9.5 The Product and Quotient Rules

Sunday, May 22, 2022 7:23 AM

**Def** Assume $g(x)$ and $h(x)$ are differentiables. Then if

قابلين للاشتقاق

① $f(x) = g(x)\,h(x)$ then "The Product Rule"

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$$f'(x) = g(x)\,h'(x) + h(x)\,g'(x)$$

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② $f(x) = \dfrac{g(x)}{h(x)}$ then $f'(x) = \dfrac{h(x)\,g'(x) - g(x)\,h'(x)}{(h(x))^2}$

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"The Quotient Rule"

قاعدة القسمة

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**Exp** Find $f'(x)$ if $f(x) = (3x^2+4x+5)(6-x^2)$

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$$f'(x) = (3x^2+4x+5)(0-2x) + (6-x^2)(6x+4)$$

$$= -6x^3 - 8x^2 - 10x + 36x + 24 - 6x^3 - 4x^2$$

$$f'(x) = -12x^3 - 12x^2 + 26x + 24$$

$$f'(1) = -12(1)^3 - 12(1)^2 + 26(1) + 24$$

$$= -12 - 12 + 26 + 24$$

$$= -12 - 12 + 26 + 24$$

$$= 26$$

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**Exp** Find slope of $g(x) = (1+2x^3)(1-x+x^4)$ at $x = 1$ and find tangent line

$$y - y_0 = m(x - x_0)$$

$x_0 = 1$

$$y_0 = g(x_0) = g(1) = (1+2\boxed{1}^3)(1-\boxed{1}+\boxed{1}^4)$$

$$= (1+2)(1)$$

$$= 3$$

$$m = \text{slope} = g'(x_0) = g'(1) = 15$$

$$g(x) = (1+2x^3)(1-x+x^4)$$

$$g'(x) = (1+2x^3)(0-1+4x^3) + (1-x+x^4)(0+6x^2)$$

$$m = g'(1) = (1+2\boxed{1}^3)(-1+4\boxed{1}^3) + (1-\boxed{1}+\boxed{1}^4)(6\boxed{1}^2)$$

$$= (1+2)(-1+4) + (1)(6)$$

$$= (3)(3) + 6$$

$$= 9 + 6$$

$$= 15$$

$= 15$

$= \text{slope}$

Tangent line :

$$y - y_0 = m(x - x_0)$$

$$y - \boxed{3} = 15(x - 1)$$

$$y - 3 = 15x - 15$$

$$+3 \qquad +3$$

$$\boxed{y = 15x - 12}$$

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Exp Find $R'(x)$ if $R(x) = \dfrac{x+5}{x^2 - 4x}$

$$R'(x) = \frac{(x^2-4x)(1+0) - (x+5)(2x-4)}{(x^2-4x)^2}$$

$$= \frac{x^2 + 4x - [2x^2 - 4x + 10x - 20]}{(x^2-4x)^2}$$

$$= \frac{x^2 + 4x - (2x^2 + 6x - 20)}{(\quad)^2}$$

$$= \frac{\text{[illegible]}}{(x^2-4x)^2}$$

$$= \frac{x^2 + 4x - 2x^2 - 6x + 20}{(x^2-4x)^2}$$

$$= \frac{-x^2 - 2x + 20}{(x^2-4x)^2}$$

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Exp Find the instantaneous rate of change of $f(x) = \frac{x^3 - 3x^2 + 2}{x^2 - 4}$ at $x = -1$ ← $f'(-1)$

$$f'(x) = \frac{(x^2-4)(3x^2-6x+0) - (x^3-3x^2+2)(2x-0)}{(x^2-4)^2}$$

$$f'(-1) = \frac{(\boxed{-1}^2-4)(3\boxed{-1}^2-6\boxed{-1}) - (\boxed{-1}^3-3\boxed{-1}^2+2)(2\boxed{-1})}{(\boxed{-1}^2-4)^2}$$

$$= \frac{(1-4)(3+6) - (-1-3+2)(-2)}{(1-4)^2}$$

$$(1-4)'$$

$$= \frac{(-3)(9) - (-2)(-2)}{(-3)^2}$$

$$= \frac{-27 - 4}{9}$$

$$= \frac{-31}{9}$$