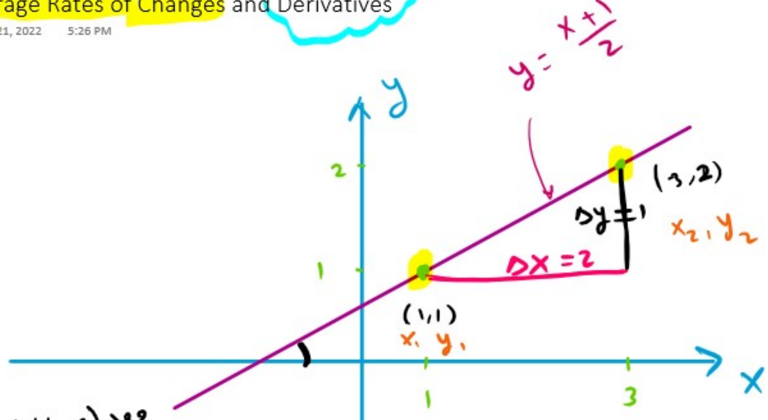


$m$ (الميل) $= \text{slope} = \dfrac{\Delta y}{\Delta x}$

$= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{2-1}{3-1}$

$= \dfrac{1}{2} > 0$

معادلة الخط المستقيم

$$y - y_1 = m(x - x_1)$$

$$y - \boxed{1} = \boxed{\tfrac{1}{2}}(x - \boxed{1})$$

$$\cancel{y - 1} = \tfrac{1}{2}x - \tfrac{1}{2}$$

$$+1 \qquad\qquad +1$$

$$\boxed{y = \frac{x}{2} + \frac{1}{2}} = \frac{x+1}{2}$$

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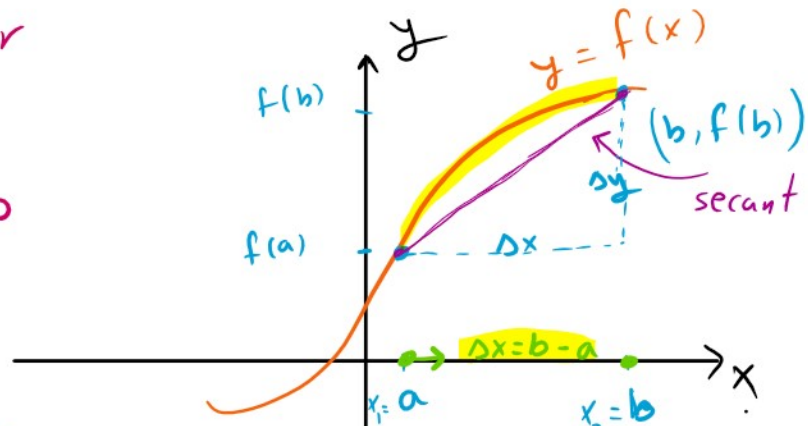
Average Rate of Change for
(معدل التغير)

$f(x)$ from $x = a$ to $x = b$

is defined by




Av. Rate Change $= \dfrac{\Delta y}{\Delta x}$ ✓

$= \dfrac{y_2 - y_1}{\quad}$

$y = f(x)$

$y_2 = f(x_2) = f(b)$

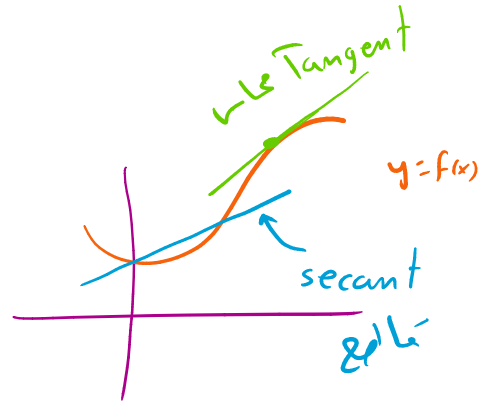
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(b) - f(a)}{b - a}$$

$y = f(x)$

$y_2 = f(x_2) = f(b)$

$y_1 = f(x_1) = f(a)$




Exp Find the average rate of change for the function $f(x) = \sqrt{3x+1}$ from $x = 0$ to $x = 5$

$$\text{av. rate of change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(5) - f(0)}{5 - 0}$$

$$= \frac{\sqrt{3\boxed{5}+1} - \sqrt{3\boxed{0}+1}}{5}$$

$$= \frac{\sqrt{16} - \sqrt{1}}{5}$$

$$= \frac{4 - 1}{5}$$

$$= \frac{3}{5}$$

---

Exp Assume the profit function is given by $P(x) = x^2 + 3x + 10$. Find average rate of change for the profite if $x$ changes from $x = 2$ to $x = 5$

$$\text{av. rate of change} = \frac{\Delta P}{\Delta X} = \frac{P(x_2) - P(x_1)}{x_2 - x_1} = \frac{P(5) - P(2)}{5 - 2}$$

$$= \frac{(\boxed{5}^2 + 3\boxed{5} + 10) - (\boxed{2}^2 + 3\boxed{2} + 10)}{3}$$

$$= \frac{(25 + 15 + 10) - (4 + 6 + 10)}{3}$$

$$= \frac{50 - 20}{3} = \frac{30}{3} = 10$$

---

Exp Assume cost function is given

$$C(x) = x^2 + \overset{5}{c}\,x + 100$$

If the average rate of change is 35 when $x$ changes from $x = 10$ to $x = 20$. Find the constant $\boxed{c}$

$$\text{av. rate of change} = \frac{\Delta C}{\Delta x} = \frac{C(x_2) - C(x_1)}{x_2 - x_1}$$

$$35 = \frac{C(20) - C(10)}{20 - 10}$$

$$35 = \frac{\left[\boxed{20}^2 + c\boxed{20} + 100\right] - \left[\boxed{10}^2 + c\boxed{10} + 100\right]}{10}$$

$$35 = \frac{400 + 20c \cancel{+100} - 100 - 10c \cancel{-100}}{10}$$

$$\frac{35}{1} = \frac{300 + 10c}{10}$$

$$\cancel{300} + 10c = 350$$
$$\cancel{-300} \qquad\qquad -300$$

$$\frac{\cancel{10}c}{\cancel{10}} = \frac{50}{10}$$

$$\boxed{c = 5}$$

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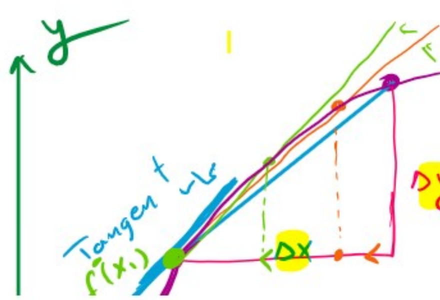
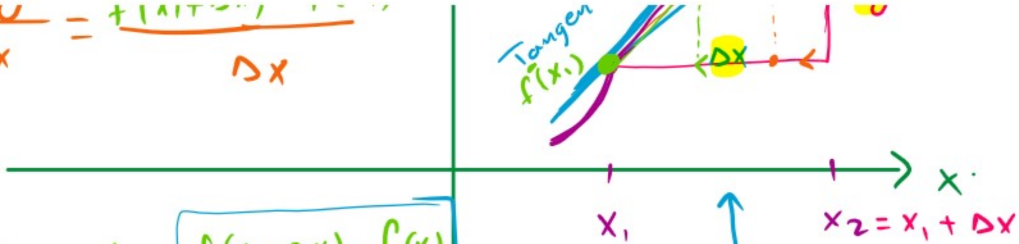
$$\text{av. rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$




$$\Delta y = y_2 - y_1 = f(x_2) - f(x_1)$$
$$= f(x_1 + \Delta x) - f(x_1)$$

$$= \frac{\quad}{\Delta x} = \frac{f(x_1+\Delta x) - f(x_1)}{\Delta x}$$




$= f(x_1+\Delta x) - f(x_1)$

$$f'(x_1) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \boxed{\frac{f(x_1+\Delta x) - f(x_1)}{\Delta x}}$$ ✓

$\Delta x = x_2 - x_1$

$\boxed{\Delta x + x_1 = x_2}$

Replace $\Delta x = h$

$$f'(x_1) = \frac{dy}{dx}\bigg|_{x=x_1} = \lim_{h \to 0} \frac{f(x_1+h) - f(x_1)}{h}$$ ✓

$1^{st}$ Derivative $= \lim_{\Delta x \to 0}$ (av. rate of change)

↳ if it exist

= slope for the tangent line

= instantinvous rate of change

$$= y' = \frac{dy}{dx} = f'(x)$$

$\frac{dv}{dt} = v' = D'(t)$ : instantenous velocity

$\frac{dP}{dx} = P' = \overline{MP}$ : margical profit

$\frac{dc}{dx} = C' = \overline{MC}$ : marginal cost

$\frac{dR}{dx} = R'(x) = \overline{MR}$ : marginal revenue

$\Delta P = P'$

- av. rate

$\frac{dH}{dx}$

$\frac{dP}{dq} = P'$

$$f'(c) = \lim_{h \to 0} \boxed{\frac{f(c+h) - f(c)}{h}}$$

av. rate of change

If limit exists then f is differentiable at x = c

قابل للإشتقاق

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Exp Find the slope of $f(x) = 3x + 4$ at $x = 1$

$$\text{slope} = m = f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

slope of tangent line

$$= \lim_{h \to 0} \frac{\left(3\boxed{1+h} + 4\right) - \left(3\boxed{1} + 4\right)}{h}$$

$$= \lim_{h \to 0} \frac{(3 + 3h + 4) - (3 + 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{7} + 3h - \cancel{7}}{h}$$

$$= \lim_{h \to 0} \qquad\qquad h$$

$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 3$$

$$= \boxed{3}$$

$$f(x) = 7x + 1 \Rightarrow f' = 7$$