

$$1) \lim_{n \rightarrow \infty} \frac{4 + (-1)^n}{n}$$

$$\frac{3}{n} < \frac{4 + (-1)^n}{n} < \frac{5}{n}$$

first 2:

$$\lim_{n \rightarrow \infty} a_n = 0$$

$$2) \lim_{n \rightarrow \infty} \frac{2n+1}{6n+1} = \frac{1}{3}$$

$$a_n = \frac{2n+1}{6n+1}$$

$$a_1 = \frac{3}{7}, a_2 = \frac{5}{13}, a_3 = \frac{7}{19}, a_4 = \frac{9}{25}$$

$$3) a_n = (-1)^n \left(1 - \frac{6}{n}\right)$$

$$\lim_{n \rightarrow \infty} (-1)^n \left(1 - \frac{6}{n}\right) = \text{DNE div}$$

$$4) \sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0 < 1 \text{ conv}$$

$$5) \{b_n \text{ conv} \rightarrow \{a_n \text{ conv}$$

$$6) \sum_{n=0}^{\infty} \left(\frac{1}{4^n} - \frac{(-1)^n}{4^{n+1}} \right) = \left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{16}\right) + \left(\frac{1}{16} - \frac{1}{64}\right) + \left(\frac{1}{64} + \frac{1}{4}\right) + \dots$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{4^n} \right) - \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{1}{4^n} \right)$$

$$= \left(1 \cdot \frac{4}{3}\right) - \frac{1}{4} \left(1 \cdot \frac{4}{3}\right) = \frac{5 \times 4}{5 \times 3} - \frac{1 \times 3}{5 \times 3} = \frac{17}{15}$$

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$$\sum_{n=1}^{\infty} \frac{n}{(lnn+10)^n}$$

$$\lim_{n \rightarrow \infty} \frac{n^{\frac{1}{n}}}{lnn+10} = 0 < 1 \quad \text{Conv by Root test}$$

$$8) \sum_{n=1}^{\infty} \frac{5}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{5}{n} - \frac{5}{n+1} \right)$$

$$\frac{5}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} = \frac{5}{n} - \frac{5}{n+1}$$

$$S_n = \left(5 - \frac{5}{2} \right) + \left(\frac{5}{2} - \frac{5}{3} \right) + \left(\frac{5}{3} - \frac{5}{4} \right)$$

$$S_n = 5 - \frac{5}{n+1} = \frac{5n}{n+1}$$

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left[5 - \frac{5}{n+1} \right] = 5$$

$$9) \sum_{n=1}^{\infty} \frac{(x+6)^n}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{(x+6)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x+6)^n} = |x+6| \sqrt{\lim_{n \rightarrow \infty} \frac{n}{n+1}} = |x+6| < 1$$

$$-1 < x+6 < 1$$

$$-7 < x < -5$$

$$x = -7 \rightarrow \sum (-1)^n \frac{1}{\sqrt{n}} \rightarrow \text{Conv}$$

$$x = -5 \rightarrow \sum \frac{1}{\sqrt{n}} \rightarrow \text{div} \quad \begin{matrix} \text{conv} \\ \text{cond} \end{matrix} x = -7$$

$$10) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{5}{4}}+3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{5}{4}}+3}$$

conv Abs

$$\frac{1}{n^{\frac{5}{4}}+3} < \frac{1}{n^{\frac{5}{4}}} \downarrow \text{conv P-series}$$

$$11) a_1 = 4, a_{n+1} = \sqrt[n]{n} a_n$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

$$12) \sum_{n=1}^{\infty} \frac{5n}{n^2+1}$$

$$\lim_{n \rightarrow \infty} \frac{5n}{n^2+1} \cdot \frac{n}{1} = 5 \cdot \text{LCT}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div}$$

$$13) \sum_{n=1}^{\infty} b_n \text{ conv} \rightarrow \sum_{n=1}^{\infty} a_n \text{ conv}$$

$$14) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2n-1)}{(2n-1)!}$$

$$\sum_{n=1}^{\infty} \frac{1}{2n!}$$

$$\lim_{n \rightarrow \infty} \frac{1}{(2n+2)!} \cdot 2n! = \lim_{n \rightarrow \infty} \frac{1}{(2n+2)(2n+1)} = 0 < 1 \text{ conv Abs}$$

$$15) a_1 = 3, a_{n+1} = \frac{n}{n+1} a_n$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{n}{n+1} = 1$$

conv to 0

$$a_2 = \frac{2}{3} \cdot 3 = 2, a_5 = \frac{5}{6} \cdot \frac{6}{5} = 1$$

$$a_3 = \frac{3}{4} \cdot 2 = \frac{3}{2}, a_7 = \frac{7}{8} \cdot 1 = \frac{7}{8}$$

$$a_4 = \frac{4}{5} \cdot \frac{3}{2} = \frac{6}{5}, a_8 = \frac{8}{9} \cdot \frac{7}{8} = \frac{7}{9}$$

$$16) \sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$$

$$\text{conv } r = \frac{1}{e} \in (-1, 1)$$

$$\text{Sum} = \frac{\frac{1}{e}}{1 - \frac{1}{e}} = \frac{1}{e} \cdot \frac{e}{e-1} = \frac{1}{e-1}$$

$$\sum_{n=1}^{\infty} \frac{(x+4)^n}{n 3^n}$$

$$\lim_{n \rightarrow \infty} \frac{(x+4)^{n+1}}{(n+1) 3^{n+1}} \cdot \frac{n 3^n}{(x+4)^n} = \lim_{n \rightarrow \infty} \frac{(x+4) n}{3 (n+1)} = \frac{|x+4|}{3} \lim_{n \rightarrow \infty} \frac{n}{n+1} = \frac{|x+4|}{3} < 1$$

$$x = -7 \rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \rightarrow \text{conv}$$

$$-1 < \frac{x+4}{3} < 1$$

$$x = -1 \rightarrow \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \text{div}$$

$$-3 < x+4 < 3$$

$$-7 < x < -1 \rightarrow \text{conv Abs}$$

$$\text{IC conv} \rightarrow [-7, -1)$$

18) div by n^{th} term test

$$19) 1.\bar{7} = 1 + \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \dots$$

$$\text{Sum} = 1 + \left[\frac{7}{10} \cdot \frac{10}{9} \right] = 1 + \frac{7}{9} = \frac{16}{9}$$

$$20) \sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+1}} = -\frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \dots$$

$$0 < E < \frac{1}{32}$$

Bonus $(2^{2^n})^2 = 2^{2^{n+1}}$

Question 2: a- $\sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1}$

$U_n = \frac{3n^2}{n^3+1} > 0$, $\downarrow$ for large n

$\lim_{n \rightarrow \infty} U_n = 0$ conv

$\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$ div
 $\lim_{n \rightarrow \infty} \frac{3n^2}{n^3+1} \cdot \frac{n}{1} = 3$

$\sum_{n=1}^{\infty} \frac{1}{n}$ div

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1}$ conv cond

b- $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$

$\lim_{n \rightarrow \infty} \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^n} = |x| \lim_{n \rightarrow \infty} \left[\frac{n}{n+1} \cdot \frac{(\ln n)^2}{(\ln(n+1))^2} \right] = |x| < 1$

$-1 < x < 1 \rightarrow$ conv Abs

$\therefore -1 \leq x \leq 1 \rightarrow$ conv Abs

$x = -1 \rightarrow \sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2} \rightarrow$ conv

$x = 1 \rightarrow \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \rightarrow$ conv by Integral test

$\lim_{b \rightarrow \infty} \int_2^b \frac{dn}{n(\ln n)^2}$
 $u = \ln n$
 $du = \frac{1}{n} dn$
 $\int \frac{du}{u^2} = -\frac{1}{\ln n}$

$f(x) = \frac{1}{x(\ln x)^2}$ $+, \downarrow$

$\lim_{b \rightarrow \infty} \left[-\frac{1}{\ln n} \right]_2^b = \lim_{b \rightarrow \infty} \left[-\frac{1}{\ln b} + \frac{1}{\ln 2} \right] = \frac{1}{\ln 2}$

So $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$ conv Abs $-1 \leq x \leq 1$

Question 3:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-2)^{n+1}}{n 4^n}$$

$$\lim_{n \rightarrow \infty} \frac{(x-2)^{n+2}}{(n+1) 4^{n+1}} \cdot \frac{n 4^n}{(x-2)^{n+1}} = \frac{|x-2|}{4} \lim_{n \rightarrow \infty} \frac{n}{n+1} = \frac{|x-2|}{4} < 1$$

$$x=2 \rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (-4)^{n+1}}{n 4^n}$$

$$-1 < \frac{|x-2|}{4} < 1$$

$$-4 < x-2 < 4$$

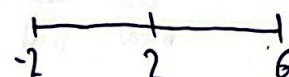
$$\boxed{-2 < x < 6}$$

Conv Abs

$$= \sum_{n=1}^{\infty} \frac{(-1)^n (-1) (-4)^n (-4)}{n 4^n}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \text{div}$$

$$x=6 \rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n (-1) (4)^n (4)}{n 4^n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \rightarrow \text{Conv cond}$$



IC Conv $\rightarrow -2 < x \leq 6$

Radius conv $\rightarrow R=4$

$x=6 \rightarrow \text{Conv cond}$

Question 4:

$$\sum_{n=0}^{\infty} (\log_2 x)^n$$

$$r = \log_2 x$$

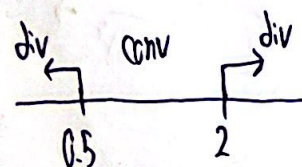
Conv $x \in (-1, 1)$

$$-1 < \log_2 x < 1$$

$$2^{-1} < x < 2$$

Conv

$$\boxed{\frac{1}{2} < x < 2}$$



div $x \geq 2$ or $x \leq \frac{1}{2}$

Radius of conv $R =$

Bonus: $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$ (Hint $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, for $-1 < x < 1$)

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

differentiate

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} n x^{n-1}$$

$$x = \frac{1}{2} \rightarrow \sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^{n-1} = \sum_{n=0}^{\infty} \frac{n}{2^{n-1}} = \frac{1}{\left(1 - \frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$$

$$\text{So } \sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = 4$$