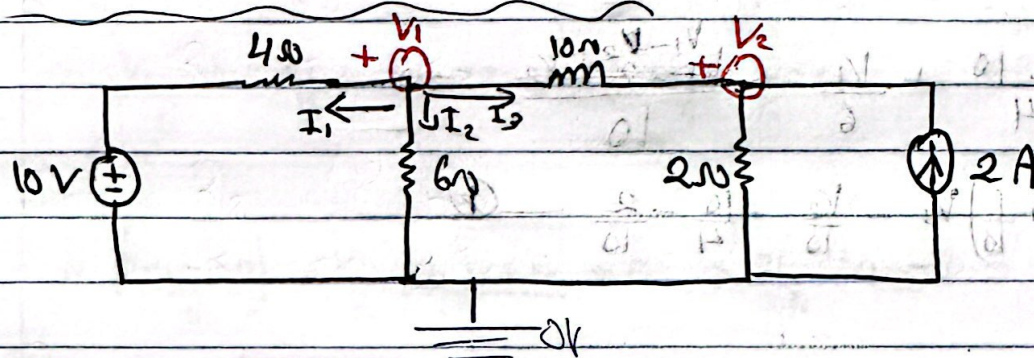


Chapter 4:

→ Nodal Method

- Node Voltage Method



steps

- ① reference node, ground 1
- ② node voltages V_1, V_2, \dots, V_n essential nodes only
- ③ at each node, write KCL eq's in terms of node voltages V_1, V_2, \dots

$$\sum I_{in} = 0$$

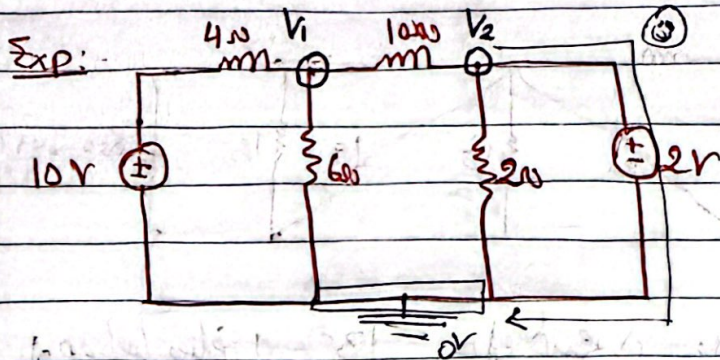
$$\boxed{\sum I_{out} = 0}$$

at V_1 $I_1 + I_2 + I_3 = 0$

$$\frac{V_1 - 10}{4} + \frac{V_1}{6} + \frac{V_1 - V_2}{10} = 0$$

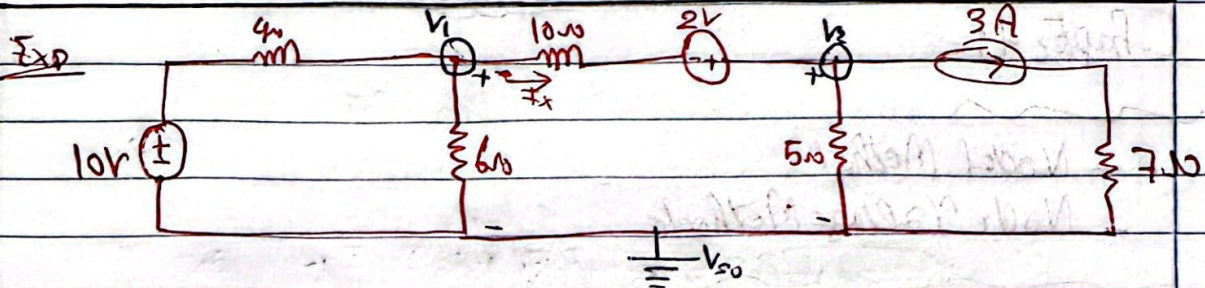
at V_2 $\frac{V_2 - V_1}{10} + \frac{V_2}{2} + 2 = 0$ ②

$$\Rightarrow \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{10} \right) V_1 - \frac{1}{10} V_2 = \frac{10}{4}$$



Sol: $V_2 = 2V$

① $V_1: \frac{V_1 - 10}{4} + \frac{V_1}{6} + \frac{V_1 - 2}{10} = 0$

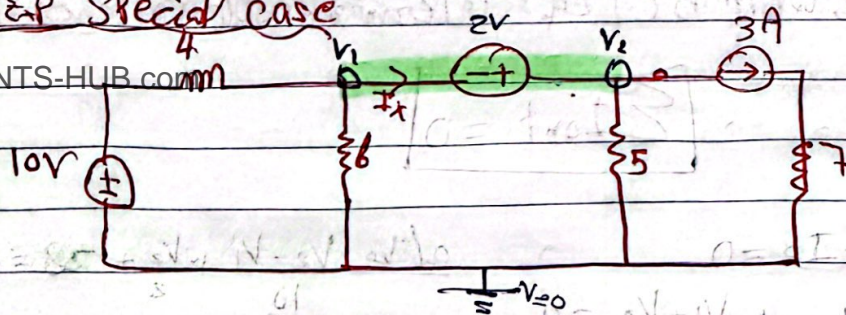


① $V_1: \frac{V_1 - 10}{4} + \frac{V_1}{6} + \frac{V_1 - V_2 + 2}{10} = 0$

$\left(\frac{1}{4} + \frac{1}{6} + \frac{1}{10}\right)V_1 - \frac{V_2}{10} = \frac{10}{4} - \frac{2}{10}$ — (1)

② $V_2: \frac{V_2 - V_1 + 2}{10} + \frac{V_2}{5} + 3 = 0$ — (2)

Exp Special case

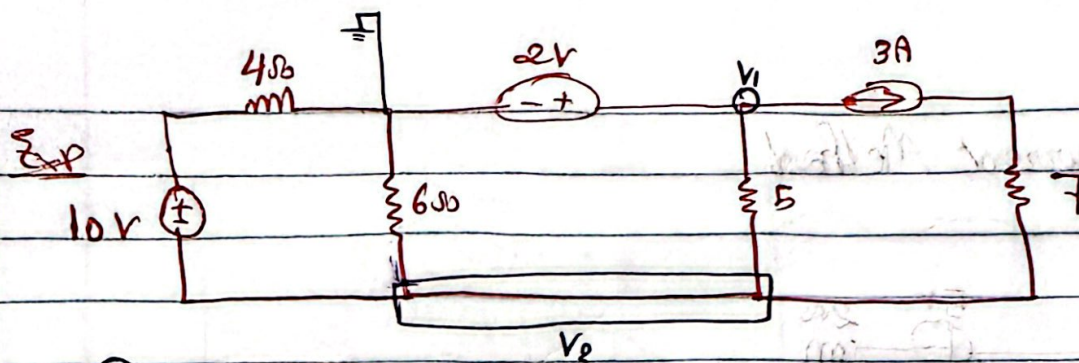


$V_2 - V_1 = 2V$

Special case
SUPER Node

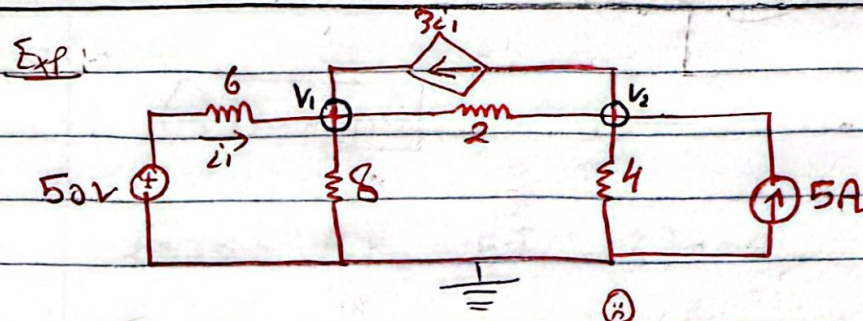
① $V_1: \frac{V_1 - 10}{4} + \frac{V_1}{6} + \frac{V_1 - V_2 + 2}{10} = 0$

$\Rightarrow \frac{V_1 - 10}{4} + \frac{V_1}{6} + I_x = 0 \Rightarrow \frac{V_1 - 10}{4} + \frac{V_1}{6} + \left(\frac{V_2}{5} + 3\right) = 0$



Exp: $V_1 = 2V$

① V_2 : $\frac{V_2 + 10}{4} + \frac{V_2}{6} + \frac{V_2 - 2}{5} - 3 = 0$



② $\frac{V_1 - 50}{6} + \frac{V_1}{8} + \frac{V_1 - V_2}{2} - 3i_1 = 0$, But $i_1 = \frac{50 - V_1}{6}$

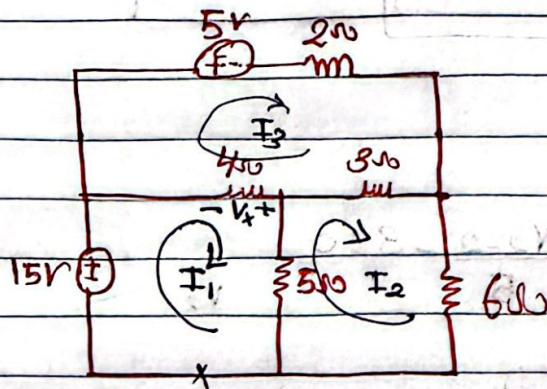
$\Rightarrow \frac{V_1 - 50}{6} + \frac{V_1}{8} + \frac{V_1 - V_2}{2} - 3\left(\frac{50 - V_1}{6}\right) = 0$

$4\left(\frac{V_1 - 50}{6}\right) + \frac{V_1}{8} + \frac{V_1 - V_2}{2} = 0 \Rightarrow \frac{31}{24}V_1 - 0.5V_2 = \frac{100}{3}$ - (1)

③ V_2 : $-5 + \frac{V_2}{4} + \frac{V_2 - V_1}{2} + 3\left(\frac{50 - V_1}{6}\right) = 0$

$-V_1 + \frac{3}{4}V_2 = -20$ - (2)

* Mesh current Method



mesh current

↳ all CW/CCW.

↳ write KVL eq's.

@ I_1

$$\rightarrow -15 + 4(I_1 - I_3) + 5(I_1 - I_2) = 0$$

$$\rightarrow \boxed{9I_1 - 5I_2 - 4I_3 = 15} \quad (1)$$

@ I_2

$$\rightarrow \boxed{-5I_1 + (5+6+3)I_2 - 3I_3 = 0} \quad (2)$$

@ I_3

$$\rightarrow \boxed{-4I_1 - 3I_2 + (2+3+4)I_3 = -5} \quad (3)$$

or

$$\rightarrow 5 + 2I_3 + 3(I_3 - I_2) + 4(I_3 - I_1) = 0$$

$$I_1 \quad I_2 \quad I_3$$

$$\rightarrow V_x = 4(I_3 - I_1)$$

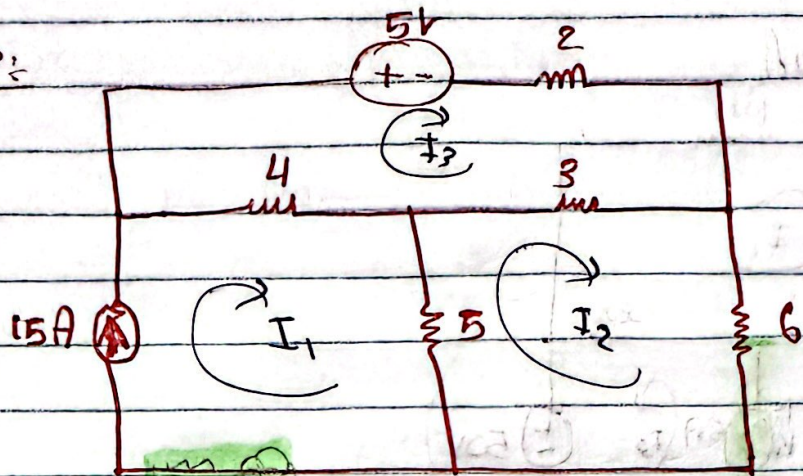
$$I_1 = 3.359 \text{ A}$$

$$I_2 = 1.589 \text{ A}$$

$$I_3 = 1.820 \text{ A}$$

$$V_x = 6.156 \text{ V}$$

Exp:



③ $I_1 = 15A$

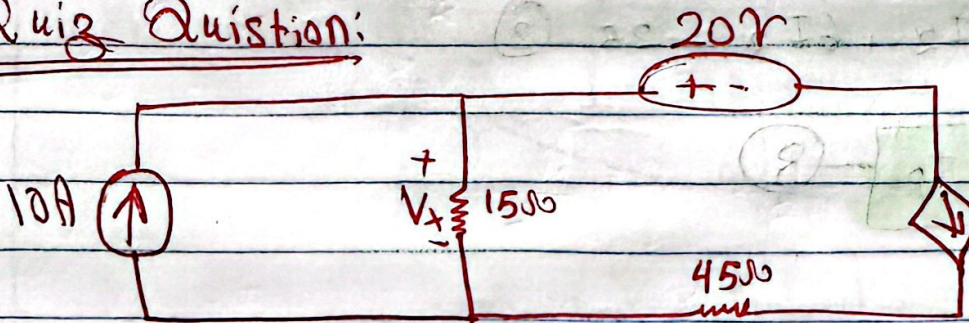
① $4I_2 - 5I_1 - 3I_3 = 0$ — (1)

② $9I_3 - 4I_1 - 3I_2 = -5$ — (2)

I_2 ✓

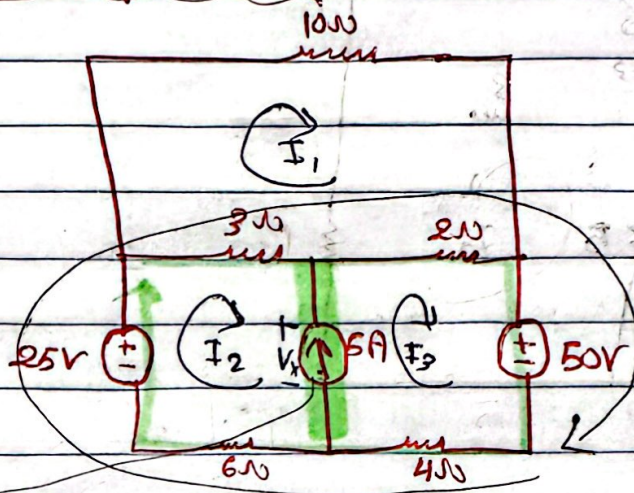
I_2 ✓

Quiz Question:



Find:
 P_{10A} , P_{20V}
 P_{150}

Ex Page 125 Super mesh



$$\textcircled{1} I_1 \quad 15I_1 - 3I_2 - 2I_3 = 0 \quad \text{--- (1)}$$

$$\textcircled{2} I_2 \quad -25 + 3(I_2 - I_1) - 5 + 6I_2 = 0 \quad \text{X}$$

$V \quad + \quad V \quad - \quad V \quad + \quad V$

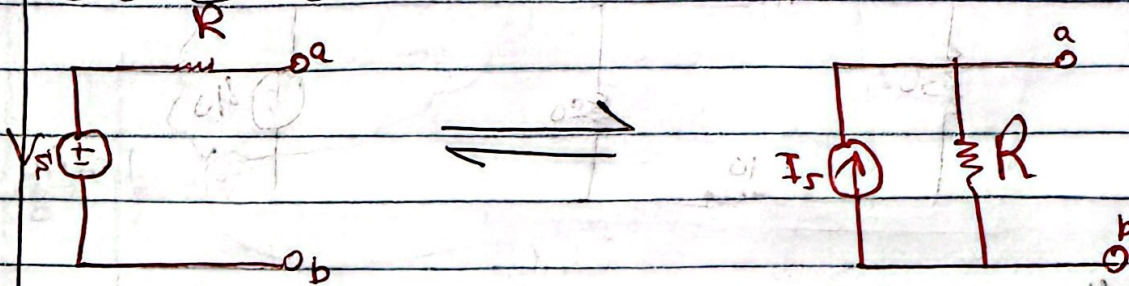
$$\textcircled{3} I_3 \quad 50 + 4I_3 - V_x + 2(I_3 - I_1) = 0 \quad \text{X}$$

$$\rightarrow -25 + 3(I_2 - I_1) + 2(I_3 - I_1) + 50 + 4I_3 + 6I_2 = 0$$

$$\rightarrow -5I_1 + 9I_2 + 6I_3 = -25 \quad \text{--- (2)}$$

$$\rightarrow 5 = I_3 - I_2 \quad \text{--- (3)}$$

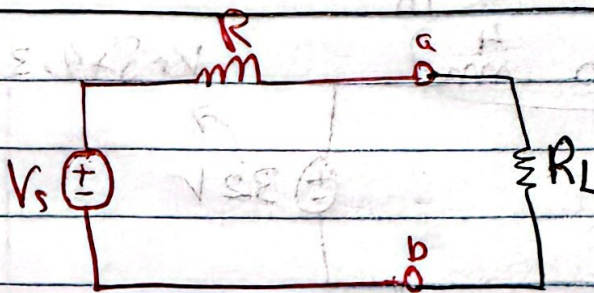
Source Transformation



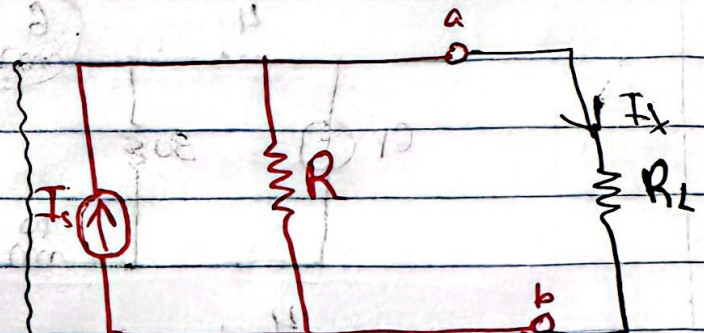
$$V_{o.c} = V_{a,b} = V_s$$

$$V_{a,b} = R I_s$$

$$V_s = R I_s$$



$$V_{RL} = V_{a,b} = \frac{R_L}{R_L + R} \cdot V_s$$

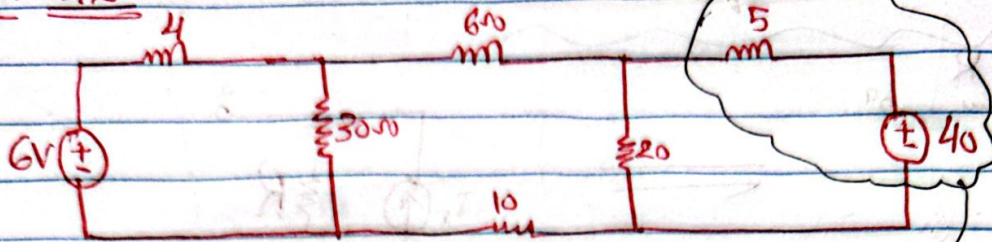


$$V_{RL} = V_{a,b} = R_L I_x$$

$$= R_L \left(\frac{R}{R + R_L} \cdot I_s \right)$$

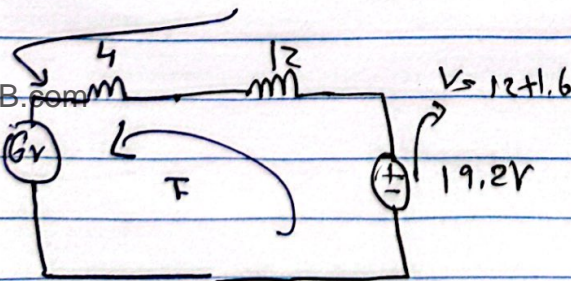
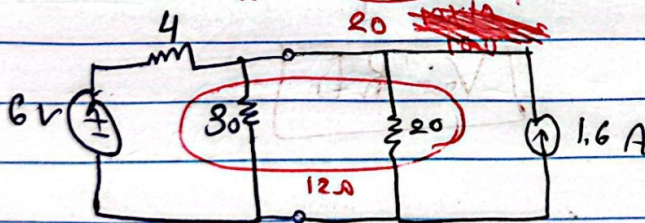
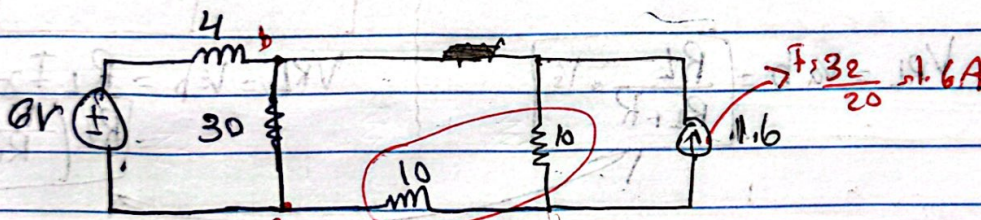
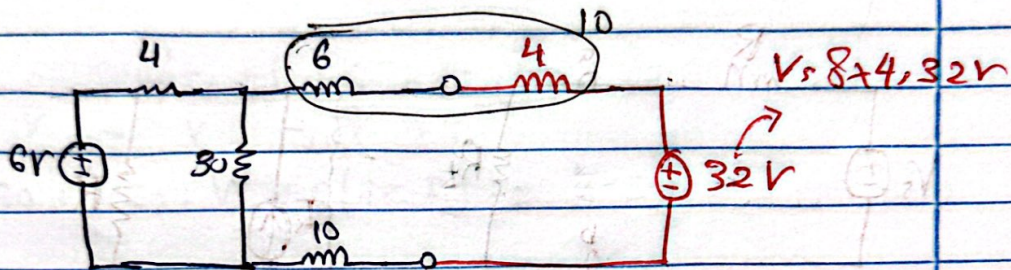
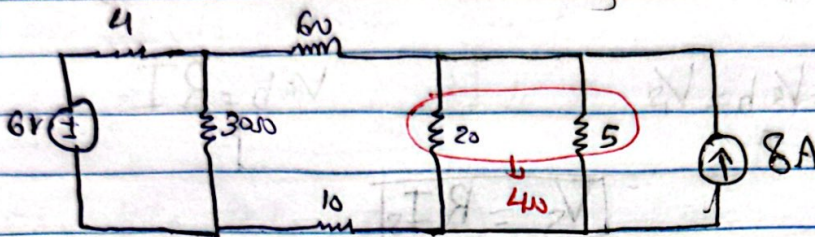
$$V_s = R I_s$$

Expt 48



$$I = \frac{40}{5} = 8A$$

Power



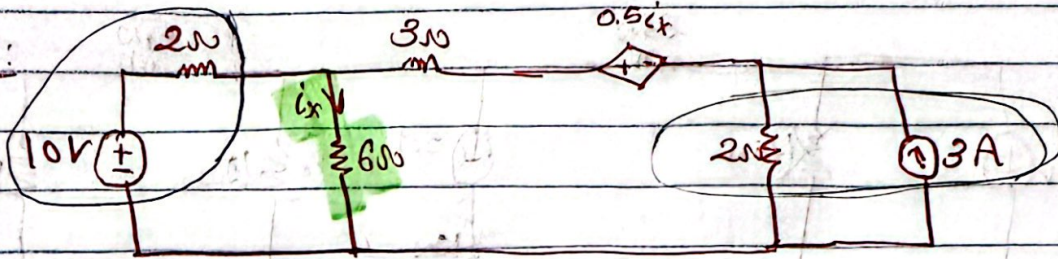
$$\therefore I = \frac{19.2 - 6}{4 + 12} = 0.825A$$

$$\rightarrow P = IV = 6 \times 0.825$$

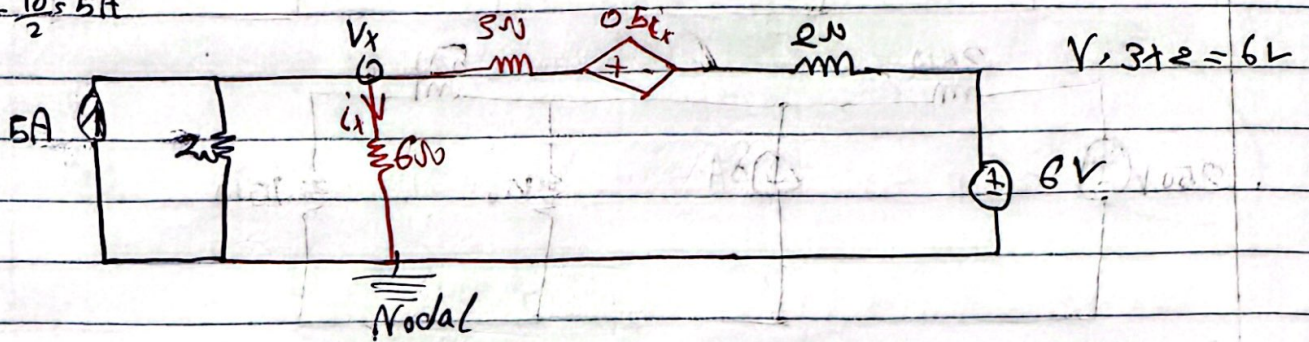
$$= 4.95W (approx.)$$

Handwritten text: Handwritten text

Exp:

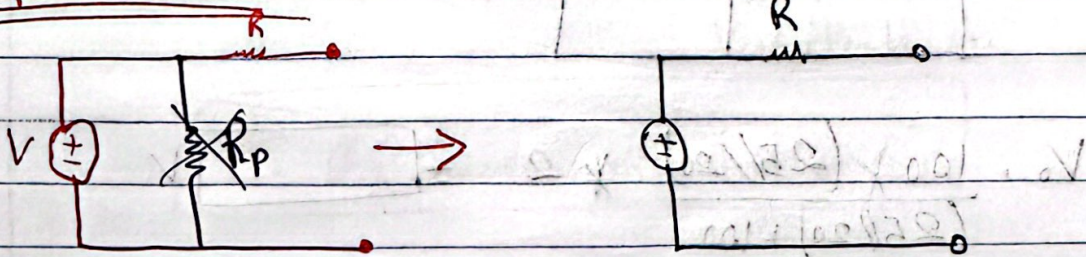


$\Delta I = \frac{10}{2} = 5A$



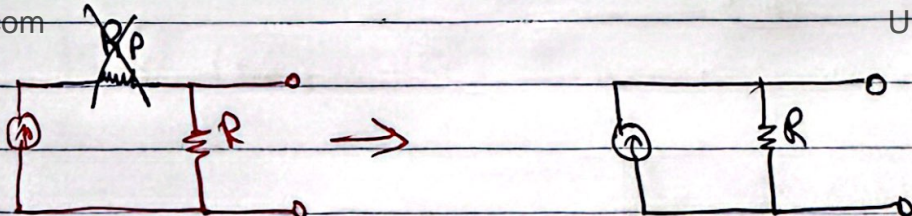
$V_{3+2} = 6V$

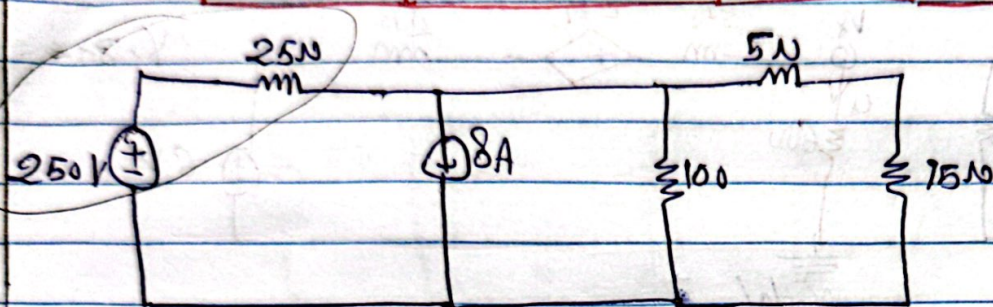
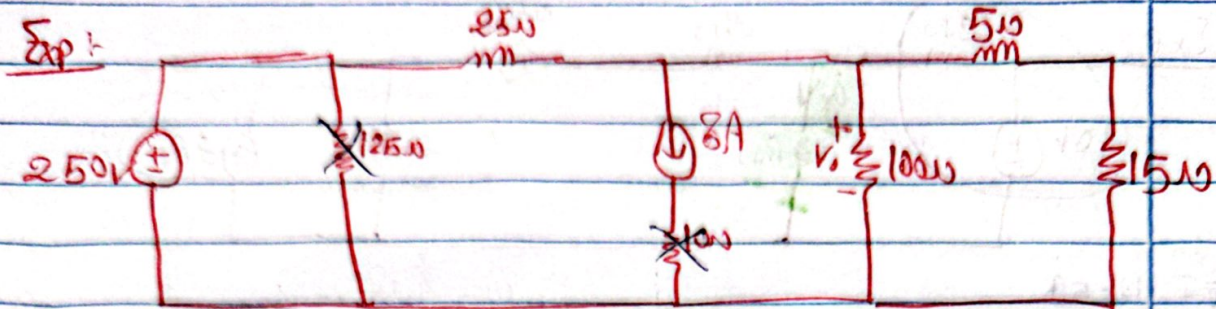
Special case



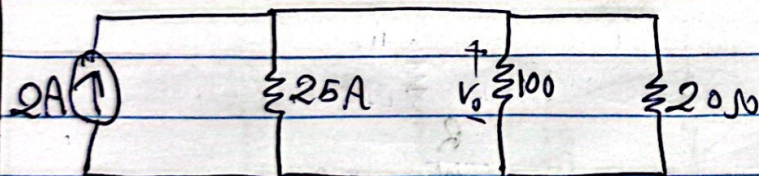
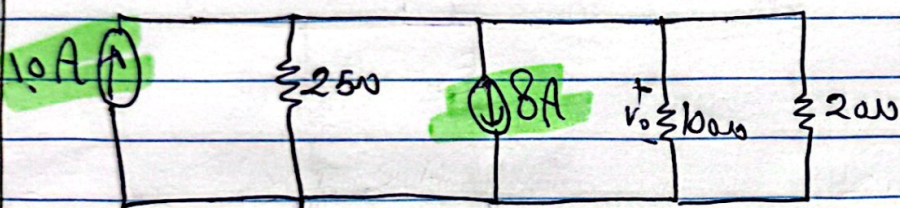
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$$A = \frac{250}{25} = 10A$$



$$V_0 = \frac{100 \times (25/20)}{(25/20) + 100} \times 2 = \boxed{9.4} V$$

* superposition theorem :-

Linear circuit

↳ you can calculate the voltage or current by finding the contribution of each individual indep. source acting alone.

↳ dep. source \rightarrow are left intact

Steps:-

Page 101/Notes

1. Kill all indep. sources ~~except~~ one indep. source
find the output V_1
 I_1

2. Repeat step one for each other indep sources V_2 V_3
 I_2 I_3

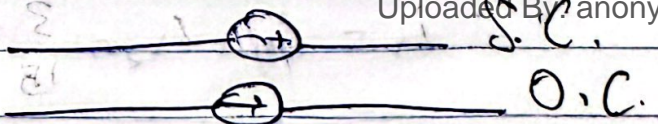
3. find the total contribution by ~~adding~~ adding:

$$I_{tot} = I_1 + I_2 + I_3 + \dots$$

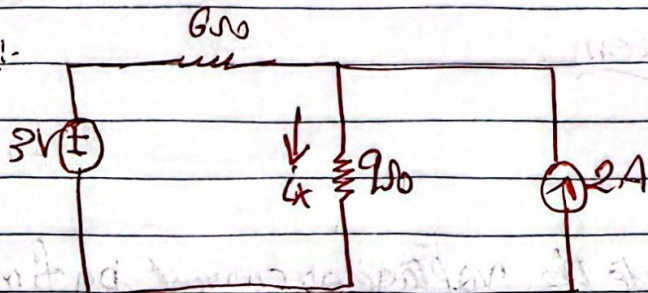
$$V_{tot} = V_1 + V_2 + V_3 + \dots$$

! dep. sources are left intact

Kill indep sources



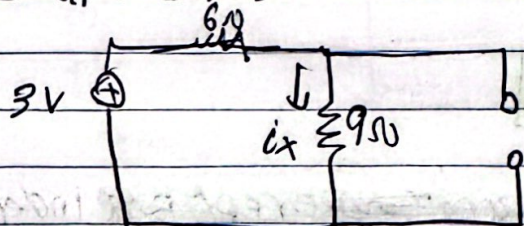
Exp:-



Find i_x

① i_{x1} due to 3V

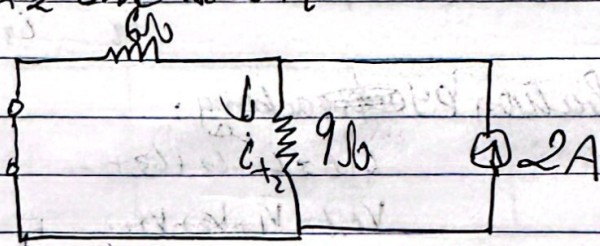
kill 2A



$$i_{x1} = \frac{3}{15} \text{ A}$$

② i_{x2} due to 2A

kill 3V



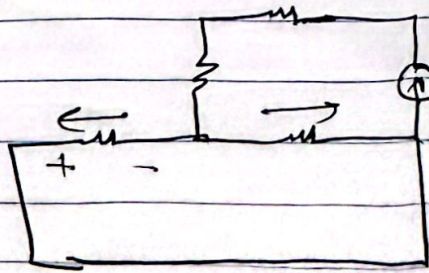
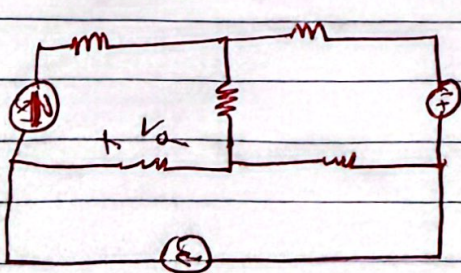
$$i_{x2} = \frac{6}{6+9} \times 2 = \frac{12}{15} \text{ A}$$

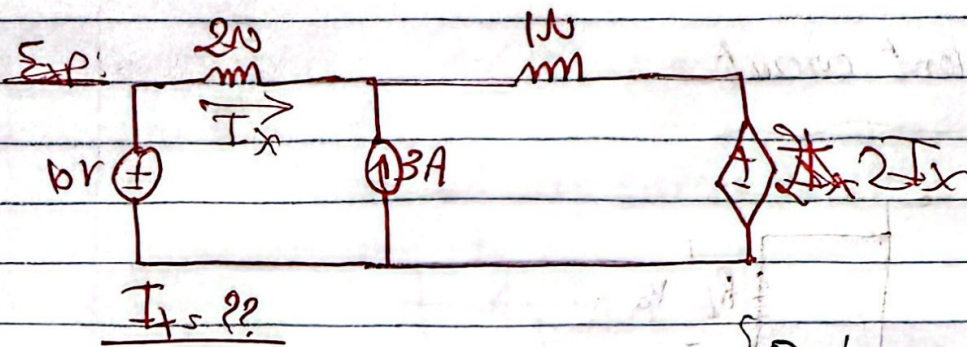
$$\therefore i_x = i_{x1} + i_{x2} = \frac{3}{15} + \frac{12}{15} = 1 \text{ A}$$

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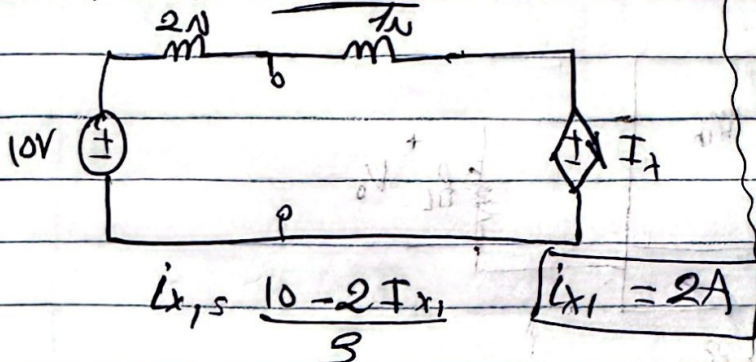
Problem 96

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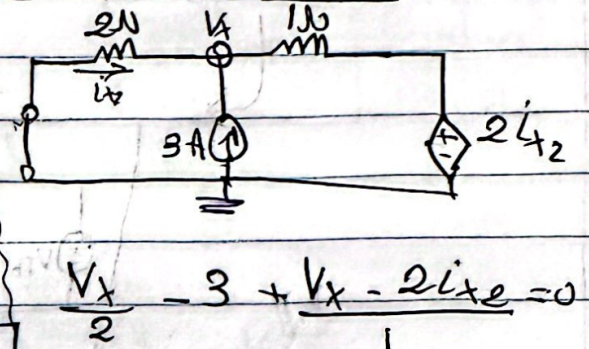




① i_{x1} due to 10V



② i_{x2} due to 3A



But $i_{x2} = \frac{V_x}{2}$

$\rightarrow \frac{V_x}{2} - 3 - \frac{V_x - 2 \cdot \frac{V_x}{2}}{1} = 0$

$\rightarrow \frac{V_x}{2} - 3 - \frac{V_x - V_x}{1} = 0$

$\rightarrow \frac{V_x}{2} - 3 = 0$

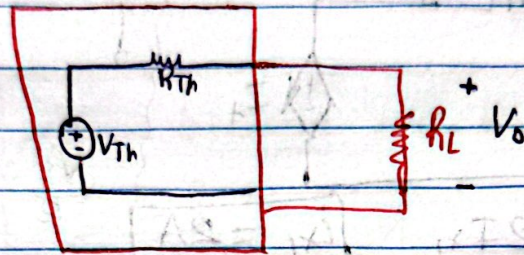
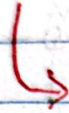
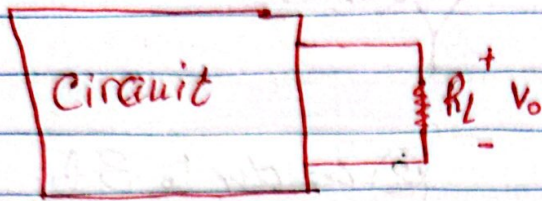
$V_x = 6V$

$\therefore i_{x2} = \frac{V_x}{2} = \frac{6}{2} = 3A$

$\therefore I_x = i_{x1} + i_{x2}$

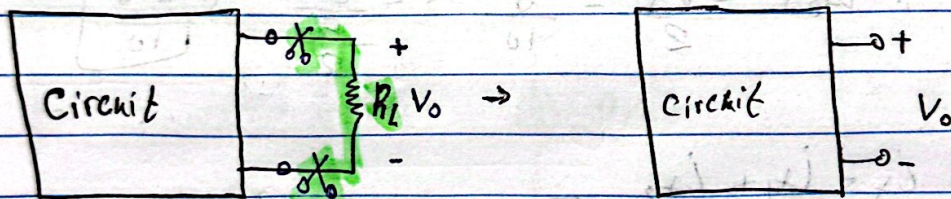
$= 2 - 0.6 = 1.4A$

Theveninequivalent circuit



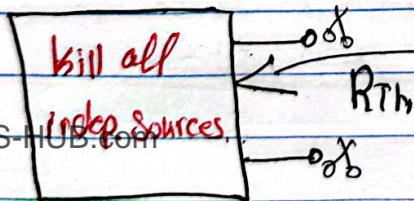
How to find V_{Th} and R_{Th} ??

1) $V_{Th} = V_{o.c.}$



2) R_{Th} ?

→ **Case 1** No dep. Sources:

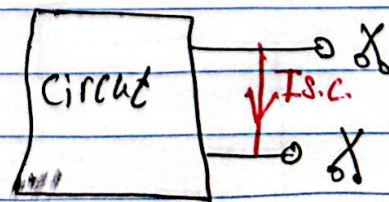


$R_{Th} \rightarrow$ series/parallel in the circuit

→ **Case 2** with dep. sources:

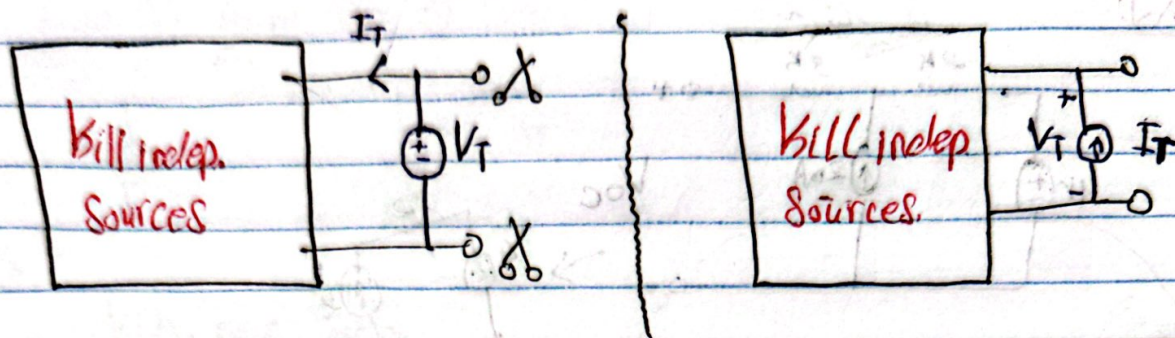
↳ method A

$$R_{Th} = \frac{V_{o.c.}}{I_{s.c.}}$$



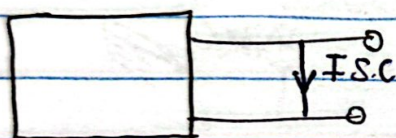
→ method B: Test source method

$$R_{Th} = \frac{V_T}{I_T} \quad \text{all indep. sources set to zero.}$$

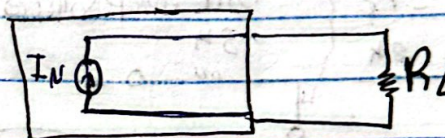


Norton eq. circuit

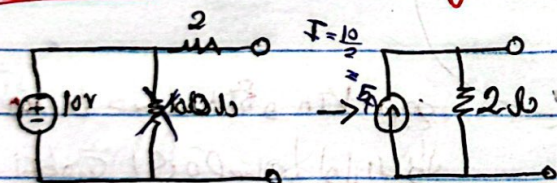
$$I_{S.C.} = I_N$$



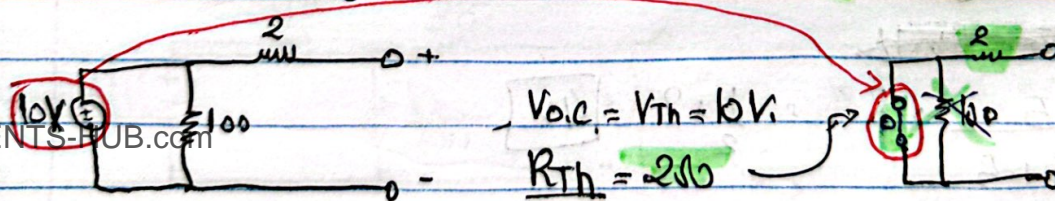
$$R_N = R_{Th}$$



* Note: Source Transformation



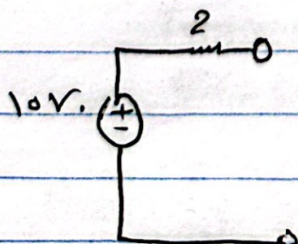
$$V_{Th} = V_{oc}$$



$$V_{oc} = V_{Th} = 10V$$

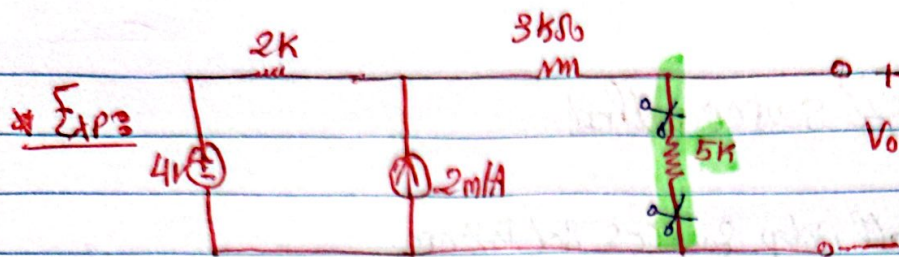
$$R_{Th} = 2\Omega$$

Series/Parallel in the circuit

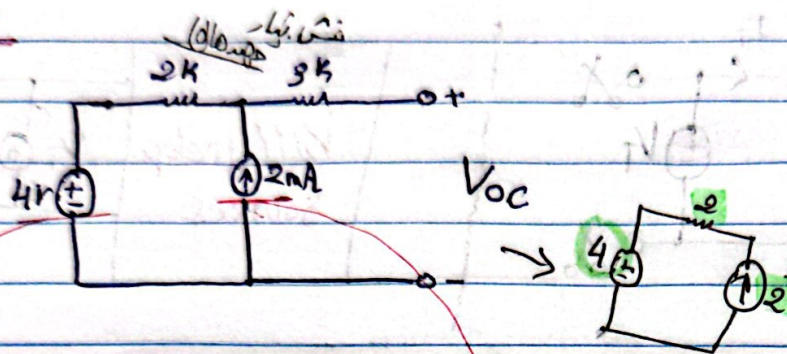


10/10 = 0

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Sol:

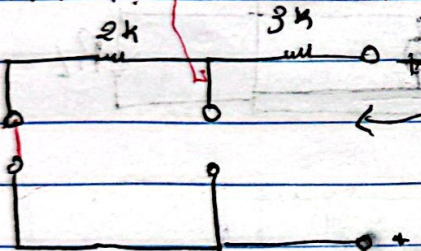


$$V_{Th} = V_{oc}$$

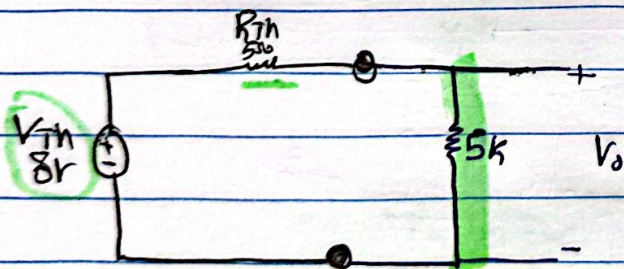
$$= \left(\frac{4V}{2k + 5k} \right) \times 5k + 4 = \boxed{8V}$$

$$R_{Th} = ??$$

kill indep. sources



$$R_{Th} = 2 + 3 = 5k\Omega$$

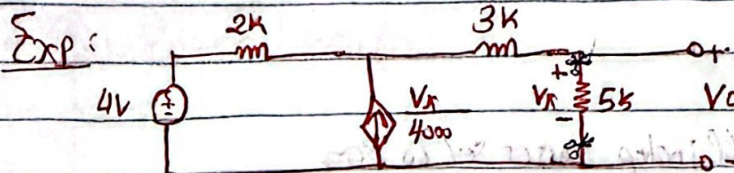


for V_{Th} & R_{Th} find the open circuit voltage & resistance
 V_o is the voltage across the load

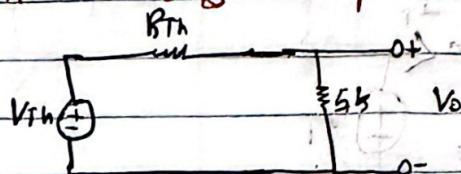
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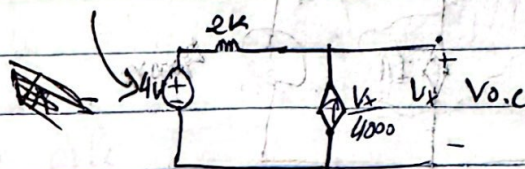
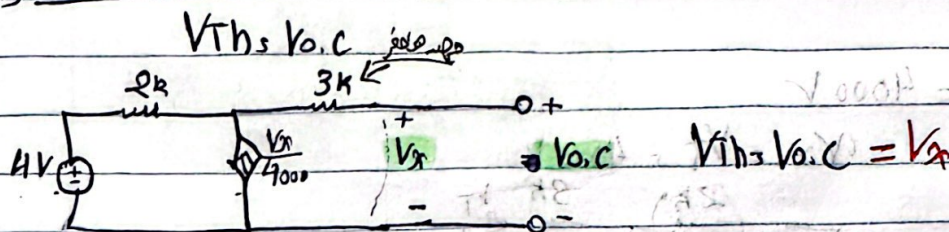
$$\rightarrow \frac{5}{5+5} \times 8 = \frac{1}{2} \times 8 = \boxed{4V}$$



Find V_o using the eq. circuit.



→ V_{Th}



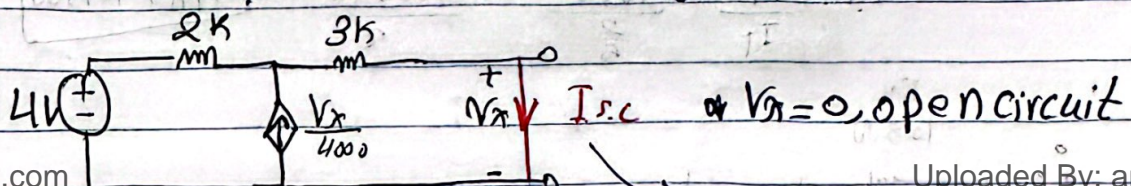
$$V_x = 4 + 2k \left(\frac{V_x}{4000} \right)$$

$$V_x = 4 + \frac{2V_x}{2000} \rightarrow V_x = 4 + \frac{V_x}{1000} \left(1 - \frac{1}{2} \right) V_x = 4 \rightarrow V_x = \frac{4}{\frac{1}{2}} = 8V$$

→ R_{Th}

method A

$$R_{Th} = \frac{V_{o.c}}{I_{s.c}} = \frac{8}{0.8} = 10k\Omega$$



So $\frac{V_x}{4000} = 0$

~~$I_{s.c} = 8$~~

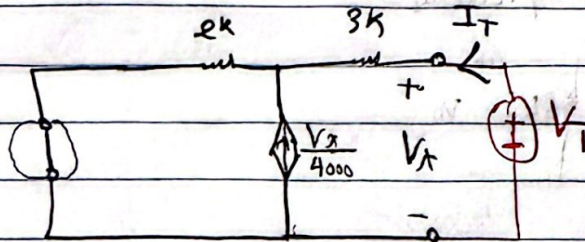
~~$I_{s.c} = \frac{4}{3+2} = \frac{4}{5} = 0.8mA$~~

$R_{Th} = \frac{8}{0.8} = 10k\Omega$

OR

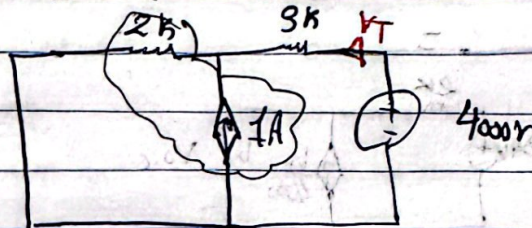
method B

$$R_{Th} = \frac{V_T}{I_T} \quad \text{all indep. sources set to zero.}$$



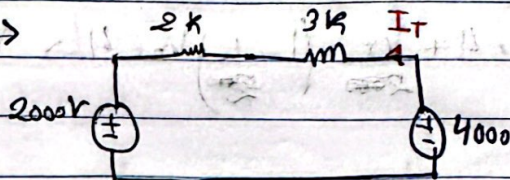
Let $V_T = 4000V$

$$V_T = V_x = V_T = 4000V$$



Source
Trans.

$$V = \frac{2000 \times 1}{5} = 2000V$$

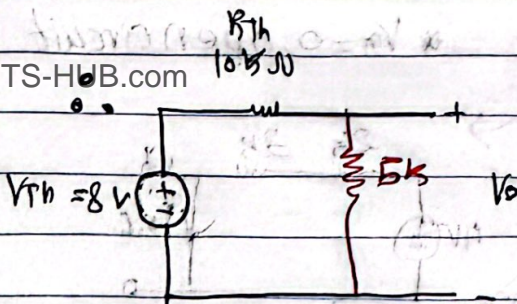


$$I_T = \frac{4000 - 2000}{5k} = \frac{2}{5} A$$

$$R_{Th} = \frac{V_T}{I_T} = \frac{4000}{\frac{2}{5}} = \frac{4000 \times 5}{2} = R_{Th} = 10k\Omega$$

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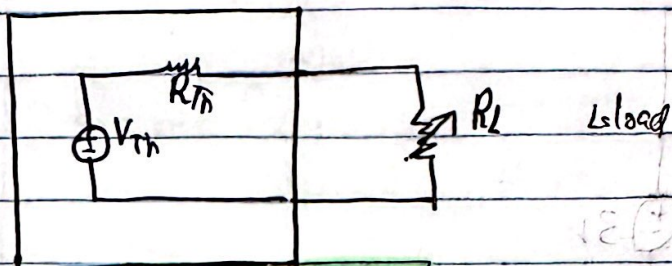


$$V_0 = \frac{5}{5+10} \times 8$$

$$V_0 = \frac{8 \times 5}{3}$$

$$V_0 = 2.666V$$

Max. Power Transfer:



$$\rightarrow V_L = \frac{R_L}{R_L + R_{th}} V_{th}$$

$$\rightarrow P_L = \frac{V_L^2}{R_L} = \frac{R_L}{(R_L + R_{th})^2} V_{th}^2$$

$$\frac{\partial P_L}{\partial R_L} = \frac{V_{th}^2 (R_L + R_{th})^2 - 2 R_L (R_L + R_{th})}{(R_L + R_{th})^2}$$

$$\text{for } \frac{\partial P_L}{\partial R_L} = 0$$

$$\rightarrow \frac{\partial R_L}{(R_L + R_{th})^2} = 2 R_L (R_L + R_{th})$$

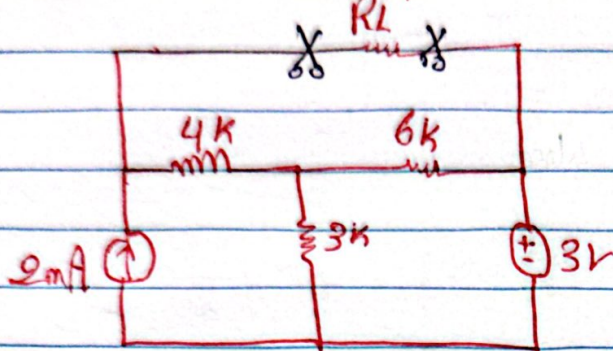
$$\rightarrow \boxed{R_L = R_{th}}$$

∴ for max power Transfer

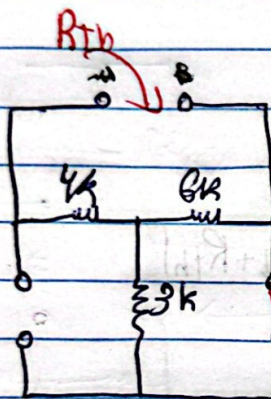
$$\boxed{R_L = R_{th}}$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4 R_{th}}}$$

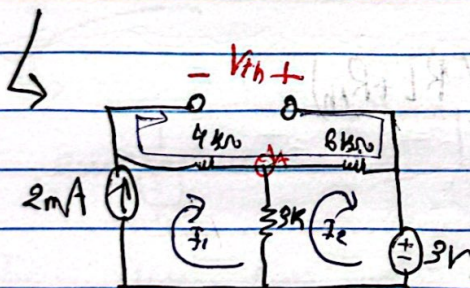
Exp: $R_{L=0}$ for P_{max}
find P_{max} :



Sol:



$$R_{Th} = 4 + \left(\frac{6 \times 3}{3} \right) = 6 \Omega$$



Node

$$V_1 - 3 + \frac{V_2}{3} = 0$$

$$V_1 - 3 + 2V_2 = 0$$

$$3V_1 = 15$$

$$V_1 = 5V$$

$$@ I_1 = 2mA$$

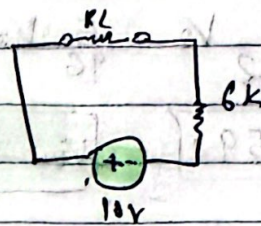
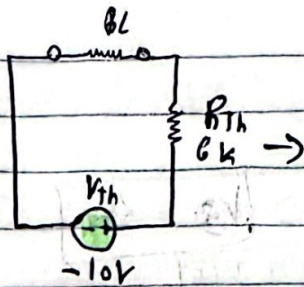
$$@ I_2 \rightarrow 3 + 3(I_2 - 2mA) + 6kI_2 = 0$$

$$+ 3I_2 - 2mA + 6kI_2 = -3$$

$$3 + 3I_2 - 6 + 6I_2 = 0$$

$$9I_2 = 3 \rightarrow I_2 = \frac{3}{9} A$$

$$V_{Th} = -\left(6k \times \frac{3}{9} \right) - 4k \times 2mA = -10V$$

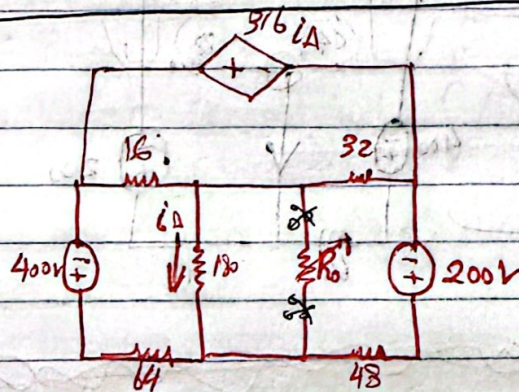


For max Power

$$R_L = R_{Th} = 6k\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{10^2}{4(6)} = \boxed{\frac{25}{6} \text{ mW}}$$

4/9/1



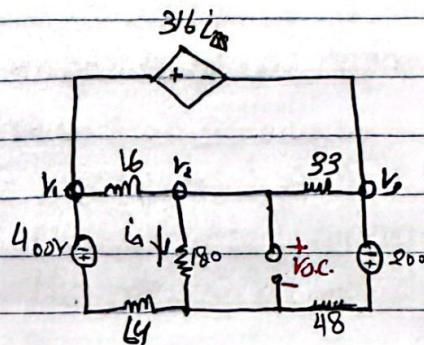
$P_{max} = ??$

$R_L = R_{Th}$ for max Power

$$R_{Th} = \frac{V_{O.C.}}{I_{S.C.}}$$

$$V_2 = V_{O.C.}$$

$$I_{S.C.} = \frac{V_2}{180}$$



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Super Node.

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$$\begin{aligned} V_1 - V_3 &= 316 \text{ mA} \\ \Rightarrow V_1 - V_3 &= \frac{316}{V_2} \\ V_1 - \frac{39}{45} V_2 - V_3 &= \frac{180}{V_2} \end{aligned}$$

$$\textcircled{a} V_2 \rightarrow \frac{V_2 - V_1}{16} + \frac{V_1}{180} + \frac{V_2 - V_3}{32} = 0 \Rightarrow \boxed{-\frac{1}{16} V_1 + \frac{143}{1440} V_2 - \frac{1}{32} V_3 = 0} \quad \textcircled{1}$$

$$\textcircled{a} V_1 \rightarrow \frac{V_1 + 400}{64} + \frac{V_1 - V_2}{16} + \frac{V_2 - V_3}{32} + \frac{V_3 + 200}{48} = 0$$



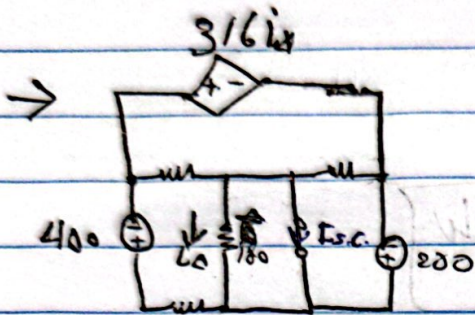
$$\frac{5}{64} V_1 - \frac{3}{32} V_2 + \frac{5}{96} V_3 = \frac{-125}{12}$$

$$V_1 = 592$$

$$V_2 = 360$$

$$V_3 = 400$$

$$= V_{oc} \text{ s.k.th}$$



$I_A = 0$

