7.3

4 / 14 / 21 / 23 / 26 / 38 / 41 / 48 / 50 / 52
65 / 73 / 82 / 91 / 92 / 98 / 106 / 108 / 118

(4)

$$e^{x^2} e^{(2x+1)} = e^t$$

$$e^{x^2+2x+1} = e^t \Rightarrow t = x^2+2x+1$$

(14) $y = \ln(3\theta e^{-\theta})$

$$y' = \frac{3\theta e^{-\theta} \cdot -1 + e^{-\theta} \cdot 3}{3\theta e^{-\theta}} = \frac{-\theta+1}{\theta} = -1 + \frac{1}{\theta}$$

(21) $y = e^{(\cos t + \ln t)}$

$$y' = e^{(\cos t + \ln t)} \cdot \left(-\sin t + \frac{1}{t}\right)$$

(23) $y = \int_0^{\ln x} \sin e^t \, dt$

$$y' = \sin e^{\ln x} \cdot \frac{1}{x} = \frac{\sin x}{x}$$

(26) $\ln xy = e^{x+y}$

$\Rightarrow \ln x + \ln y = e^x \cdot e^y$

$\Rightarrow \frac{1}{x} + \frac{y'}{y} = e^x \cdot e^y \cdot y' + e^y e^x$

$$y' = \frac{e^{x+y} - \frac{1}{x}}{\left(\frac{1}{y} - e^{x+y}\right)}$$

(38) $\int \frac{e^{-\sqrt{r}}}{\sqrt{r}}\, dr$

$u = -\sqrt{r}$

$du = \frac{-1}{2\sqrt{r}}\, dr$

$= \int -2e^u\, du$

$= -2e^{-\sqrt{r}} + C$

(41) $\int \frac{e^{1/x}}{x^2}\, dx$

$u = \frac{1}{x}$

$du = \frac{-1}{x^2}\, dx$

$= -\int e^u\, du$

$= -e^{\frac{1}{x}} + C$

(48) $\int_0^{\sqrt{\ln\pi}} 2x e^{x^2} \cos(e^{x^2})\, dx$

$u = e^{x^2}$

$du = e^{x^2} \cdot 2x\, dx$

$= \int \cos u\, du = \sin u$

$= \sin e^{x^2} \Big]_0^{\sqrt{\ln\pi}} = \sin e^{\ln\pi} - \sin e^0$

$= \sin\pi - \sin 1$

$= 0 - \sin 1$

$= -\sin 1$

(50) $\int \frac{dx}{1+e^x}$

$= \int \frac{(1) \cdot e^{-x}}{(1+e^x)e^{-x}}\, dx = \int \frac{e^{-x}}{e^{-x}+1}\, dx$

$u = e^{-x}+1$

$du = -e^{-x}\, dx$

$= \int \frac{-du}{u} = -\ln|u|$

$= -\ln|e^{-x}+1|$

52) $\dfrac{dy}{dt} = e^{-t}\sec^2(\pi e^{-t})$, $y(\ln 4) = \dfrac{2}{\pi}$

intial value problem

$$\int dy = \int e^{-t}\sec^2(\pi e^{-t})\,dt$$

$u = \pi e^{-t}$

$du = -\pi e^{-t}\,dt$

$$\int \frac{-1}{\pi}\sec^2(u)\,du$$

$$= \frac{-1}{\pi}\tan u + C$$

$$y(t) = \cancel{\int} \frac{-1}{\pi}\tan(\pi e^{-t}) + C$$

$$y(\ln 4) = \frac{2}{\pi}$$

$$\Rightarrow y(\ln 4) = \frac{2}{\pi} = \frac{-1}{\pi}\tan(\pi e^{-\ln 4}) + C$$

$$\frac{2}{\pi} = \frac{-1}{\pi}\tan(\pi e^{\ln\frac{1}{4}}) + C$$

$$\frac{2}{\pi} = \frac{-1}{\pi} + C \Rightarrow C = \frac{3}{\pi}$$

$$y(t) = \frac{-1}{\pi}\tan(\pi e^{-t}) + \frac{3}{\pi}$$

(65) $y = 2^{\sin 3t}$

$y' = 2^{\sin 3t} \cdot 3\cos 3t \cdot \ln 2$

(73) $y = \log_3 \left(\left(\frac{x+1}{x-1} \right)^{\ln 3} \right)$

$y = \ln 3 \ \log_3 \left(\frac{x+1}{x-1} \right)$ $\left(\log_a x = \frac{\ln x}{\ln a} \right.$

$y = \cancel{\ln 3} \cdot \frac{\ln \left(\frac{x+1}{x-1} \right)}{\cancel{\ln 3}}$

$y = \ln(x+1) - \ln(x-1)$

$y' = \frac{1}{x+1} - \frac{1}{x-1}$

(82) $y = t \log_3 \left(e^{(\sin t)(\ln 3)} \right)$

$y = t \cdot (\sin t)(\ln 3) \cdot \log_3 e$

$y = t (\sin t) \cancel{(\ln 3)} \cdot \frac{\overbrace{\ln e}^{1}}{\cancel{\ln 3}}$

$y = t \cdot \sin t$

$y' = t \cdot \cos t + \sin t$

(91) $$\int_2^4 x^{2x}(1+\ln x)\,dx$$

$$y = x^{2x}$$

$$\ln u = \ln x^{2x}$$

$$\ln u = 2x \cdot \ln x$$

$$\frac{u'}{u} = 2x \cdot \frac{1}{x} + \ln x \cdot 2$$

$$u' = 2 + 2\ln x$$

$$du = 2(1+\ln x)\,dx$$

$$x=2 \longrightarrow u=16$$

$$x=4 \rightarrow u=65536$$

$$= \int_{16}^{65536} \frac{u}{2}$$

$$= \frac{1}{2}\,\frac{u^2}{2}\Big]_{16 \rightarrow 2^4}^{65536 \rightarrow 4^8}$$

$$= \frac{1}{2}\left(4^8 - 2^4\right)$$

(92) $$\int \frac{x\,2^{x^2}}{1+2^{x^2}}$$

$$u = 1 + 2^{x^2}$$

$$du = 2^{x^2} \cdot 2x \cdot \ln 2\,dx$$

$$= \frac{1}{\ln 2}\int \frac{du}{u}$$

$$= \frac{1}{\ln 2} \cdot \ln\left|1+2^{x^2}\right| + C$$

(98) $\displaystyle\int_1^4 \frac{\log_2 x}{x}\,dx$ $\qquad \left\{\begin{array}{l} u = \log_2 x \\ du = \dfrac{1}{\ln 2 . x}\,dx \end{array}\right.$

$= \ln 2 \int u\,du$

$= \ln 2 . \dfrac{u^2}{2} = \dfrac{\ln 2}{2} . \left(\log_2 x\right)^2 \Big]_1^4 = \dfrac{\ln 2}{2}\left(2^2 - 0\right)$

$= 2\ln 2 = \ln 4$

(106) $\displaystyle\int \frac{dx}{x(\log_8 x)^2}$ $\qquad \left\{\begin{array}{l} u = \log_8 x \\ du = \dfrac{1}{\ln 8 . x}\,dx \end{array}\right.$

$= \displaystyle\int \frac{\ln 8\,dx}{u^2}$

$= \ln 8 \cdot \dfrac{u^{-1}}{-1} + C$

$= -\ln 8 \left(\log_8 x\right)^{-1} + C$

(108) $\displaystyle\int_1^{e^x} \frac{1}{t}\,dt = \ln|t| \Big]_1^{e^x} = \ln e^x - \ln 1$

$= x$

118 $y=(\ln x)^{\ln x}$ $\quad ?y'$

$\ln y = \ln(\ln x)^{\ln x}$

$\ln y = \ln x \cdot \ln(\ln x)$

$\frac{y'}{y} = \cancel{\ln x} \cdot \frac{1}{x\cancel{\ln x}} + \ln(\ln x) \cdot \frac{1}{x}$

$y' = \left(\frac{1+\ln(\ln x)}{x}\right) \cdot (\ln x)^{\ln x}$