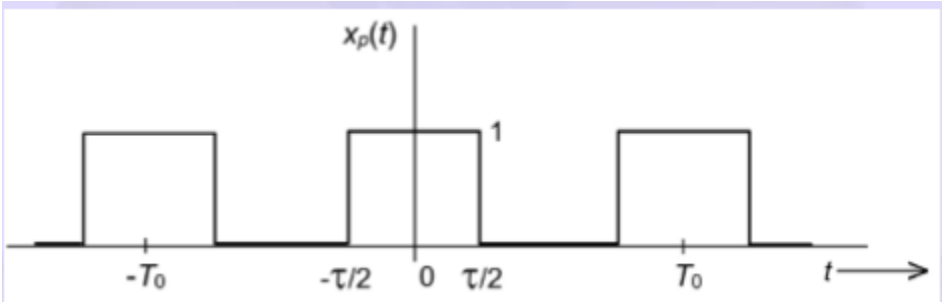
Problem Set 1

Signals and Systems

1. For the periodic signal,
   a. show that

$$x_n = \left(\frac{\tau}{T_0}\right)\text{sinc}(nf_0\tau)$$

   b. Find the average power in the signal
   c. Find the discrete power spectral density
   d. Find the fraction of the total power contained in the first five harmonics assuming $\tau = T_0/4$.
   e. Use matlab to plot both $x_p(t)$ and the first five harmonics assuming $\tau = T_0/4$.



2. A. Use the duality property, to find the Fourier transform of

$$z(t) = A\,\text{sinc}\,2Wt$$

   B. Find the total energy in z(t).

3. Make use of the result of Problem 2 to find the energy in the signal

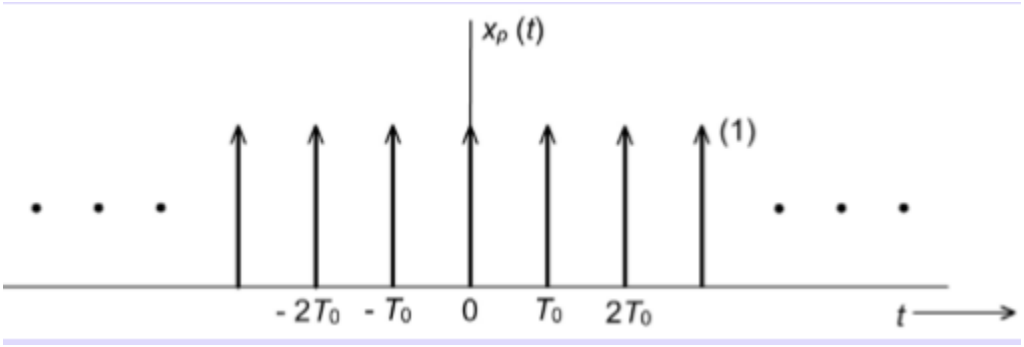
$$x(t) = 2AW\,\text{sinc}\,(2Wt).$$

4. Evaluate the integrals:

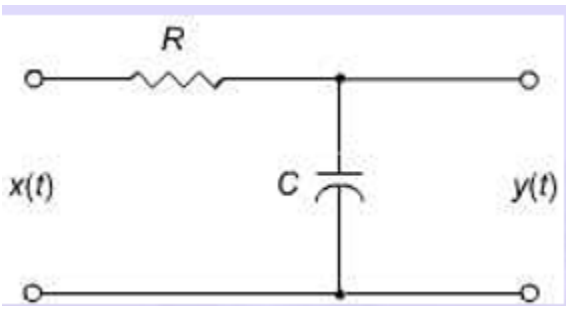
$$\text{(a)} \quad \int_{-4}^{4} t^3\,\delta(t-5)\,dt$$

$$\text{(b)} \quad \int_{4.9}^{5.1} t^3\,\delta(t-5)\,dt$$

5. Find the Fourier transform of the uniform impulse train

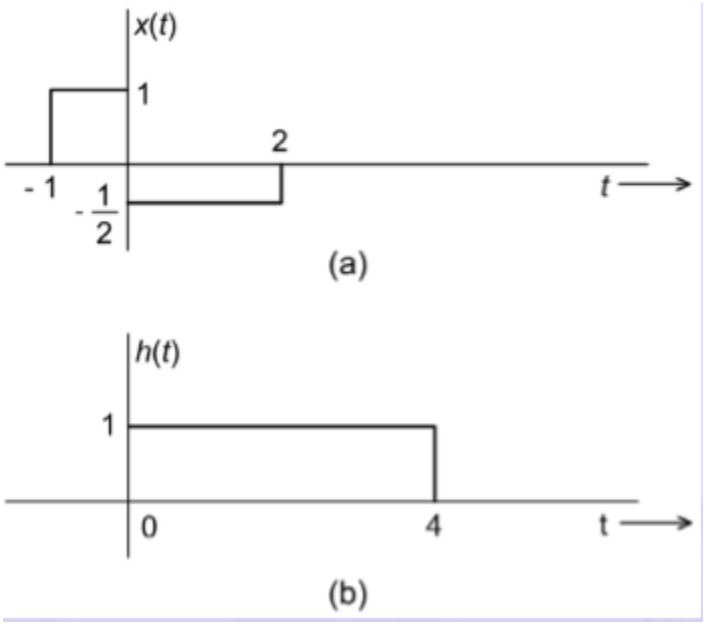


6. A. Find the frequency response H(f) and impulse response h(t) of the circuit:



B. Find the 3-dB bandwidth of the circuit

7. The input x(t) and the impulse response h(t) of a LTI system are as shown in the figure below, find the output y(t)



a. Find the Fourier transfor of x(t) and h(t)
b. Use the convolution intergral to find y(t)

c. Find the equivalent time duration of of x(t)
d. Find the equivalent rectangular bandwidth of x(t)

8. The input x(t) and the impulse response h(t) of a LTI system are

$$x(t) = \begin{cases} e^{-\alpha t}, & t \geq 0 \\ 0, & otherwise \end{cases}$$

$$h(t) = \begin{cases} e^{-\beta t}, & t \geq 0 \\ 0, & otherwise \end{cases}$$

a. Find X(f) and H(f)
b. Find the output energy spectral density
c. Use the convolution integral to find y(t)

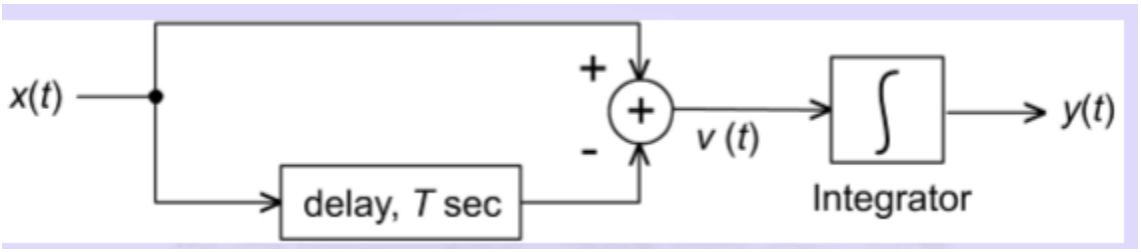
9. The input x(t) and the impulse response h(t) of a LTI system are

$$x(t) = \begin{cases} 2, & |t| < 2 \\ 0, & outside \end{cases}$$

$$h(t) = \begin{cases} 2e^{-t}, & t \geq 0 \\ 0, & outside \end{cases}$$

find the output y(t)

10. Find the impulse response of the system:



11. Fnd the period, and find the Fourier series representation of the following periodic signals

a) $x(t) = 2\cos(200\pi t) + 5\sin(400\pi t)$

b) $x(t) = 2\cos(200\pi t) + 5\sin(300\pi t)$

c) $x(t) = 2\cos(150\pi t) + 5\sin(250\pi t)$

12. Consider the following signal

$$\begin{aligned} x(t) &= \cos(2\pi f_1 t) + a\sin(2\pi f_1 t) \\ &= X_A(a)\cos(2\pi f_1 t + X_p(a)) \end{aligned}$$

a) Find $X_A(a)$.

b) Find $X_p(a)$.

c) What is the power of $x(t)$, P_x?

d) Is $x(t)$ periodic? If so what is the period and the Fourier series representation of $x(t)$?

13. The input-output characteristic of a channel is described by the differential equation:

$dy(t)/dt + 2y(t) = 4x(t)$

a. Find the transfer function, $H(f)$, of the channel.
b. Find the 3-dB bandwidth of the channel.

14. Let m(t) be a baseband signal with Fourier transform

$$M(f) = \begin{cases} m_0 & -f_m \le f \le f_m \\ 0 & otherwise \end{cases}$$

Let $\hat{m}(t)$ be the Hilbert transform of m(t), find the energy in $\hat{m}(t)$.

15. The impulse response of a linear time-invariant system is given by:

$h(t) = e^{-2\pi Bt}u(t)$

a. Is this system causal? Explain
b. Is this system stable? Explain
c. Find $\int_0^5 h(t)\delta(t-1)dt$

16. The Fourier transform of a time signal $m(t)$ is given by:

$M(f) = \frac{1}{1+j(f/B)}$

a. Find the 6-dB bandwidth of the message
b. Find $M(f)\delta(f-B)$

17. Consider the signal $g(t) = e^{-a|t|}$.

a. Explain why this signal is an energy signal.
b. Find and sketch the energy spectral density of $g(t)$.
c. Find the total energy in $g(t)$.
d. Find the 3-dB bandwidth of $g(t)$.
e. Find the fraction of the signal energy contained in the bandwidth of Part d relative to the total signal energy.

18. A periodic signal x(t) defined over one period is:

$$x(t) = \begin{cases} a|t| & -T_0/2 \le t \le -T_0/2 \\ 0 & |f| > W \end{cases};$$

Find the Fourier series coefficient an, n = 1, 3, 5.

19. Consider the signal $g(t) = e^{-2\pi Bt}u(t)$.

a. Find the autocorrelation function $R_g(\tau)$.

b. Find the energy spectral density.

c. Find the energy in the signal.

d. Find the 3-dB bandwidth of the signal.