Varying center distance between gears:

Increasing center distance between gears, results:

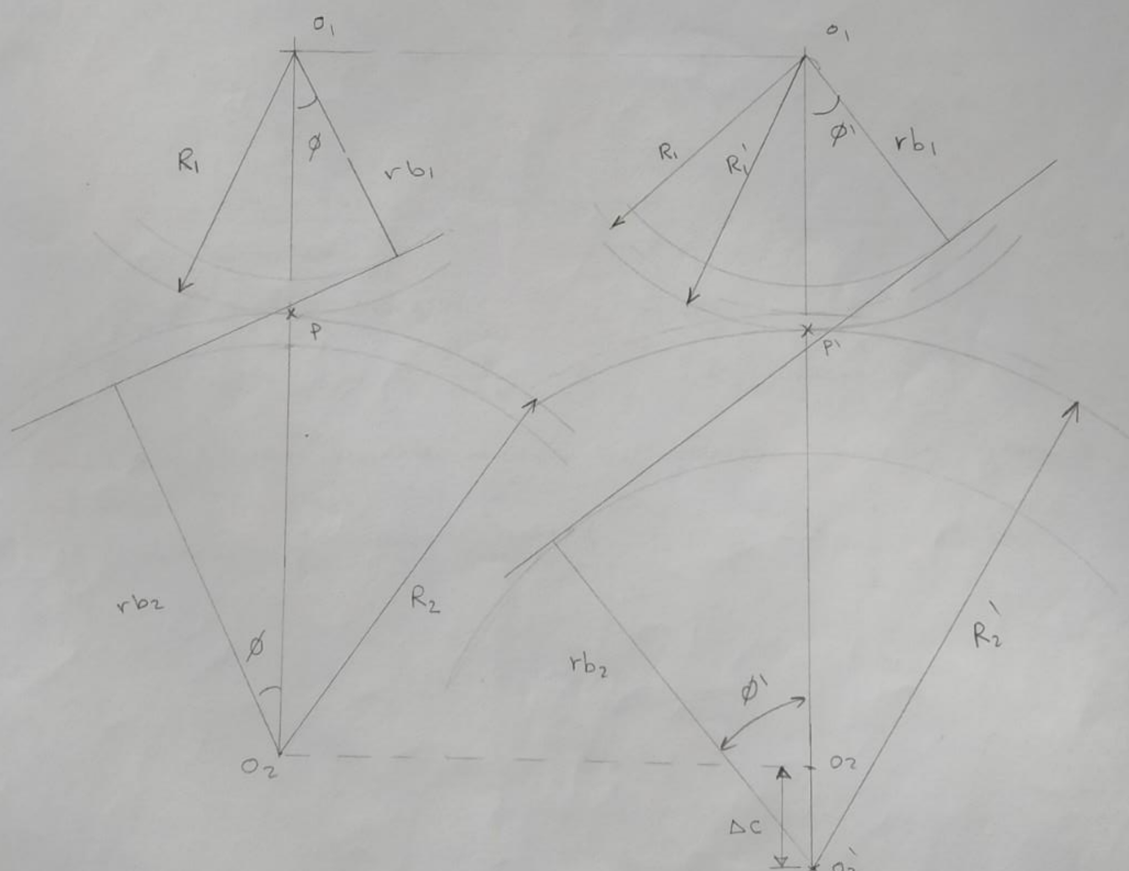
1- Clearance [Backlash] produced between the teeth.

2- Pitch radius increase [pitch radius always tangent to each other]

3- Increase the center distance increases the pressure angle since base circle is constant.

4- Increasing center distance → does not change velocity ratio

5- Increasing center distance reduces the path of contact.



Center distance between gears:

$$C = \frac{d_1 + d_2}{2} = \frac{N_1 + N_2}{2P} = \left(\frac{N_1 + N_2}{2}\right) m$$

$R_1 = \frac{d_1}{2}$

α = pressure angle cutting or standard pressure angle and pitch

...asing center distance to C' by ΔC

$\Delta C = C' - C$

pitch point changes to P'

R_1 and R_2 are not tangent

New pitch radius R_1', R_2' such that

$$\frac{\omega_1}{\omega_2} = \frac{R_2'}{R_1'} = \frac{N_2}{N_1}$$

$$C' = R_1' + R_2'$$

$$R_1' = \left(\frac{N_1}{N_1 + N_2}\right) C' \qquad R_2' = \left(\frac{N_2}{N_1 + N_2}\right) C'$$

ϕ' = operating pressure angle

ϕ = cutting " "

$$\phi' > \phi$$

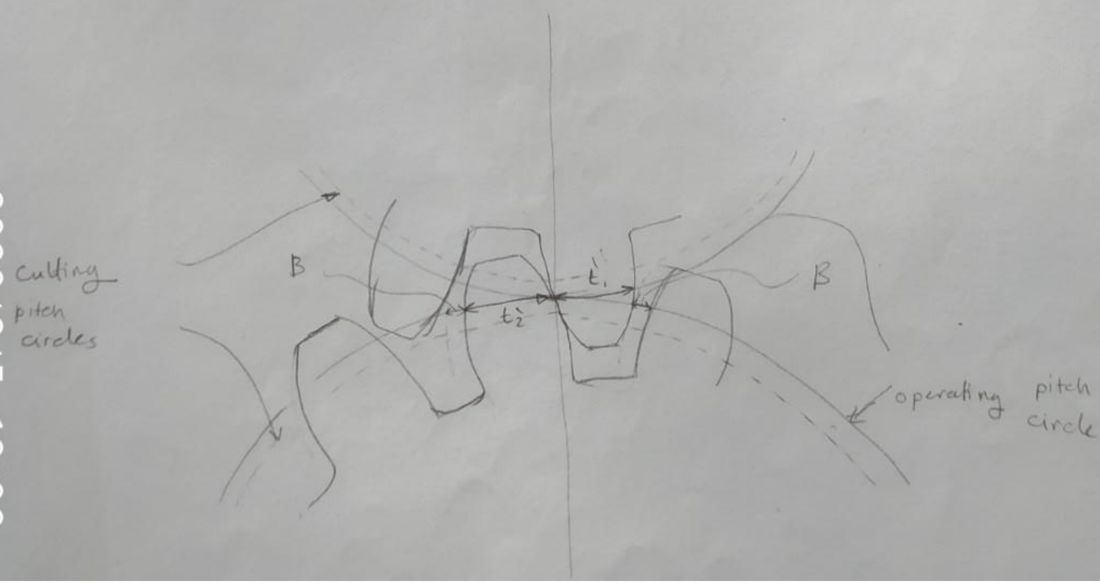
$$C' = \frac{R_{b1} + R_{b2}}{\cos\phi'} = (R_1 + R_2)\frac{\cos\phi}{\cos\phi'} = C\,\frac{\cos\phi}{\cos\phi'}$$

$$\cos\phi' = \frac{C}{C'}\cos\phi$$

$$\Delta C = C' - C = C\,\frac{\cos\phi}{\cos\phi'} - C = C\left[\frac{\cos\phi}{\cos\phi'} - 1\right]$$

when the center distance increase the clearance between gears increases [Backlash]

If direction of rotation reversed lost of motion encountered.



on operating pitch circle:

$$t_1' + t_2' + B = \frac{2\pi R_2'}{N_2}$$

B = Backlash

$$t' = 2R'\left[\frac{t}{2R} + inv\phi - inv\phi'\right]$$

$$inv\phi = \tan\phi - \phi$$

t = thickness of standard gear $\left(t = \frac{P}{2} = \frac{\pi m}{2} = \frac{\pi}{2P}\right)$

R = pitch radius of standard gear

R' = " " of operating gear

$$B = 2c'(inv\phi' - inv\phi)$$

Example:

$m = 3$, $\phi = 20°$ $N_p = 24$, $N_g = 60$ teeth

If center distance increased by 0.5 mm find backlash.

Solution:

$$C = \left(\frac{N_p + N_g}{2}\right)m = 126 \text{ mm.}$$

$$c' = C + \Delta C = 126.5 \text{ mm}$$

$$R_1' = \left(\frac{N_1}{N_1 + N_2}\right)c' = 36.143 \text{ mm}$$

$$R_2' = c' - R_1' = 90.357 \text{ mm.}$$

$$\cos\phi' = \frac{c}{c'}\cos\phi = 20.61°$$

$$B = 2c'(inv\phi' - inv\phi)$$

$$= 2 \times 126.5(0.01636 - 0.0149) = 0.3689 \text{ mm.}$$

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Gear - tooth Bending Stress

lewis formula:

Basic assumption.

1- The full load is applied at tip of single tooth.

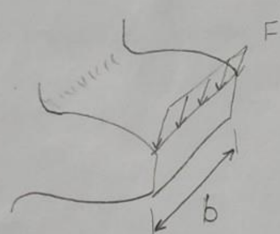
(for precision gears load is carried by more tooths)

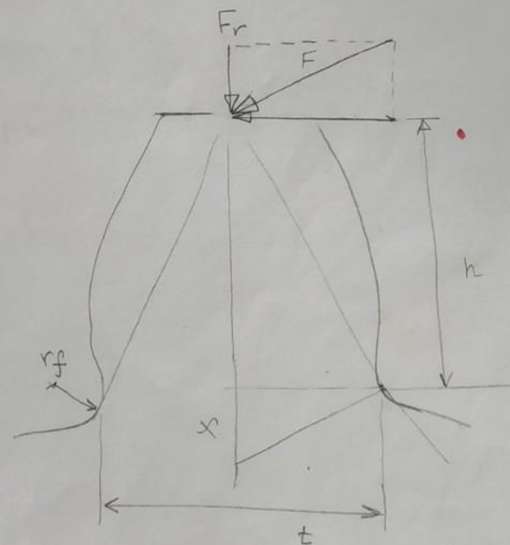
2- Radial component is negligible.

$F_r \rightarrow$ produces compressive stress opposite to tensile stress produced by F_t.

3- load is distributed uniformly across face width.

4- Stress concentration was neglected by lewis





Bending formula:

$$\sigma = \frac{Mc}{I} = \frac{6 F_t h}{b t^2} \quad \text{—(a)}$$

By similar triangle

$$\frac{t/2}{x} = \frac{h}{t/2} \quad , \text{ or } \quad \frac{t^2}{h} = 4x \quad \text{—(b)}$$

Subst. (b) into (a) :

$$\Rightarrow \sigma = \frac{6 F_t}{4 b x}$$

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Define lewis form factor Y : $Y = \frac{2x}{\ldots}$ —(c)

Subst. (d) in (c):

$$\sigma = \frac{F_t}{b p y} \quad \text{[Lewis equation].}$$

Gears are made to standard value of (P).

$$p = \frac{\pi}{P} \quad , \quad y = \frac{Y}{\pi}$$

$$\Rightarrow \sigma = \frac{F_t P}{b Y} \quad \text{— English units}$$

$$\sigma = \frac{F_t}{m b Y} \quad \text{— SI Units.}$$

Y = Lewis form factor function of P.

Y, y function of tooth shape

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Gear-tooth bending stress:

The factors that affect gear-tooth bending stress

1- Pitch-line velocity: The greater V → the greater the impact loading

V ↑ → Increases the no. of teeth that come in contact. (teeth can never be made with absolute perfection).

2- Manufacturing accuracy: affect the contact ratio → Impact loading.

3- Contact ratio: precision gears → [1 < C.R. < 2]

The load transmitted is divided among two pairs of teeth.

⇒ Two condition occurs:

a- Half the load is applied at the tip of single tooth.

b- full load is carried at tip of single tooth.

4- Stress concentration: at the base of the tooth.

5- Degree of shock loading [load factor].

6- Accuracy and rigidity of mounting.

7- Moment of inertia of gears and attached rotating member.

AGMA: American Gear Manufacturing Ass.

Suggest a modification factors to be applied to lewis formula In order to take these factors into account.

AGMA Stress formula:

$$\sigma = \frac{F_t P}{b J K_v} K_o K_m \quad [\text{U.S. Units}], \quad \sigma = \frac{F_t}{b J m} \frac{K_o K_m}{K_v} \quad [\text{SI Units}]$$

b = face width of the tooth

J = Spur gear geometry factor

Include Lewis form factor (Y) and stress concentration factor based on fillet radius $r_f = \frac{0.35}{P}$ $\quad J = \frac{Y}{K_f m_N}$

J → depends on number of teeth in mating gears → controls C.R.

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$J = \dfrac{Y}{K_f \, m_N}$, m_N = load sharing ratio. = portion of total load carried by the most heavily loaded tooth.

K_v = velocity or dynamic factor → Indicates the severity of impact loading.

$K_v = f$ (V = pitch line velocity , manufacturing accuracy).

$$K_v = \left(\frac{A}{A+\sqrt{V}}\right)^B \quad , \; V = ft/min.$$

$$K_v = \left(\frac{A}{A+\sqrt{200\,V}}\right)^B \quad , \; V = m/s.$$

Where: $A = 50 + 56(1-B)$, Q_v = AGMA quality no.

$$B = \frac{(12 - Q_v)^{2/3}}{4}$$

$Q_v = 3-7$ → comertial - quality gears

$Q_v = 8-12$ → precision - quality.

Dynamic factor (K_v) accounts for.

- Error in tooth spacing.
- Vibration of tooth due to tooth stiffness.
- Pitch line velocity , Inertia.
- Wear and deformation of contacting portions of teeth.
- Gear shaft mis alignment.
- Tooth friction.

Over load factor (K_o): Fig.(14-7)

Reflect the degree of: non-unifomity of driving and load torques.

Driver (Source of power)	Driven Machine.		
	Uniform	Moderate shock	Heavy shock
Uniform.	1.0	1.25	1.75
Light shock	1.25	1.5	2.0
Medium shock	1.5	1.75	2.25

Load distribution factor:

Reflect degree of accuracy of mating gears alignment

K_m affect load distribution, and accounts for misalignment and deflection of rotational axis.

K_m is applied under the following conditions:

1- $\frac{b}{d} \leq 2"$

2- Gears mounted between Brgs.

3- $b \leq 40"$

4- Contact when load across full width of narrowest member.

$$K_m = 1 + C_{mc}\left[C_{pf}\, C_{pm} + C_{ma}\, C_e\right]$$

$$C_{mc} = \begin{cases} 1, & \text{uncrowned teeth} \\ 0.8, & \text{for crowned teeth.} \end{cases}$$

$$C_{pf} = \begin{cases} \frac{b}{10d} - 0.025 & b \leq 1" \\ \frac{b}{10d} - 0.0375 + 0.0125b & 1 \leq b \leq 17" \\ \frac{b}{10d} - 0.1109 + 0.0207\, b - 0.000228\, b^2, & 17 < b \leq 40" \end{cases}$$

$$\frac{b}{10d} = 0.05 \quad \text{for} \quad \frac{b}{10d} < 0.05$$

$$C_{pm} = \begin{cases} 1, & \text{for straddle-mounted pinion with } \frac{S_1}{S} < 0.175 \\ 1.1, & \text{" " " " } \frac{S_1}{S} \geq 0.175 \end{cases}$$

$$C_e = \begin{cases} 0.8, & \text{for gearing adjusted at assembly, or Compatibility is improved by lapping, or both} \\ 1, & \text{for all other conditions.} \end{cases}$$

$$C_{ma} = A + B\, b + C\, b^2$$

Fig. [14-11] C_{ma} Vs b, Table [14-9] A, B, C

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