$|x| = \frac{\sqrt{x}}{\sqrt{x}} = -1$, $\pm\sqrt{3}$

① $\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{x+1}$ $\quad x^1 \quad (-1) \quad x^1$

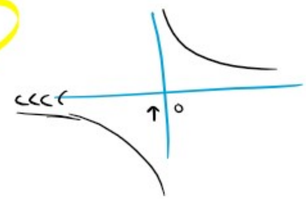
$\lim_{x \to -\infty} \dfrac{\frac{\sqrt{x^2+1}}{\sqrt{x^2}}}{\frac{x+1}{\sqrt{x^2}}}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $\frac{a}{b} > 0$

$\sqrt{x^2} = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$\lim_{x \to -\infty} \dfrac{\sqrt{\frac{x^2+1}{x^2}}}{\frac{x+1}{|x|}} = \lim_{x \to -\infty} \dfrac{\sqrt{1 + \frac{1}{x^2}}}{\frac{x+1}{-x}} = \dfrac{1}{\lim_{x \to -\infty} \left(-1 - \frac{1}{x}\right)}$ $\quad \frac{1}{x} \to 0$

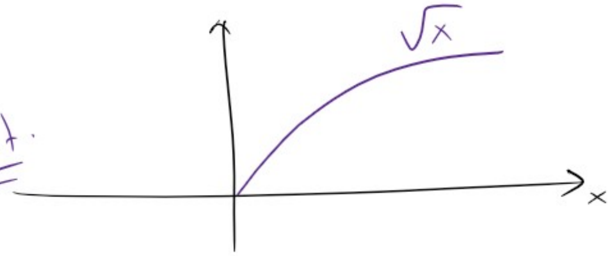
$= \frac{1}{-1} = -1$



$\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x+1} = 1$

$\lim_{x \to -\infty} \sqrt{1 + \frac{1}{x^2}}$ $\quad$ cont. $=$

$\sqrt{1 + \lim_{x \to -\infty} \frac{1}{x^2}}$



$(x-y)(x+y) = x^2 - y^2$

② $\lim_{x \to \infty} \left(\sqrt{x^2+1} - \sqrt{x^2 - x}\right) \dfrac{\left(\sqrt{x^2+1} + \sqrt{x^2 - x}\right)}{\sqrt{x^2+1} + \sqrt{x^2 - x}}$ $\quad \infty - \infty$ undef

$\infty^x \quad x(x-1)^y \quad \infty$

$\lim_{x \to \infty} \dfrac{\cancel{x^2} + 1 - (\cancel{x^2} - x)}{\sqrt{x^2+1} + \sqrt{x^2 - x}} = \lim_{x \to \infty} \dfrac{x + 1}{\sqrt{x^2+1} + \sqrt{x^2 - x}}$ $\quad \frac{1}{2}$

$1 \quad 1$

$\lim \dfrac{\frac{x+1}{\sqrt{x^2}}}{}$

$\sqrt{x^2} = |x| = x$

$$\lim_{x \to \infty} \frac{\sqrt{x^2}}{\dfrac{\sqrt{x^2+1} + \sqrt{x^2-x}}{\sqrt{x^2}}}$$

$\sqrt{x^2} = |x| = x$

$$\lim_{x \to \infty} \frac{\dfrac{x+1}{x}}{\dfrac{\sqrt{x^2+1}}{\sqrt{x^2}} + \dfrac{\sqrt{x^2-x}}{\sqrt{x^2}}} = \lim_{x \to \infty} \frac{1 + \frac{1}{x}}{\sqrt{1+\frac{1}{x^2}} + \sqrt{1-\frac{1}{x}}}$$

$\frac{1}{x} \to 0$, $\frac{1}{x^2} \to 0$, $\frac{1}{x} \to 0$

$$= \frac{1}{\sqrt{1}+\sqrt{1}} = \frac{1}{2}$$

$$\sqrt{\frac{x^2+1}{x^2}} \qquad \sqrt{\frac{x^2-x}{x^2}}$$