Functions

- The set of ordered pairs $\{(1,3),(1,6),(2,6),(3,9),(3,12),(4,12)\}$ represents relation between the 1^{st} compenent $x \in \{1,2,3,4\}$ and the 2^{nd} component $y \in \{3,6,9,12\}$.
- The set of first components is called the domain (input)
- The set of second components is called the range (output)

Def. • A relation is set of ordered pairs or is a rule that determines how the ordered pairs are found

- A relation is also defined by a table or graph or equation or inequality

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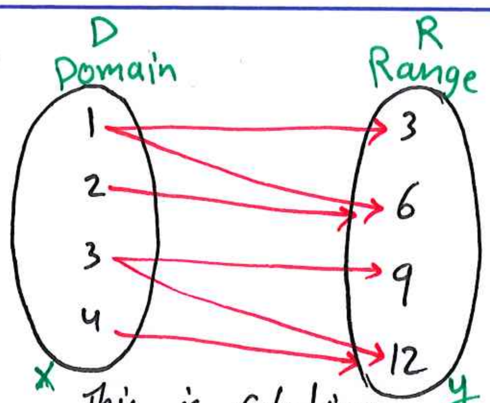
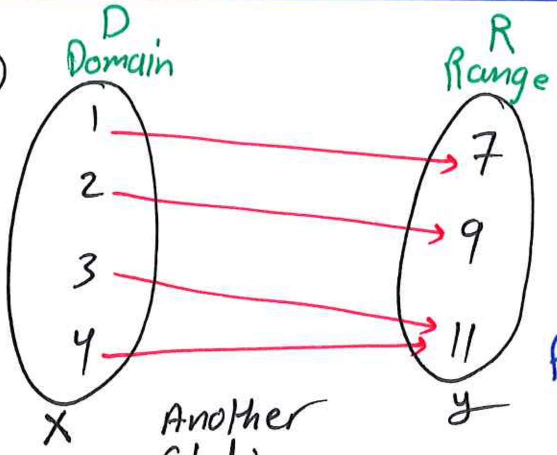
Exp

①



This is relation between x and y (not function)

②



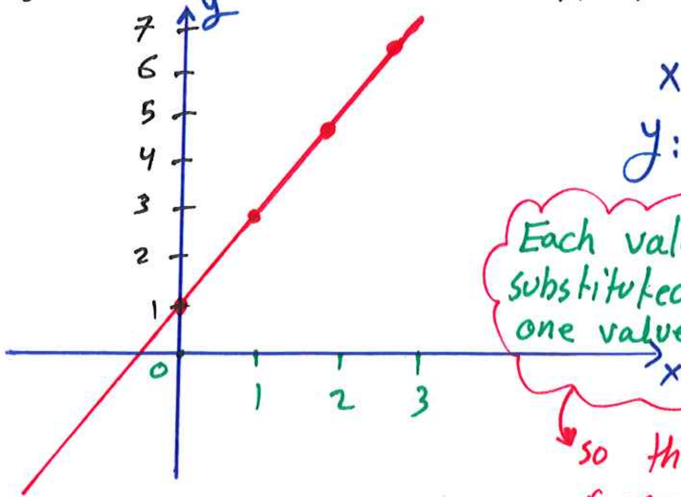
Another relation between x and y

here the relation is function

③ The equation $y = 2x + 1$ represents relation between x and y

x	y
0	1
1	3
2	5
3	7
⋮	⋮

Table



x: independent variable

y: dependent variable

Each value of x is substituted to give only one value of y

so this relation represents function

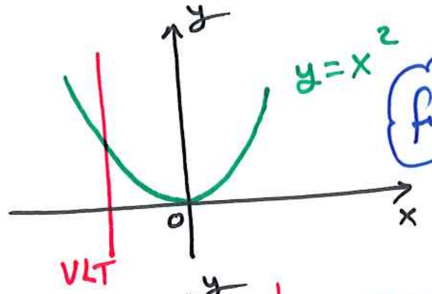
Def A function is relation between two sets such that each element in domain is assigned to a unique element in range.

Remark: VLTest If any vertical line intersects the graph at most once, then this graph represents function.

Otherwise, the graph is not function

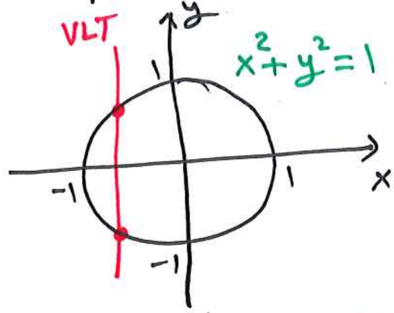
Exp* which of the following graphs represent function? Justify

[1]



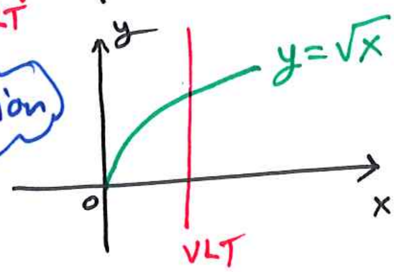
function

[2]



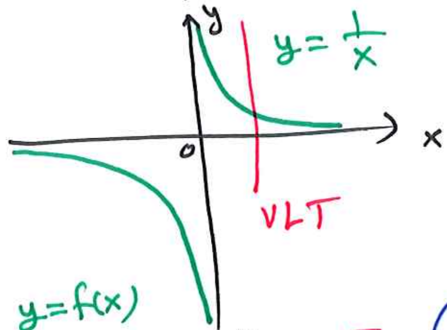
not function

[3]



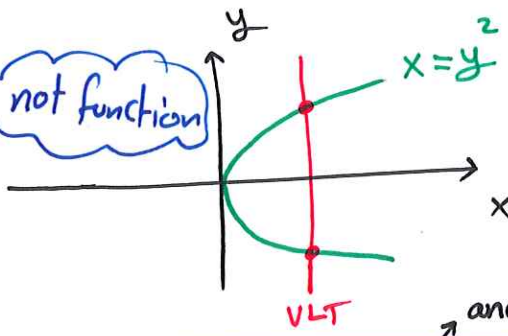
function

[4]



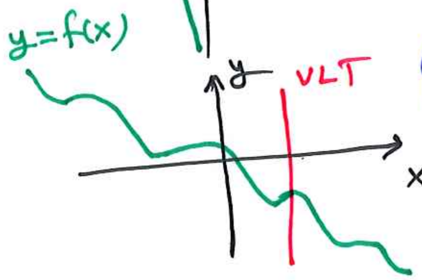
function

[5]



not function

[6]



function

Exp Find the domain and Range of functions in Exp*

[1] Domain (values of x) is the set of all real numbers $\mathbb{R}$

[3] Domain (values of x) is $[0,\infty)$

[4] Domain (values of x) $= \mathbb{R}\setminus\{0\} = (-\infty,0)\cup(0,\infty)$

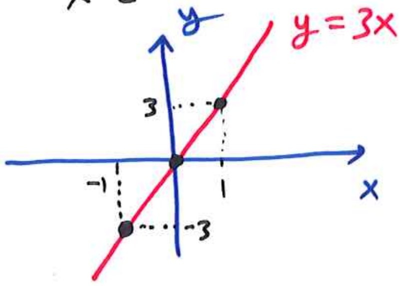
[6] Domain $= \mathbb{R} = (-\infty,\infty)$

Range (values of y) $\Rightarrow$ [1] Range $=[0,\infty)$ [3] Range $=[0,\infty)$

[4] Range $=\mathbb{R}\setminus\{0\}$ [6] Range $=\mathbb{R}$

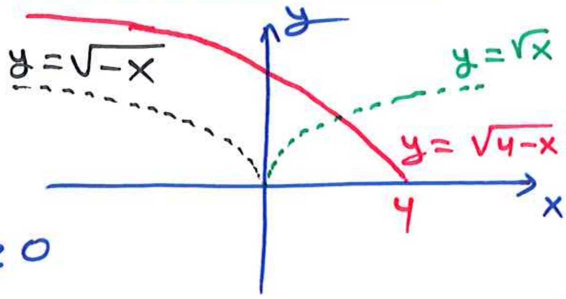
Exp Find domain and Range of the functions

[1] $y = 3x$ [2] $y = \sqrt{4-x}$ [3] $y = 1 + \dfrac{1}{x-2}$



[1] $y = 3x \Rightarrow D = \mathbb{R} = (-\infty, \infty)$

$R = \mathbb{R} = (-\infty, \infty)$



[2] $y = \sqrt{4-x}$

Domain $\Rightarrow 4 - x \geq 0$

$4 \geq x \quad \Rightarrow$ Domain $= (-\infty, 4]$

Range $= [0, \infty)$ "values of y"

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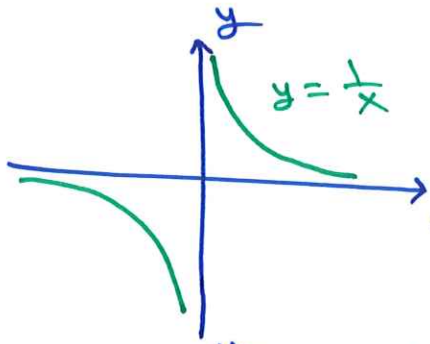
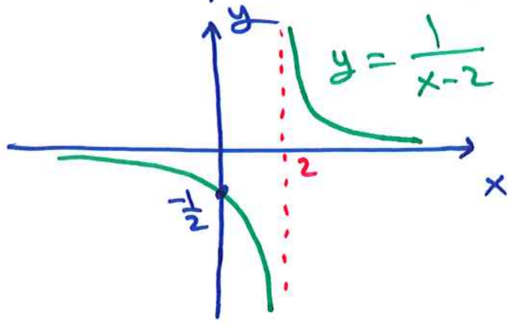
[3] $y = 1 + \dfrac{1}{x-2}$



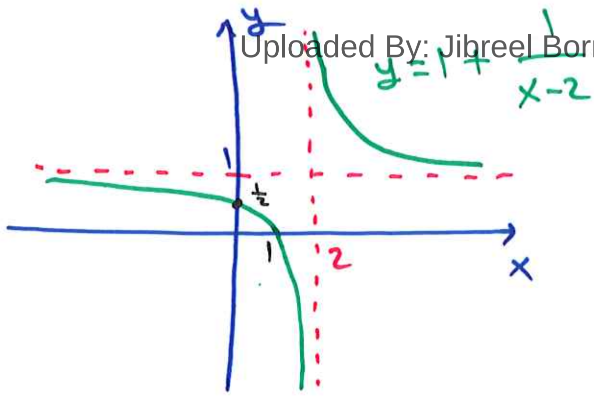
Domain $\Rightarrow x - 2 \neq 0$

$x \neq 2$

$\Rightarrow$ Domain $= \mathbb{R} \setminus \{2\}$



Range $= \mathbb{R} \setminus \{1\}$



Exp Suppose $g(x) = 8x - 10$ Find

1. $g(0) = 8(0) - 10 = 0 - 10 = -10$

2. $g(1) = 8(1) - 10 = 8 - 10 = -2$

3. $g(-\frac{3}{4}) = \cancel{8}^{2}(-\frac{3}{\cancel{4}}) - 10 = -6 - 10 = -16$

4. $g(3) = 8(3) - 10 = 24 - 10 = 14$

5. $$\frac{g(x+h) - g(x)}{h} = \frac{8(x+h) - 10 - (8x - 10)}{h}$$

$$= \frac{\cancel{8x} + 8h \cancel{-10} \cancel{-8x} \cancel{+10}}{h} = \frac{8h}{h} = 8 \quad , h \neq 0$$

Operations with Functions

Let f and g be functions of x

Sum : $(f+g)(x) = f(x) + g(x)$

Difference : $(f-g)(x) = f(x) - g(x)$

Product : $(f \cdot g)(x) = f(x) \cdot g(x)$

Quotient : $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad , \quad g(x) \neq 0$

Exp Let $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$. Find

1. $(f+g)(4) = f(4) + g(4) = \sqrt{4} + 4^2 - 1 = 2 + 16 - 1 = 17$

2. $(f \cdot g)(1) = f(1) \cdot g(1) = \sqrt{1} \cdot (1^2 - 1) = 1 \cdot 0 = 0$

3. $\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{\sqrt{4}}{4^2 - 1} = \frac{2}{16-1} = \frac{2}{15}$

4. $(f \cdot f)(4) = f(4)\, f(4) = \sqrt{4} \cdot \sqrt{4} = 2 \cdot 2 = 4 \quad = [f(4)]^2$

Def. Let f and g be any functions.

The composite function g of f is defined by

$$(g \circ f)(x) = g\big(f(x)\big)$$

- Similarly we define the composit function f of g:

$$(f \circ g)(x) = f\big(g(x)\big)$$

Exp** Let $f(x) = 1-2x$ and $g(x) = 3x^2$. Find

$\boxed{1}$ $(g-f)(x) = g(x) - f(x) = 3x^2 - (1-2x) = 3x^2 + 2x - 1$

$\boxed{2}$ $(f \cdot g)(1) = f(1)\, g(1) = [1-2(1)][3(1)^2] = (1-2)(3) = -3$

$\boxed{3}$ $(f \circ g)(0) = f\big(g(0)\big) = f\big(3(0)^2\big) = f(0) = 1-2(0) = 1-0 = 1$

$\boxed{4}$ $(g \circ f)(x) = g\big(f(x)\big) = g\big(1-2x\big) = 3(1-2x)^2$

$\boxed{5}$ $(f \circ f)(x) = f\big(f(x)\big) = f\big(1-2x\big) = 1-2(1-2x) = 1-2+4x$
$= 4x-1$

$\boxed{6}$ $(f \circ g)(x) = f\big(g(x)\big) = f\big(3x^2\big) = 1-2(3x^2) = 1-6x^2$

Note $(f \circ g)(x)$ not necessarily equals to $(g \circ f)(x)$

see $\boxed{4}$ and $\boxed{6}$ in Exp**