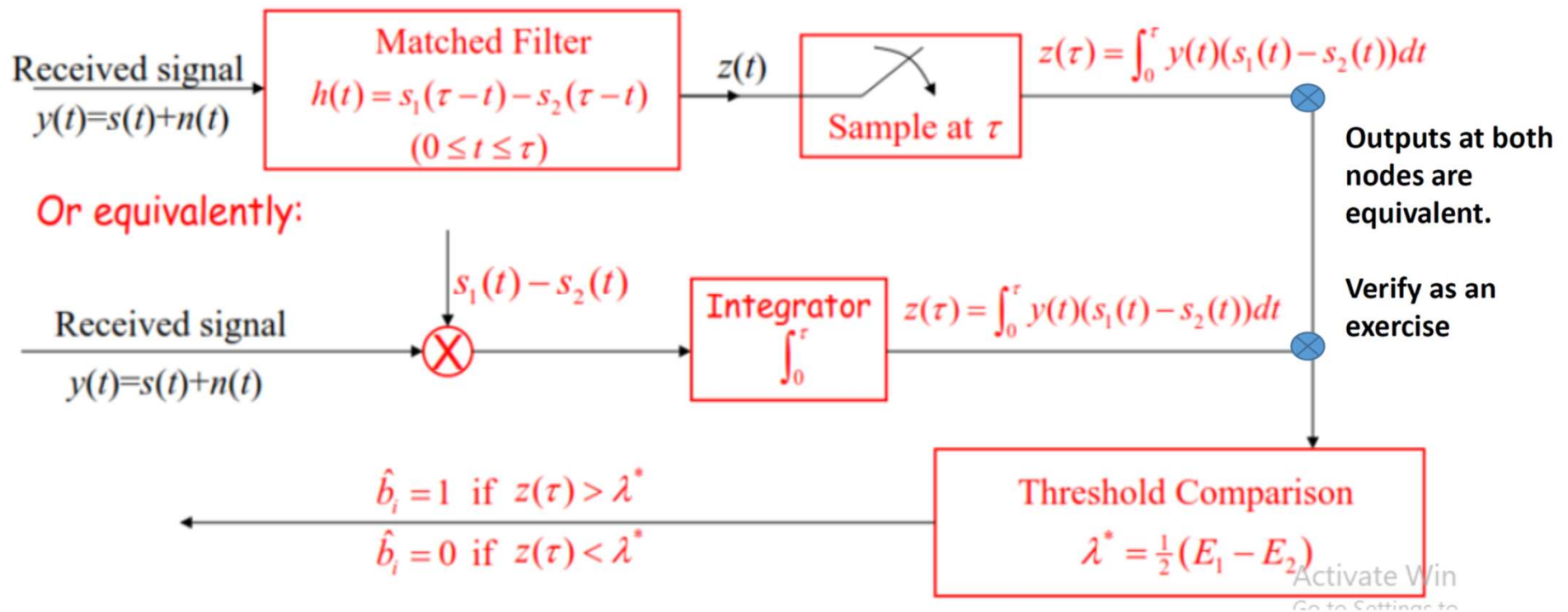


# Equivalent Implementations of the Optimum Receiver

The structure of the optimum receiver is depicted in the figure below. Note that the receiver can be implemented in terms of the matched filter and, equivalently, in terms of a correlator (a multiplier followed by an integrator).



## Example: Antipodal Binary Transmission

Let us consider a digital binary communication system where bits 1 and 0 are represented by the signals  $s_1(t)$  and  $-s_1(t)$ , respectively.

For this case,  $E_1 = E_2 = E = \int_0^\tau (s_1(t))^2 dt$

Therefore, the threshold is  $\lambda^* = (E_1 - E_2) = 0$

The probability of error is:

$$P_b^* = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) - s_2(t))^2 dt}{2N_0}}\right) \quad P_b^* = Q\left(\sqrt{\frac{4 \int_0^\tau (s_1(t))^2 dt}{2N_0}}\right)$$
$$P_b^* = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) + s_1(t))^2 dt}{2N_0}}\right) \quad P_b^* = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

**Probability of error decreases as the signal to noise ratio increases**

# Baseband Data Transmission

Binary data transmission by means of two voltage levels is referred to as baseband signaling. Manchester encoding, for example, is used in the Ethernet local area network as the signaling scheme. Here, we consider polar non-return to zero baseband transmission scheme, in terms of probability of error, optimum receiver structure, power spectral density and bandwidth.

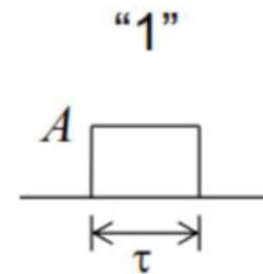
**Polar nonreturn to zero (also known as binary pulse amplitude modulation)**

## Signal Representation

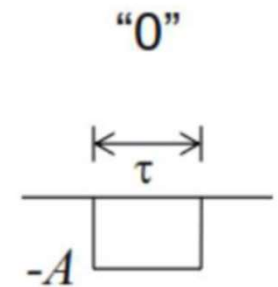
The baseband signals representing digits 1 and 0 are:

$$s_1(t) = A, \quad 0 \leq t \leq \tau$$

$$s_0(t) = -A, \quad 0 \leq t \leq \tau$$



$$s_1(t) = A \\ 0 \leq t \leq \tau$$

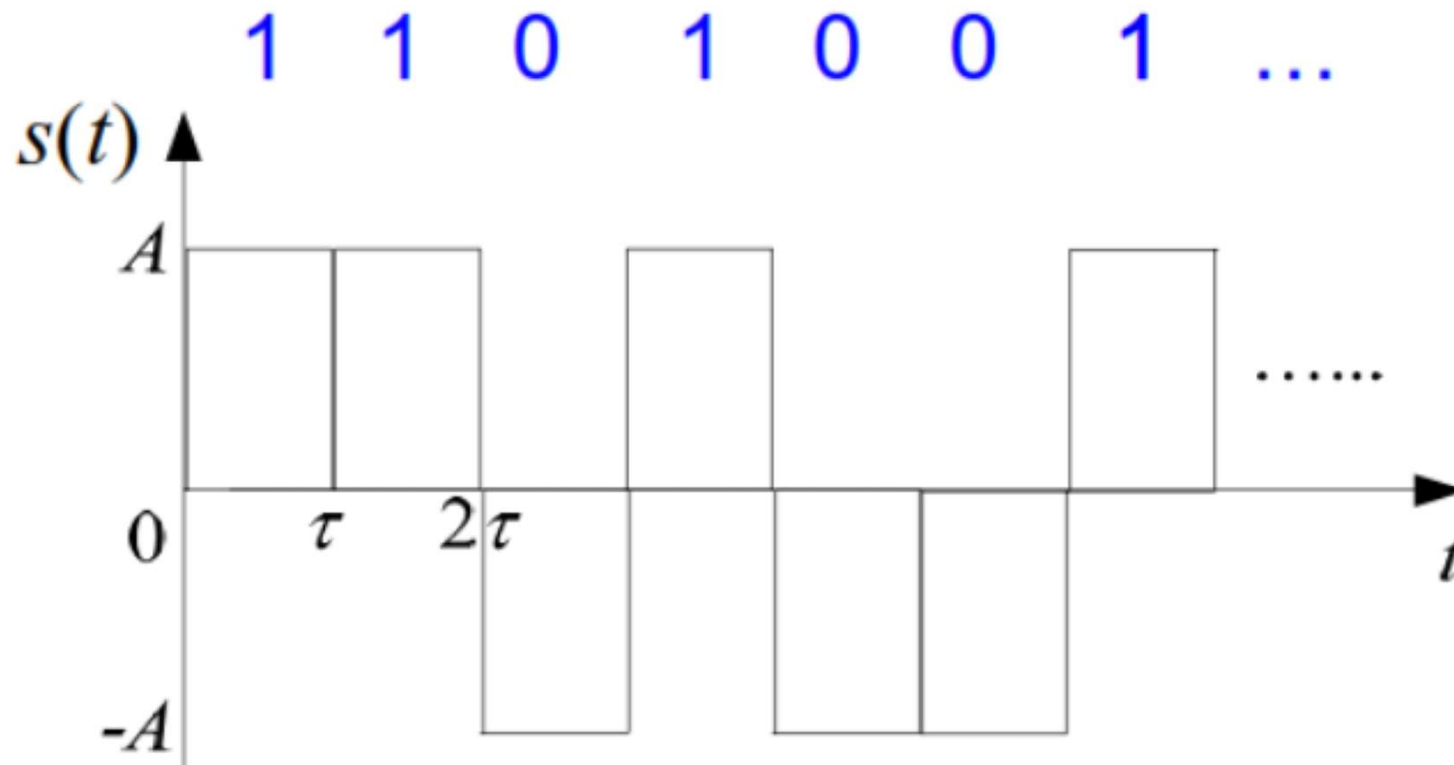


$$s_2(t) = -A \\ 0 \leq t \leq \tau$$

where,  $\tau$  is the symbol duration and  $R_b = 1/\tau$  is the data rate in bits/sec.

# Baseband Data Transmission

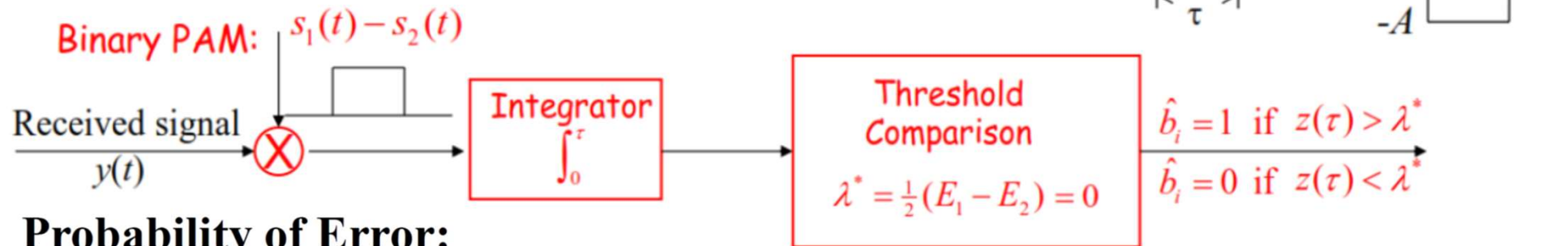
- **Generation: Convert data into polar non-return to zero format**



# Baseband Data Transmission

## Optimum Receiver

The optimum receiver is, of course, the matched filter, also implemented as a correlator, as shown in this figure.



## Probability of Error:

$$P_b^* = Q\left(\sqrt{\frac{\int_0^\tau (s_1(t) - s_2(t))^2 dt}{2N_0}}\right)$$

Receiver Implemented  
as a Correlator

## Optimal BER:

Note that:  $E_1 = E_2 = \int_0^\tau A^2 dt = A^2\tau \Rightarrow \lambda^* = (E_1 - E_2) = 0$

$$P_b^* = Q\left(\sqrt{\frac{2A^2\tau}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Average Energy per bit:  $E_b = \frac{1}{2}(E_1 + E_2) = A^2\tau$



## General Result on the Power Spectral Density of a digital M-ary baseband signal

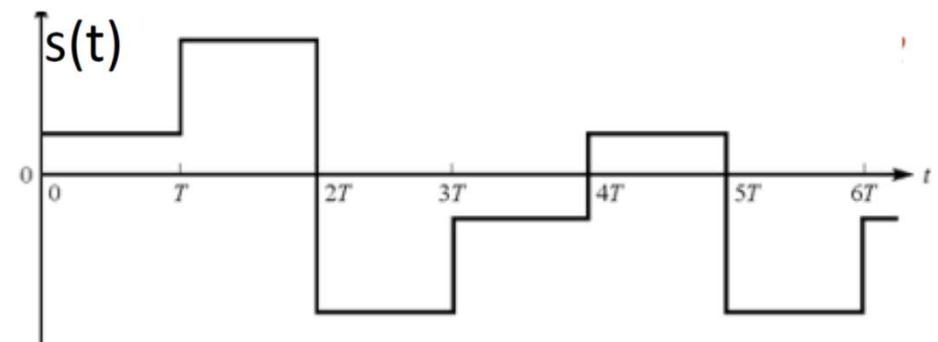
The time-domain representation of a digital M-ary baseband signal is

$$s(t) = \sum_{n=-\infty}^{\infty} Z_n \cdot v(t - n\tau)$$

where  $Z_n$  is a discrete random variable with  $\Pr\{Z_n = a_i\} = 1/M, i = 1, \dots, M,$

$v(t)$  is a unit-baseband signal, and symbols in different time slots are assumed independent. Under these assumptions, the power spectral density of  $s(t)$  is given by

$$G_s(f) = \frac{1}{\tau} |V(f)|^2 \cdot \left( \sigma_Z^2 + \frac{\mu_Z^2}{\tau} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{\tau}\right) \right)$$



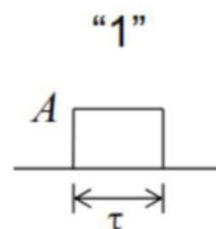
## Power Spectral Density of the Polar Non-return to Zero baseband signal

The general result stated above for the M-ary baseband signal can be specialized to the polar nonreturn to zero transmission as follows

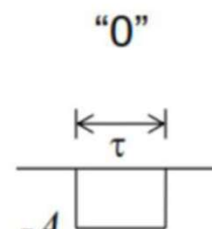
- The signal amplitude assumes two equally likely values. i.e.,  $P\{Z_n = \pm 1\} = 1/2$
- The basic unit pulse is  $v(t) = \begin{cases} A, & 0 \leq t \leq \tau \\ 0, & \text{otherwise} \end{cases}$
- The Fourier transform of the basic unit pulse is  $V(f) = A\tau \text{sinc}(f\tau)$
- The mean and variance of Z are:  $\mu_Z = 0, \sigma_Z^2 = 1$

$$E(Z) = \sum_{\text{all } z_i} z_i P(Z = z_i)$$

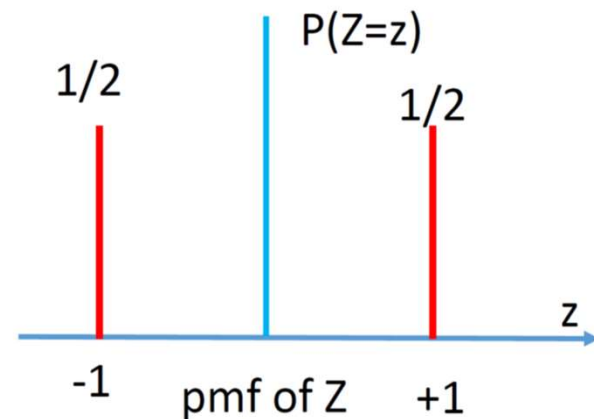
$$\text{Var}(Z) = \sigma_Z^2 = \sum_{\text{all } z_i} (z_i - E(Z))^2 P(Z = z_i)$$



$$s_1(t) = A \\ 0 \leq t \leq \tau$$



$$s_2(t) = -A \\ 0 \leq t \leq \tau$$



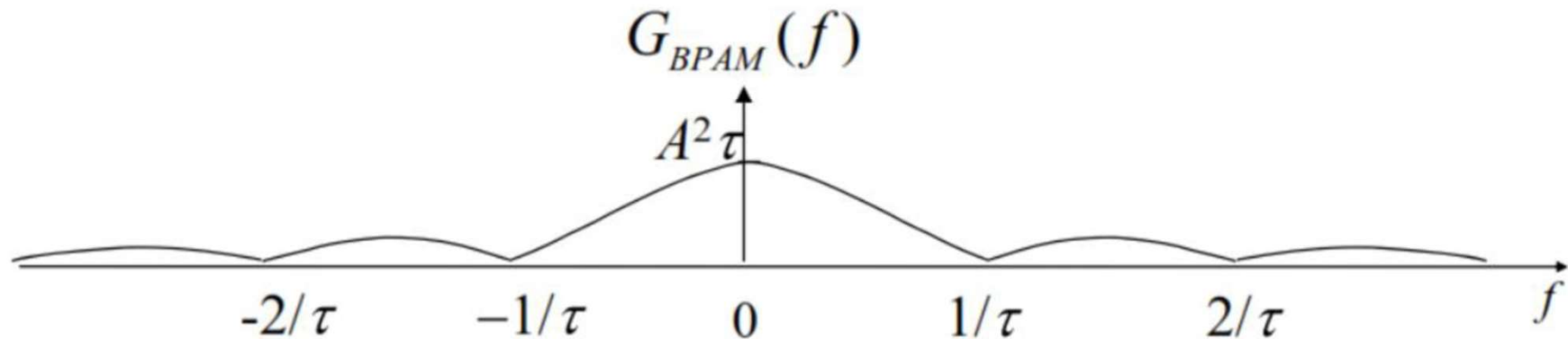
## Power Spectral Density of the Polar Non-return to Zero baseband signal

- The general power spectral density formula

$$G_s(f) = \frac{1}{\tau} |V(f)|^2 \cdot \left( \sigma_Z^2 + \frac{\mu_Z^2}{\tau} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{\tau}\right) \right)$$

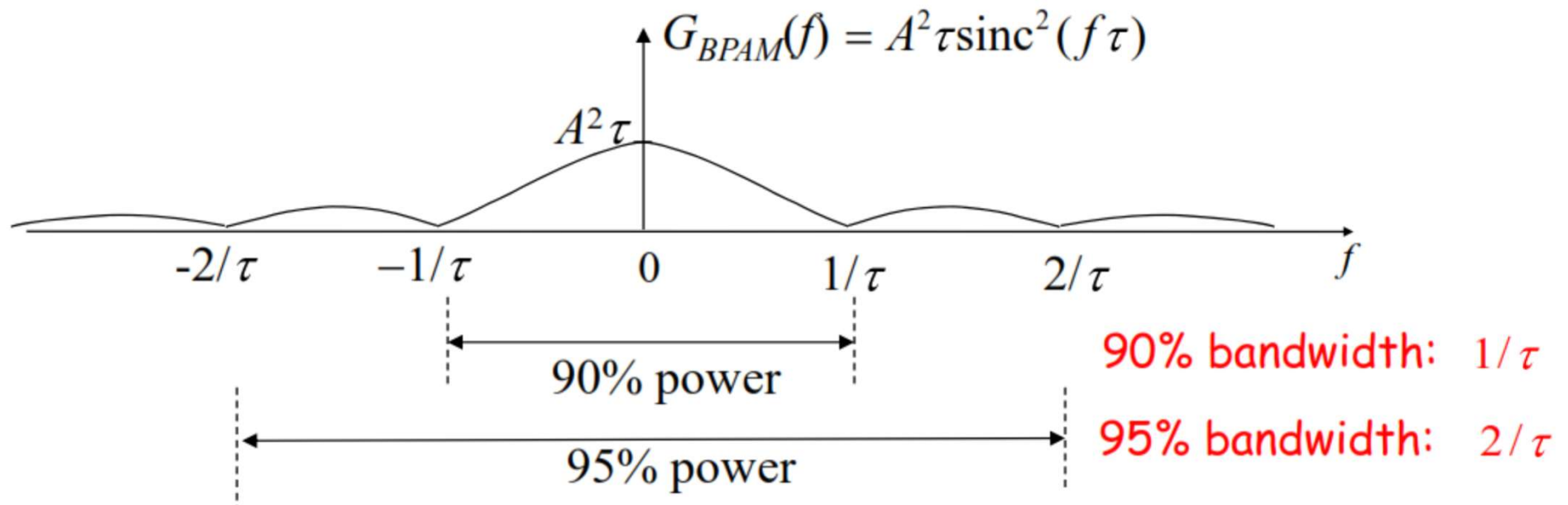
- Substituting:  $E(Z)=0$ ,  $\text{Var}(Z)=1$ , and the Fourier transform of the rectangular pulse  $v(t)$ , we get:

$$G_{BPAM}(f) = A^2 \tau \text{sinc}^2(f\tau)$$





## Bandwidth of the Polar Non-return to Zero baseband signal



The 90% power bandwidth  $= \frac{1}{\tau} = R_b$  (data rate)

The 95% power bandwidth  $= \frac{2}{\tau} = 2R_b$  (twice the data rate)