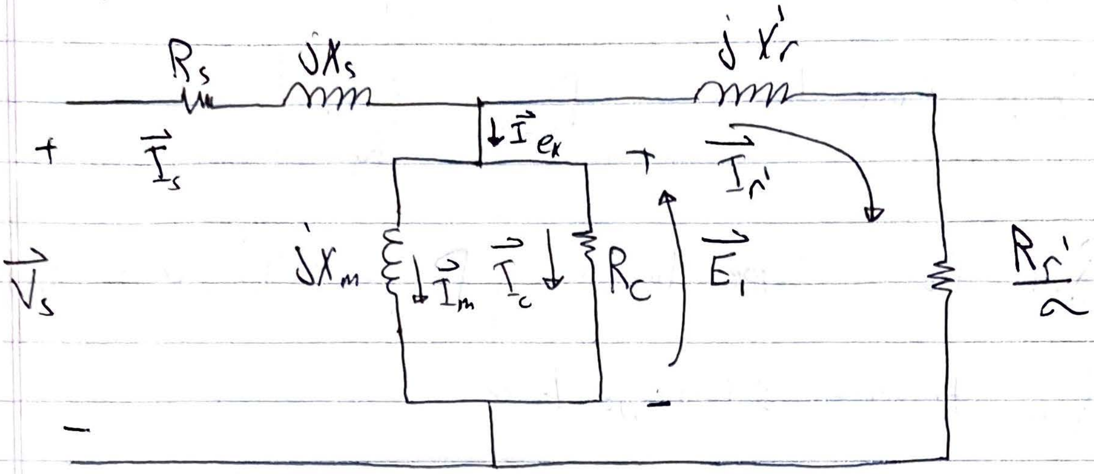
Final equivalent circuit:

$jX_r' = jX_{ro} = a^2_{eff} X_{ro}$

$R_r' = a^2_{eff} R_r$



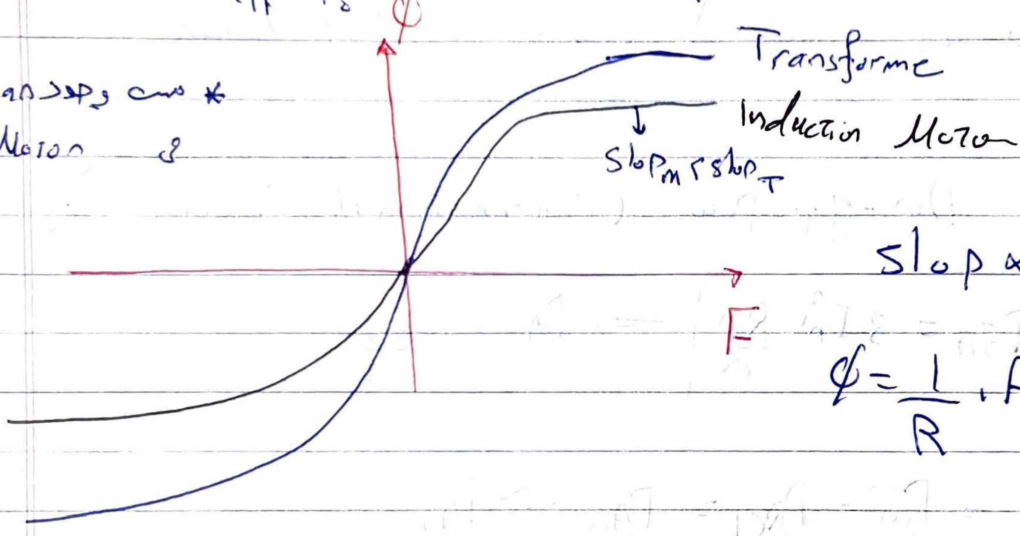
Equivalent circuit of Induction Motor.



$$R_r' = a_{eff}^2 R_r , \quad X_r' = a_{eff}^2 X_{r_0} \quad \vec{I}_r' = \frac{1}{a_{eff}} \vec{I}_r$$

$$\vec{E}_1 = a_{eff} \vec{E}_{r_0} \qquad \alpha : slip$$

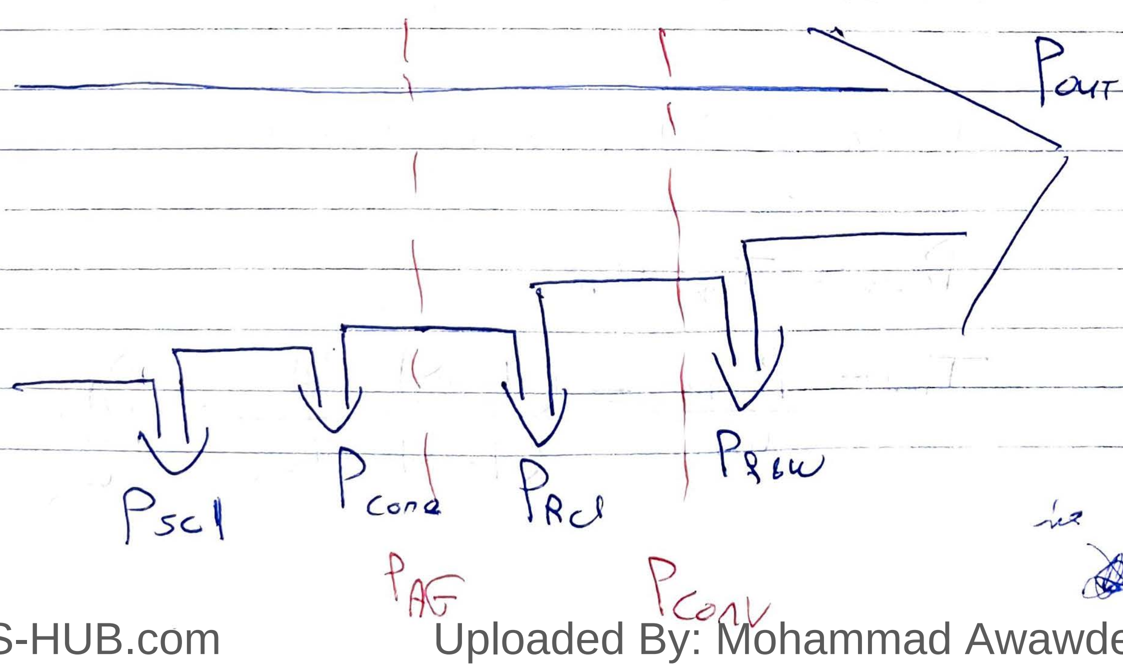
* بس وجود Air gap
Motor ع



$$slop \propto \frac{1}{R}$$

$$\phi = \frac{1}{R} \cdot F$$

Power calculations



P_{scl} : Stator copper losses $\boxed{P_{Scl} = 3I_s^2 R_s}$

P_{core}: Core losses $\boxed{P_{core} = \frac{3E_1^2}{R_C}}$

P_{Rcl} : Rotor Copper losses $\boxed{P_{Rcl} = 3I_r'^2 R_r' = 3I_r^2 R_r}$

$P_{F\&W}$ = Friction & windage losses

P_{conv} : Converted power to Mechanical

$$P_{conv} = T_{ind}\, \omega_m$$

Induced Torque ← T_{ind} ; ω_m → Rotor (Mechanical speed)

P_{AG} : Air-gap power (Consumed in the Resistor $\frac{R_r'}{\alpha}$)

$\boxed{P_{GA} = 3I_r'^2 \frac{R_r'}{\alpha}} \Rightarrow P_{AG} = \frac{P_{Rcl}}{\alpha}$

$$P_{conv} = P_{AG} - P_{Rcl} = P_{AG} - \alpha P_{AG}$$

$\boxed{P_{conv} = (1-\alpha) P_{AG}}$

$P_{out} = T_{load}\, \omega_m$

$P_{out} = P_{conv} - P_{F\&W}$

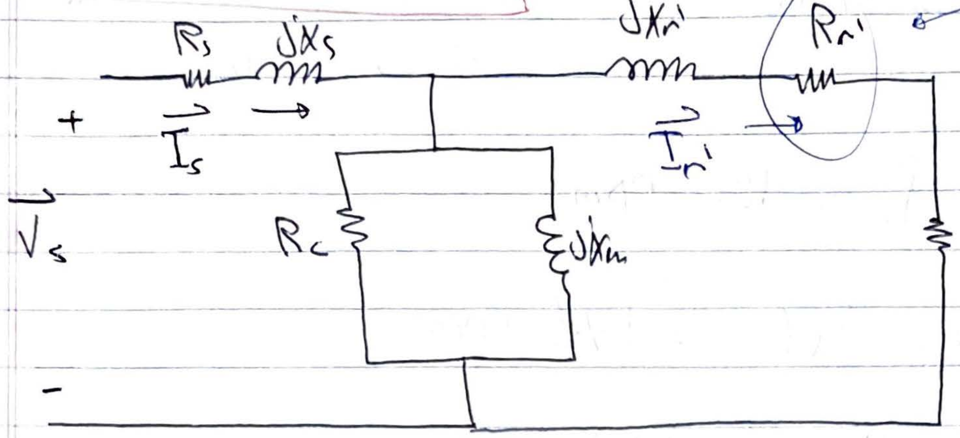
Induced Torque equation

$$P_{conv} = T_{ind}\, \omega_m .$$

$$T_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-\alpha)}{(1-\alpha)} \frac{P_{AG}}{\omega_s} = \frac{P_{AG}}{\omega_s}$$

$$\boxed{T_{ind} = \frac{3 I_r'^2 R_r'}{a \omega_s}}$$

$$\frac{R_r'}{a} = \frac{R_r'}{a} - R_r' + R_r'$$



or Con. Res

$$\left(\frac{1-a}{a}\right) R_r' = R_{CON}'$$

Converted to power

$$P_{con} = (1-a) P_{AG} = (1-a) \frac{3 I_r'^2 R_r'}{a}$$

$$= 3 I_r'^2 R_{conv} , \quad \boxed{R_{conv} = \frac{1-a}{a} R_r'}$$

Motor efficiency

$$\eta = \frac{P_{out}}{P_{in}} , \quad P_{in} = 3 V_s I_s PF = \sqrt{3}\, V_L I_L PF$$

Rotor efficiency

$$\eta_r = \frac{P_{conv}}{P_{AG}} = \frac{(1-a) P_{AG}}{P_{AG}} = 1 - a$$

Example A 460V, 25 hp, 60 Hz, 4 poles, Y-connected induction motor has the following impedances in ohms, per phase referred to the stator circuit.

$R_s = 0.641\ \Omega$, $R_r' = 0.332\ \Omega$

$X_s = 1.106\ \Omega$, $X_r' = 0.464\ \Omega$, $X_m = 26.3\ \Omega$

The Total Rotational losses are 1.1 kW and assumed to be constant, for a rotor slip of 2.2% at the rated voltage & rated frequency, find the motor's

a) speed b) current c) PF d) P_{conv} & P_{AG}

e) T_{ind} & T_{load} f) (η)

a) speed, n_m

$$a = \frac{n_s - n_m}{n_s} = 1 - \frac{n_m}{n_s}$$

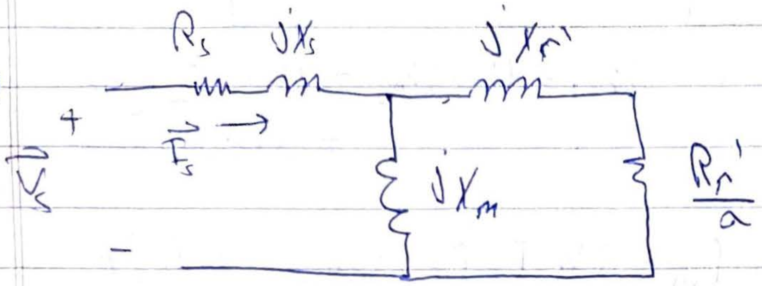
$$n_m = (1-a)n_s$$

$$n_m = (1-a)n_e$$

$$n_s = \frac{120}{P} f_e = 1800\ rpm$$

$$n_m = (1-0.022) \times 1800 = 1760\ rpm$$

b)



Per-phase

$$\vec{I_s} = \frac{\vec{V_s}}{Z_{eq}}, \quad Z_{eq} = 14.07 \angle 33.6^\circ\ \Omega$$

$$\vec{I_s} = \frac{(460/\sqrt{3}) \angle 0^\circ}{14.07 \angle 33.6^\circ} \Rightarrow \vec{I_s} = 18.88 \angle -33.6^\circ\ A$$

c) $PF = \cos(\theta_{V_s} - \theta_{ol}) = \cos(0 - (-33.6))$

$$= \cos(33.6) = 0.833\ \text{lagging}$$

d) we can use current divide to find I_r'
but there is easiest way.

$$P_{in} = 3 V_s I_s PF = (3)\left(\frac{460}{\sqrt{3}}\right)(18.88)(0.833) = 12.53\ kW$$

$$P_{SCl} = 3 I_s^2 R_s = (3)(18.88)^2(0.641)$$

$$P_{SCl} = 0.685\ kW$$

$P_{AG} = P_{in} - \cancel{P_{core}} - P_{SCL} = 12.53 - 0.641 \Rightarrow P_{AG} = 11.845\ KW$

$P_{conv} = (1-\alpha)\, P_{AG} = 11.585\ KW$

e) $P_{conv} = T_{ind}\, \omega_m$

$$T_{ind} = \frac{P_{conv}}{\omega_m} = \frac{11585}{1760\,\frac{\pi}{30}} = \cancel{56.9}\ N.m$$

62.8

$P_{out} = T_{Load}\, \omega_m$

$P_{out} = P_{conv} - P_{f\&w} = 11.585 - 1.1 = 10.485\ KW$

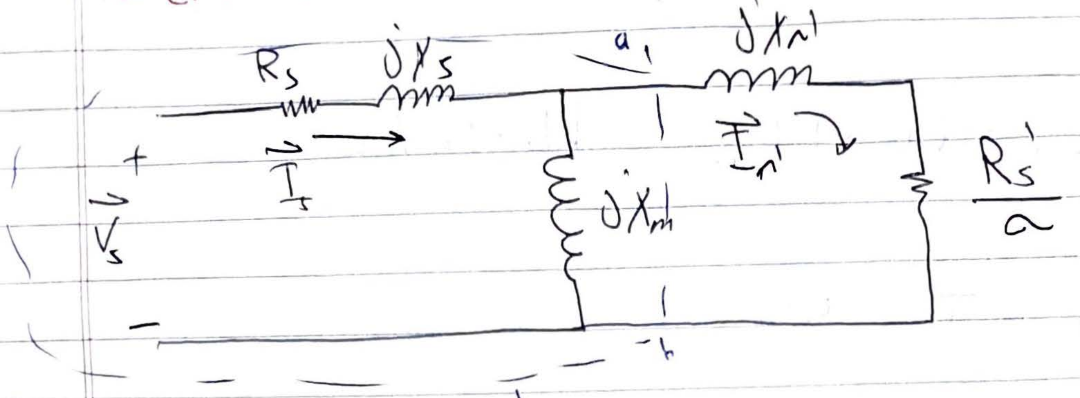
$$T_{Load} = \frac{10.485}{1760\left(\frac{\pi}{30}\right)} = 56.9\ N.m$$

f) $$\eta = \frac{P_{out}}{P_{in}} = \frac{10.425}{12.53} = 83.7\%$$

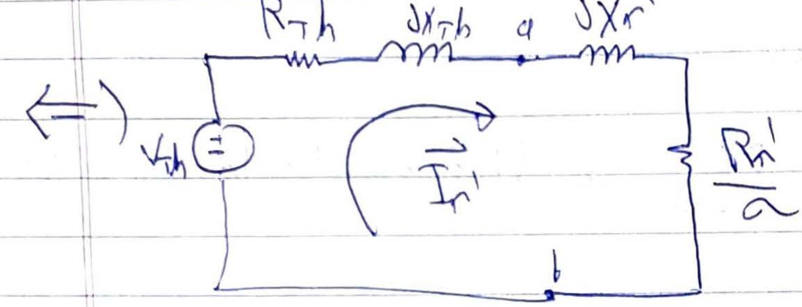
$\eta_r = 1 - \alpha = 1 - 0.022$

لدراسة إيجاد معادلة لل Torque بدلالة ال Parameters الموجودة بالدائرة.

Derivation of Induced Torque equation



$$T_{ind} = \frac{3 I_r'^2 R_r'}{a\omega_s} = \frac{P_{AG}}{\omega_s}$$



$$\vec{V}_{Th} = \vec{V}_{oc} = \frac{jX_m}{R_s + j(X_m + X_s)} \cdot \vec{V}_s$$

$$X_m + X_s >> R_s \Rightarrow \boxed{\vec{V}_{Th} \approx \frac{X_m}{X_m + X_s} \cdot \vec{V}_s}$$

$$Z_{Th} = (R_s + jX_s) \,//\, jX_m$$

$$Z_{Th} = \frac{(R_s + jX_s)(jX_m)}{R_s + j(X_s + X_m)}$$

$$X_s + X_m >> R_s \Rightarrow Z_{Th} \approx \frac{(R_s + jX_s)(\cancel{j}X_m)}{\cancel{j}(X_s + X_m)}$$

$$Z_{Th} = R_s\left(\frac{X_m}{X_m+X_s}\right) + jX_s\left(\frac{X_m}{X_s+X_m}\right) = R_{Th} + jX_{Th}$$

$$\boxed{R_{Th} = R_s\left(\frac{X_m}{X_m+X_s}\right)} \;, \quad \boxed{X_{Th} = X_s\left(\frac{X_m}{X_m+X_s}\right) \simeq X_s}$$

$\approx R_s$

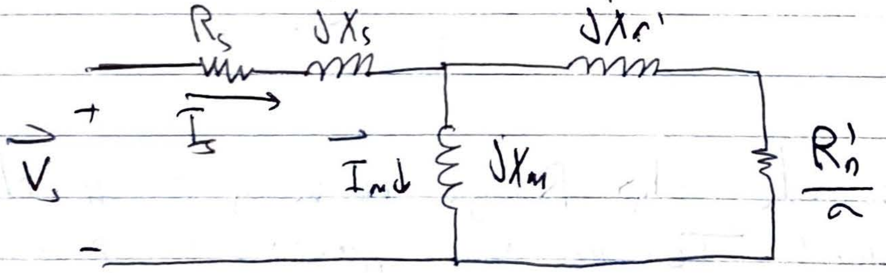
$$\vec{I}_r' = \frac{\vec{V}_{Th}}{\left(R_{Th}+\frac{R_r'}{a}\right) + j\left(X_{Th}+X_r'\right)}$$

$$I_r' = \frac{V_{Th}}{\sqrt{\left(R_{Th}+\frac{R_r'}{a}\right)^2 + \left(X_{Th}+X_r'\right)^2}}$$

مهم ⊛

$$\boxed{T_{ind} = \frac{(3)\,V_{Th}^2}{\omega_s\left[\left(R_{Th}+\frac{R_r'}{a}\right)^2 + \left(X_{Th}+X_r'\right)^2\right]} \cdot \frac{R_r'}{a}}$$

Induced Torque equation



per-phase equivalent circuit

$$T_{ind} = \frac{P_{AG}}{\omega_s} \;, \quad P_{AG} = 3I_r'^2\,\frac{R_r'}{a}$$

$$T_{ind} = \frac{3I_r'^2\,R_r'}{a\,\omega_s}$$

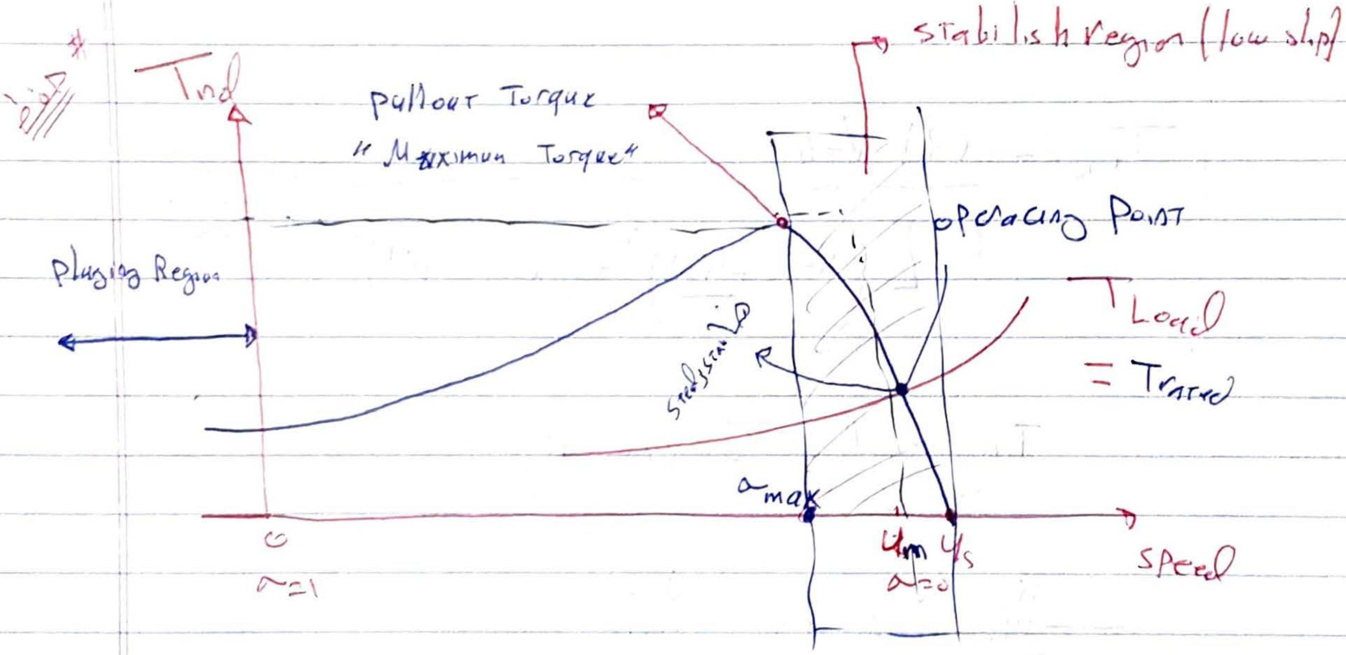
$$\vec{V}_{Th} = \frac{jX_m}{R_s + j(X_m + X_s)} \cdot \vec{V}_s$$

$$V_{Th} = X_m$$

$$T_{ind} = \frac{3V_{Th}^2}{\omega_s\left[\left(R_{Th} + \frac{R_r'}{s}\right)^2 + (X_{Th} + X_r')^2\right]} \cdot \frac{R_r'}{s}$$

$$T_{ind} = f(s) = g(\omega_m)$$

$$\boxed{\omega_m = (1-s)\omega_s}$$



Note: when the slip Range is very small, the Induced Torque is approximately proportional to the slip (linear relationship)

$$T_{ind} \approx \frac{3V_{Th}^2}{\omega_s\left[\left(\frac{R_r'}{s}\right)^2\right]} \cdot \frac{R_r'}{s} = \frac{3V_{Th}^2}{\omega_s R_r'} \cdot s = Ks$$

$$T_{ind} - T_{Load} = J \frac{d\omega_m}{dt}$$

under steady state (speed = constant)

$$\Rightarrow T_{ind} = T_{Load}$$

Plugging Region

its used to stop the motor rapidly by switching any two of 3 phases.

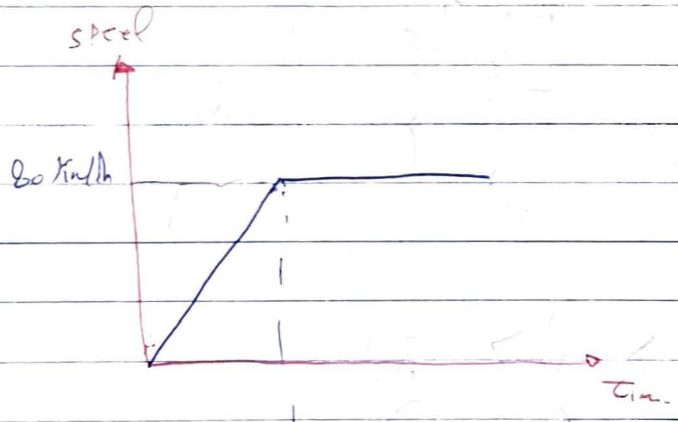
المشكلة بـ عند التشغيل بكون مار تيار كبير جدا؟

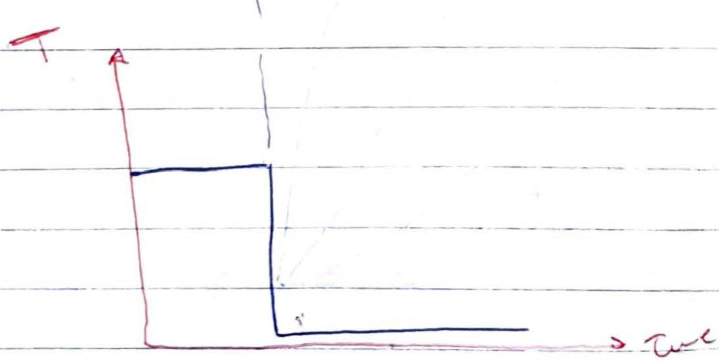
T_{start} : Starting Torque

T_{start} is slightly higher than the motor rated Torque

T_{max} = Pullout or maximum Torque.

$$T_{max} = (2-3)\, T_{rated}$$





$$\frac{dT_{ind}}{da} = 0$$

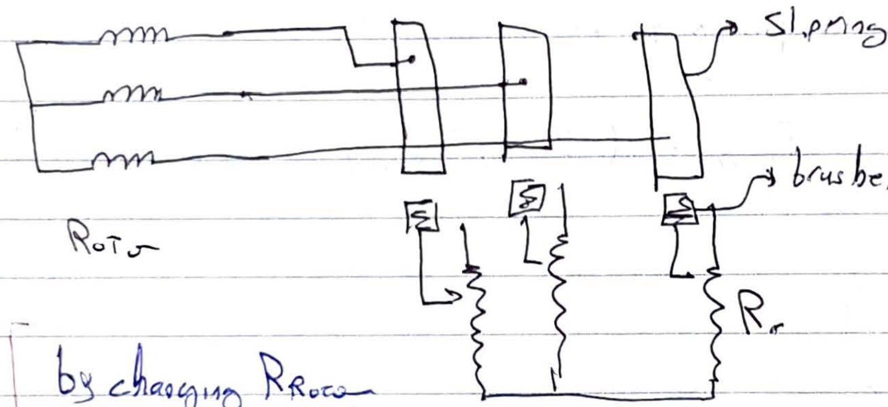
$$\boxed{a_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}} \quad \text{--- (1)}$$

مهم

$$\boxed{T_{max} = \frac{3V_{Th}^2}{2\omega_s\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}\right]}}$$

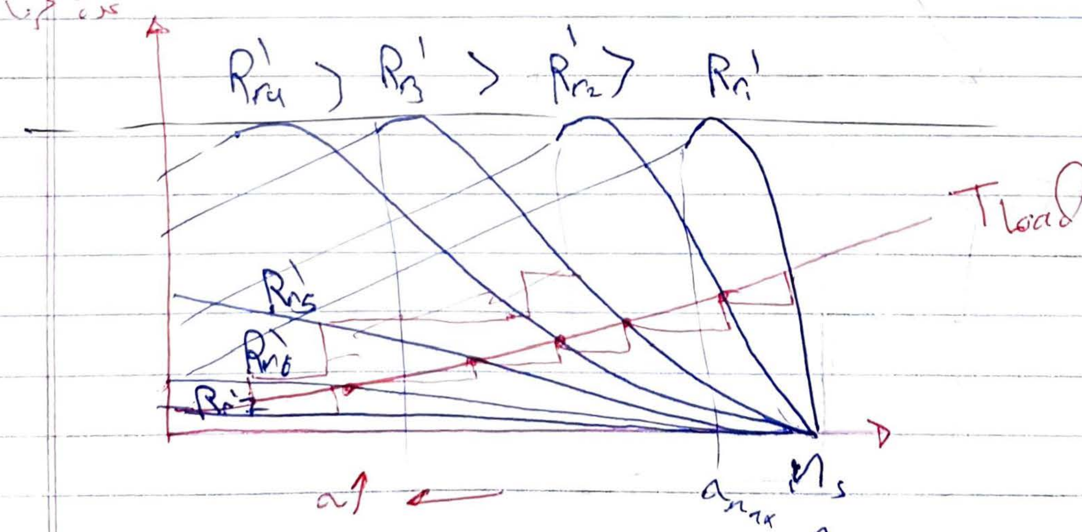
Wound Rotor Induction Motor.

تتميز هذا النوع بذلك لاني بستطيع ان اغير R_r فيه.



غير ما غير a_{max} حسب العلاقة و Y_s ... ثابتة وذلك كله ... مرة كبرنا

by changing R_{Rotor}



$Y_{Reff} = \text{Load}$ Y_{eff} ... R_r' ... اطلقنا

① $R_{r}' < R_{r}' < R_{ru}' \Rightarrow R_r' \uparrow \Rightarrow a_{max} \uparrow$

$a \uparrow \Rightarrow$ speed $\downarrow \Rightarrow (\eta_r = 1-a) \downarrow$

efficie $\Rightarrow \eta \downarrow \Rightarrow T_{START} \uparrow$

② $R_r' > R_{ru}' \Rightarrow T_{START} \downarrow \Rightarrow \eta \downarrow$ efficiency

Example A 460 V, 25 hp, 4 pole, Y-connected, wound Rotor IM has The following impedances

$R_s = 0.641\,\Omega$, $R_r' = 0.332\,\Omega$

$X_r = 1.106\,\Omega$, $X_r' = 0.464\,\Omega$, $X_m = 26.1\,\Omega$.

a) what is T_{max}? At what speed & slip does it occurs?
b) what is T_{start}
c) if R_r' is doubled, what is the speed at which T_{max} occurs? what is the new start.

Solution

a) $$T_{max} = \frac{3V_{Th}^2}{2\omega_s\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}\right]}$$

$$R_{Th} = \left(\frac{X_m}{X_m + X_s}\right) R_s \simeq R_s, \quad X_{Th} \simeq X_s$$

$$V_{Th} = \frac{X_m}{\sqrt{R_s^2 + (X_s + X_m)^2}} \cdot V_s$$

$f = 60\ Hz$

$R_{Th} = 0.59\,\Omega \qquad X_{Th} = X_s = 1.06\,\Omega \qquad V_{Th} = \frac{\ldots}{\sqrt{\ldots}} \cdot \frac{480}{\sqrt{3}}$

$= 255.2\ Volt$

$\omega_s = \left(\frac{2}{P}\right)\omega_e = 188.5\ rad/s$

$T_{max} = 229\ N.m$

$s_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}} = \frac{0.332}{\sqrt{\ }} = 0.198$

$n_m = (1 - s_{max})\, n_s = 1444\ rpm$

b) max when $\boxed{s=1}$ $\quad T_{start} = \frac{3 V_{Th}^2}{\omega_s\left[\left(\frac{R_r'}{s} + R_{Th}\right)^2 + (X_{Th} + X_r')^2\right]} \cdot \frac{R_r'}{s}$

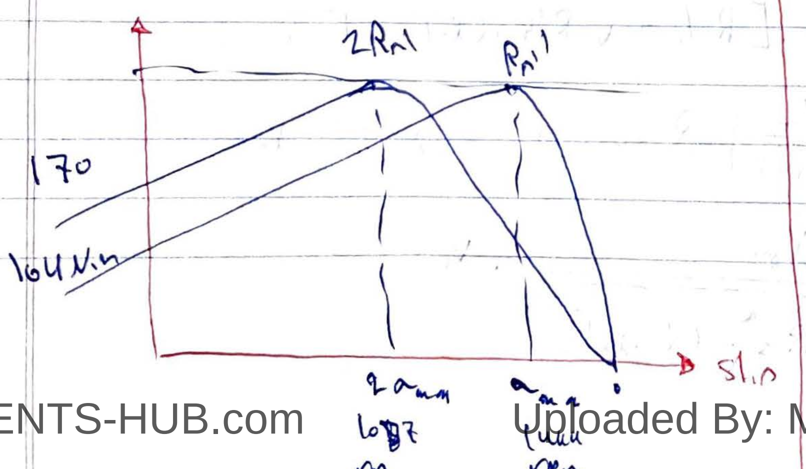
$T_{start} = 164\ N.m$

c) $s_{max} = (2)(0.198) = 0.396$ since $R_{r2}' = 2R_{r1}'$

$n_m = (1 - s_{max})\, n_s = 1087\ rpm$

only $R_r' = 2R_{r1}'$

$T_{start} = \frac{(3)(255.2)^2}{188.5\left[(0.332 \times 2) + (0.59)\right]^2 + \cdots} = 170\ N.m$



Matlab

$X = 0.00000001 : 1$

$R_{Th} =$

$X_{Th} =$

$\vdots$

$n_s =$

Rr' = 2X

$g =$

Plot (X, g)

Plot (X, g)

hold on زكي كان لازم يجيب

Calculation of starting current

$$I_{L,start} = \frac{S_{start}}{\sqrt{3}\, V_L}$$

S_{start} = Rated power (hp) * code letter factor.

Nominal code letter	Locked Rotor [KVA/hp]
A	0 - 3.15
B	3.15 - 3.55
C	3.55 - 4.00
D	4.00 - 4.50
e	4.50 - 5
F	5 - 5.6
⋮	⋮

Ex What is the starting current of a 15 hp, 208 V, code letter 3ϕ induction Motor?

$S_{start} = (15)(5.6) = 84\ kVA$

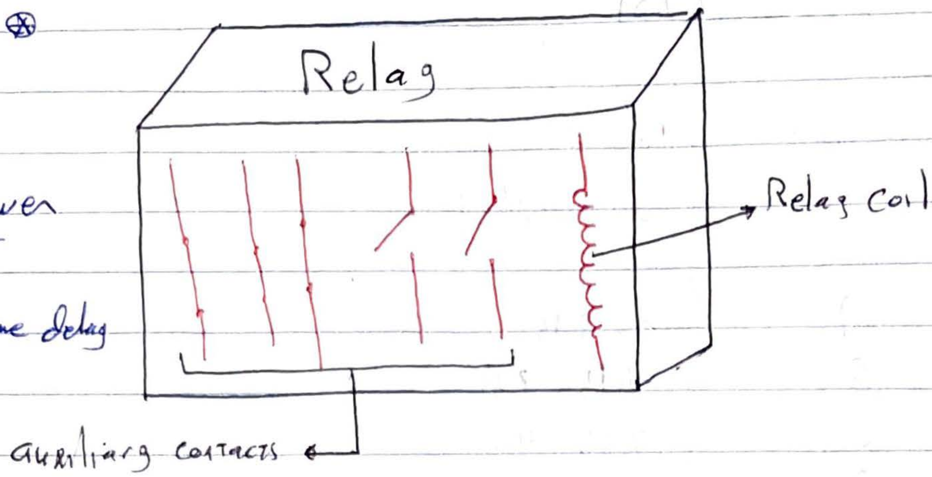
$$I_{L,start} = \frac{S_{start}}{\sqrt{3}\, V_L} = 233 A$$ [Rang of starting current the value will be closed to it]

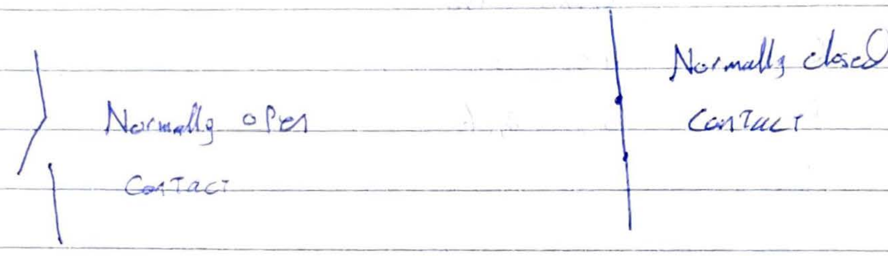
Relay logic control of 3-Ø Induction motor (ON-off control)

Relay: its an electro-magnet switch that has a coil and a set of auxiliary contacts, the contacts can be either normally open or normally closed.

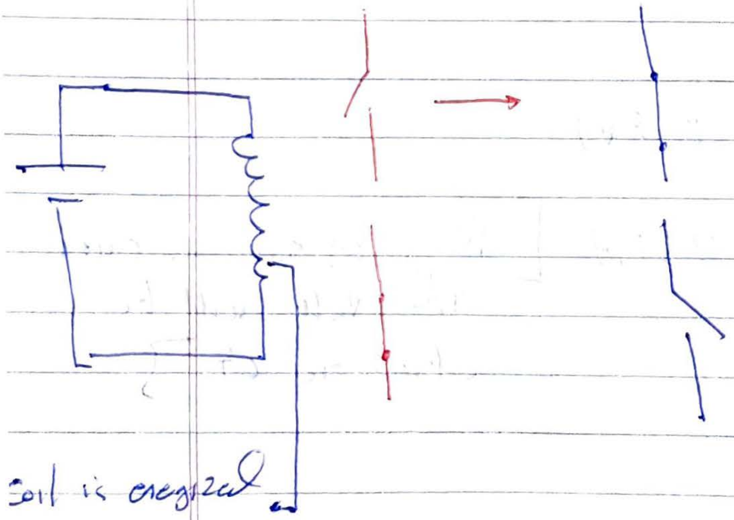
For low power

Relay base Time delay



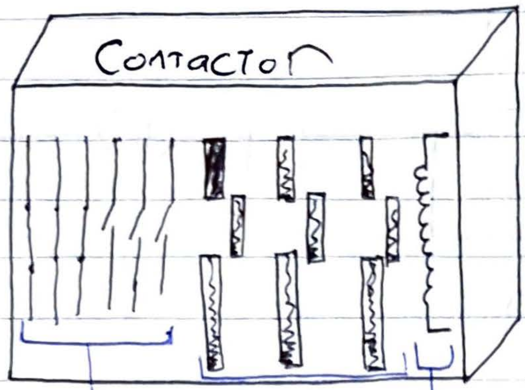


when the coil is energized, the normally open contact will close and the normally close contact will open.



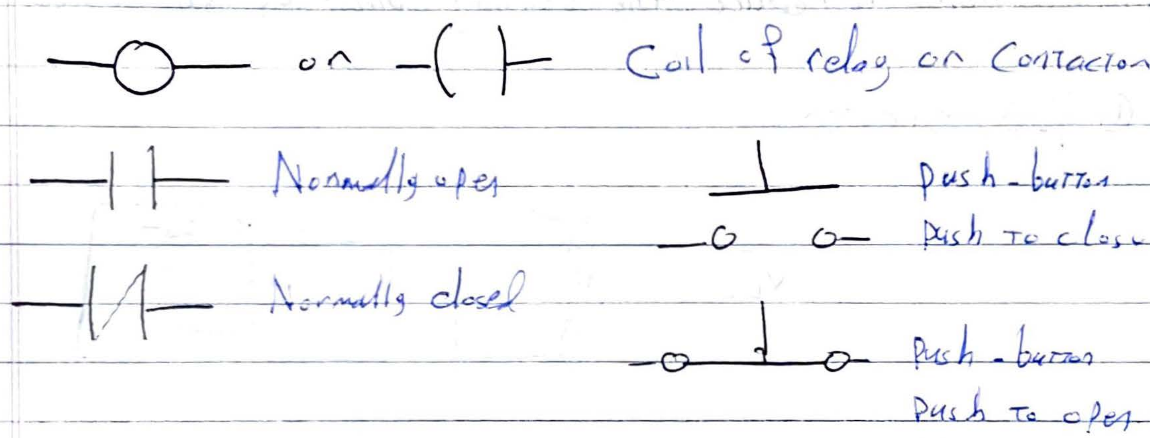
Contactor: its an electro-magnet switch that has three main contacts with set of auxiliary contacts

For 3-φ power
3-φ IM

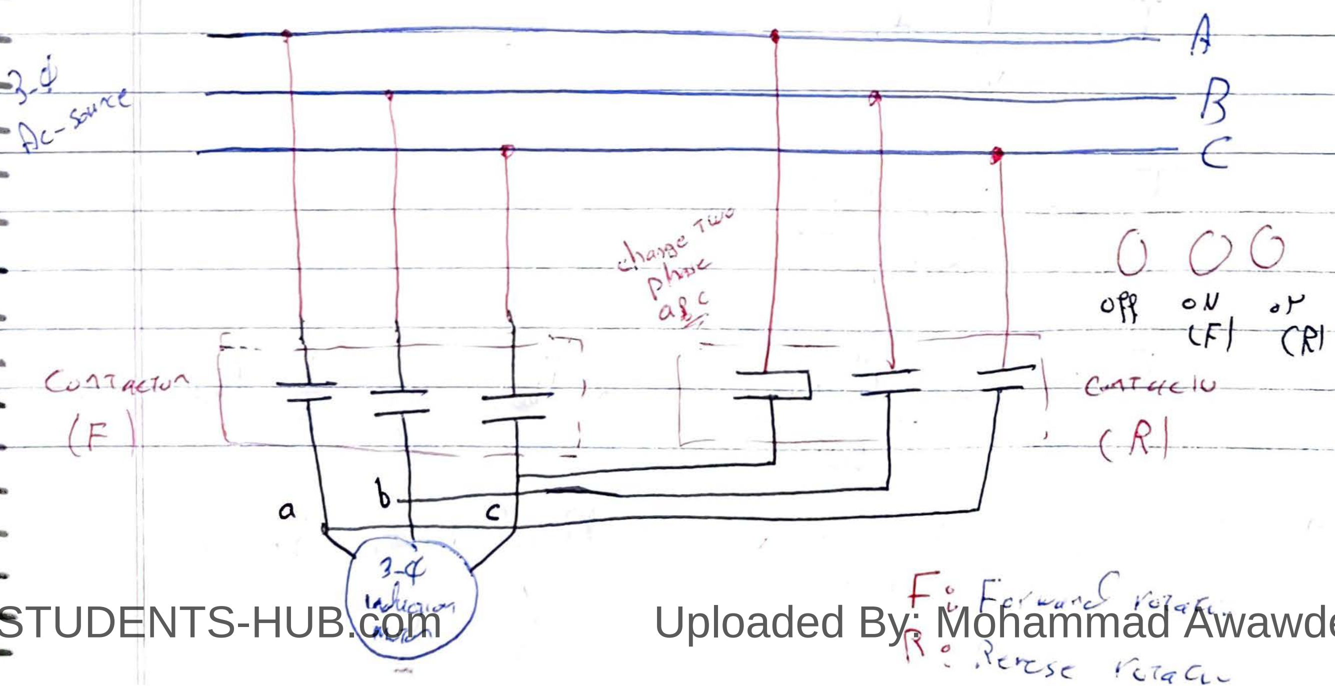


⇒ The auxiliary contact can be either normally open or normally closed

Symbols:

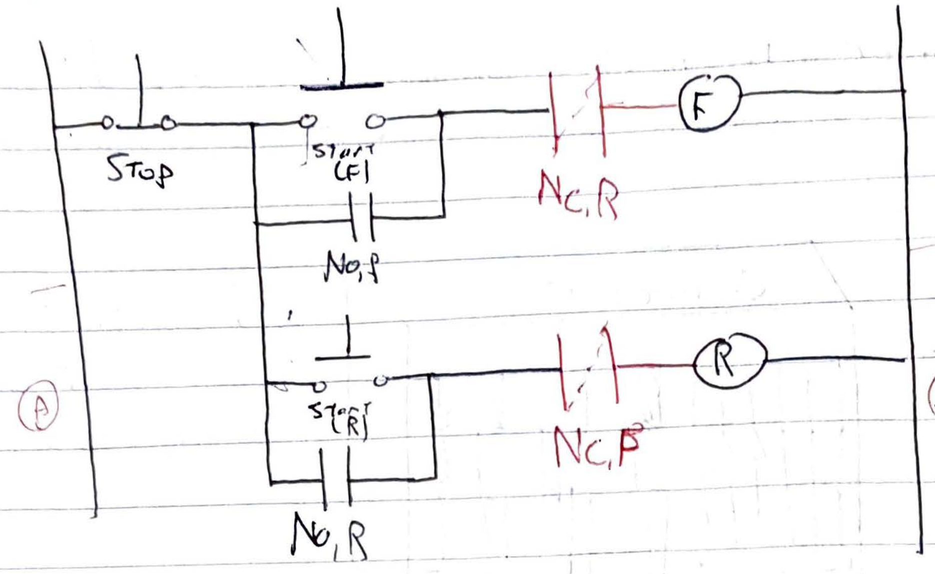


Example Control & power circuit to reverse the direction of motor



F: Forward rotation
R: Reverse rotation

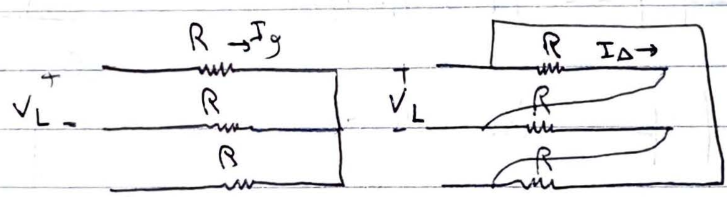
Ladder Diagram



Control Circuit

* Method to reduce the starting current of 3ϕ induction motor.

① Y-Δ starter



$$I_\Delta = \frac{V_L}{R} \quad , \quad I_Y = \frac{V_L/\sqrt{3}}{R} = \frac{1}{\sqrt{3}}\frac{V_L}{R}$$

$$I_Y = \frac{1}{\sqrt{3}} I_\Delta$$

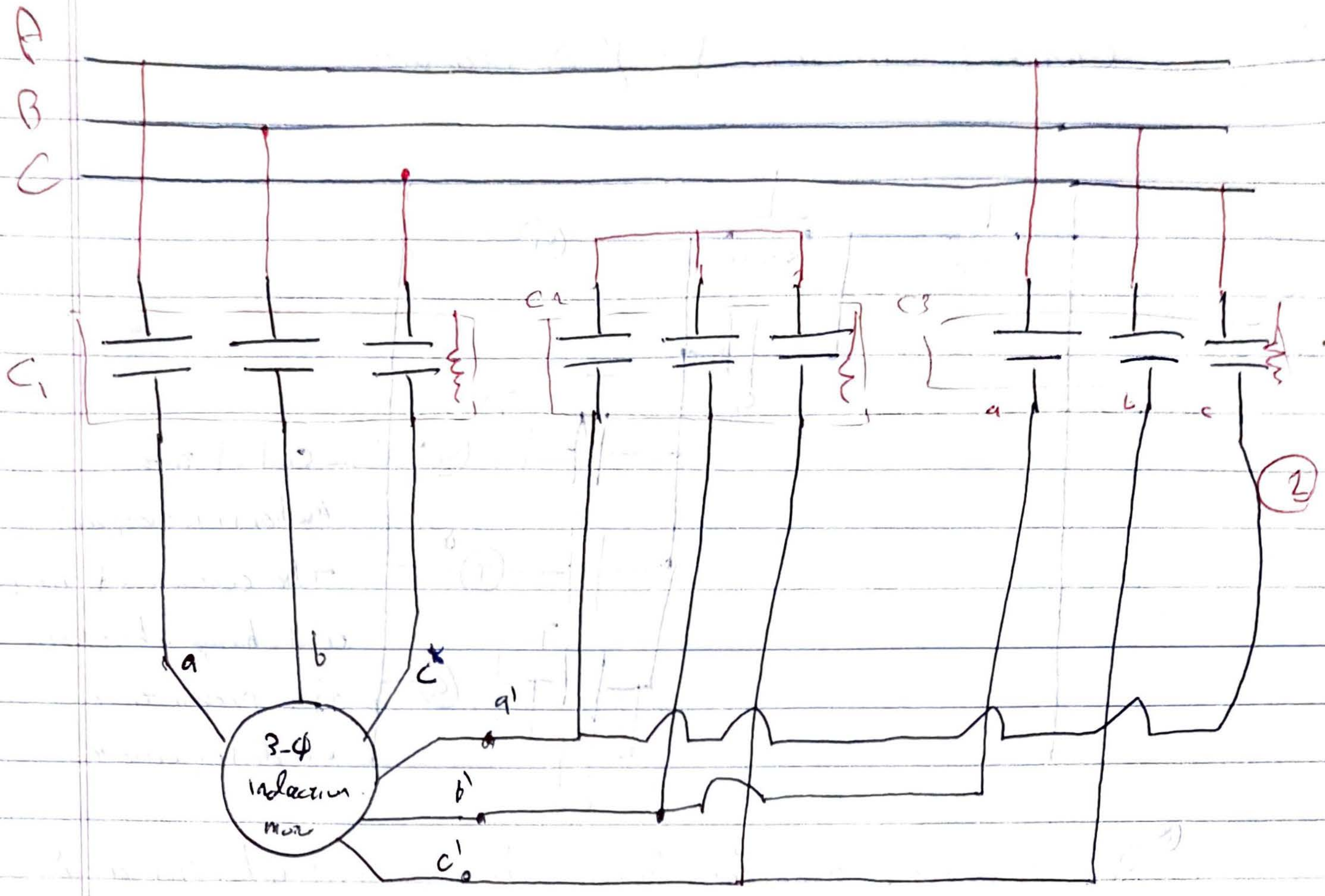
$$T_{ind} = \frac{3 I_r'^2 R_r'}{s\omega_s}$$

$$T_{ind,start} = \frac{3 I_r'^2 R_r'}{\omega_s} \quad , \quad T_{ind,start,Y} = \frac{1}{3} T_{ind,start,\Delta}$$

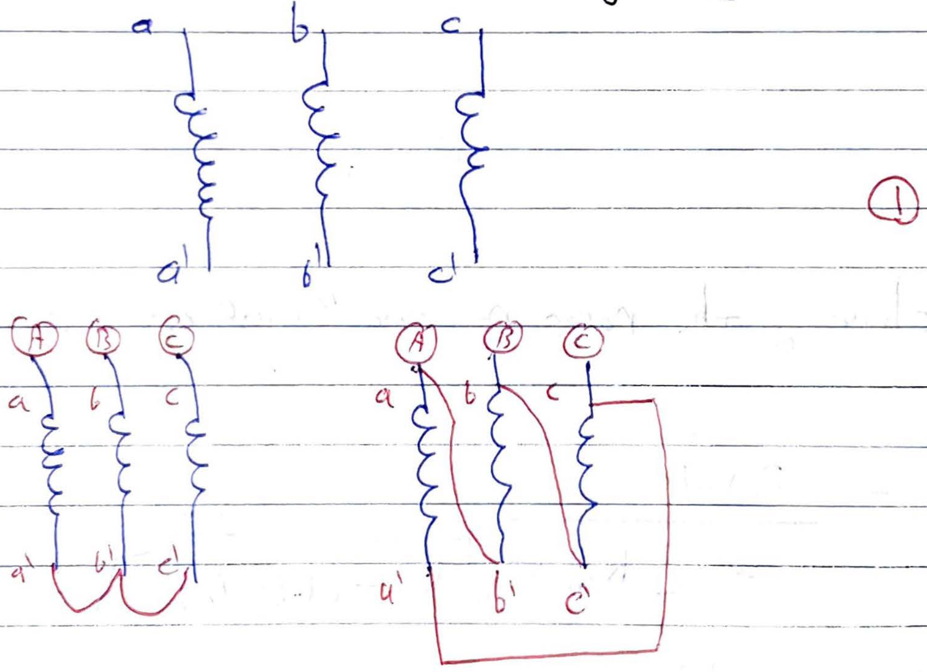
* هنا لما نبلش ال starting current كبير و
نقدر نعرض

Rated voltage of motor

Y Δ



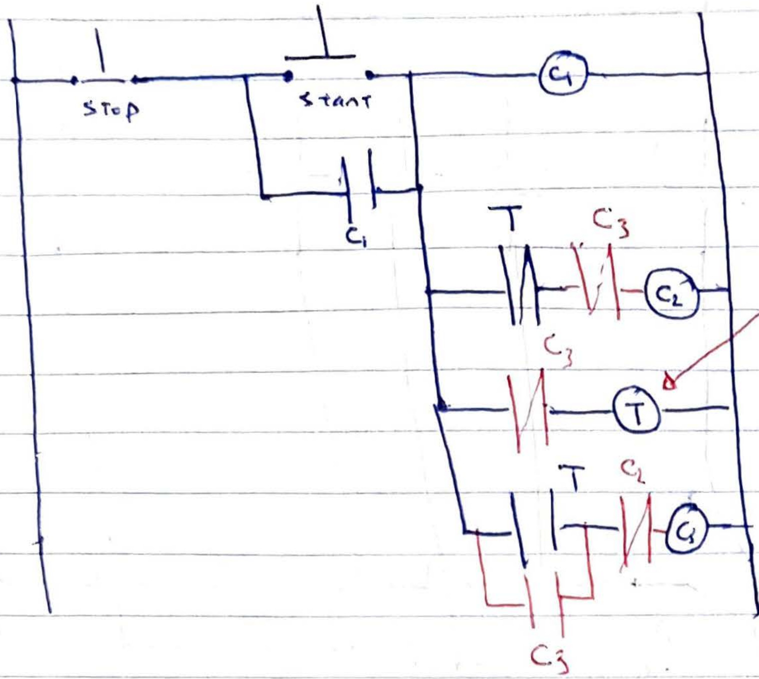
Stator windings (Must be below)



Video

Control circuit for Y-Δ starter

3



→ Coil of Timer "when it is energized the contacts of Timer will change their state after preset time on delay time"

* Red Parameters using for Remove the Timer Coil when it's is at Δ
* Try to add Reversed switch

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② changing the rotor resistance "wound rotor induction machine"

$$T_{max} = \frac{3 V_{Th}^2}{2\omega_s \left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_r')^2} \right]}$$

$$S_{max} = \frac{R_r'}{\sqrt{R_{Th}^2 + (X_{Th} + X_r')^2}}$$