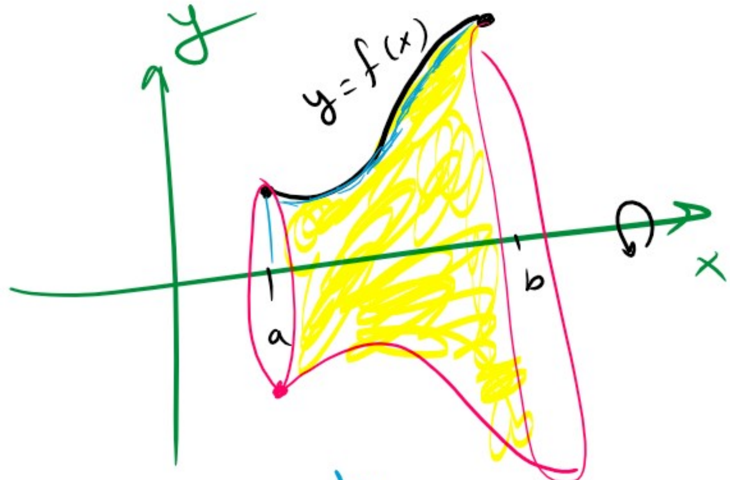
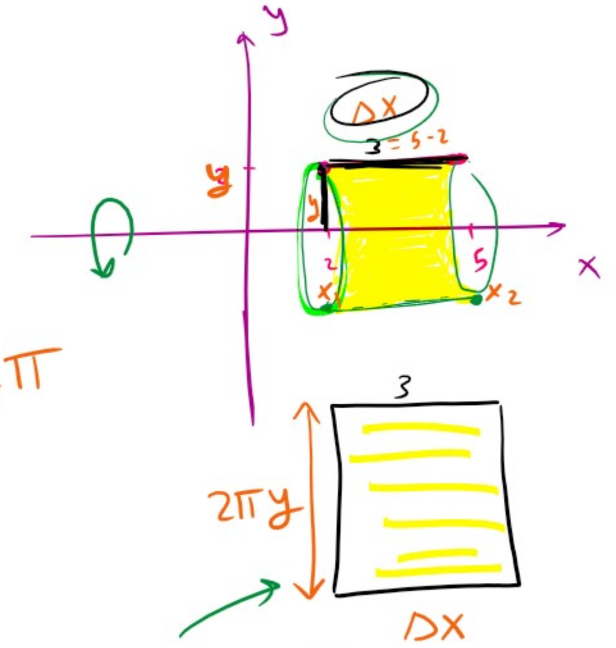
حجم الاسطوانة = مساحة القاعدة × الارتفاع

SA : Surface Area





$2y\pi$

$$SA = 2\pi y\,\Delta X$$

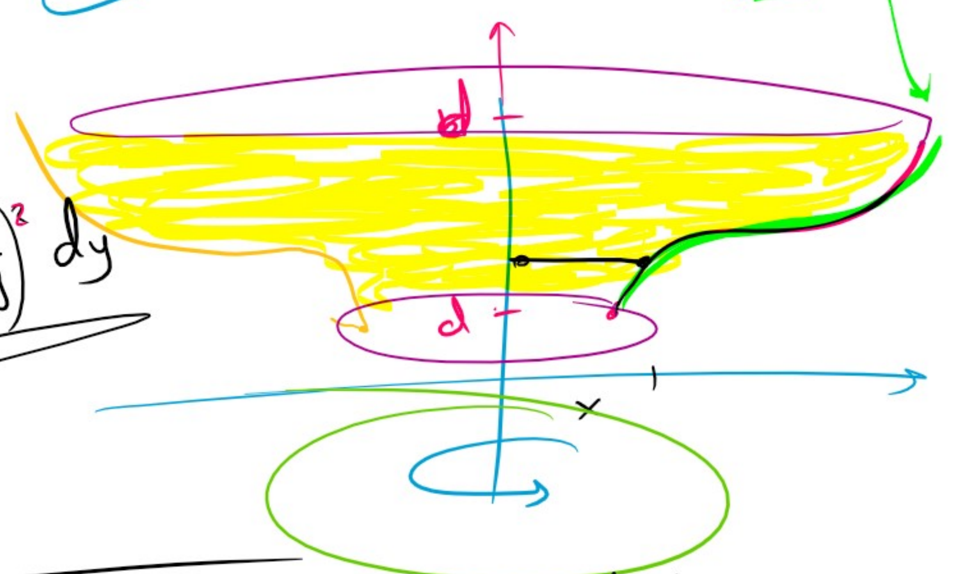
$$SA = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$y > 0$

$x \geq 0$

$$SA = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$x \geq 0$



Exp Find Surface Area generated by revolving the curve

Exp Find ... revolving the curve about x-axis

(1) $y = 2\sqrt{x}$, $1 \le x \le 2$,

$y \ge 0$ on $[1,2]$

$\frac{dy}{dx} = \cancel{2}\,\frac{1}{\cancel{2}\sqrt{x}}$

$\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$

$$SA = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_1^2 2\pi (2\sqrt{x}) \sqrt{1+\frac{1}{x}}\,dx$$

$$= 4\pi \int_1^2 \sqrt{x} \sqrt{\frac{x+1}{x}}\,dx$$

$$= 4\pi \int_1^2 \cancel{\sqrt{x}}\,\frac{\sqrt{x+1}}{\cancel{\sqrt{x}}}\,dx$$

$u = x+1$

$du = dx$

$x = 1 \Rightarrow u = 2$

$x = 2 \Rightarrow u = 3$

$$= 4\pi \int_1^2 \sqrt{x+1}\,dx$$

$$= 4\pi \int_2^3 \sqrt{u}\,du$$

$$= 4\pi \left.\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right|_2^3 = \dots = \frac{8\pi}{3}\left(3\sqrt{3}-2\sqrt{2}\right)$$

... generated by

Q18 Find area of surface generated by revolving $x = \frac{1}{3} y^{\frac{3}{2}} - \sqrt{y}$, $1 \le y \le 3$ about y-axis

$\frac{1}{3} - 1$

$-x = -\left(\frac{1}{3} y^{\frac{3}{2}} - \sqrt{y}\right)$

$x = \frac{1}{3}\sqrt{y^3} - \sqrt{y} < 0$

$SA = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\Rightarrow$ take $x = \sqrt{y} - \frac{1}{3}\sqrt{y^3}$

$x > 0$ on $[1,3]$

$\frac{dx}{dy} = \frac{1}{2\sqrt{y}} - \frac{1}{3}\cdot\frac{3}{2} y^{\frac{1}{2}}$

$= \frac{1}{2\sqrt{y}} - \frac{1}{2}\sqrt{y}$

$= \frac{1}{2}\left(\frac{1}{\sqrt{y}} - \sqrt{y}\right)$

$= \int_1^3 2\pi\left(\sqrt{y} - \frac{1}{3}\sqrt{y^3}\right) \sqrt{1 + \frac{1}{4}\left(\frac{1}{\sqrt{y}} - \sqrt{y}\right)^2}\, dy$

$\sqrt{1 + \frac{1}{4}\left(\frac{1}{y} - 2 + y\right)}$

$\sqrt{1 + \frac{1}{4y} - \frac{1}{2} + \frac{1}{4}y}$

$\sqrt{\frac{1}{4y} + \frac{1}{2} + \frac{1}{4}y}$

$\sqrt{\frac{1}{4}\left(\frac{1}{y} + 2 + y\right)}$

$\sqrt{\left(\frac{1}{\sqrt{y}} + \sqrt{y}\right)^2}$

$$\text{———}\ \frac{1}{2}\cancel{\sqrt{\left(\frac{1}{\sqrt{y}}+\sqrt{y}\right)^{2}}}$$

$$SA=\int_{1}^{3}\cancel{2}\pi\left(\sqrt{y}-\frac{1}{3}y^{\frac{3}{2}}\right)\cancel{\frac{1}{2}}\left(\frac{1}{\sqrt{y}}+\sqrt{y}\right)dy$$

$$\frac{16\pi}{9}$$

$$\underset{\downarrow}{\underline{\underline{SA}}}=\int_{a}^{b}\underset{\downarrow +}{2\pi}\ \boxed{x}\ \underset{\downarrow +}{\boxed{\sqrt{1+\left(\frac{dy}{dx}\right)^{2}}}}\,dx$$

مَ تكون سالبة

$$+\qquad +$$

$$\boxed{x>0}$$

$$x<0$$

$$\boxed{-x>0}$$

Q20 Find the area for the surface generated by revolving surface generated by revolving the curve

$$x = \sqrt{2y-1}\ , \quad \frac{5}{8} \le y \le 1$$

about y-axis

$x = \sqrt{2\left(\frac{5}{8}\right) - 1} = \sqrt{\frac{10}{8} - 1} > 0$

$x = \sqrt{2(1) - 1} = \sqrt{1} > 0$

$x \geq 0$

$$L = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= 2\pi \int_{5/8}^{1} \sqrt{2y-1} \sqrt{1 + \frac{1}{2y-1}}\, dy$$

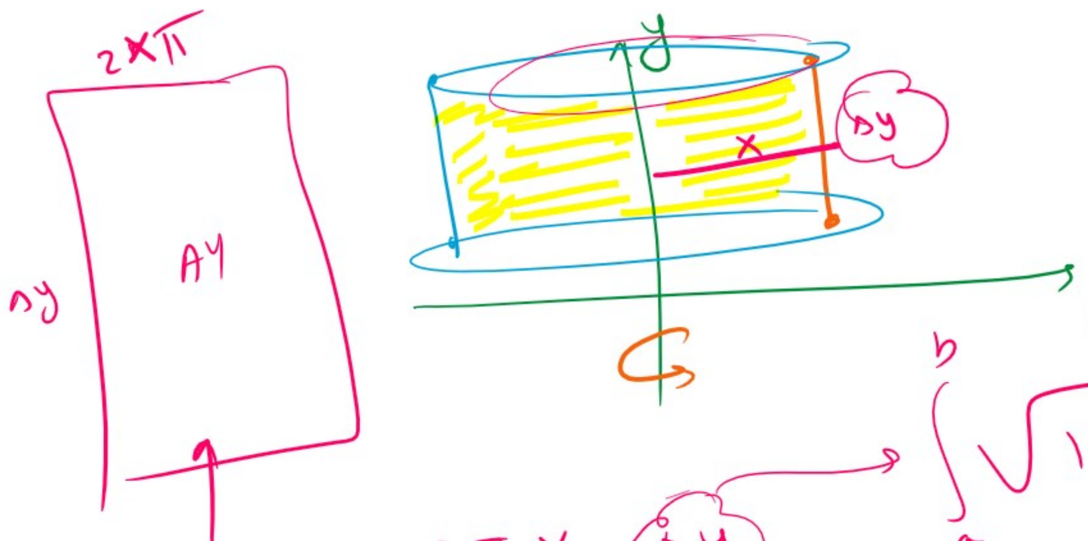
$\frac{dx}{dy} = \frac{1 \times \cancel{2}}{\cancel{2}\sqrt{2y-1}}$

$$= 2\pi \int_{5/8}^{1} \sqrt{2y-1} \sqrt{\frac{2y \cancel{-1} \cancel{+1}}{2y-1}}\, dy$$

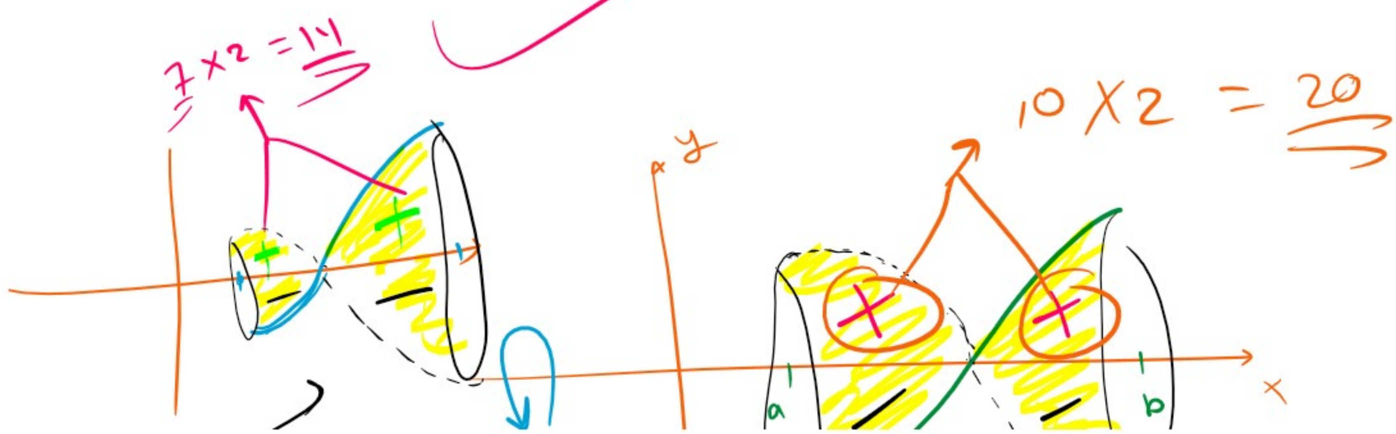
$$= 2\pi \int_{5/8}^{1} \cancel{\sqrt{2y-1}}\, \frac{\sqrt{2y}}{\cancel{\sqrt{2y-1}}}\, dy$$

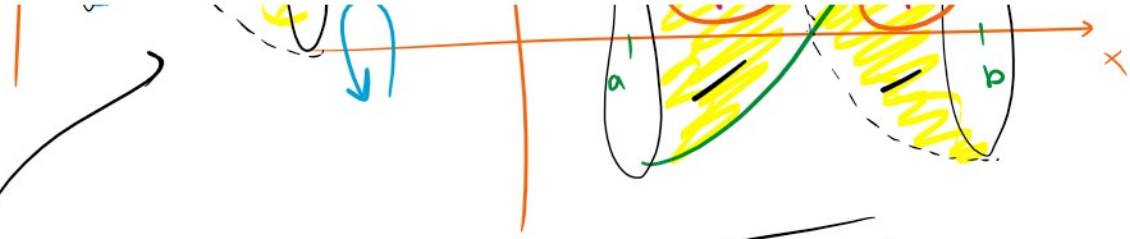
$$= 2\pi \int_{5/8}^{1} \sqrt{2}\,\sqrt{y}\,dy$$

$$= 2\sqrt{2}\,\pi \left.\frac{y^{3/2}}{\frac{3}{2}}\right|_{5/8}^{1} = \dots = \frac{\pi}{12}\left(16\sqrt{2} - 5\sqrt{2}\right)$$



$$SA = \underline{\underline{2\pi x}}\ \Delta y \longrightarrow \int_a^b \sqrt{1 + \left(f'(x)\right)^2}$$





عليه $\int = 0$

عليه $\int = \boxed{0}$