# $\boxed{4.3}$ Monotonic Functions and the First Derivative Test



* A function that is increasing or decreasing on an interval is called monotonic on the interval.

**Corollary3** اللازمة Suppose $f$ is continuous on $[a,b]$ and differentiable on $(a,b)$.

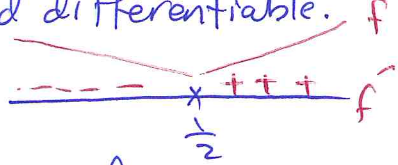
→ If $f'(x) > 0$ for every $x \in (a,b)$, then $f$ is increasing on $[a,b]$

→ If $f'(x) < 0$ for every $x \in (a,b)$, then $f$ is decreasing on $[a,b]$

**Example:** Find the critical points of $f(x) = x^2 - x$ and identify the intervals on which $f$ is increasing or decreasing.

$f(x)$ is everywhere continuous and differentiable.

$f'(x) = 2x - 1 = 0 \Leftrightarrow x = \frac{1}{2}$




$f$ is increasing on $(\frac{1}{2}, \infty)$ and decreasing on $(-\infty, \frac{1}{2})$

## First Derivative Test for Local Extrema:

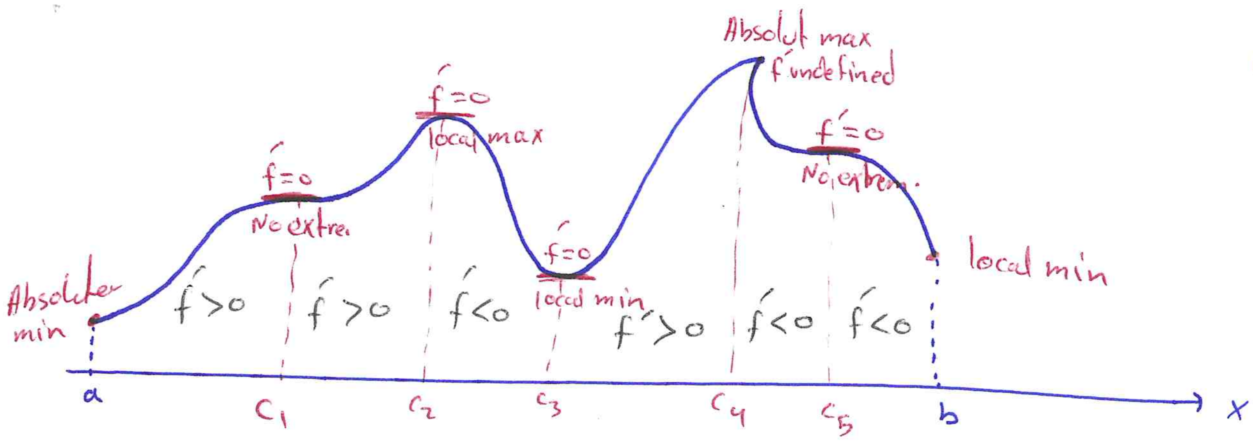
Suppose $c$ is a critical point of a continuous function $f$, and $f$ is differentiable on $(a,b)$ except possible at $c$, where $c \in (a,b)$ then:

$\boxed{1}$ If $f'$ changes from negative to positive at $c$, then $f$ has a local min at $c$.




$\boxed{2}$ If $f'$ " " positive to negative at $c$, " " " " " max at $c$.

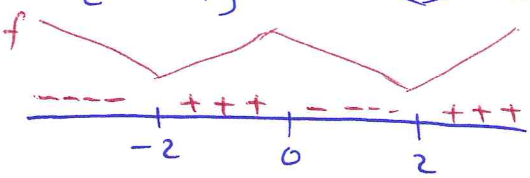
$\boxed{3}$ If $f'$ does not change sign at $c$, then $f$ has no local extremum at $c$.




**Example**: Find the critical points of $f(x) = x^4 - 8x^2 + 16$
Identify the intervals on which $f$ is increasing or decreasing
Find the local max/min and absolute max/min

$f$ is continuous for all $x \Rightarrow f'(x) = 4x^3 - 16x = 0$

$\Leftrightarrow 4x[x^2 - 4] = 0 \Leftrightarrow x = 0, 2, -2$ critical points




- $f$ is increasing on $(-2, 0)$ and on $(2, \infty)$
- $f$ is decreasing on $(-\infty, -2)$ and on $(0, 2)$

- $f$ has local min at $(-2, f(-2)) = (-2, 0)$
  $(2, f(2)) = (2, 0)$
- $f$ has local max at $(0, f(0)) = (0, 16)$
- $f$ has absolute min at $0$ when $x = \pm 2$
- $f$ has no absolute max