INVESTMENT MANAGEMENT

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FINN431

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Chapter 7

Optimal Risky Portfolios

CHAPTER 7

OPTIMAL RISKY PORTFOLIOS

Investment can be viewed as a top-down process:

يمكن النظر إلى الاستثمار على أنه عملية من الأعلى إلى الأسفل:

STEP1) Capital allocation between risky portfolio (P)& Risk-free portfolio (F).

الخطوة الأولى: تخصيص رأس المال بين محفظة المخاطرة (P) والمحفظة الخالية من المخاطر.(F)

STEP2) Asset allocation in the risky portfolio (P) across board asset classes.

الخطوة الثانية: توزيع الأصول ضمن محفظة المخاطرة (P) عبر فئات الأصول الواسعة.

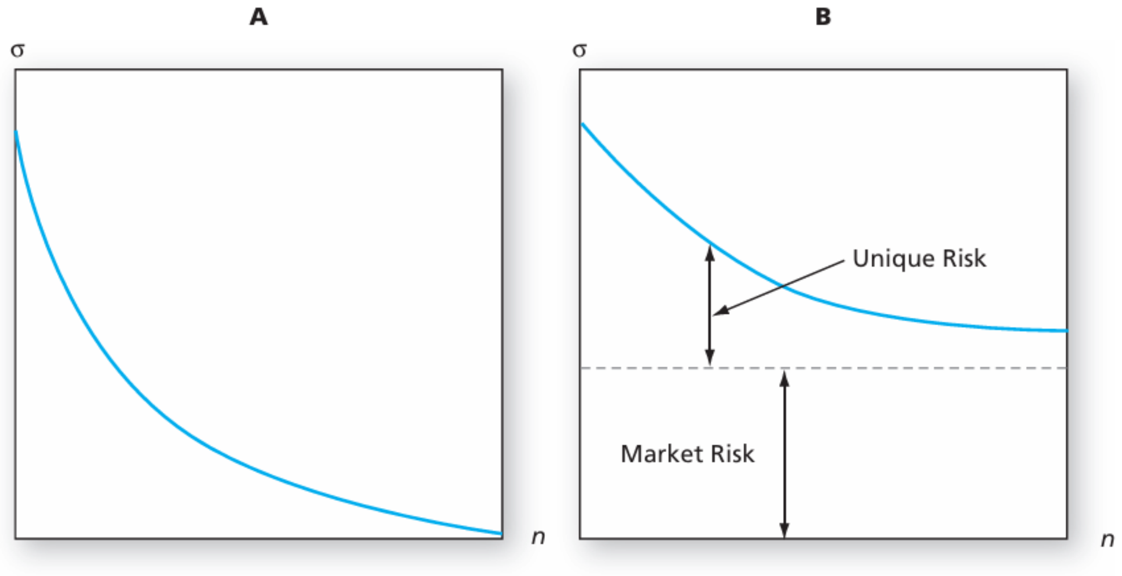
STEP3) Security allocation of individual assets within each asset class.

الخطوة الثالثة: تخصيص الأوراق المالية الفردية داخل كل فئة من فئات الأصول.

Diversification & portfolio risk:

1) Non diversifiable risk → Systematic risk → Market risk, it is a risk that cannot be eliminated through diversification.

2) Diversifiable risk → Unsystematic risk → Firm specific risk → Unique risk. It is a risk that can be eliminated through diversification (we use correlation coefficient as a rule).



Naïve diversification strategy [Equally weighted portfolio]

Example: Place half of your fund in asset A& another half in asset B, diversification should reduce risk. If asset A price falls, asset B price might rise. 2 effects are off-setting.

Now we'll consider an efficient diversification strategy. In other words, we want to construct risky portfolios to provide lowest risk for any given level of expected return.

Example page 208:

Portfolio constructed from 2 assets which are:

Debt fund → Invest in bonds → D

Equity fund → Invest in stocks → E

Portfolio	R (return)	σ (Risk)	Proportion (Weight)
D	r_d	σ_d	W_d
E	r_e	σ_e	W_e

$$r_p = \sum r * w \quad = r_d * W_d + r_e * W_e$$

$$E(r_p) = \sum E(r) * w \quad = E(r_d) * W_d + E(r_e) * W_e$$

Variance (σ^2) of a portfolio

$$\sigma_p^2 = (W_d * \sigma_d)^2 + (W_e * \sigma_e)^2 + 2 * W_e * W_d * COV(r_e, r_d)$$

$$COV(r_e, r_d) = \rho_{d,e} * \sigma_d * \sigma_e$$

Correlation coefficient = ρ "Rho"

$$-1 \leq \rho_{(d,e)} \leq +1$$

- ➔ Variance of a portfolio is reduced if the covariance is negative.
- ➔ When correlation coefficient is -1 a perfect hedge is possible (The standard deviation of the portfolio is 0).

ρ	Description
-1	Perfectly negative correlated assets
$-1 < \rho < +1$	Negatively correlated assets
0	Uncorrelated assets
$0 < \rho < +1$	Positively correlated assets
+1	Perfectly positive correlated assets

If correlation or $\rho = -1$ → theoretically σ_p^2 could be zero, but practically not possible; because we have a market risk.

STEP1) $\sigma_p^2 = (W_d * \sigma_d)^2 + (W_e * \sigma_e)^2 + 2 * W_e * W_d * COV(r_e, r_d)$

STEP2) $COV(r_e, r_d) = \rho_{d,e} * \sigma_d * \sigma_e$

STEP3) $\sigma_p^2 = (W_d^2 * \sigma_d^2) + (W_e^2 * \sigma_e^2) + 2W_d * W_e * \rho_{d,e} * \sigma_d * \sigma_e$

Apply when Correl (ρ)=-1

STEP4) $\sigma_p^2 = (W_d^2 * \sigma_d^2) + (W_e^2 * \sigma_e^2) + 2W_d * W_e * \rho_{d,e} * \sigma_d * \sigma_e$

STEP5) $\sigma_p^2 = (W_d^2 * \sigma_d^2) + (W_e^2 * \sigma_e^2) - 2W_d * W_e * \sigma_d * \sigma_e$

STEP6) $\sigma_p^2 = (W_d^2 * \sigma_d^2) - 2W_d * W_e * \sigma_d * \sigma_e + (W_e^2 * \sigma_e^2)$

Foiling the equation

STEP7) $\sigma_p^2 = (W_d * \sigma_d - W_e * \sigma_e)^2$

$W_e + W_d = 1 \rightarrow W_e = 1 - W_d$

STEP8) $\sigma_p^2 = (W_d * \sigma_d - (1 - W_d) * \sigma_e)^2$

STEP9) $\sqrt[2]{0^2} = \sqrt[2]{(W_d * \sigma_d - (1 - W_d) * \sigma_e)^2}$

STEP10) $0 = (W_d * \sigma_d - (1 - W_d) * \sigma_e)$

STEP11) $0 = W_d * \sigma_d - 1 * \sigma_e + W_d * \sigma_e$

STEP12) $0 = W_d * \sigma_d - \sigma_e + W_d * \sigma_e$

STEP13) $\sigma_e = W_d * \sigma_d + W_d * \sigma_e$

STEP14) $\sigma_e = W_d * (\sigma_d + \sigma_e)$

STEP15) $\frac{\sigma_e}{(\sigma_d + \sigma_e)} = W_d$

If correlation= +1 → The right hand in the portfolio variance equation would be a perfect would be a perfect square. There is no benefit from diversification in this case. As a result, the standard deviation of the portfolio is the weighted average of components standard deviation.

$$\sigma_p^2 = (W_d^2 * \sigma_d^2) + (W_e^2 * \sigma_e^2) + 2W_d * W_e * \rho_{d,e} * \ \sigma_d * \sigma_e$$

$$\sigma_p^2 = (W_d^2 * \sigma_d^2) + 2W_d * W_e * \sigma_d * \sigma_e \ + (W_e^2 * \sigma_e^2)$$

$$\sigma_p^2 = (W_d \ * \sigma_d + W_e \ * \sigma_e)^2$$

$$\sigma_p = W_d \ * \sigma_d + W_e \ * \sigma_e$$

We always seek negatively correlated assets or low correlated assets to reduce risk. The lower the correlation between assets, the GREATER the gain in efficiency.

Example Page 211:

$E(r_d)$=8% or 0.08

$E(r_e)$=18% or 0.18

σ_d = 12% or 0.12

σ_e= 20% or 0.20

Equation that you should know:

$$W_d + W_e = 1$$

$$E(r_p) = W_d * E(r_d) + W_e * E(r_e)$$

W_d	W_e	E (r_p)	ρ= -1	ρ= 0	ρ= +0.3	ρ= +1
0	1	0.13	0.2	0.2	0.2	0.2
0.1	0.9	0.125	0.168	0.1804	0.183957	0.192
0.2	0.8	0.12	0.136	0.16179	0.16876	0.184
0.3	0.7	0.115	0.104	0.144554	0.154661	0.176

0.4	0.6	0.11	0.072	0.129244	0.141986	0.168
0.5	0.5	0.105	0.04	0.116619	0.131149	0.16
0.6	0.4	0.10	0.008	0.107629	0.122638	0.152
0.7	0.3	0.095	0.024	0.103228	0.116962	0.144
0.8	0.2	0.09	0.056	0.104	0.114543	0.136
0.9	0.1	0.085	0.088	0.109836	0.115585	0.128
1	0	0.08	0.12	0.12	0.12	0.12

$$\sigma_p = \sqrt{(W_d * \sigma_d)^2 + (W_e * \sigma_e)^2 + 2 * W_d * W_e * \rho * \sigma_d * \sigma_e}$$

Example take the 5th row at (W_d =0.5 and W_e=0.5) and ρ=-1

$$\sigma_p = \sqrt{(0.5 * 0.12)^2 + (0.5 * 0.2)^2 + 2 * 0.5 * 0.5 * -1 * 0.12 * 0.2}$$

$$\sigma_p = 0.04$$

How to find $W_{\min d\ or\ e}$ from risky portfolio equation

Step 1 → $\sigma_p^2 = (W_d * \sigma_d)^2 + (W_e * \sigma_e)^2 + 2 * W_e * W_d * COV(r_e, r_d)$

Know that $W_e + W_d = 1$ →Solve for Debt → $W_e = (1 - W_d)$

Step 2 → $\sigma_p^2 = (W_d * \sigma_d)^2 + ((1 - W_d) * \sigma_e)^2 + 2 * (1 - W_d) * W_d * COV(r_e, r_d)$

Step 3→ $\sigma_p^2 = (W_d * \sigma_d)^2 + (1 - W_d)^2 * (\sigma_e)^2 + (2W_d - 2W_d^2) * COV(r_e, r_d)$

Step4 →$\frac{d\sigma_p^2}{d\,W_d} = 2W_d * 2\sigma_d^2 + (2 - 2W_d) * (-1) * (\sigma_e)^2 + (2 - 4W_d) * COV(r_e, r_d)$

Step 5→ $0 = 2W_d * 2\sigma_d^2 - (2 + 2W_d) * \sigma_e^2 + (2 - 4W_d) * COV(r_e, r_d)$

Step 6→ $\frac{0}{2} = \frac{2(W_d * \sigma_d^2 - 1 + W_d * \sigma_e^2 + (1 - 2W_d) * COV(r_e, r_d))}{2}$

Step 7→ $0 = W_d * \sigma_d^2 - \sigma_e^2 + \sigma_e^2 W_d + COV(r_e, r_d) - 2W_d COV(r_e, r_d)$

Step 8→ $0 = -\sigma_e^2 + COV(r_e, r_d) + W_d * (\sigma_d^2 + \sigma_e^2 - 2COV(r_e, r_d))$

Step 8→ $\sigma_e^2 - COV(r_e, r_d) = W_d * (\sigma_d^2 + \sigma_e^2 - 2COV(r_e, r_d))$

Step 9 → $W_{MIN_d} = \frac{\sigma_e^2 - COV(r_e, r_d)}{(\sigma_d^2 + \sigma_e^2 - 2COV(r_e, r_d))}$

And to find W_{MIN_e} → $W_{MIN_e} = (1 - W_{MIN_d})$

EXAMPLE)

Givens: $E(r_d) = 8\%$ / $\sigma_d = 12\%$/ $E(r_e) = 13\%$/ $\sigma_e = 20\%$/ $\rho_{d,e} = +0.3$

Weight of the MINIMUM VARIANCE PORTFOLIO?

Required: $E(r_p) = ?$/ $\sigma_p = ?$

Solution:

Step1) Find the W_{MIN_d} & W_{MIN_e}

$$W_{MIN_d} = \frac{\sigma_e^2 - COV_{(r_e, r_d)}}{\sigma_d^2 + \sigma_e^2 - 2COV_{(r_e, r_d)}} = \frac{(0.2)^2 - (0.3 * 0.12 * 0.2)}{(0.12)^2 + (0.2)^2 - 2(0.3 * 0.12 * 0.2)}$$

$$W_{MIN_d} = 0.82$$

$$W_{MIN_e} = 1 - W_{MIN_d}$$

$$W_{MIN_e} = 1 - 0.82 \rightarrow W_{MIN_e} = 0.18$$

Step2) Find $E(r_{MIN_p})$

$$E\left(r_{MIN_p}\right) = \sum E(r\quad) * W_{MIN}$$

$$E\left(r_{MIN_p}\right) = E(r_d) * W_{MIN\,d} + E(r_e) * W_{MIN\,e}$$

$$E\left(r_{MIN_p}\right) = (0.08 * 0.82) + (0.13 * 0.18)$$

$$E\left(r_{MIN_p}\right) = 0.089$$

Step3) Find σ_{MIN_p}

$$\sigma^2_{MIN_p} = W^2_{MIN_d} * \sigma^2_{MIN_d} + W^2_{MIN_e} * \sigma^2_{MIN_e} + 2 * W_{MIN_d} * W_{MIN_e} * COV_{(r_e,r_d)}$$

$$\sigma^2_{MIN_p} = W^2_{MIN_d} * \sigma^2_{MIN_d} + W^2_{MIN_e} * \sigma^2_{MIN_e} + 2 * W_{MIN_d} * W_{MIN_e} * \rho_{(r_e,r_d)} * \sigma_{MIN_d} * \sigma_{MIN_e}$$

$$\sigma^2_{MIN_p} = (0.82)^2 * (0.12)^2 + (0.18)^2 * (0.20)^2 + (2 * 0.82 * 0.18 * 0.3 * 0.12 * 0.2)$$

$$\sigma^2_{MIN_p} = 0.013104$$

$$\sigma_{MIN_p} = 0.1145 \ or\ 11.45\%$$

- Assume that the investment universe has ONLY 2 risky assets, and we are searching for the optimal portfolio.
- Optimal portfolio → Maximization for utility.

  And we know that $U = E(r_p) - \frac{1}{2} * A * \sigma_p^2$

  What are the 2 risky assets? The 2 risky assets are Debt and Equity, remind that, Equity is much riskier than Debt; therefore. We could recognize that the expected return, as well as risk, for the equity is higher than debt.

$$MAX\ U = E(r_p) - \frac{1}{2} * A * \sigma_p^2$$

We know that $E(r_p) = \sum E(r\ \ ) * W$ → $E(r_p) = E(r_d) * W_d + E(r_e) * W_e$

$\sigma_p^2 = W_d^2 * \sigma_d^2 + W_e^2 * \sigma_e^2 + 2 * W_d * W_e * COV_{(r_e,r_d)}$

Apply it to the Utility Equation:

$$MAX\ U = [E(r_d) * W_d + E(r_e) * W_e] - \frac{1}{2} * A * [W_d^2 * \sigma_d^2 + W_e^2 * \sigma_e^2 + 2 * W_d * W_e * COV_{(r_e,r_d)}].$$

We know that $W_d + W_e = 1 \rightarrow W_e = 1 - W_d$; therefore.

$$U = [E(r_d) * W_d + E(r_e) * (1 - W_d)] - \frac{1}{2} * A * [W_d^2 * \sigma_d^2 + (1 - W_d)^2 * \sigma_e^2 + 2 * W_d * (1 - W_d) * COV_{(r_e,r_d)}].$$

$$U = [E(r_d) * W_d + E(r_e) - E(r_e) * W_d] - \frac{1}{2} * A * [W_d^2 * \sigma_d^2 + (1 - 2W_d + W_d^2) * \sigma_e^2 + 2 * W_d * (1 - W_d) * COV_{(r_e,r_d)}].$$

$$U = E(r_d) * W_d + E(r_e) - E(r_e) * W_d - \frac{1}{2} * A[W_d^2 * \sigma_d^2 + \sigma_e^2 - 2W_d * \sigma_e^2 + W_d^2 * \sigma_e^2 + 2W_d * COV_{(r_e,r_d)} - 2W_d^2 COV_{(r_e,r_d)}].$$

$$U = E(r_d) * W_d + E(r_e) - E(r_e) * W_d - \frac{1}{2} * A[W_d^2 * \sigma_d^2 + \sigma_e^2 - 2W_d * \sigma_e^2 + W_d^2 * \sigma_e^2 + 2W_d * COV_{(r_e,r_d)} - 2W_d^2 * COV_{(r_e,r_d)}].$$

$$\frac{d_u}{d_{w_d}} = E(r_d) * 1W_d^0 + 0 - E(r_e) * 1 - \frac{1}{2} * A[2 * W_d^1 * \sigma_d^2 - 2W_d^0 * \sigma_e^2 + 2W_d^1 * \sigma_e^2 + 2W_d^0 * COV_{(r_e,r_d)} - 4W_d^1 COV_{(r_e,r_d)}].$$

$$\frac{d_u}{d_{w_d}} = [E(r_d) - E(r_e)] - \frac{1}{2} * A[2W_d\sigma_d^2 - 2\sigma_e^2 + 2W_d\sigma_e^2 + 2COV_{(r_e,r_d)} - 4W_d COV_{(r_e,r_d)}]$$

Take out “2” as a common factor

$$\frac{d_u}{d_{w_d}} = [E(r_d) - E(r_e)] - A * \frac{1}{\cancel{2}} * \cancel{2}[W_d\sigma_d^2 - \sigma_e^2 + W_d\sigma_e^2 + COV_{(r_e,r_d)} - 2W_d COV_{(r_e,r_d)}]$$

$$\frac{d_u}{d_{w_d}} = [E(r_d) - E(r_e)] - A * [W_d\sigma_d^2 - \sigma_e^2 + W_d\sigma_e^2 + COV_{(r_e,r_d)} - 2W_d COV_{(r_e,r_d)}]$$

Let $\frac{d_u}{d_{w_d}} = 0$ "$Zero$", then solve for W_d

$$0 = [E(r_d) - E(r_e)] - A * [W_d\sigma_d^2 - \sigma_e^2 + W_d\sigma_e^2 + COV_{(r_e,r_d)} - 2W_d COV_{(r_e,r_d)}]$$

$$0 = [E(r_d) - E(r_e)] - AW_d\sigma_d^2 + A\sigma_e^2 - AW_d\sigma_e^2 - ACOV_{(r_e,r_d)} + 2AW_d COV_{(r_e,r_d)}$$

$$AW_d\sigma_d^2 + AW_d\sigma_e^2 - 2AW_d COV_{(r_e,r_d)} = [E(r_d) - E(r_e)] + A\sigma_e^2 - ACOV_{(r_e,r_d)}$$

$$W_d * \left[A\sigma_d^2 + A\sigma_e^2 - 2ACOV_{(r_e,r_d)}\right] = [E(r_d) - E(r_e)] + A\sigma_e^2 - ACOV_{(r_e,r_d)}$$

$$W_d = \frac{[E(r_d) - E(r_e)] + A * [\sigma_e^2 - COV_{(r_e,r_d)}]}{A * \left[\sigma_d^2 + \sigma_e^2 - 2COV_{(r_e,r_d)}\right]}$$

Applying the $COV_{(r_e,r_d)} = \rho_{(r_e,r_d)} * \sigma_d * \sigma_e$

$$W_d = \frac{[E(r_d) - E(r_e)] + A * [\sigma_e^2 - \rho_{(r_e,r_d)} * \sigma_d * \sigma_e]}{A * \left[\sigma_d^2 + \sigma_e^2 - 2 * \rho_{(r_e,r_d)} * \sigma_d * \sigma_e\right]}$$

And $1 - W_d = W_e$

Notice that we calculate the weights for the risky portfolios <u>ONLY</u> there is no risk-free at all.

Therefore, you can calculate the optimal portfolio if you invest in risky assets only.

Example:

Givens: $E(r_d) = 8\%/\ E(r_e) = 13\%/\ \sigma_d = 12\%/\ \sigma_e = 20\%/\ A = 4/\ \rho_{(r_e,r_d)} = +0.3$

Question: Find the optimal portfolio for 2 risky assets Investment UNIVERSE

Required: $W_d = ?/\ W_e = ?/\ E(r_p) = ?/\ \sigma_p = ?$

STEP1) Find W_d & W_e

$$W_d = \frac{[E(r_d) - E(r_e)] + A * [\sigma_e^2 - \rho_{(r_e,r_d)} * \sigma_d * \sigma_e]}{A * [\sigma_d^2 + \sigma_e^2 - 2 * \rho_{(r_e,r_d)} * \sigma_d * \sigma_e]}$$

$$W_d = \frac{[0.08 - 0.13] + 4 * [(0.2)^2 - (0.3 * 0.12 * 0.20)]}{4 * [(0.12)^2 + (0.20)^2 - 2 * 0.3 * 0.12 * 0.2]}$$

$$W_d = \frac{0.0812}{0.16}$$

$$W_d = 0.5075 \rightarrow 50.75\%$$

$$W_e = 1 - W_d \rightarrow W_e = 1 - 0.5075 \rightarrow W_e = 0.4925 \rightarrow W_e = 49.25\%$$

STEP2) Find $E(r_p)$

$$E(r_p) = \sum[E(r\ \ ) * W\ \ ]$$

$$E(r_p) = E(r_d) * W_d + E(r_e) * W_e$$

$$E(r_p) = 0.08 * 0.5075 + 0.13 * 0.4925$$

$$E(r_p) = 0.1046 \rightarrow 10.46\%$$

STEP3) Find σ_p

$$\sigma_p^2 = W_d^2 * \sigma_d^2 + W_e^2 * \sigma_e^2 + 2 * W_d * W_e * \rho_{(r_e,r_d)} * \sigma_d * \sigma_e$$

$$\sigma_p^2 = (0.5075)^2 * (0.12)^2 + (0.4925)^2 * (0.20)^2 + 2 * (0.5075) * (0.4925) * (0.3) * (0.12) * (0.20)$$

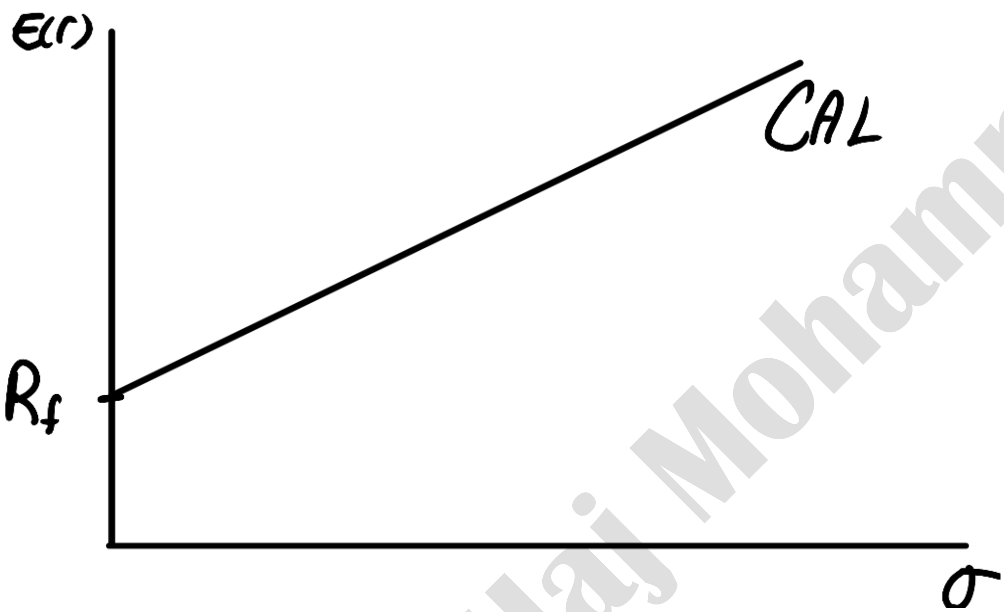
$$\sqrt{\sigma_p^2} = \sqrt{0.01701025}$$

$$\sigma_p = 0.13042 \rightarrow \sigma_p = 13.04\%$$

ASSET ALLOCATION WITH STOCKS, BONDS and T-BILLS:

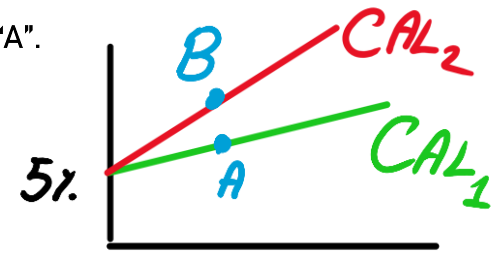
We know the Capital Allocation Line “CAL” equation or Best fit line equation.

$$CAL \rightarrow E(r_c) = R_f + \left(\frac{E(r_p) - R_f}{\sigma_p}\right) * \sigma_c$$



Example Page 215: Two possible CAL are drawn from the risk-free rate (R_f=5%) to 2 feasible portfolios.

→ CAL is drawn through the minimum variance portfolios “A”.



$E(r_d) = 8\%/\ \sigma_d = 12\%/\ E(r_e) = 13\%/\ \sigma_e = 0.20/\rho_{(r_e,r_d)} = 0.3$

STEP1) Find W_d & W_e for both CAL 1 AND CAL 2

$$W_d = \frac{\sigma_e^2 - [\rho_{(r_e,r_d)} * \sigma_d * \sigma_e]}{\sigma_d^2 + \sigma_e^2 - 2 * [\rho_{(r_e,r_d)} * \sigma_d * \sigma_e]}$$

$$W_d = \frac{(0.2)^2 - [0.3 * 0.12 * 0.2]}{(0.12)^2 + (0.2)^2 - 2 * [0.3 * 0.12 * 0.2]}$$

$$W_d = 0.82$$

$$1 - W_d \rightarrow W_e = 1 - 0.82 \rightarrow W_e = 0.18$$

STEP2) Find $E(r_p)$ for CAL 1

$$E(r_p) = E(r_d) * W_d + E(r_e) * W_e$$

$$E(r_p) = (0.08 * 0.82) + (0.13 * 0.18)$$

$$E(r_p) = 0.0234 \rightarrow 2.34\%$$

STEP3) Find σ_p for CAL 1

$$\sigma_p^2 = W_d^2 * \sigma_d^2 + W_e^2 * \sigma_e^2 + 2 * W_d * W_e * \rho_{(r_e,r_d)} * \sigma_d * \sigma_e$$

$$\sigma_p^2 = (0.82)^2 * (0.12)^2 + (0.18)^2 * (0.20)^2 + 2 * (0.82) * (0.18) * (0.3) * (0.12) * (0.20)$$

$$\sigma_p^2 = 0.013104$$

$$\sigma_p = \sqrt{0.013104} \rightarrow \ \sigma_p = 0.1145 \ or \ 11.45\%$$

STEP4) Find the reward-to-volatility ratio "Slope of CAL 1"

$$S_p = \frac{E(r_p) - R_f}{\sigma_p} = \frac{0.0234 - 0.05}{0.1145} = 0.34$$

CAL_2 is drawn through portfolio (B) which invests 70% in bonds and 30% in stocks.

STEP1) Find $E(r_p)$ for CAL 2

$$E(r_p) = E(r_d) * W_d + E(r_e) * W_e$$

$$E(r_p) = (0.08 * 0.70) + (0.13 * 0.30)$$

$$E(r_p) = 0.095 \rightarrow 9.5\%$$

STEP2) Find σ_p for CAL 2

$$\sigma_p^2 = W_d^2 * \sigma_d^2 + W_e^2 * \sigma_e^2 + 2 * W_d * W_e * \rho_{(r_e,r_d)} * \sigma_d * \sigma_e$$

$$\sigma_p^2 = (0.70)^2 * (0.12)^2 + (0.30)^2 * (0.20)^2 + 2 * (0.70) * (0.30) * (0.3) * (0.12) * (0.20)$$

$$\sigma_p^2 = 0.01368$$

$$\sigma_p = \sqrt{0.01368} \rightarrow \ \sigma_p = 0.1170 \ or \ 11.70\%$$

STEP3) Find the reward-to-volatility ratio "Slope of CAL 2"

$$S_p = \frac{E(r_p) - R_f}{\sigma_p} = \frac{0.095 - 0.05}{0.117} = 0.38$$

Question: If I am investing in both risky assets and risk-free assets, and I want to find the optimal portfolio, should I use the investment UNIVERSE equation above?

سؤال: إذا كنت أستثمر في كل من الأصول الخطرة والأصول الخالية من المخاطر، وأريد العثور على المحفظة المثالية، فهل يجب أن أستخدم معادلة INVESTMENT UNIVERSE أعلاه؟

Answer: NO. You can't use this equation because you now invest in risk-free asset. So, we should make another equation.

الإجابة: لا. لا يمكنك استخدام هذه المعادلة لأنك تستثمر الآن في أصول خالية من المخاطر. لذا، ينبغي علينا أن نصنع معادلة أخرى.

$$MAX\ S_P = \frac{E(r_p) - R_f}{\sigma_p}$$

*بغض النظر عن A

Steps to follow to arrive at the complete portfolio

Step1) Specify the return characteristic of all securities (Such as Expected return, Standard deviation, variances, and covariance)

Step2) Establish the risky portfolio

a) Calculate the weights of each risky asset that should be included in the P "risk portfolio". Asset Allocation

*ما رح الخص الاشتقاق لمعادلة Max S_p لانه رح يكون طويل ومعقد جدا.

$$W_d = \frac{E(R_D) * \sigma_e^2 - E(R_E) * COV(R_E, R_D)}{E(R_D) * \sigma_e^2 + E(R_E) * \sigma_d^2 - [E(R_D) + E(R_E)] * COV(R_E, R_D)}$$

Optimal portfolio for ALL INVESTORS.

$$E(R_D) = \left[E(R_d) - R_f\right] \quad , E(R_f) = [E(r_e) - R_f]$$

b) Calculate the properties "Capital Allocation" (Expected return & risk (SD) of P)

c) Allocate the funds between R_f & P

d) Calculate the fraction of the complete portfolio allocated to P & to treasury bills.

e) Calculate the share of the complete portfolio (Expected return and risk).

EXAMPLE)

$$E(r_d) = 8\%, \qquad E(r_e) = 13\%, \qquad \rho_{d,e} = 0.3$$

$$\sigma_d = 12\%, \qquad \sigma_e = 20\%, \qquad R_f = 5\%$$

Calculate optimal complete portfolio "Weights, E(r), Risk (SD)"

$$W_d = \frac{E(R_D) * \sigma_e^2 - E(R_E) * COV(R_E, R_D)}{E(R_D) * \sigma_e^2 + E(R_E) * \sigma_d^2 - [E(R_D) + E(R_E)] * COV(R_E, R_D)}$$

$$W_d = \frac{0.03 * (0.2)^2 - (0.08) * (0.3) * 0.12 * 0.2}{0.03 * (0.2)^2 + (0.08)(0.12)^2 - [0.03 + 0.08] * 0.3 * 0.12 * 0.2}$$

$$W_d = 0.40, \qquad W_e = 1 - 0.40 = 0.60$$

$$E(r_p) = (0.40 * 0.08) + (0.60 * 0.13) = 0.11$$

$$\sigma_P = \sqrt{(0.4)^2(0.12)^2 + (0.2)^2 + (2 * 0.4 * 0.6 * 0.3 * 0.12 * 0.2)}$$

$$\sigma_p = 0.142 \rightarrow 14.2\%$$

$$S_p = \frac{E(r_p) - R_f}{\sigma_p} = \frac{0.11 - 0.05}{0.142} = 0.42$$

$$y^* = \frac{E(r_p) - R_f}{A * \sigma_p^2} = \frac{0.11 - 0.05}{4 * (0.142)^2} = 74.39\%$$

$$WF = 1 - y^* = 1 - 0.7439 = 0.2561 = 25.61\%$$

$$E(r_c) = R_f + y^*[E(r_p) - R_f]$$

$$E(r_c) = 0.05 + 0.7439 * [0.11 - 0.05] = 0.0946 \; or \; 9.46\%$$

$$\sigma_C = ?$$

$$\sigma_c = y^* * \sigma_p = 0.7439 * 0.142 = 0.1056 \; or \; 10.56\%$$

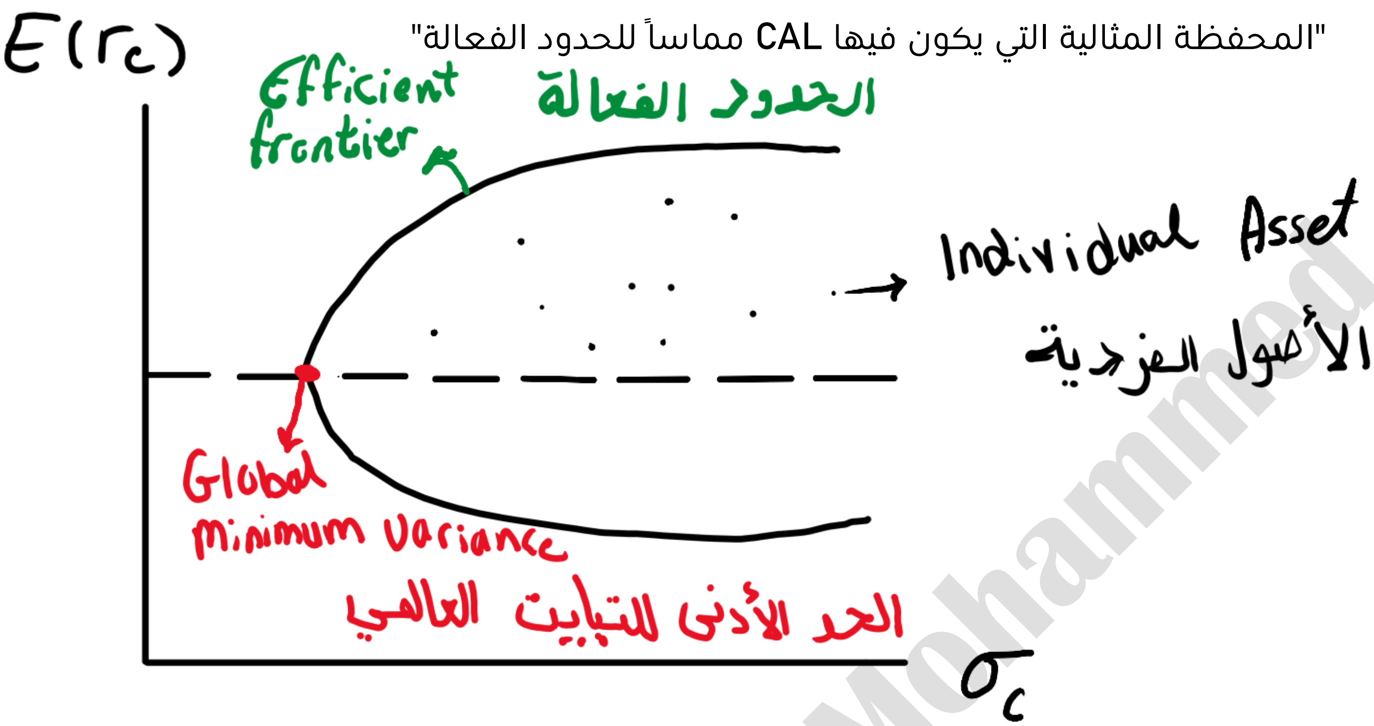
Complete Portfolio	Weight
T-Bills	0.2561
Debt Fund	0.4* 0.7439 = 0.29756
Equity Fund	0.4* 0.7439 = 0.44634

Optimal P "Tangent to efficient frontier"

P→ For a complete portfolio

"Optimal Portfolio at which the CAL is tangent to the efficient frontier"

"المحفظة المثالية التي يكون فيها CAL مماساً للحدود الفعالة"



"Efficient frontier is the part of the frontier that lies above the global minimum variance portfolio"

"الحدود الفعالة هي الجزء من الحدود الذي يقع فوق محفظة التباين الأدنى العالمي"