Information and Coding Theory

ENEE 5304

Problem Set 4

Linear Block Codes

1. Consider the (7, 4) linear block code with a generator matrix G:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

    a. Find the codeword corresponding to the message (1011)
    b. Find the parity check matrix H^T such that $GH^T = 0$
    c. Construct the Syndrome table for error correction
    d. Can you identify and correct the error in the received sequence (0001101)

2. Is it possible to design a (15, 11) single error correcting code? Verify using the Hamming bound.
3. Consider the (5, 1) repetition code
    a. Find the minimum distance of the code
    b. How many errors can this code detect?
    c. How many errors can this code correct
    d. If the bit error probability is p, find the error probability when this code is employed.
4. Consider the (6, 3) linear block code with the generating matrix

$$G = \begin{bmatrix} 1 0 0 1 1 0 \\ 0 1 0 0 1 1 \\ 0 0 1 1 0 1 \end{bmatrix}$$

    a. Find all codewords of the code
    b. Find the minimum distance of the code
    c. Find the error correcting capability of the code
    d. Find the codewords corresponding to the messages (001), (010) and (100).

5. Find the error correcting capability of the linear block code (23, 12).