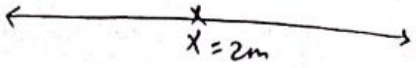
Chapter 2

Motion along a straight line
الحركة في بعد واحد

1 Position $[\vec{x}]$ الموقع



position of the Body = 2m

$[x] \equiv m$ متر

2 Displacement الازاحة

لا تعتمد على تفاصيل الحركة من نقطة البداية والنهاية

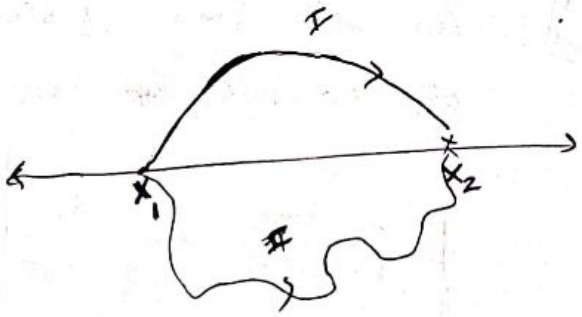
متجه vector $\Delta\vec{x} = \vec{x}_2 - \vec{x}_1$ $[\Delta x] = m$

خصائص: ① vector متجه ② does not depend on the Path. لا تعتمد على المسار

3 distance المسافة

① scalar quantity كمية قياسية

② it depends on the Path, تعتمد على المسار

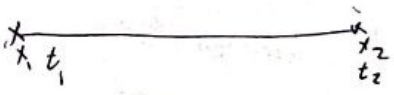


4 average Velocity معدل السرعة

V_{avg} ← vector متجه ← Displacement

$$V_{avg} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1}$$

$[V_{avg}] = \frac{m}{s}$

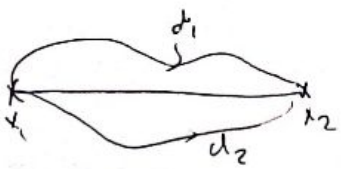


5 speed السرعة

* scalar كمية قياسية
* distance

* $S = \frac{\text{total distance}}{\text{total time.}}$

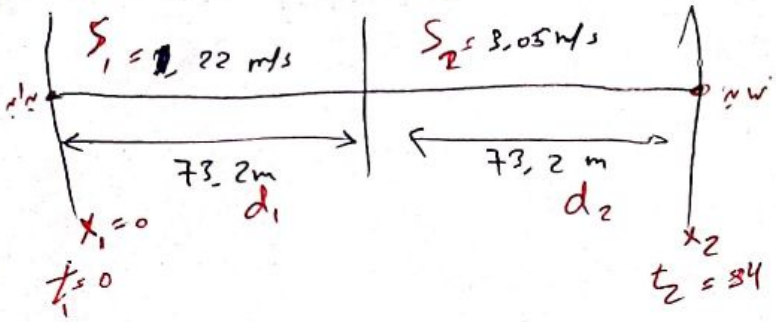
$S = d_1 + d_2$



Q2 / p 32 Compute V_{avg} in these two cases!

(I) You walk 73,2 m at speed 1,22 m/s and then you run 73, at speed 3,05 m/s

$$V_{avg} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_2 - \vec{x}_2}{t_2 - t_1}$$

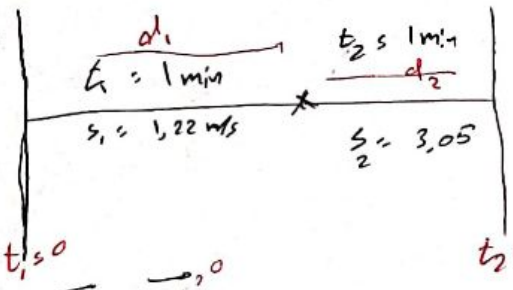


$$S_1 = \frac{d_1}{t_2} \Rightarrow t_1 = \frac{d_1}{S_1} = \frac{73,2}{1,22} = 60 \text{ sec}$$

$$S_2 = \frac{d_2}{t_2} \Rightarrow t_2 = \frac{73,2}{3,05} = 24 \text{ sec}$$

$$V_{avg} = \frac{146,4 - 0}{84 - 0} = \frac{146,4}{84} = 1,74 \text{ m/s}$$

(II) You walk 1 min at speed 1,22 m/s and then run for 1 min at 3,05 m/s find V_{avg}



$t_2 = 1\text{min} + 1\text{min} = 2\text{min} = 120 \text{ sec}$

$$S_1 = \frac{d_1}{t_1} \Rightarrow d_1 = S_1 t_1 = 73,2\text{m}$$

$$S_2 = \frac{d_2}{t_2} \Rightarrow d_2 = S_2 t_2 = 183 \text{ m}$$

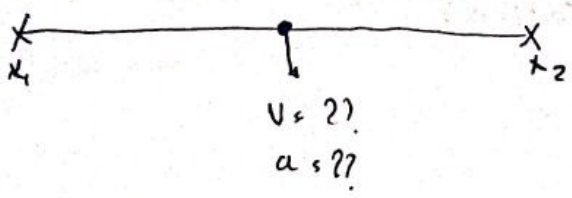
$x_2 = d_1 + d_2 = 256,2$ m

$$V_{avg} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} = \frac{256,2 - 0}{120 - 0} = 2,14 \text{ m/s}$$

(6) Avg acceleration

* vector also
* depends on the velocity [change in velocity] $\Rightarrow [\vec{a}] = \frac{m}{s^2}$

$$\vec{a}_{avg} = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1}$$



V_{avg}
a_{avg}

$V_{avg} = \frac{\Delta x}{\Delta t}$

$V \equiv$ instantaneous velocity

$a \equiv$ instantaneous acceleration

$V_{inst} = \frac{dx}{dt}$

$x = \int v\,dt$

Exp. if $x(t) = t^2 - 2t$ find v at $t = 2$ sec?

$V_{inst} = \frac{dx}{dt} = 2t - 2$

$V\big|_{t=2} = 2(2) - 2 = 2$ m/s

Exp2 if $x(t) = t^2 - t$

find V_{avg} between $t_1 = 3$sec and $t_2 = 5$ secs

$V_{avg} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} =$

Exp3: if $v(t) = t + 1$ and $x(t=0) = 2$ find $x(t) = ??$.

$x(t) = \int v(t)\,dt$

$= \int t + 1\,dt \Rightarrow x = \frac{t^2}{2} + t + C$

$x\big|_{t=0} = 2 \Rightarrow C = 2$

$x(t) = \frac{t^2}{2} + t + 2$ ✱

* Instantaneous acceleration:

$$a_{inst} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

تكامل $x(t) \xrightarrow{\frac{dx}{dt}} v(t) \xrightarrow{\frac{dv}{dt} = \frac{d^2x}{dt^2}} a(t)$

$x(t) \xleftarrow{\int v\,dt} v(t) \xleftarrow{\int a\,dt} a(t)$

Q14 / P33

electron $x(t) = 16\,t\,e^{-t}$ (m) بعد (s) How far is the electron from the origin (start) when it stops? ($v = 0$)

stop $\Rightarrow$ $v = 0$

$$v = \frac{dx}{dt} = -16te^{-t} + 16e^{-t} = 0$$

$$16e^{-t}[-t + 1] = 0$$

$16e^{-t} \neq 0$

$-t + 1 = 0$

$t = 1$ sec $\Rightarrow x = 16e^{-1} = \frac{16}{e}$ m

* Constant acceleration :-

① $a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0}$

$v = v_0 + at$ ①

x_0 ≡ initial Position

v_0 ≡ initial velocity

x ≡ final Position

v ≡ final velocity

② $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0}$

$x - x_0 = (v_{avg})\,t$, $v_{avg} = \frac{v + v_0}{2}$ تعويض

$x - x_0 = \left(\frac{v + v_0}{2}\right)t$

$x - x_0 = v_0 t + \frac{1}{2}at^2$ ②

$v^2 = v_0^2 + 2a\Delta x$ ③

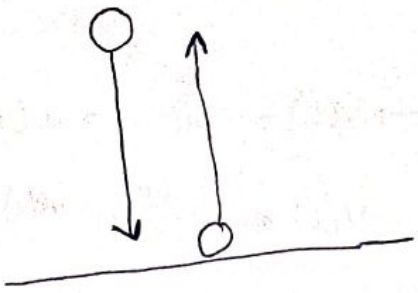
* free falling

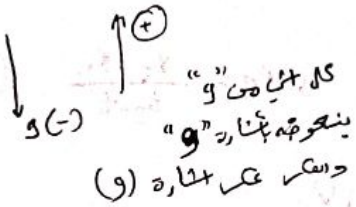
السقوط الحر

$a \to g$

$ox \to oy$

$a = g = 9{,}8\ m/s^2$





كل اتجاه لـ "g" بنعوضه بإشارة "g" وبعكس عكس اتجاه (g)

① $V = V_0 + gt$

② $y - y_0 = V_0 t + \frac{1}{2} g t^2$

③ $V^2 = V_0^2 + 2g\Delta y$

P58: An object falls a distance h from rest if it travels 0,5 h in the last 1 sec find (a) the time (b) the height of it is fall

أوجد الزمن

$\Delta y = V_0 t + \frac{1}{2} g t^2$

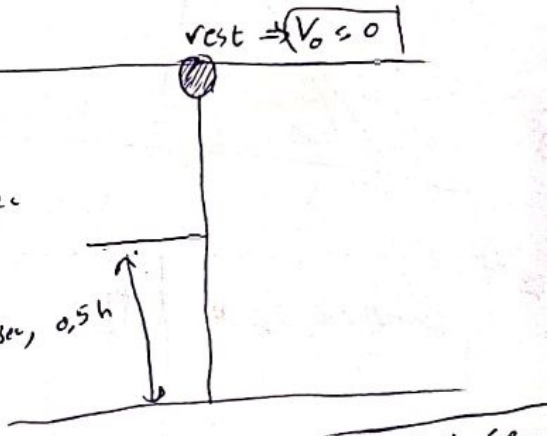
$h = \frac{1}{2} g t^2$ ②

$V^2 = V_0^2 + 2g\Delta y$

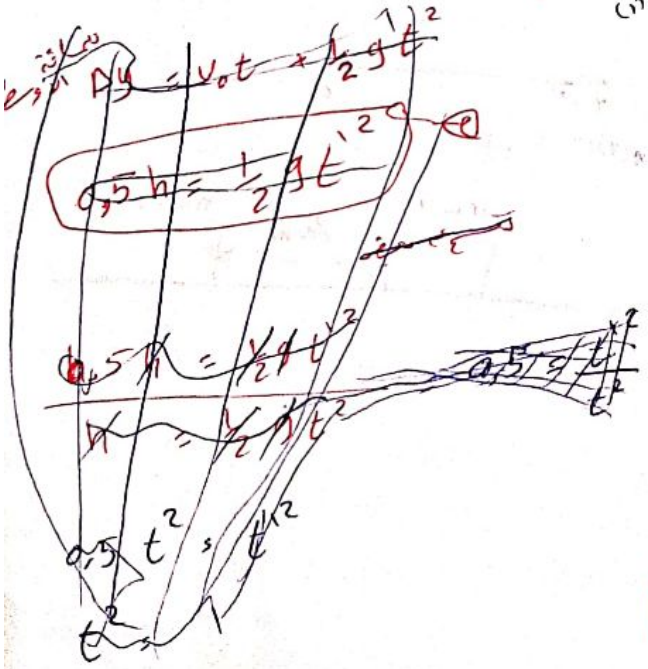
$V = \sqrt{2g(0,5h)}$ ①

$0,5h = V_0 t + \frac{1}{2} g t^2$ ← 1sec

نعوض H وبنعوض بالمعادلة (1)



$t = 3{,}4$ sec



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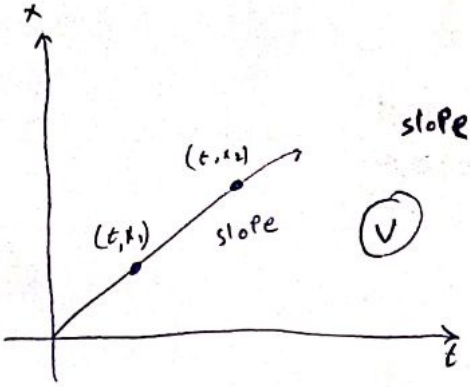
Graphical integration in motion analysis.

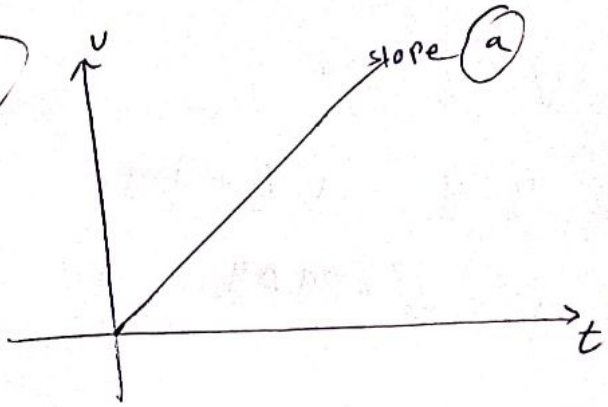
slope ← ميل

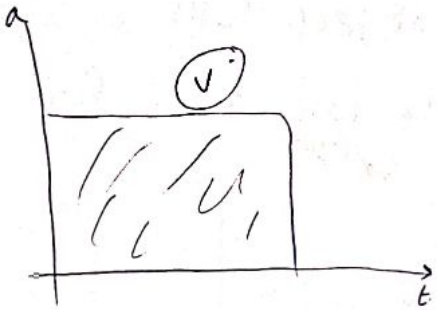
$$x(t) \xrightarrow{\frac{dx}{dt}} v(t) \xrightarrow{\frac{dv}{dt}} a(t)$$

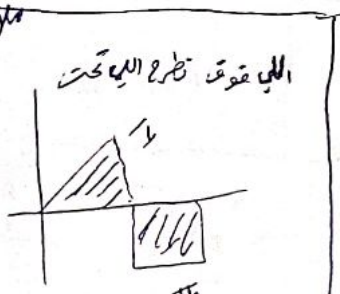
$$x(t) \xleftarrow{\int v\,dt} v(t) \xleftarrow{\int a\,dt} a(t)$$



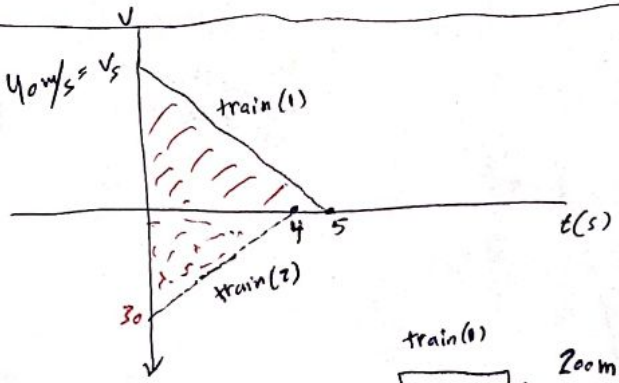
$$\text{slope} = \frac{x_2 - x_1}{t_2 - t_1}$$ ← (v)

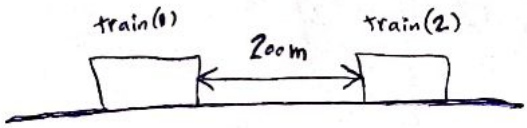






P35





$x(\text{train (1)}) = \frac{1}{2}[5][40] = 100\ m$

$x(\text{train (2)}) = \frac{1}{2}[4][30] = 60\ m$

$\Delta x = 40\ m$

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Chapter 2

① Variables

- distance , displacement
- speed , avg velocity
- avg acceleration
- Inst , a, v,

Constant a

- x-motion
- y-motion
  a = g
  Dx = Dy

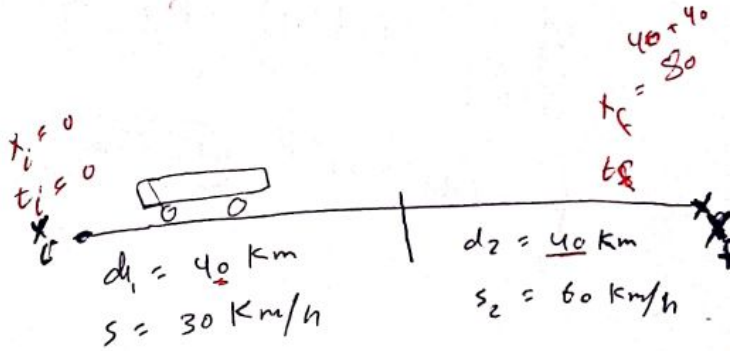
Graphical

Chapter 2

Discation

Q3

$V_{avg} = ??$ ⓐ $V_{Avg} = \frac{x_f - x_i}{t_f - t_i}$ ($x_i = 0$, $t_i = 0$)



$t_1 = \frac{d_1}{s_1} = \frac{40\ km}{30\ km/h} = 1,33h$

$t_2 = \frac{d_2}{s_2} = \frac{40}{60} = 0,67h$

$t_f = 2h$

$V_{avg} = \frac{80 - 0}{2 - 0} = 40\ km/h$

ⓑ $S = \frac{\text{total } d}{\text{total } t} = \frac{80}{2} = 40\ km/h$

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Q17 : $x = 9,75 + 1,5t^3$ [2, 3]

Find (a) the v_{avg} during $t = 2$ sec to $t = 3$ sec

(b) instantaneous v at $t = 2$ sec

(c) " " $t = 3$ sec

(d) when the partical is midway at $t = 2$ sec and 3 sec

(a) $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{x(t=3) - x(t=2)}{3-2}$

$= \frac{50,25\ cm - 21,75\ cm}{3-2} = 28,5\ cm/s$

(b) $v_{ins} = \frac{dx}{dt} = 4,5t^2$

$v|_{t=2} = (4,5)(2)^2 = 18\ cm/s$

(c) $v|_{t=3} = (4,5)(3)^2 = 40,5\ cm/s$

(d) $x_{mid} = \frac{x_1 + x_2}{2} = 36\ cm$

$36 = 9,75 + 1,5t^3$

$t = 2,546$

بعوض بالمشتقة $v|_t = 30,3\ m/s$

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Q22

$x(t) = ct^2 - bt^3$

(a) what unit, y, c, and b? $c \left[\frac{m}{s^2}\right]$, $b \left[\frac{m}{s^3}\right]$

(c) $C = 3$, $b = 2$ at what time the Partical need the maximum ~~Position~~ positive x-Position from $x = 0$ to $t = 4$?

(d) what distance does the Particals move.

(e) " " displacement

(f) find V at $t = 1$ sec

① ..... $t = 2, 3s, 4s$

② find a at $t = 1, 2s, 3s, 4s$.

(c) when partical reach max $\Rightarrow V = 0$

$V = \frac{dx}{dt} = 2ct - 3bt^2 = 0$, $t[2c - 3bt] = 0$

$t \neq 0$

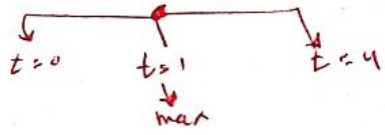
$2c - 3bt = 0$

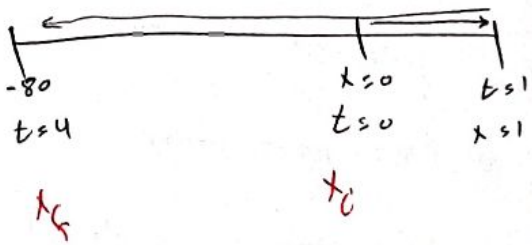
$t = \frac{2c}{3b} = 1 \text{ sec}$

(d) $x(t=4) = -80 \text{ m}$

move to $x = 1$ then

total Path $= |+1| + 80 \text{ m} = 82 \text{ m}$





(e) displacement

$x(t=0) = 0$

$x(t=4) = -80$

$\Delta x = -80 - 0 = -80 \text{ m}$

(f) $V = \frac{dx}{dt} = 2ct - 3bt^2$

$a = \frac{dv}{dt} = 2c - 6bt$

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Q35 $x_{go} = 270\ m$

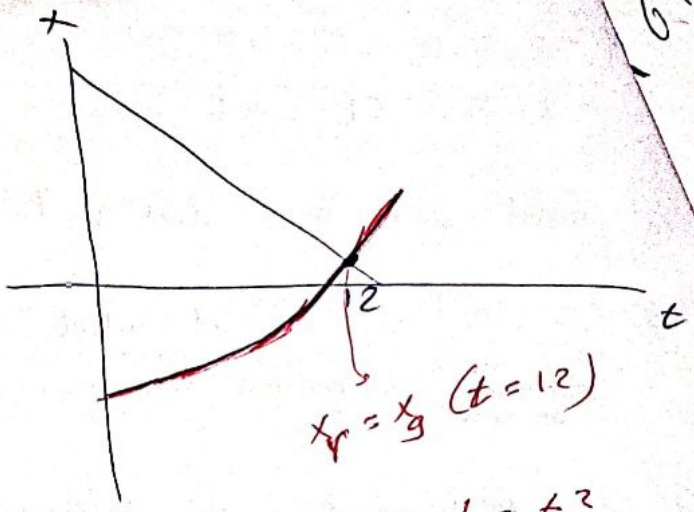
$x_{ro} = -35\ m$

green-car (speed constant)

$= v_{go} = 20\ m/s$ $(a_g = 0)$

red-car begin from rest $[v_{ro} = 0]$

what is a of red car ??



$x_r = x_g \ (t = 12)$

$\Delta x = v_0 t + \frac{1}{2} a t^2$

$x - x_0 = v_0 t + \frac{1}{2} a t^2$

$x = x_0 + \frac{1}{2} a t^2 + v_0 t$

$x_r(t) = x_{r_0} + \frac{1}{2} a_r t^2 + v_0 t$ (the $v_0 t$ term $= 0$)

$x_r(t) = x_{r_0} + \frac{1}{2} a_r t^2$

$x_g = x_{g_0} + v_g t$, $a = 0$

$x_r = x_g$ (when $t = 12$)

$270 - 20(12) = -35 + \frac{1}{2} a_r (12)^2$

$a = 0,9\ m/s^2$

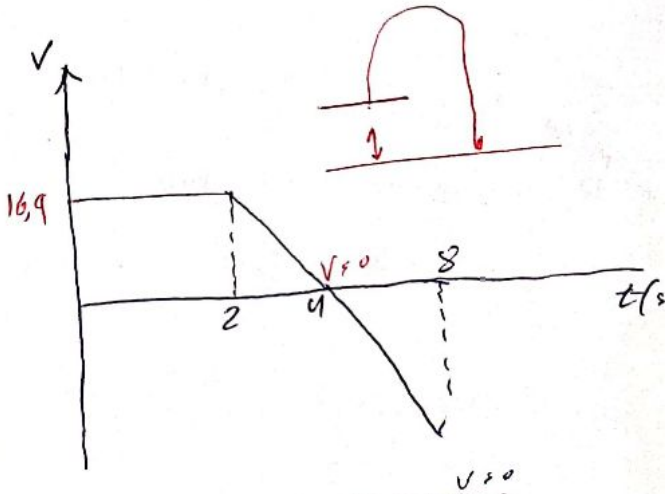
Q51

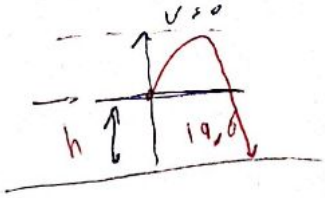
$V = 19,6\ m/s$

vertical velocity

@ max-hight above

(b) how high is breake-face point above the ground.





@ $V = V_0 + gt$

$0 = V_0 - (9,8)(2) = 19,6\ m/s$

$dy = v_0 t + \frac{1}{2} g t^2$

$(19,6)(2) + \frac{1}{2}(-9,8)(2)^2 \simeq 20\ m$

(b) $dy = v_0 t + \frac{1}{2} g t^2$

$= (19,6)(6) + \frac{1}{2}(-9,8)(6)^2 \simeq -59\ m$

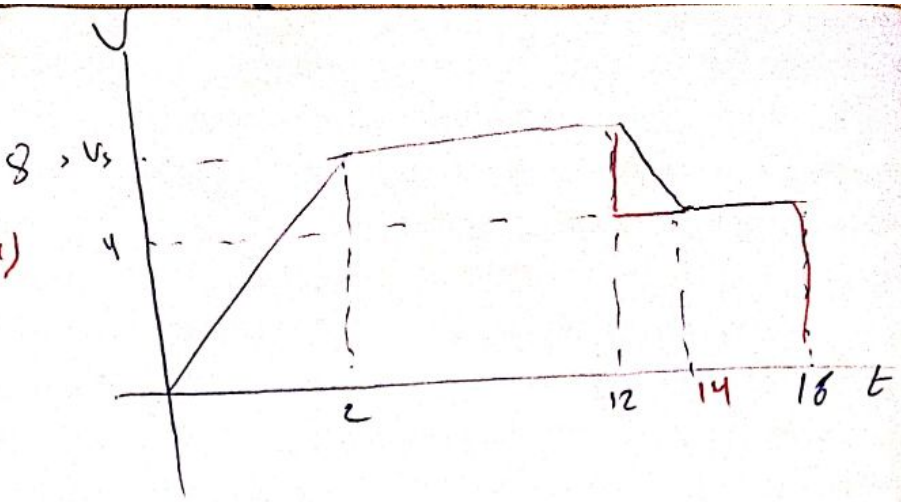
$|dy| = 59,m$

Q9

ΔX = Area (v-t)

$\therefore X = \frac{1}{2}(2)(8) + 8(8) + \frac{1}{2}(7)(4) + 4(4)$

$= 100$ m



Good Luck

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Chapter 3
Vectors.

Physical quantities

- **Scalar**
  - magnitude
  - unit
  - mass, distance
- **Vector**
  - magnitude
  - unit
  - direction
  - Displacement, velocity, acceleration

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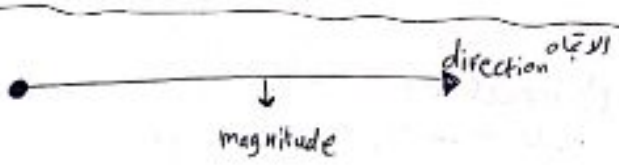
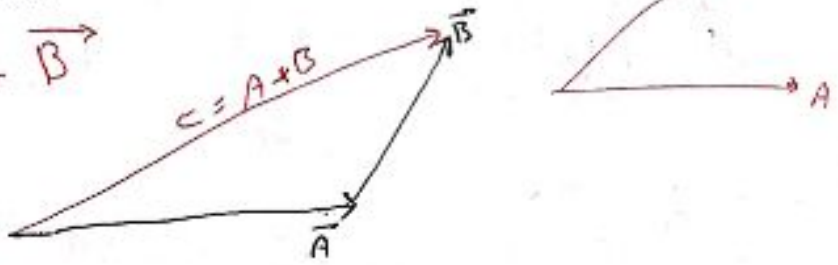
* **Vectors**



vector ① زرع ①
② زرع ② vector ③
③

* **Adding vectors Geometrically.**

$\vec{C} = \vec{A} + \vec{B}$



Exmp: Find $\vec{C} = \vec{A} + \vec{B}$

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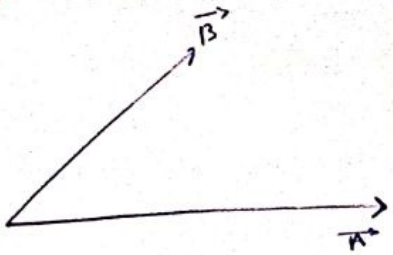
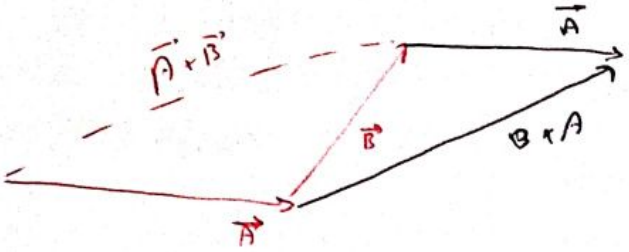
متجهات

① Commutitave التبديلية

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$





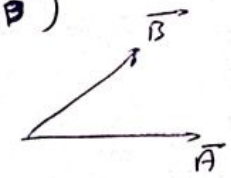
② associative Law : العلاقة التجميعية

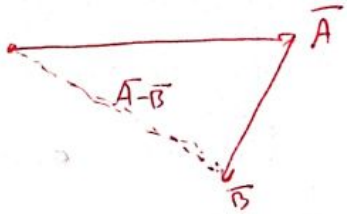
$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

* Subtraction .

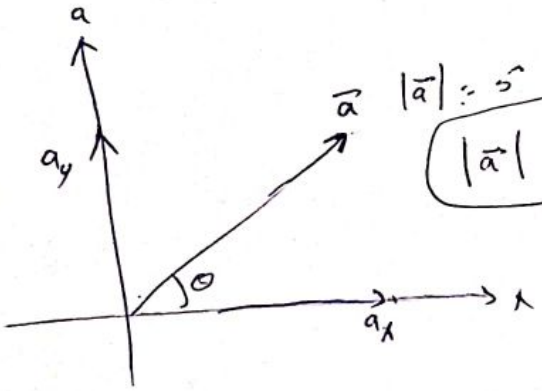
$$\vec{C} = \vec{A} - \vec{B}$$

$$= \vec{A} + (-\vec{B})$$





* Components of vectors مركبات المتجهات



$|\vec{a}| = ?$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

الاشتقاق

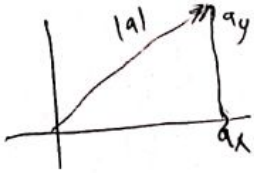
$$a_x = |\vec{a}| \cos\theta$$

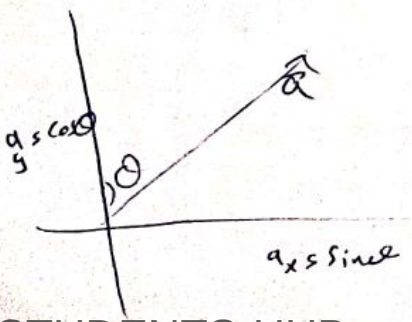
$$a_y = |\vec{a}| \sin\theta$$

$$\sqrt{a_x^2 + a_y^2} = \sqrt{|a|^2 \cos^2\theta + |a|^2 \sin^2\theta}$$

$$= \sqrt{|a|^2 [\cos^2\theta + \sin^2\theta]} = \sqrt{|a|^2} = |\vec{a}|$$

$$\tan\theta = \frac{a_y}{a_x}$$





$$|\vec{a}| = \sqrt{|a_x|^2 + |a_y|^2}$$

$$\tan\theta = \frac{a_x}{a_y}$$

$$\sin(\theta + 2\theta) = 2\sin\theta$$

$$AB\cos\theta = A \cdot B$$

$$\cos B = \cos(B + \theta)$$

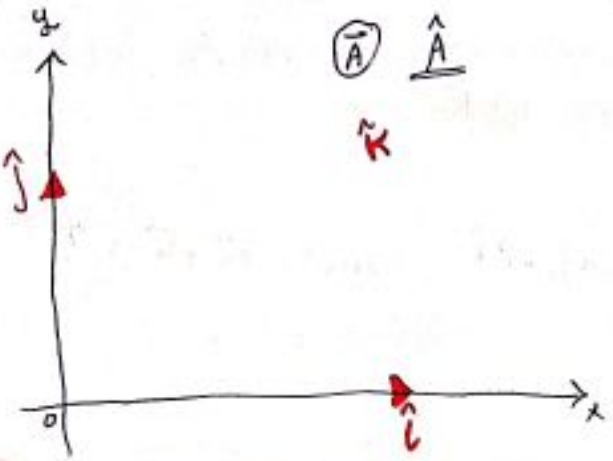
نشتقاق مرتين

$$AB\cos\theta = A \cdot B$$

* Unit vector : Magnitude = 1

متجه الوحدة

direction = in the same direction of the orignal vector



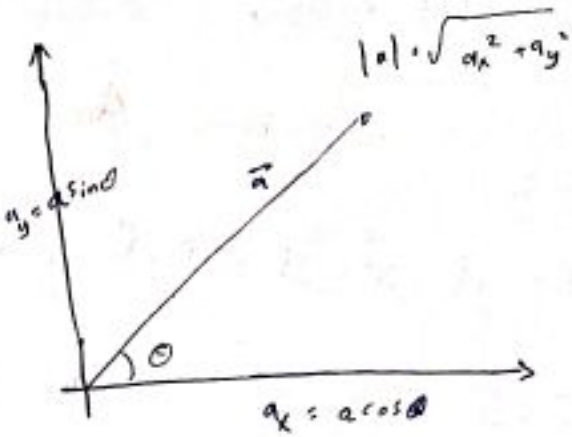
$\vec{A}$ $\hat{A}$

$\hat{k}$

$\hat{i}$ ≡ unite vector in the same direction of X

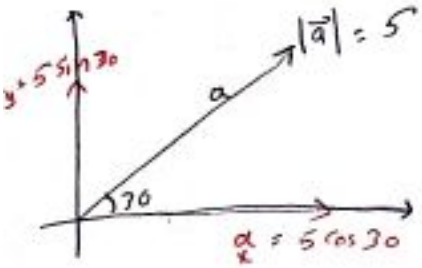
$\hat{j}$ ≡ " " " " " " " " -y

$\hat{k}$ ≡ " " " " " " " " -z



$\vec{a} = a_x\hat{i} + a_y\hat{j}$

Exp :



$\vec{a}$ in the unit vector notation.

$\vec{a} = 5\cos 30\,\hat{i} + 5\sin 30\,\hat{j}$

$\vec{a} = 4.33\,\hat{i} + 2.5\,\hat{j}$

بشكل عام

$A = A_x\hat{i} + A_y\hat{j}$

Exp :- $\vec{A} = 2\hat{i} + 3\hat{j}$ ① find $|\vec{A}| = ??$ ② find $\hat{A} = ??$

① $|\vec{A}| = \sqrt{(2)^2 + (3)^2} = \sqrt{13} = 3.6$

② $\hat{A}$ = unite vector in the same direction of $\vec{A}$

$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 3\hat{j}}{3.6}$

$= \frac{2}{3.6}\hat{i} + \frac{3}{3.6}\hat{j} = 0.55\,\hat{i} + 0.83\,\hat{j}$

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* Adding vectors :-

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ Find $\vec{c} = \vec{a} + \vec{b}$:?!

$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$

$\vec{c} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$

Exp: if $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 2\hat{k}$ Find $\vec{a} + \vec{b}$ = ?!

$\vec{a} + \vec{b} = (2+1)\hat{i} + (3-4)\hat{j} + (-1+2)\hat{k}$

$\vec{a}+\vec{b} = 3\hat{i} - \hat{j} + \hat{k}$ $\Rightarrow |\vec{a}+\vec{b}| = \sqrt{(3)^2 + (-1)^2 + (1)^2} = \sqrt{9+1+1} = \sqrt{11}$

Chapter 3 Part 2

$|\vec{a}|$, $|\vec{b}|$, θ ; addition

$|\vec{c}| = |\vec{a}+\vec{b}| = \sqrt{|a|^2 + |b|^2 + 2|a||b|\cos\theta}$

حالات خاصة ① $\vec{a}$, $\vec{b}$ in the same direction

$|\vec{a}| = 5$, $|\vec{b}| = 6$ $\Rightarrow$ $|\vec{c}| = |\vec{a}| + |\vec{b}|$

② $\vec{a}$, $\vec{b}$ in opposite direction in the same direction of the largest vector

$\vec{b} = 7$ ← $\vec{a} = 5$ $|\vec{c}| = |\vec{b} - \vec{a}|$ (القيمة المطلقة لأكبر - أصغر)

③ $\vec{a}$, $\vec{b}$, Normal ($\perp$) $\theta = 90$

$|\vec{c}| = \sqrt{|a|^2 + |b|^2}$

$\tan\theta = \frac{|a|}{|b|}$

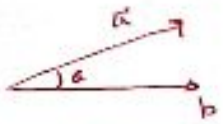
* Multiplying Vectors: ضرب المتجهات

① dot Product الضرب النقطي (يعطي رقم)

② cross Prodicut الضرب التقاطعي (يعطي متجه)

* dot product

① $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$



if $(\vec{a} \perp \vec{b}) \Rightarrow \vec{a} \cdot \vec{b} = 0$

② $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$

find $\vec{a} \cdot \vec{b} = ??$

$\hat{i} \cdot \hat{i} = |\hat{i}||\hat{i}| \cos 0 = 1$

$\hat{j} \cdot \hat{j} = 1$

$\hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = 0$

$\hat{j} \cdot \hat{k} = 0$

$\hat{i} \cdot \hat{k} = 0$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

EXP: if $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, ① find $\vec{a} \cdot \vec{b}$?

$\vec{b} = 3\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{a} \cdot \vec{b} = 6 + 12 - 5 = 13$

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لايجاد $\vec{a} \cdot \vec{b}$

Case I: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

Case II: $a \cdot b = a_x b_x + a_y b_y + a_z b_z$

② find θ between $\vec{a}$ and $\vec{b}$?

$|a| = \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{14}$

$|b| = \sqrt{9 + 16 + 25} = \sqrt{50}$

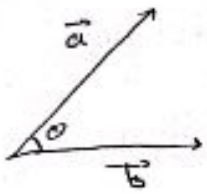
$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$13 = \sqrt{14}\sqrt{50} \cos\theta \Rightarrow \cos\theta = \frac{13}{\sqrt{14(50)}} = 0.49 \Rightarrow \theta \approx 60$

* Cross Product.

Case I..

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta$



$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

Case II :- $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{b} = b_x\hat{i} + b_y\hat{j}$

$\vec{a} \times \vec{b} = (a_x\hat{i} + a_y\hat{j}) \times (b_x\hat{i} + b_y\hat{j}) \Rightarrow a_x b_y \hat{k} - a_y b_x \hat{k}$

Exp. : if $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ find $\vec{a} \times \vec{b} = ??$

$\vec{b} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = (3\times2 - 4\times-1)\hat{i} - (2\times2 - 3\times-1)\hat{i} + (2\times4 - 3\times3)\hat{k}$$

$\vec{a} \times \vec{b} = 10\hat{i} - 7\hat{j} - \hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(10)^2 + (-7)^2 + (-1)^2}$

Also $\vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{c} \perp \vec{a}$, $\vec{c} \perp \vec{b}$

P34 $\vec{a} = 3\hat{i} + 5\hat{j}$ Find (a) $\vec{a} \times \vec{b}$ (b) $\vec{a} \cdot \vec{b}$ (c) $(\vec{a}+\vec{b}) \cdot \vec{b}$

$\vec{b} = 2\hat{i} + 4\hat{j}$ (d) the component of $\vec{a}$ along the direction of $\vec{b}$

(a) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 0 \\ 2 & 4 & 0 \end{vmatrix} = (\ 0\ )\hat{i} - (\ 0\ )\hat{j} + (\ 2\ )\hat{k} = \boxed{2\hat{k}}$

(b) $\vec{a} \cdot \vec{b} = 6 + 20 = \boxed{26}$

(c) $\vec{a} + \vec{b} = 5\hat{i} + 9\hat{j}$

$(\vec{a}+\vec{b}) \cdot \vec{b} = 5 \times 2 + 9 \times 4 = \boxed{46}$

(d) $\vec{a} \cdot \hat{b}$

$\hat{b} = \dfrac{\vec{b}}{|b|} = \dfrac{2\hat{i} + 4\hat{j}}{\sqrt{4+16}} = \boxed{\dfrac{2}{\sqrt{20}}\hat{i} + \dfrac{4}{\sqrt{20}}\hat{j}}$

$\vec{a} \cdot \hat{b}$

$(3\hat{i} + 3\hat{j}) \cdot \left(\dfrac{2}{\sqrt{20}}\hat{i} + \dfrac{4}{\sqrt{20}}\hat{j}\right)$, $\boxed{1,34 + 2,68 = 4}$

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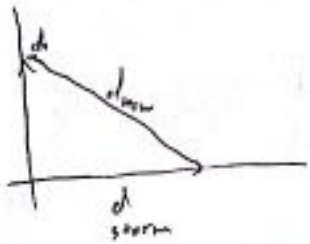
Chapter 3
Discution

Q5 : $\vec{d_o} = 120\ km\ \hat{j}$

$d_{storm} = 100\ \hat{i}$



$\vec{d_o} = \vec{d}_{new} + \vec{d}_{storm}$

$\vec{d}_{new} = \vec{d_o} - \vec{d}_{storm}$

$d_{new} = 120\hat{j} - 100\hat{i}$

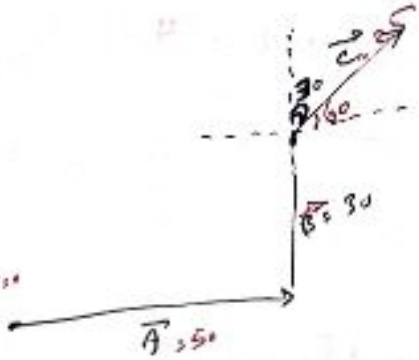
$d_{new} = -100\hat{i} + 120\hat{j}$

$|d_{new}| = \sqrt{(-100)^2 + (120)^2} = 156\ km$

(b) direction $= \tan\theta = \frac{120}{-100} \Rightarrow \theta = -50,2$

Q12 :- $\vec{A} = 50\hat{i}$

$\vec{B} = 30\hat{j}$



$\vec{C} = 25\cos 60\,\hat{i} + 25\sin 60\,\hat{j}$

(a) $\vec{r} = \vec{A} + \vec{B} + \vec{C} = 62.5\,\hat{i} + 51,7\,\hat{j}$

$|\vec{r}| = \sqrt{(62.5)^2 + (51,7)^2} = 81\ km$

(b) $\tan\theta = \frac{51,7}{62,5} \Rightarrow \theta = 40^\circ$ north of east

Q36 If $\vec{d_1} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ what is $\underbrace{(d_1 + d_2)}_{A} \cdot \underbrace{(d_1 \times 4d_2)}_{B}$?

$d_2 = -5\hat{i} + 2\hat{j} - \hat{k}$

(a) $\vec{d_1} + \vec{d_o} = -2\hat{i} + 3\hat{k}$

(B) $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ -20 & 8 & -4 \end{vmatrix} = -24\hat{i} - 68\hat{j} - 16\hat{k}$

$= (A \cdot B) = 0$

d_1, d_2 in the plane

d_{plane} ⊥ d perpendicular to

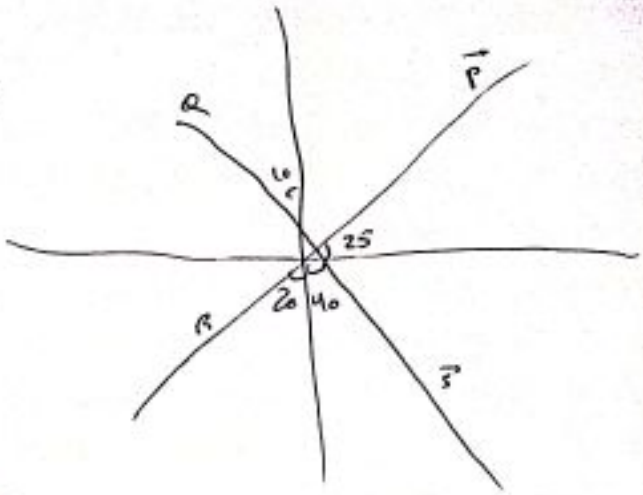
P56
C.

$\vec{P}$: 10 m at 25° counter clock wise +x

$\vec{Q}$: 12 m at 10 clock wise +y

$\vec{R}$: 8 m at 20 colock wise −y

$\vec{S}$: 9 m at 40 counter clock wise −y



$\vec{P} = 10\cos 25\,\hat{i} + 10\sin 25\,\hat{j}$

$\vec{Q} = 12\cos 100\,\hat{j} + 12\sin 100\,\hat{j}$

$\vec{R} = 8\cos 250\,\hat{i} + 8\sin 250\,\hat{j}$

$S = 9\cos 310\,\hat{i} + 9\sin 310\,\hat{j}$

$\vec{P}+\vec{Q}+\vec{R}+\vec{S} = \boxed{10\hat{i} + 1{,}63\,\hat{j}}$

$\sqrt{(10)^2 + (1{,}63)^2} = 10{,}2\ m$

$\tan\theta = \frac{1.63}{10} \Rightarrow \theta = 9{,}24°$

P73) two vectors $\vec{a} = 3\hat{i} + 5\hat{j}$, $\vec{b} = 2\hat{i} + 4\hat{j}$ find ⓐ $\vec{a}\times\vec{b}$

ⓑ $\vec{a}\cdot\vec{b}$ ⓒ $(\vec{a}+\vec{b})\cdot\vec{b}$ ⓓ the component of $\vec{a}$ along $\vec{b}$?

ⓐ $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 5 & 0 \\ 2 & 4 & 0 \end{vmatrix} = 2\hat{k}$

ⓑ $\vec{a}\cdot\vec{b} = 3(2) + 5(4) = 26$

ⓒ $(\vec{a}+\vec{b})\cdot\vec{b}$

$5\hat{i} + 9\hat{j} \cdot b = 10 + 36 = \boxed{46}$

ⓓ $\hat{b} = \frac{\vec{b}}{|b|} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{(2)^2 + (4)^2}} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{20}} = \frac{2}{\sqrt{20}}\hat{i} + \frac{4}{\sqrt{20}}\hat{j}$

$\vec{a}\cdot\hat{b} = \frac{3(2) + 5(4)}{\sqrt{20}} = 5{,}81$

Chapter 4

motion in two and three dimensions

① Position Vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Example: $\vec{r} = -3\hat{i} + 2\hat{j} + 5\hat{k}$

② Displacement Vector

If the objects is displaced from position r_1 to r_2

$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

Sample problem 4.1

$\vec{r_1} = -3\hat{i} + 2\hat{j} + 5\hat{k}$, $\vec{r_2} = 9\hat{i} + 2\hat{j} + 8\hat{k}$ find $\Delta\vec{r}$

$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$

$= (9 - (-3))\hat{i} + (2 - 2)\hat{j} + (8 - 5)\hat{k} = 12\hat{i} + 3\hat{k}$

Q1: A Partical goes from $x = -2$, $y = 3$, $z = 1$ to $x = 3$, $y = -1$, $z = 4$

It's displacement is.

$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (3 - -2)\hat{i} + (-1 - 3)\hat{j} + (4 - 1)\hat{k} = 5\hat{i} - 4\hat{j} + 3\hat{k}$

ⓐ $9\hat{i} + 2\hat{j} + 5\hat{k}$

ⓑ $5\hat{i} - 4\hat{j} + 3\hat{k}$

ⓒ $-5\hat{i} + 4\hat{j} - 3\hat{k}$

ⓓ $-5\hat{i} - 2\hat{j} = 3\hat{k}$

سؤال 16 نسخة بلاك

$x = v_0\sqrt{\frac{2y}{g}}$

Q2 $r_1 = 2\hat{i} + 5\hat{j} - 12\hat{k}$

$r_2 = 2\hat{i} + 5\hat{j} - 8\hat{k}$

$\Delta r = r_2 - r_1$

$= (2 - 2)\hat{i} + (5 - 5)\hat{j} + (-8 - -12)\hat{k} = 4\hat{k}$

* Average velocity

$$v_{avg} = \frac{\text{displacement}}{\text{interval time}} = \frac{\Delta \vec{r}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$= v_{avg}\hat{i} + v_{avg}\hat{j} + v_{avg}\hat{k}$$

* Average acceleration

$$\vec{a}_{avg} = \frac{\text{Change in velocity}}{\text{interval time}} = \frac{\Delta v}{\Delta t}$$

$$= \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\hat{k}$$

$$= a_{avg}\hat{i} + a_{avg}\hat{j} + a_{avg}\hat{k}$$

* instantaneous velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

* instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

* Sample Problem 4, 2

$x = -0,31t^2 + 7,2t + 28$

$y = 0,22t^2 - 9,1t + 30$

@ At $t = 15\ s$, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

$$\vec{r} = x\hat{i} + y\hat{j} = (-0,31t^2 + 7,2t + 28) + (0,22t^2 - 9,1t + 30)$$

$$\vec{r}(15) = (-0,31(15)^2 + 7,2(15) + 28) + (0,22(15)^2 + 9,1(15) + 30)$$

$$= (66\,m)\hat{i} + (-57\,m)\hat{j}$$

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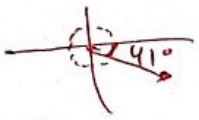
$$r = \sqrt{x^2 + y^2} = \sqrt{(66)^2 + (-57)^2} = 87\,m$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-57}{66} = -41^\circ$$



(b) find the velocity of the rabbit at the instant $t = 15\ s$?

$$\vec{V} = V_x \hat{i} + V_y \hat{j}$$

$$V_x = \frac{dx}{dt} = -0,62t + 7,2 \Rightarrow V_x(t=15) = -3\ m/s$$

$$V_y = \frac{dy}{dt} = 0,44t - 9,1 \Rightarrow V_y(t=15) = -2,5\ m/s$$

$$\vec{V} = (-2,1)\hat{i} + (-2,5m)\hat{j}$$

(c) find the acceleration at $t = 15\ s$

$$\vec{V_x} = -0,62t + 7,2 \Rightarrow a_x = \frac{dv}{dt} = -0,62t + 7,2 = -0,62\ m/s^2$$

$$\vec{V_y} = 0,44t - 9,1 \Rightarrow a_y = \frac{dv_y}{dt} = 0,44t - 9,1 = 0,44\ m/s^2$$

$$\vec{a} = -0,62\hat{i} + 0,44\hat{j}$$

$$\vec{a} = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0,62)^2 + (0,44)^2} = 0,76\ m/s^2$$

$$\theta = \tan^{-1}\frac{0,44}{-0,62} = -35°$$

$-35 + 180 = 145°$

1) $\Delta\vec{r} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\vec{r_f} = 3\hat{j} - 4\hat{k}$

$\Delta r = \vec{r_f} - \vec{r_i} \Rightarrow r_0 = r_f - \Delta r$

$2\hat{i} - 3\hat{j} + 6\hat{k} = 3\hat{j} - 4\hat{k} - \vec{r_0}$

$r_0 = r - \Delta r$

$= (3\hat{j} - 4\hat{k}) - (2\hat{i} - 3\hat{j} + 6\hat{k}) = -2\hat{i} + 6\hat{j} + (-10)\hat{k}$

(5) $\vec{r_0} = 5\hat{i} - 6\hat{j} + 2\hat{k}$ $t = 10s$ $r = -2\hat{i} + 8\hat{j} - 2\hat{k}$ $\vec{v}_{avg} = ?$

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r_2} - \vec{r_1}}{\Delta t} = \frac{(-2\hat{i} + 8\hat{j} - 2\hat{k}) - (5\hat{i} - 6\hat{j} + 2\hat{k})}{10s}$$

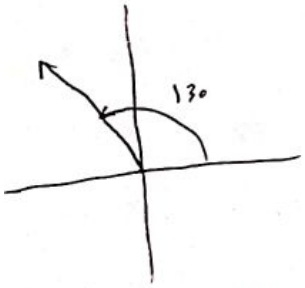
$$= \boxed{-0.7\hat{i} + 1.40\hat{j} - 0.4\hat{k}} \ m/s$$

(11) $\vec{r} = \hat{i} + 4t^2\hat{j} + t\hat{k}$ (a) velocity (b) acceleration.

(a) $\vec{v} = \frac{d\vec{r}}{dt} = \boxed{8t\hat{j} + \hat{k}}$ (b) $\vec{a} = \frac{d\vec{v}}{dt} = 8\hat{j} + \hat{k} = \boxed{8\hat{j}}$

Sample Problem 4.5

$\vec{v_0} = -2\hat{i} + 4\hat{j}$ at $t = 0$ $(a = 3 m/s^2)$ $\theta = 130°$ $\vec{v}$ at $t = 5s$ = ??



$a_x = a\cos\theta = 3\cos 130 = \boxed{-1.93 \ m/s^2}$

$a_y = a\sin\theta = 3\sin 130 = \boxed{2.30 \ m/s^2}$

$\vec{a} = -1.93\hat{i} + 2.30\hat{j}$

$v = v_0 + at$

$v_x = -2 + (-1.93)(5) = \boxed{-11.65 \ m/s}$

$v_y = 4 + (2.30)(5) = \boxed{15.50 \ m/s}$

$\vec{v} = -12\hat{i} + 16\hat{j}$

$v = \sqrt{(-12)^2 + (16)^2} = 19.4 \approx 19$

$\theta = \tan^{-1}\frac{v_y}{v_x} = 127 = \boxed{130°}$

Q1: At $t = 0$ a car moves with velocity $v_0 = 2\hat{i} + \hat{j}$ and $\vec{a} = 2\hat{i}$ the velocity of the car at $t = 2$ is:

$v_{0x} = -2$ $v_{0y} = 4$ $a_x = 0$ $a_y = 2$

$v_x = v_{0x} + a_x t = 2 + 0 \times 2 = \boxed{2}$ $v_y = v_{0y} + at = 1 + 2 \times 2 = 5$

$\vec{v} = v_{0x}\hat{i} + v_{0x}\hat{j} = 2\hat{i} + 5\hat{j}$

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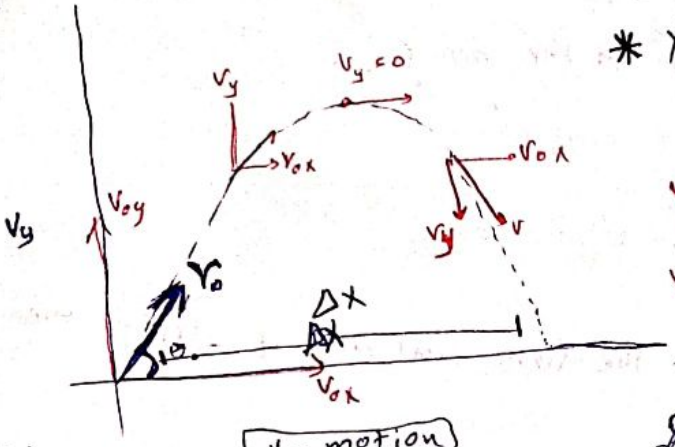
$X = V_0 \sqrt{\frac{2y}{g}}$

Chapter 4 Part 2

Projectile motion
حركة المقذوفات



* X-motion , y-motion are indepen d on each other
ما يعتمد على بعض

$V_{0x} = V_{0x} \cos\theta$

$V_{0y} = V_{0y} \sin\theta = (V_0 \cos\theta)(t)$

ثابتة

X- motion

① $V_{0x} \Longrightarrow$ Constant $(a_x = 0)$
V_x تبقى ثابتة

② $\Delta X = (V_{0x})(t)$

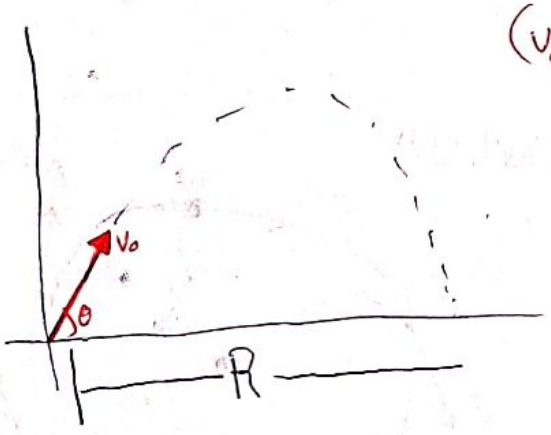
y- motion :-

[1] $V_y = V_{0y} + gt$

[2] $V_y^2 = V_{0y}^2 + 2a\Delta y$

[3] $\Delta y = V_{0y} t + \frac{1}{2} g t^2$

* Range المدى



① $\Delta X = R$

$(V_0 \cos\theta)(t)$ ——①

② $\Delta y = 0$

$V_{0y} t + \frac{1}{2} g t^2 = 0$

$V_0 \sin\theta (t) + \frac{1}{2} g t^2 = 0$ ——②

② في ① نعوض

$$R = \frac{V_0^2 \sin 2\theta_0}{g}$$

* Range max $\longrightarrow$ $\theta_0 = 45^\circ$

* Equation of the path
معادلة المسار

$$y = (\tan\theta_x) X - \frac{g X^2}{2(V_0 \cos\theta_0)^2}$$

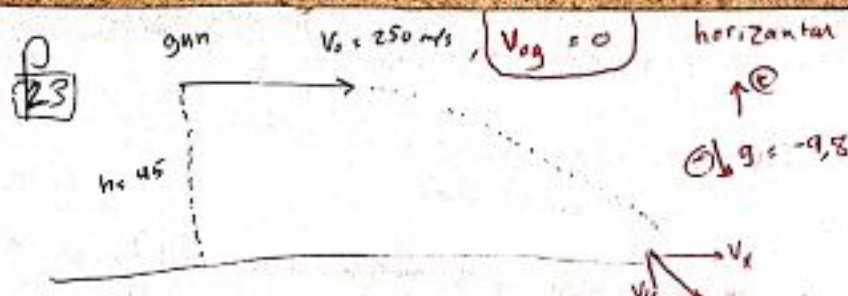
$t = \sqrt{\frac{2y}{g}}$

$X = V_0 t$

$R = V_x t$

$V_f = V_0 \sin\theta - gt$

$y = \frac{1}{2} g t^2$



@ how long does the projectile remain in the air? $(t = ?)$

$$\Delta y = \cancel{V_{0y}}\,t + \tfrac{1}{2}gt^2 \Rightarrow -h = \tfrac{1}{2}(-9.8)t^2$$

$$t = \sqrt{\frac{2h}{g}} = \boxed{3.03 \text{ sec}}$$

(b) At what horizontal distance from the firing point does it strike the ground.

$$\Delta X = \overbrace{V_0 \cos\theta_0}^{V_{0x}}(t) = 250(3.03) = \underline{758 \text{ m}}$$

(c) what is the magnitude of the vertical velocity when it strike the ground

$$V_x = V_{0x} \Rightarrow V_y = \cancel{V_{0y}}^{0} + gt$$

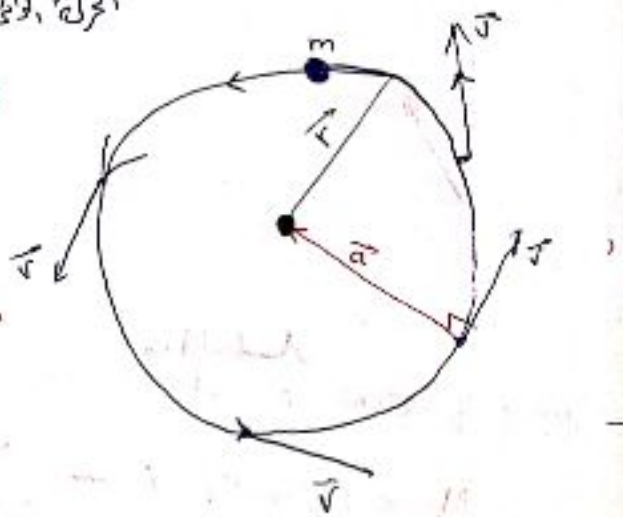
$$V_y = gt = (-9.8)(3.03) = \boxed{-29.6} \Rightarrow |V_y| = \underline{29.6}$$

(d) $|\vec{V}| = \sqrt{v_x^2 + v_y^2}$

(2) Uniform Circular motion

$\vec{V}$ in the direction of the tangent

Uniform $\Rightarrow |\vec{V}| = \text{constant}$



$|a_t| = 0$

$|\vec{V}| = \dfrac{2\pi r}{\text{Time}}$

T ≡ Period

$a_r \equiv$ centeracle acceleration

$(a_r \perp V)$ $\boxed{a_r = \dfrac{v^2}{r}}$

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P 60

$a_{avg} = \frac{v_2 - v_1}{\Delta t}$

$v = \frac{2\pi r}{T}$

$a_r = \frac{v^2}{r}$

$\vec{a} = 6\hat{i} - 4\hat{j}$

ⓐ $\vec{v} \cdot \vec{a}$ → $= 0$ $[\vec{a} \perp \vec{v}]$

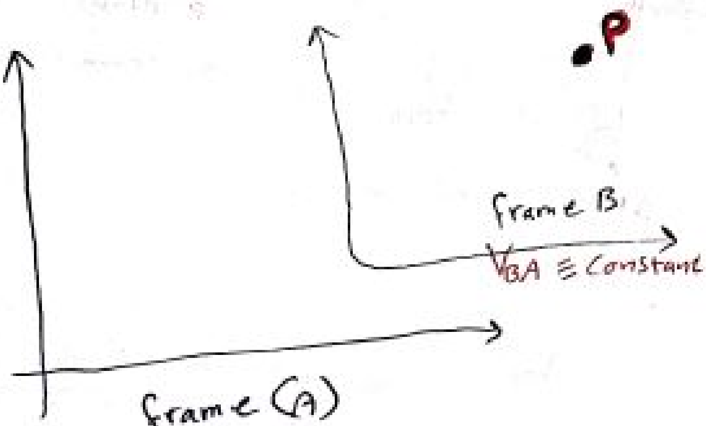
ⓑ $\vec{r} \times \vec{b}$ → 0 $[a \parallel \vec{r}]$ ($\sin 180 = 0$)

$R = vt$

$y = \frac{1}{2} g t^2$

---

* Relative motion in 1-D (الحركة النسبية بعد واحد)



① $X_{PA} = X_{PB} + X_{BA}$

① frame A

② frame B

③ ~~frame B~~ Point

$\frac{d}{dt}[X_{PA}] = \frac{d}{dt}[X_{PB} + X_{BA}]$

② $V_{PA} = V_{PB} + V_{BA}$ (constant)

③ $a_{PA} = a_{PB} + 0$

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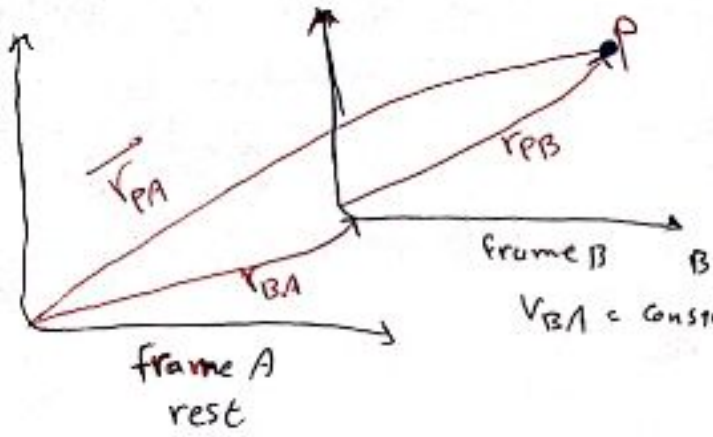
Relative motion in 2-D

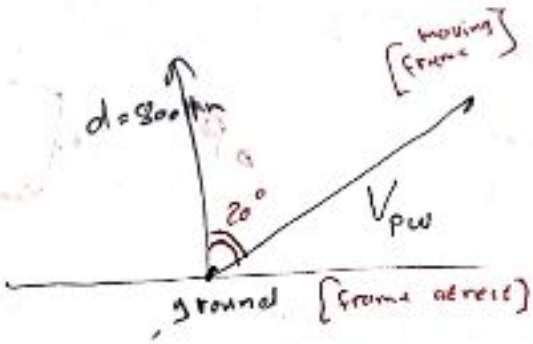


$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$
$$\vec{V}_{PA} = \vec{V}_{PB} + \vec{V}_{BA}$$
$$\vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA}$$

P76)

$\vec{V}_{Pa} = 500$ km/h

$\vec{V}_{Pg} = ??$



a: air الجو
P: plane الطيارة
g: ground الأرض

$\vec{d}_{Pg} = 800\hat{j}$ [2h]

$\vec{V}_{Pg} = \vec{V}_{Pa} + \vec{V}_{ag}$

$|V_{Pg}| = \frac{800 \text{ Km}}{2h} = 400 \text{ Km/h}\ \hat{j}$

$\vec{V}_{Pa} = 500\cos 70\,\hat{i} + 500\sin 70\,\hat{j}$

$= 171\hat{i} + 470\hat{j}$

$\vec{V}_{ag} = \vec{V}_{Pg} - \vec{V}_{Pa}$

$= 400\hat{j} - (171\hat{i} + 470\hat{j})$

$\vec{V}_{ag} = -171\hat{i} - 70\hat{j}$ ans

تابعين

Variables of motion

- $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$
- $V_{avg} = \frac{\Delta r}{\Delta t}$, $V_{ins} = \frac{dr}{dt}$
- $a_{avg} = \frac{\Delta v}{\Delta t}$, $a_{ins} = \frac{dv}{dt}$

Projectile motion

$R = \frac{v_0^2 \sin 2\theta_0}{g}$

Uniform circular motion

$v = \frac{2\pi r}{T}$

$a = \frac{v^2}{r}$

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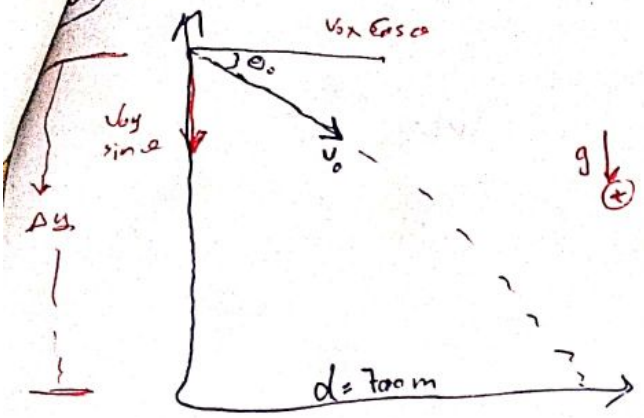
Discussion chapter 4



$\Delta x = V_{ox} t$

$t = \frac{d}{V_{ox}} = \frac{d}{V_o \cos\theta_o} = \frac{700}{80,5 (\cos 30)} = 10s$

$V_o = 290$ Km/h @ $t = ??$

$\theta_o = 30°$

$\frac{290 \times 1000}{3600} = 80,5$ m/s

(b) $\Delta y = ??$ $\quad \Delta y = V_{oy} t + \frac{1}{2} g t^2$

$\Delta y = V_o \sin\theta t + \frac{1}{2} g t^2$

$= 80,5 \sin 30 (10) + \frac{1}{2} (9,8)(10)^2 = 892,5$ m

P32

بدي @ $\Delta y = V_{oy} t + \frac{1}{2} g t^2$ | $d = V_{ox} t$

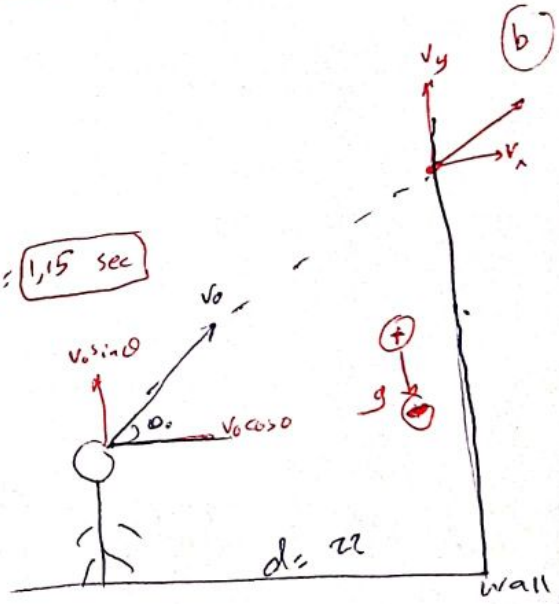
$t = \frac{d}{V_o \cos\theta_o} = \frac{22}{25 \cos 40} = 1,15$ sec

$\Delta y = 25 \sin 40 (1,15) + \frac{1}{2} (9,8)(1,15)^2$

$= 12$ m

(b) $V_x = V_{ox} = 25 \cos 40 = 14,2$ m/s

$V_y = V_{oy} + g t = 25 \sin 40 - 9,8 (1,15) = 4,8$ m/s



$V_o = 25$

$\theta_o = 40°$

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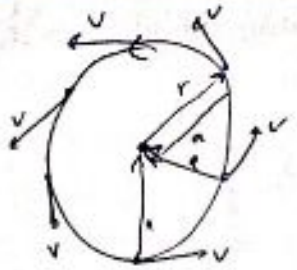
P60



uniform $|\vec{v}|$ = constant

$r = 3m$

$\vec{a} = 6\hat{i} - 4\hat{j}$ find (a) $\vec{v}\cdot\vec{a}$ (b) $\vec{r}\times\vec{a}$

$|v| = \frac{2\pi r}{T}$

$|\vec{a}_r| = \frac{v^2}{r}$

(a) $\vec{v}\cdot\vec{a} = |\vec{v}||\vec{a}|\cos\theta = 0$ ($\cos 90$)

(b) $\vec{r}\times\vec{a} = |\vec{r}||\vec{a}|\sin\theta = 0$ ($\theta = 180$)

P76 : find $\overline{V_{ag}}$ = ??

P = plan طيار
g = ground الأرض
a = air الهواء

$d_{pg} = 500\ km\ (2h)$

$V_{pg} = \frac{500}{2} = \boxed{400\hat{j}}$

$V_{pa} = 500\cos 70\hat{i} + 500\sin 70\hat{j} = \square$

$500\sin 70$

$P_{pa} = 500\ km/h$

20

70

$500\cos 70$

$\vec{V}_{pg} = \vec{V}_{pa} + \vec{V}_{ag}$

$V_{ag} = V_{pg} - V_{pa} = \boxed{-171\hat{i} - 70\hat{j}}$

$|\vec{V}_{ag}| = \sqrt{(-171)^2 + (-70)^2} = 185\ km/h$

سرعة الطائرة بالنسبة للأرض = سرعة الطائرة بالنسبة للهواء + سرعة الهواء بالنسبة للأرض

(Constant)

Good Luck

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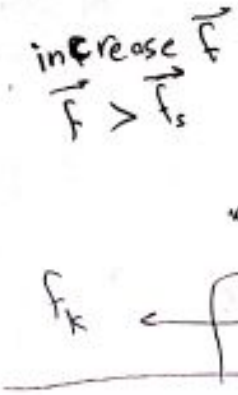
Chapter 6

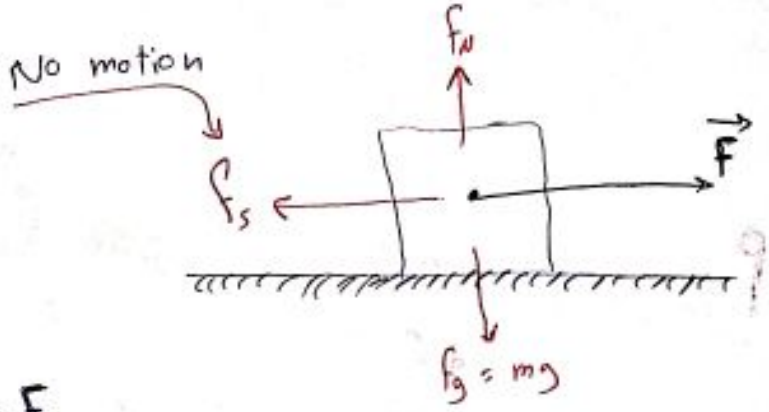
Forces and Motion II

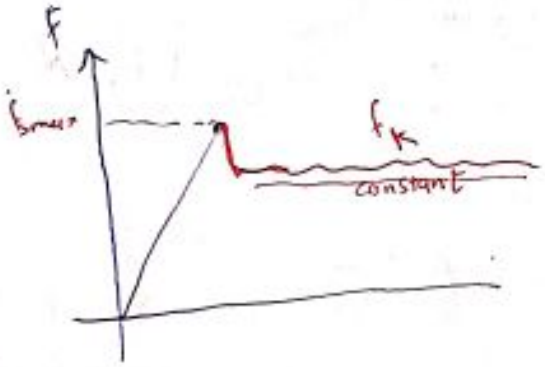
* friction

① Static frictional force الاحتكاك السكوني (f_s) $(F_{s,max})$

② Kientick frictional force الاحتكاك الحركي







① $f_{s,max} = \mu_s f_N$, μ_s = coffe static fraction

f_N = Normal fors

② $f_k = \mu_k f_N$, μ_k = Coff. Kentick Friction

$\mu_k << \mu_s$

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اذا كانت $f > f_{smax}$ تكون $f_s = \mu_k N$

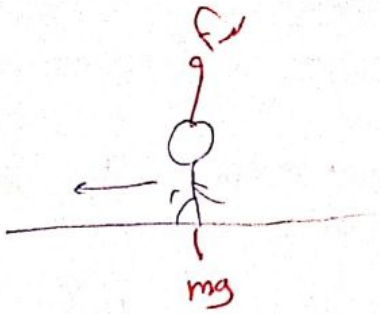
اما اذا كانت $f < f_{smax}$ جسم ثابت تكون $f_s = f$

$mg - \mu_k g = a$

P_6 : $m = 79\ kg$, $f_k = 470\ N$, $\mu_k = ??$

$f_k = \mu_k f_N$

$\mu_k = \frac{f_k}{f_N} = \frac{470}{774,2} = \boxed{0,61}$



μ_s , μ_k دائما أقل من واحد ودوما لا وحدة

$f_N - mg = 0$

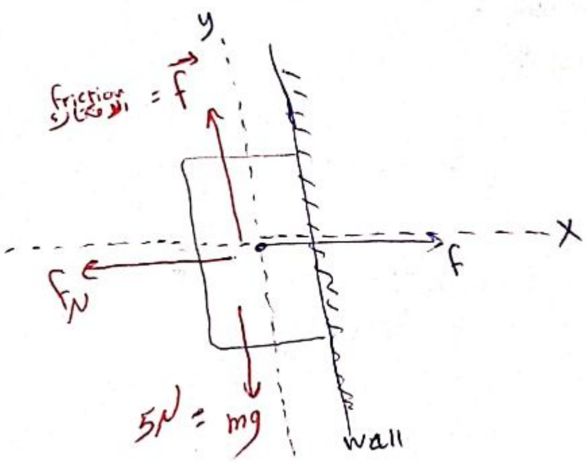
$f_N = mg$

$= 79(9,8) = \boxed{774,2\ N}$

P_{19} : $|\vec{F}| = 12N$

$\mu_s = 0,6$

$\mu_k = 0,4$



f_N عمودية على السطح دائماً

ⓐ Will the block move?

if $f >> f_{s\,max}$ [moving]

$f_{s\,max} = \mu_s f_N = 0,6(12) = \boxed{7,2N}$

$\vec{f} = mg = \boxed{5N}$

$f < f_{s\,max}$ No moving

ⓑ In unit vector notition what is the force from the wall on the block?

$\vec{F}_{wall} = -f_N \hat{i} + f \hat{j}$

$\boxed{F_{wall} = -12\hat{i} + 5\hat{j}}$

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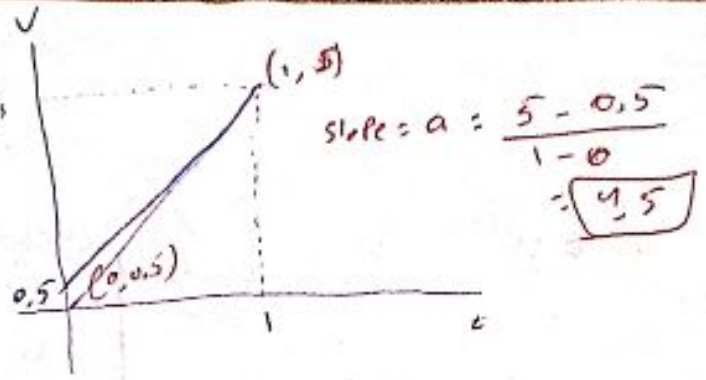
4 : $m = 4,1\ kg$

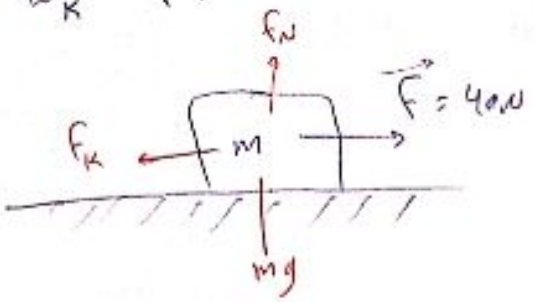
$v_s = 5\ m/s$

$f = 40\ N$

$\mu_k = ??$



slope $= a = \frac{5 - 0,5}{1 - 0} = 4,5$



$\vec{f} - \vec{f}_K = ma$

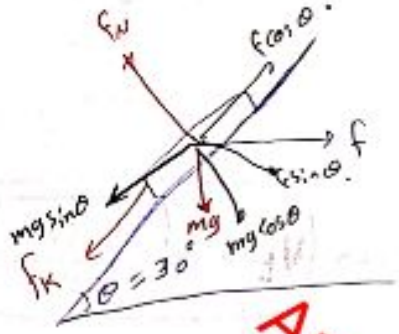
$f_K = f - ma$

$= 40 - 4,1(4,5)$

$= 21,5\ N$

$f_K = \mu_k f_N \Rightarrow \mu_k = \frac{f_K}{f_N} = \frac{21,5}{41} = 0,53$

Exp. :



$f = 12N$

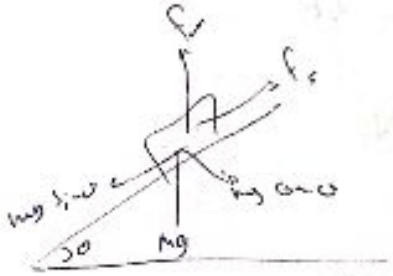
$m = 1\ kg$

$\mu_K = 0,25$

* find the acceleration of the block ?

$f\cos\theta - (mg\sin\theta + f_K) = m\ddot{x}$

$F_K = \mu_k f_N = \mu_k [f\sin\theta + mg\cos\theta]$



$mg\sin\theta = f_s = \mu_s f_N$ — (1)

$f_N = mg\cos\theta$

$mg\sin\theta = \mu_s mg\cos\theta$

$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\mu_s = \tan\theta$

Chapter 6 Part 2

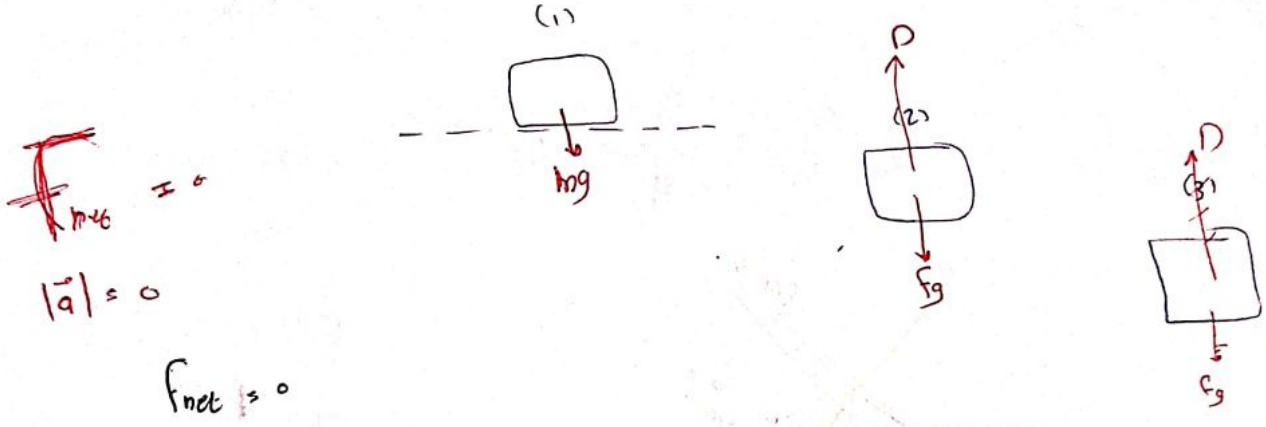
Drag force, terminal speed.

$0 < C < 1$

Fluids (موائع) → liquids (سوائل), gasses (غازات)

$$D = \frac{1}{2} C \rho A v^2$$

- D: Drage force
- C: Drag coff
- ρ: density of flouid
- A: cross sectinical Area
- v: speed



$\sum F_{net} = 0$

$|\vec{a}| = 0$

$f_{net} = 0$

$D - f_g = 0$

$D = f_g = \frac{1}{2} C \rho A v_t^2 = f_g \Rightarrow$ **$v_t = \sqrt{\frac{2 f_g}{C \rho A}}$**

Sample Problem 6, 3

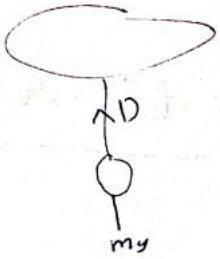
$R = 1,5$ mm $\times 10^{-6}$ (تحويل)

$h = 1200$ m

$C = 0,6$

* Assume the drop is sphere (كرة), find the terminal speed if $\rho_w = 1000$ kg/m^3

$\rho_a = 1,2$ kg/m^3



$$v_t = \sqrt{\frac{2 f_g}{C \rho A}}$$ (ρ هواء)

$f_g = mg = (\rho V) g$

$= \frac{4}{3} \pi R^3 \rho_w g$

$f_g = 0,14$ N

$A = \pi r^2$

$= (3,14)(\quad) = 7,06 \times 10^{-4}$ m/s

$v_t = 7,94$ m/s

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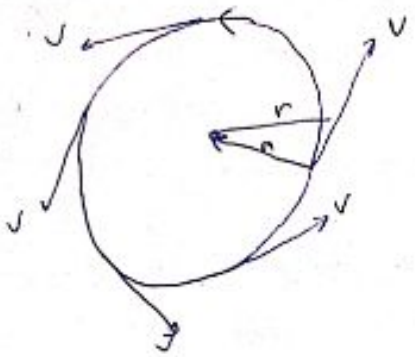
Uniform Circular motion.



$|\vec{v}|$ = Constant

$|\vec{v}| = \frac{2\pi R}{T}$

$a_r = \frac{v^2}{R}$

$f_r = m.a$

$f_r = \frac{mv^2}{R}$

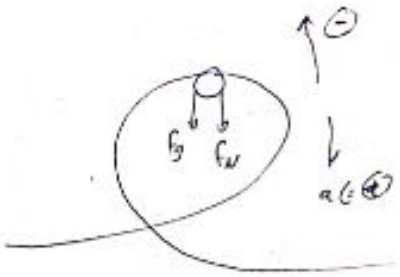
$\tan\theta = \frac{a}{g}$

Case one 1

① Vertical loop

$f_g + f_N = \frac{mv^2}{R}$

f_r

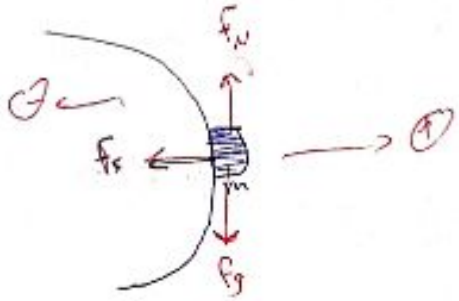


② flat circular motion :-

$-f_s = -\frac{mv^2}{R}$

$f_s = \frac{mv^2}{R}$

$f_N - f_g = 0$



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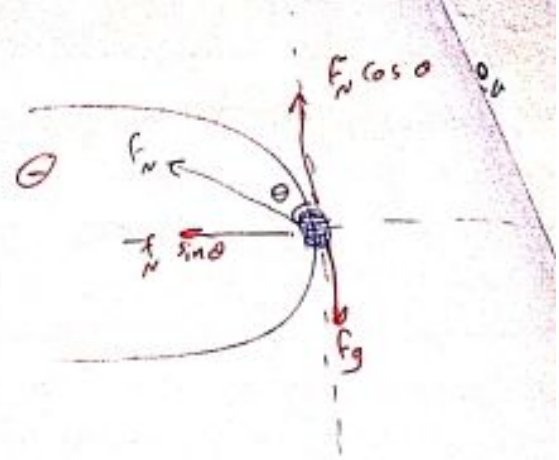
(3) Banked circular motion :-

$-f_N \sin\theta = -\frac{mv^2}{R}$

$f_N \sin\theta = \frac{mv^2}{R}$

$f_N \cos\theta - f_g = 0$

$\tan\theta = \frac{g}{v}$

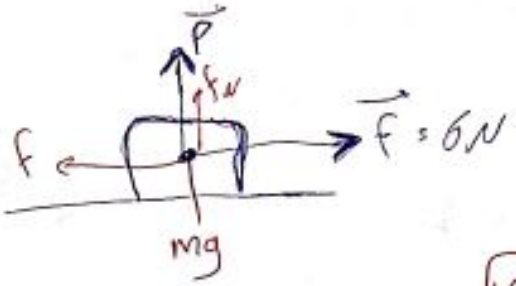


Discussion

Ex: $m = 2.5\ kg$ (at rest)

$\mu_s = 0.4$

$\mu_k = 0.25$



$f_N = mg - P = 16.5\ N$

$= 16\ N$

(a) $|P| = 8\ N$ $f = ??$

$f_{s,max} = \mu_s f_N = 0.4(16.5)$

$= 6.6\ N$

$f < f_{s,max}$ no motion

(b) $|P| = 10\ N$

$f_N = mg - P = 14.5\ N$

$f_{s,max} = \mu_s f_N = 5.8$ moving

$f_k = \mu_k f_N = 0.25(14.5) = 3.6\ N$

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(c) $P = 12\ N$

$f_N = 12.5\ N$

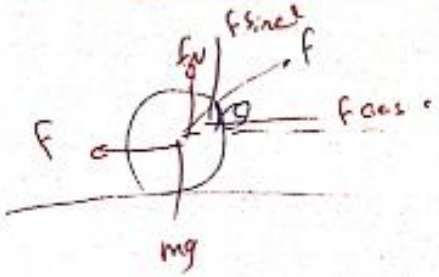
$f_{s,max} = 5\ N$

$f_k = \mu_k f_N = .25(12.5) = 3.1\ N$

10 at rest

$f = 0.5mg$

$\theta = 20$



(a) $\mu_s = 0.6$, $\mu_k = 0.5$

$f_{smax} = \mu_s f_N$

$= 0.6 \times .82mg = .49mg$

$f_{smax} < 0.46$ no moving

(a) $f\cos\theta = (0.5mg)(\cos 20)$

$= 0.46mg$

(-) $f_N = mg - f\sin\theta$

$= mg - 0.5mg \sin 20$

$= .82mg$

(b) $\mu_s = .4$, $\mu_k = 0.3$

$f_{smax} = \mu_s f_N$

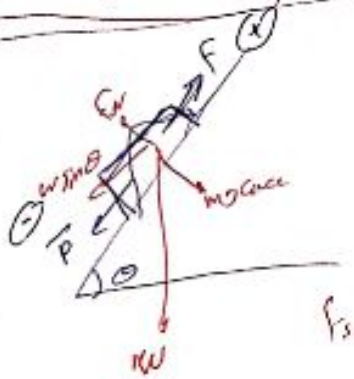
$= .4(0.82mg) = 3.28mg$ moving $f > f\cos\theta$

$f\cos\theta - f_k = ma$

$.46mg - 0.3 = .82mg = ma$

$a = 2.17 m/s$

P17



$w = 45N$ $\mu_s = 0.5$

$\theta = 15°$ $\mu_k = 0.34$

(a) $\vec{P} = -5\hat{i}$

$f_{smax} = \mu_s f_N =$

$= 0.5\, w\cos\theta = .4(45)(\cos 15) = 21.7N$

$w\sin\theta + P = 45\sin 15 + 5 = 16.6N$ no moving

$f_s = |P| + w\sin\theta$

$= 16.6N = 16.6\hat{i}$

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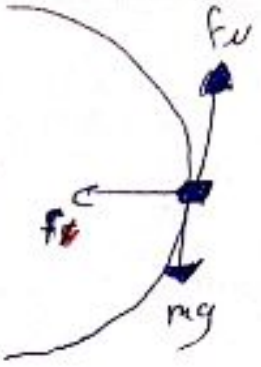
Prob: $\mu_s = 0.6$

$r = 30.5\ m$

flat circular motion



$f \leq f_{s\,max}$

$\frac{mv^2}{R} \leq \mu_s f_N$

$f = ma$

$f = \frac{mv^2}{R}$

$\frac{mv^2}{R} \leq \mu_s mg$

$v^2 \leq \mu_s g R \Rightarrow v \leq \sqrt{\mu_s g R} \Rightarrow v = \sqrt{\mu_s g r} = 13\ m/s$

Good Luck

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