Potential Energy and
8- Conservation of Energy

Forces in Nature:

1) Gravitational Force mg
2) Spring Force $-kx$
3) Normal Force F_N
4) friction Force , $f_{s,m} = \mu_s F_N$

   $f_k = \mu_k F_N$
5) Drag Force $\frac{1}{2} C \rho A v^2$
6) Tension Force

⋮

There are 2 kinds of Forces

Conservative Forces → mg, → $-kx$, → ...

Nonconservative Forces → Friction Force
→ Drag Force
→ ...
→ ...

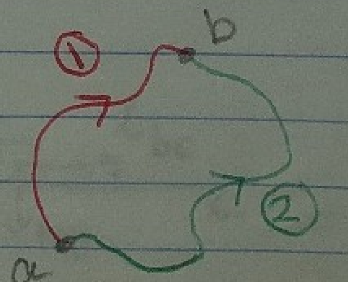
Conservative Forces like mg + $-kx$

have several Properties :-

1) Work done by a Conservative Force is

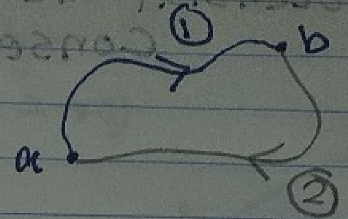
Path independent

$$W_{\substack{con\,(1) \\ a \to b}} = W_{\substack{con\,(2) \\ a \to b}}$$



2) Work done by a conservative Force around a closed Path $= 0$

$$W_{\substack{con.\\ a\to a}} = 0$$



3) Work done against Conservative Force do not dissipated (do not lost) but this Work is stored in the system as Potential Energy ...

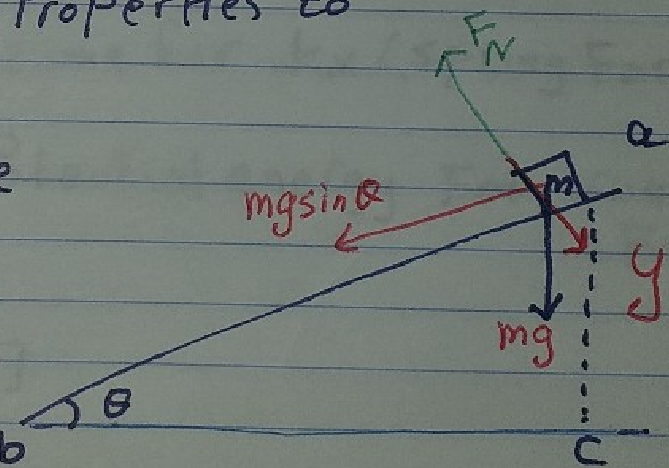
Potential Energy; stored in the system
U   due to its Position
     due to its state

4) Work done by a Conservative Force = (−) change in Potential Energy

$$W_{\substack{conservativeF\\ i\to f}} = -\Delta U \quad \text{Joul}$$
$$= -\left[U_f - U_i\right] \quad \text{Joul}$$

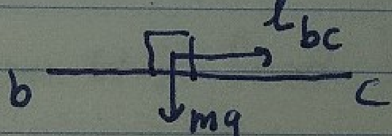
Let us show these Properties to mg Force:

1) Find the Work done by mg in moving (m) From $a \to b \to c$?



$W = \vec{F} \cdot \vec{d}$

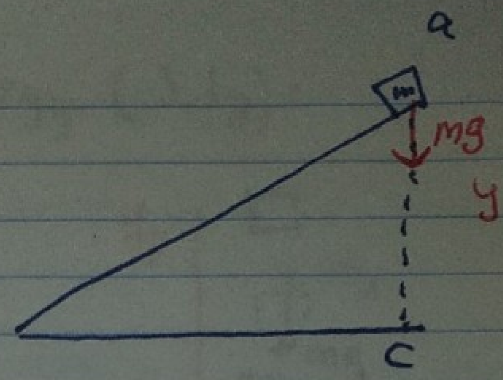
$$W_{\substack{mg\\ a\to b}} = (mg\sin\theta)\, l_{ab} \cos 0$$
$$= (mg\sin\theta)\, l_{ab}$$
$$= mg\left(\frac{y}{l_{ab}}\right) l_{ab} = mgy$$



$$W_{\substack{mg\\ b\to c}} = mg\, l_{bc} \cos 90 = 0$$

$$W_{\substack{mg\\ a\to b\to c}} = W_{mg(a\to b)} + W_{mg(b\to c)} = mgy \quad \text{Joul}$$

② Find the Work done by mg in moving m (from $a \to c$)



$$W_{mg\ a \to c} = mgy\cos 0 = mgy$$

$$\Longrightarrow W_{mg(a\to b\to c)} = W_{mg(a\to c)}$$ (It is Path Independent)

Find the Work done by mg from ($a \to b \to c \to a$)?

$$W_{mg(a\to b\to c\to a)} = W_{mg(a\to b)} + W_{mg(b\to c)} + W_{mg(c\to a)}$$

$$= mgy + 0 + mgy\cos 180$$

$$= 0$$

$W_{mg(a\to a)} = 0$ [Work done by mg around a closed path = 0]

We can do the same for $-kx$ (spring Force)

How to find the Potential energy For Conservative Force?

$$W_{\substack{\text{cons.}\\ i\to f}} = -\Delta U = -[U_f - U_i]$$

$$U_f - U_i = -W_{\text{cons.}(i\to f)}$$

$$U_f - U_i = -\int_i^f \vec{F}_{\text{cons.}} \cdot d\vec{r}$$

Do this integral for mg you will find U_g

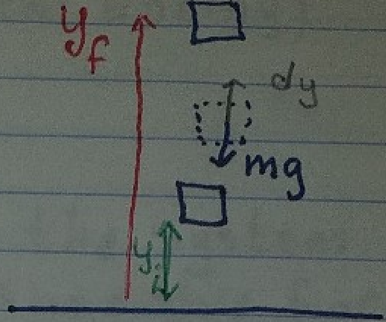
Do this integral for $-kx$ you will find U_s

Gravitational Potential Energy (U_g):

$$U_f - U_i = -\int_{y_i}^{y_f} -mg\,dy$$

$$= \int_{y_i}^{y_f} mg\,dy = mgy_f - mgy_i$$

$$U_f - U_i = mgy_f - mgy_i$$



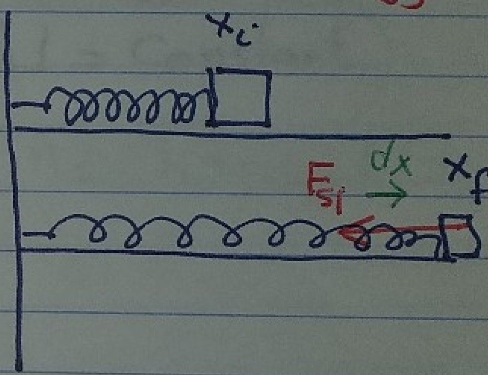
let the zero level for U_g at $y_i = 0$

$$U_f - 0 = mgy_f$$

$$U_g = mgy$$ Joul

gravitational Potential energy

Elastic Potential Energy: (U_s)
Spring Potential Energy:



$$U_f - U_i = -\int_{x_i}^{x_f} -kx\,dx = \int_{x_i}^{x_f} kx\,dx$$

$$U_f - U_i = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

let zero level for U_{spring} at $x_i = 0$

$$U_f = \frac{1}{2}kx_f^2$$

$$U_s = \frac{1}{2}kx^2$$ Joul

~~elastic~~
elastic Potential energy

Conservation of Mechanical Energy:

$$E_{mec} = K + U \quad \text{(mechanical Energy)}$$

$$W_{conservativeF} = -\Delta U$$

If the only Force acting on the System is a Conservative force, then this force is a resultant force $\Rightarrow$

$$W = \Delta K \quad \text{(Work done by } F_{net}\text{)}$$

$$\Delta K = -\Delta U$$

$$\Delta K + \Delta U = 0 \Rightarrow K + U = \text{Constant}$$

conservation of mechanical E

$$(K+U)_i = (K+U)_f$$

Work done by Nonconservative Force:

If $F_{cons.}$ + $F_{nonc.}$ are acting on the system then

$$W_{net} = \Delta K$$

$$W_{cons.} + W_{nonc} = \Delta K$$

$$-\Delta U + W_{nonc.} = \Delta K$$

$$W_{nonc.} = \Delta K + \Delta U$$

$$W_{nonc.} = \Delta E_{mec.}$$

Problem (8-6)

$m = 0.032\,kg$

frictionless loop

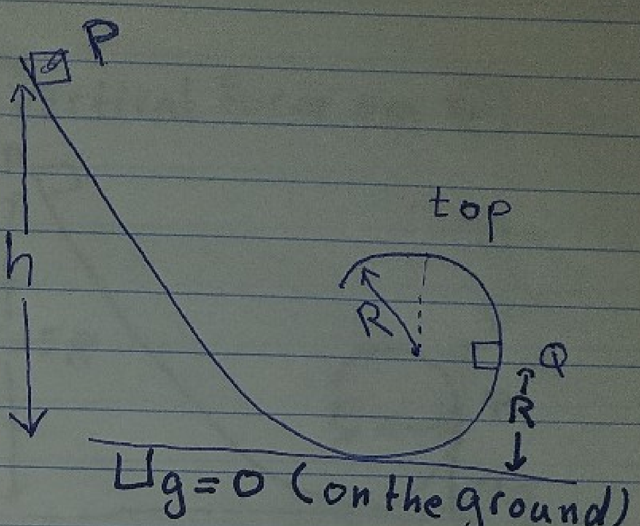
$R = 12\,cm$

At Point P: $V = 0$

$h = 5R$

a) Find $W_{g(P\to Q)}$

b) Find $W_{g(P\to top)}$



$U_{g(P)} = mgh = 5mgR$

$U_{g(Q)} = mgR$

$U_g(top) = 2mgR$

$W_{g(P\to Q)} = -\Delta U = -[U_Q - U_P]$

$= -[mgR - 5mgR] = -[-4mgR]$

a) $W_{g(P\to Q)} = 4mgR = 0.150$ Joul

b) $W_g(P\to t) = -\Delta U = -[U_{top} - U_P] = -[2mgR - 5mgR]$

$W_{g(P\to top)} = 3mgR = 0.113\,J$

If $\boxed{m}$ is given an initial ~~Push~~ speed instead of ~~feal~~ being released from rest

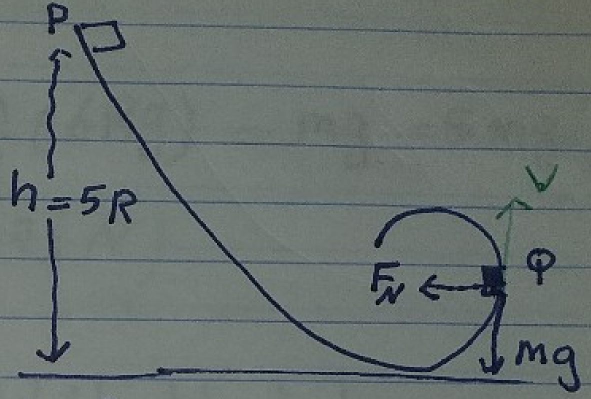
$U_g(P)$, $U_g(Q)$, $U_g(top)$, $W_g(P\to Q)$ and $W_g(P\to top)$

do not change.

Problem (8-17)

In Problem 6

a) find the horizontal force + b) vertical force on m at point Q?



At point Q

2 Forces are acting on m

$mg(-\hat{j})$

$F_N(-\hat{i})$

$\left(F_N = \frac{mv^2}{R}\right)_Q \Rightarrow$ We have to find v at Q.

$(K+U)_P = (K+U)_Q$; No nonconservative forces are acting

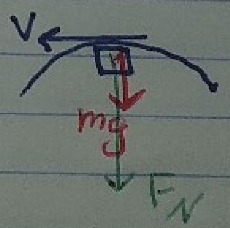
$0 + 5mg = \frac{1}{2}mv_Q^2 + mgR$

$\frac{1}{2}mv_Q^2 = 4mgR$, $v_Q = \sqrt{8Rg}$

$\left(F_N = \frac{mv^2}{R}\right)_Q = \frac{m}{R} \cdot 8Rg = 8mg$

$\vec{F}_Q = -8mg\,\hat{i} - mg\,\hat{j}$ $\quad \vec{F}_Q = -2.5\hat{i} - 0.314\hat{j}$ N

Extra Find F_N at the top?



At the top 2 Forces are acting on m $\to$ $mg(-\hat{j})$, $F_N(-\hat{j})$

$\vec{F}_{net} = mg(-\hat{j}) + F_N(-\hat{j})$

$\frac{mv^2}{R}(-\hat{j}) = -\hat{j}(mg + F_N) \Rightarrow \frac{mv^2}{R} = mg + F_N$

$\left(F_N = \frac{mv^2}{R} - mg\right)_{top}$ you have to find v_{top} from $E_{top} = E_P$

$$(K+U)_P = (K+U)_{top}$$

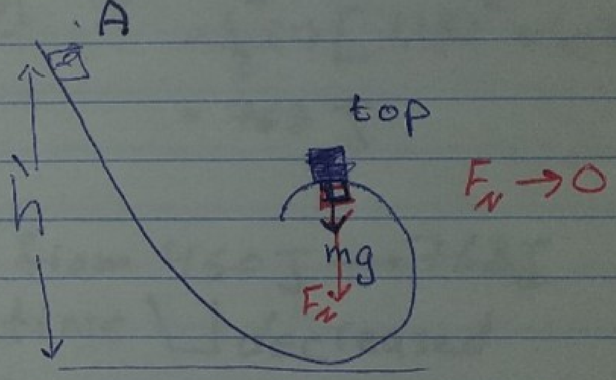
$$0 + 5mgR = \frac{1}{2}mv_t^2 + 2mgR$$

$$v_t = \sqrt{6Rg}$$

$$(F_N)_{top} = \frac{mv_t^2}{R} - mg = \frac{m}{R}(6Rg) - mg = 5mg$$

$$(\vec{F}_N)_{top} = -5mg\hat{j} = -1.57\hat{j}\ N$$

(c) At what height (h) should the block be released from rest so that it is on the verge of losing contact with the track at the top of the loop?



At the top in this case $F_N \longrightarrow 0$

This is the meaning of on the verge of losing contact with the track.

At the top in this case

$$\frac{mv^2}{R} = mg \quad , \text{remember } F_N \longrightarrow 0$$

$$v_t = \sqrt{Rg}$$

$$(K+U)_A = (K+U)_{top}$$

$$0 + mgh' = \frac{1}{2}mv_t^2 + 2mgR$$

$$mgh' = \frac{1}{2}m.Rg + 2mgR = 2.5mRg \quad , R = 12\,cm.$$

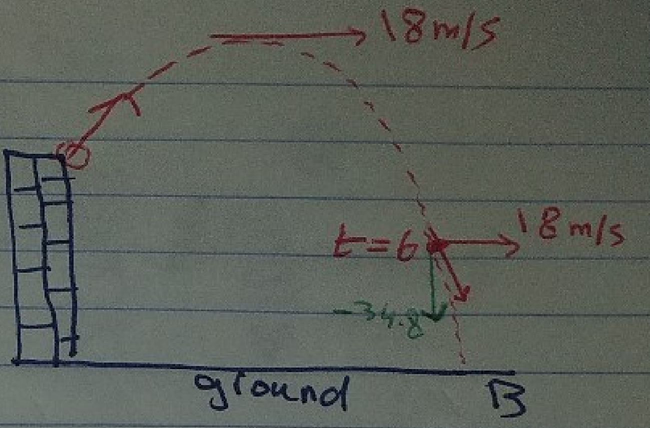
$$h' = 2.5R = 30\,cm$$

(8-25) $m = 1\,kg$

At $t=0$, $\vec{V}_0 = 18\hat{i} + 24\hat{j}$ m/s A

After 6s find

ΔU from $t_1 = 0 \longrightarrow t_2 = 6s$



At $t = 6s$: $V_x = 18$ m/s

$V_y = V_{oy} + a_y t$
$= 24 + -9.8(6)$
$= 24 - 58.8 = -34.8$ m/s

$(K+U)_0 = (K+U)_6$, $K_0 = \frac{1}{2} m V^2 = \frac{1}{2}(\cancel{\;})$

$K_0 + U_0 = K_6 + U_6$ $\qquad = \frac{1}{2}(1)(18^2 + 24^2)$

$\qquad = 450\,J$

$U_6 - U_0 = K_0 - K_6 = -[K_6 - K_0]$ $\quad K_6 = \frac{1}{2} m v^2$

$\Delta U = -\Delta K$ $\qquad = \frac{1}{2}(1)[(18)^2 + (-34.8)^2]$

$\qquad = 768\,J$

$= -[768 - 450]$

$\Delta U = -138\,J$

Note: K increased from $450J \longrightarrow 768J$

At the same time U decreased.

(Extra) Find the work done by gravity from $t=0 \longrightarrow t=6s$

$$W_{g\,(0\to 6)} = -\Delta U = -[-138] = +138\,J$$

(extra) Find ΔU from $t=0 \xrightarrow{to}$ the heighest Point

$t=0 \longrightarrow$ reach y_{max}.

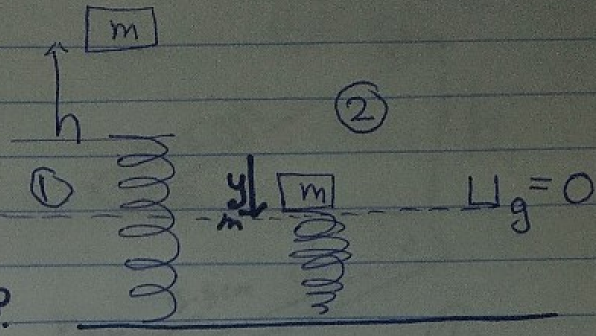
At y_{max} $\vec{v} = 18\hat{i}$ m/s , $v_y = 0$

$$K_0 + U_0 = K_{y_m} + U_{y_m} \quad , \quad U_{y_m} - U_0 = -(K_{y_m} - K_0)$$

$$U_{y_m} - U_0 = -\left[\frac{1}{2}(1)(18)^2 - 450\right] = -[162 - 450]$$

$$U_{y_m} - U_0 = 288\,J$$

(8-24) $m = 2kg$
$h = 40 cm$
$k = 1960 N/m$
Find the maximum distance the spring is compressed? Find y_m?



$$(K+U)_1 = (K+U)_2$$

in this Problem there are 2 Conservative forces mg & $-kx$

$$0 + mg(y_m + h) = 0 + \frac{1}{2}k y_m^2$$

$$2(9.8)(y_m + 0.4) = \frac{1}{2}(1960) y_m^2$$

$$19.6 y_m + 7.84 = 980 y_m^2$$

$$980 y_m^2 - 19.6 y_m - 7.84 = 0$$

$$125 y_m^2 - 2.5 y_m - 1 = 0$$

$$(12.5 y_m + 1)(10 y_m - 1) = 0$$

$$10 y_m - 1 = 0 \quad y_m = \frac{1}{10} = 0.1 m$$

(8-29) $m = 12 kg$ $\theta = 30°$
$v_0 = 0$

spring compressed 2cm by a force of 270N

$$F = kx, \quad k = \frac{F}{x} = \frac{270}{0.02}$$

$$k = 13500 N/m$$

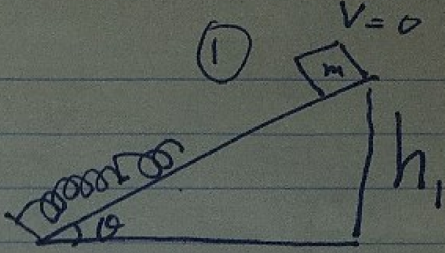
The block stops momentarily when it compress the spring 5.5 cm

a) d ?

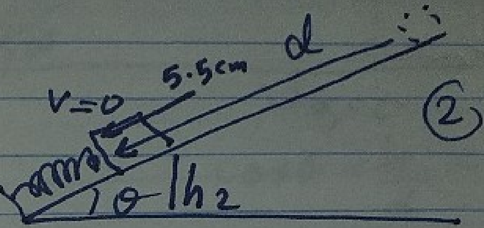
b) v_1 ?

(8-29) $mg, -kx$ are

a) conservative forces



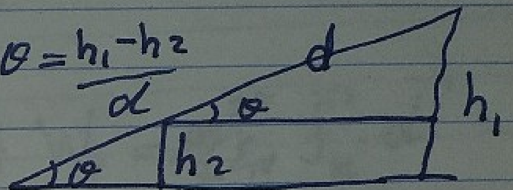
$$(K+U)_1 = (K+U)_2$$



$$0 + mgh_1 = 0 + mgh_2 + \frac{1}{2}kx^2$$

$$mgh_1 - mgh_2 = \frac{1}{2}kx^2$$

$$mg(h_1 - h_2) = \frac{1}{2}kx^2 \quad , \quad \sin\theta = \frac{h_1 - h_2}{d}$$



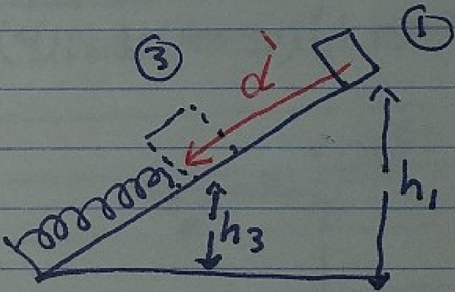
$$mgd\sin\theta = \frac{1}{2}kx^2$$

$$\sin 30 = \frac{h_1 - h_2}{d}$$

$$d = \frac{\frac{1}{2}kx^2}{(\sin\theta)mg}$$

$$d = \frac{(0.5)(13500)(5.5\times10^{-2})^2}{(12)(9.8)\sin 30} = \frac{20.41875}{58.8}$$

$$d = 0.35\,m = 35\,cm$$



b) $(K+U)_1 = (K+U)_3$

$$0 + mgh_1 = \frac{1}{2}mv_3^2 + mgh_3$$

$$mg(h_1 - h_3) = \frac{1}{2}mv_3^2$$

$$\sin\theta = \frac{h_1 - h_3}{d'}$$

$$v_3^2 = 2g(h_1 - h_3)$$

$$h_1 - h_3 = d'\sin\theta$$

$$= 2.89$$

$$= (35 - 5.5)10^{-2}(0.5)$$

$$V_3 = 1.7\,m/s$$

$$= 14.75\,cm$$

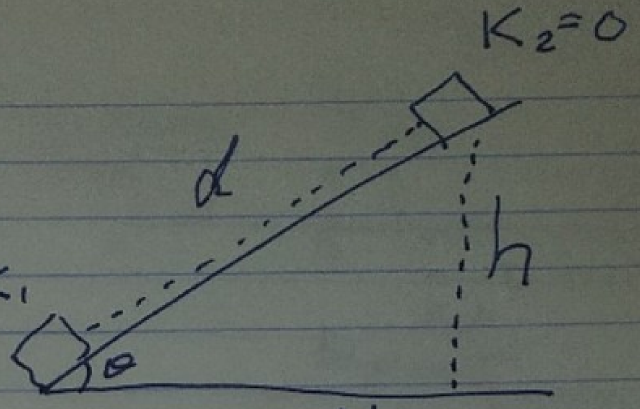
(8-60) $m = 4\,kg$

$\theta = 30°$

$K_1 = 128\,J$

$\mu_k = 0.3$

$d = ?$



$\cos\theta = \frac{h}{d}$

$W_f = \Delta E_{mec}$

$= (K_2 + U_2) - (K_1 + U_1)$

$= (0 + mgh) - (128 + 0)$

$W_f = mgd\cos\theta - 128$, but $W_f = \vec{f}_k \cdot \vec{d}$

$= \mu_k F_N d \cos 180$

$= \overset{(-)}{\mu_k} mg\cos\theta\, d$

$-\mu_k mg\cos\theta\, d = mgd\cos\theta - 128$

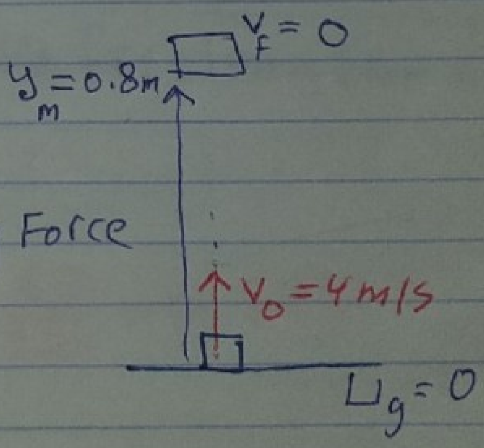
$128 = mgd\cos\theta(1 + \mu_k)$

$d = \frac{128}{mg\cos\theta(1+\mu_k)} = \frac{128}{(4)(9.8)(\cos 30)(1+0.3)}$

$d = \frac{128}{44} = 2.9\,m$

(8-97) $m = 0.5\,kg$

Find Work done by air drag Force



$W_{Drag} = \Delta E = E_f - E_i$

$= (K + U)_f - (K + U)_i$

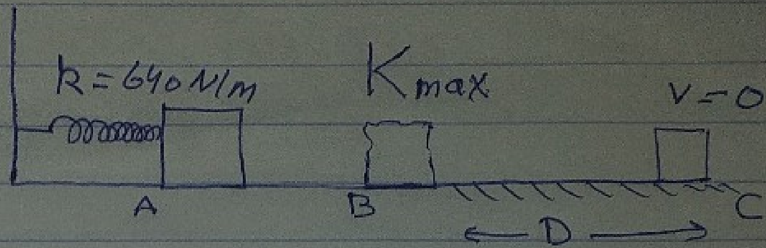
$(0 + mgy_m) - (\frac{1}{2}mv_0^2 - 0)$

$= mgy_m - \frac{1}{2}mv_0^2 = 0.5(9.8)(0.8) - \frac{1}{2}(0.5)(4)^2$

$= 3.92 - 4$

$W_{Drag} = -0.08\,J$

(8-53) $m = 3.5\,kg$
$k = 640\,N/m$
$D = 7.8\,m$, $\mu_k = 0.25$



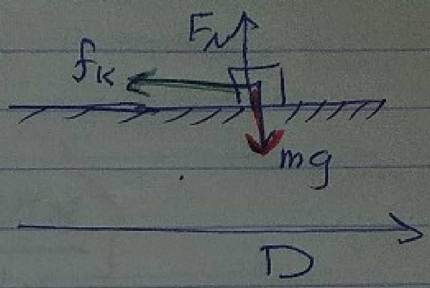
a) $W_{f_k} = \vec{f_k} \cdot \vec{D}$

$= (\mu_k mg) D \cos 180$

$= -\mu_k mgD$

$= (-)\, 0.25 (3.5)(9.8)(7.8)$

$W_{f_k} = -67\,J \Longrightarrow E_{thrmal} = 67\,J = 67\,J$



b) Find maximum kinetic energ?
from $B \longrightarrow C$

$W_f = \Delta E = (K+U)_c - (K+U)_B$

$-67 = (0+0) - (K_B + 0)$

$-67 = -K_B$

$(K_B)_{max} = 67\,J$

c) x? $(K+U)_A = (K+U)_B$ No friction from $A \to B$

$0 + \frac{1}{2} k x^2 = 67 + 0$

$\frac{1}{2}(640)(x^2) = 67$, $x^2 = \frac{67}{320} = 0.209$

$x = 0.46\,m = 46\,cm$

Solve Problem 55.

8- Potential Energy Function
Potential Energy curve

$$W_{con} = -\Delta U$$

$$\Delta U(x) = -W_{con.} = -F_{con.}(x)\Delta x$$

$$F_{cons}(x) = -\frac{\Delta U(x)}{\Delta x} \Rightarrow F_{cons}(x) = -\frac{dU(x)}{dx}$$

Problem (8-26)

A Conservative Force $\vec{F} = (6x-12)\hat{i}$ N

At $x=0$, $U_0 = 27J$

a) Write an expression for $U(x)$?

$$\Delta U = -\int \vec{F}_{con} \cdot \vec{dr}$$

$$U_f - U_i = -\int_{x_i}^{x_f} (6x-12)\,dx = -\left[\frac{6x^2}{2} - 12x\right]_{x_i}^{x_f}$$

$$U(x) - 27 = -3\left[x^2 - 4x\right]_{x_i}^{x}$$

$$U(x) - 27 = -3\left[x^2 - 4\cancel{x}^{0}\right] - 3[0]$$

$$U(x) = 27 - 3x^2 + 12x \text{ Joul}$$

b) Find U_{max}? ~~and at~~

U is max at $\frac{dU(x)}{dx} = 0$

$$-6x + 12 = 0 \Rightarrow x = 2m$$

$$U_{max} = 27 - 3(2)^2 + 12(2)$$
$$= 27 - 12 + 24$$

$U_{max} = 39J$ is at $x = 2m$

(8-26) Continuation

c) find x? for $U = 0$

$0 = 27 - 3x^2 + 12x$

$x^2 - 4x - 9 = 0$

$x = \frac{-^-4 \pm \sqrt{16 - 4(1)(-9)}}{2} = \frac{+4 \pm \sqrt{52}}{2} = \frac{+4 \pm 7.2}{2}$

$x_1 = 5.6\,m \quad ,,, \quad x_2 = -1.6\,m \Rightarrow U = 0$

(8-104) $m = 20\,kg$ is acted on by a Conservative force $F = -3x - 5x^2$ At $x = 0$, $U_0 = 0$

a) Find U at $x = 2\,m$.

letus find $U(x)$

$$U_f - U_i = -\int_{x_i}^{x_f} F_x\,dx$$

$$U(x) - 0 = -\int_0^x (-3x - 5x^2)\,dx$$

$$U(x) = \frac{3}{2}x^2 + \frac{5}{3}x^3 \text{ Joul}$$

At $x = 2\,m$, $U = \frac{3}{2}(2)^2 + \frac{5}{3}(2)^3 = 6 + 13.3$

$= 19.6\,J$

b) At $x = 5\,m$, $V_x = -4\,m/s$ find V_x? at $x = 0$

Mechanical energy is conserved

$(K + U)_{x=5} = (K + U)_{x=0}$

$\frac{1}{2}\overset{(20)}{\cancel{(2)}}(-4)^2 + \left[\frac{3}{2}(5)^2 + \frac{5}{3}(5)^3\right] = \frac{1}{2}(20)V_0^2 + 0$

$[160] + [37.5 + 208.3] = 10V_0^2$

$[160] + [245.8] = 10V_0^2$

$V_0 = \pm 6.37\,m/s$ at $x = 0 \Rightarrow V_0 = -6.37\,m/s$

c) Do it: $U = -8\,J$ at $x = 0$

Problem: A 0.2kg particle moves along the x-axis under the influence of a Conservative force, its Potential energy is given by

$$U(x) = 8x^2 + 2x^4 \text{ Joul}$$

At $x = 1m$, its speed $v = 5 m/s$

1) Find its speed at the origin?

$$(K+U)_1 = (K+U)_0$$

$$\frac{1}{2}(0.2)(5^2) + [8+2] = \frac{1}{2}(0.2)V_0^2 + [0]$$

$$2.5 + 10 = 0.1V_0^2$$

$$V_0 = 11.2 m/s$$

2) Find the Conservative Force?

$$F(x) = -\frac{dU}{dx} = -(16x + 8x^3)$$

$$F(x) = -16x - 8x^3 \; N$$

3) Find the turning Points?

At the turning Points $\to$ $K = 0$, $v = 0$; $E = U$; $E = 12.5$; $U = 12.5 = 8x^2 + 2x^4$

$$E = K + U$$

$$E_0 = E_1 = 12.5 J$$

$$2x^4 + 8x^2 - 12.5 = 0$$

$$x^2 = \frac{-8 \pm \sqrt{64 - 4(2)(-12.5)}}{2(2)} = \frac{-8 \pm 12.8}{4}$$

$$x^2 = \frac{-8 + 12.8}{4} = 1.2 \quad , \; x = \sqrt{1.2}$$

$$x = \pm 1.1 \; m$$

4) Find the equilibrium Points?

At equilibrium Points

$$F = 0$$

$$0 = -16x - 8x^3$$

$$0 = 8x(2 + x^2)$$

$$x = 0$$

Reading Potential Energy curve.

$$E_{mec} = K + U$$

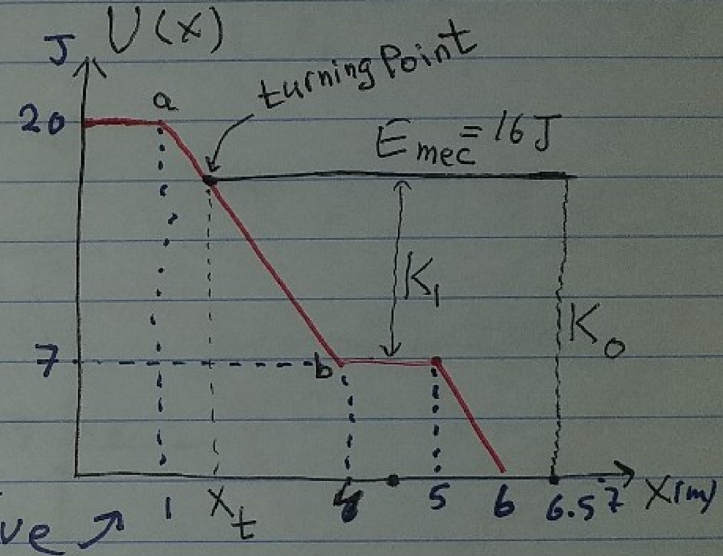
$$W_{con} = -\Delta U$$

$$F_{con}\, dx = -dU \qquad F_{con} = -\frac{dU}{dx}\ ,\ F(x)$$

At turning Points $\rightarrow K = 0,\ v = 0$
$\rightarrow E_{mec} = U$

At equilibrium Points $\longrightarrow F(x) = 0$

Sample Problem 8.04 :-



m = 2kg moves along the x-axis under the influence of $F_{con.}$, the associated Potential energy of this Conservative force is given by this curve ↗

At $x = 6.5m$, $\vec{V_0} = -4\hat{i}\ m/s$

a) determine the Particle speed at $x_1 = 4.5m$

At $x_0 = 6.5m$ $\rightarrow K_0 = \frac{1}{2} m v_0^2 = \frac{1}{2}(2)(-4)^2 = 16J$
$\rightarrow U_0 = 0$ from the curve
$\rightarrow E_0 = 16J$

At $x_1 = 4.5m$ $\rightarrow U_1 = 7J$
$\rightarrow K_1 = \frac{1}{2} m v_1^2$?
$\rightarrow E_1 = 16J$

$E_0 = E_1 = 16J$

$K_1 = E_1 - U_1 = 16 - 7 = 9J$

$\frac{1}{2} m v_1^2 = 9$

$v_1 = 3m/s$

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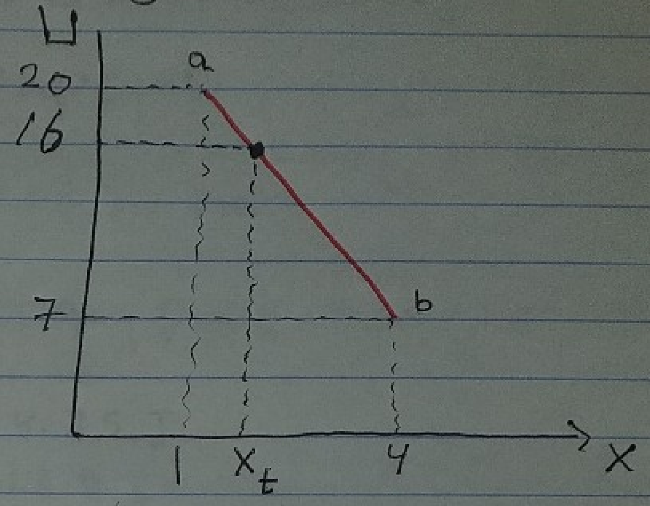
b) Where is the particle's turning Points ?

to find the turning Point X_t take the slope of the straight line

At $X_t \rightarrow K_t = 0$

$\rightarrow U_t = E = 16\,J$



$$\frac{20-7}{1-4} = \frac{16-7}{X_t - 4}$$

$$\frac{13}{-3} = \frac{9}{X_t - 4}$$

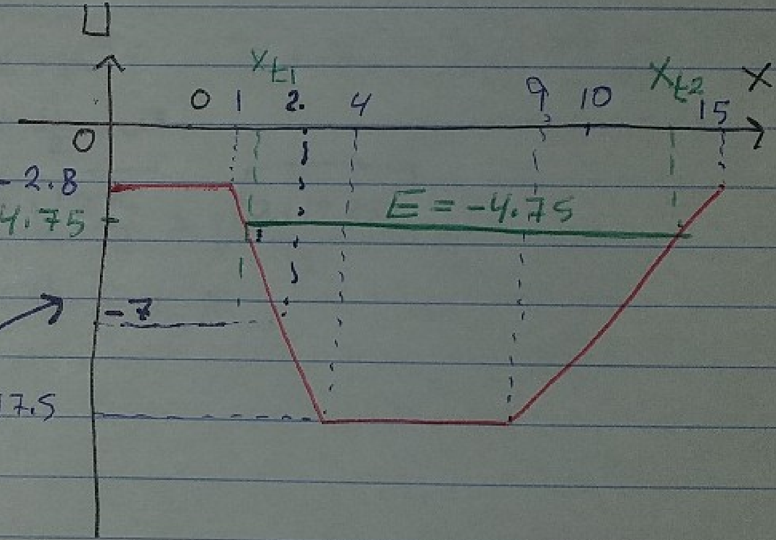
$$X_t - 4 = \frac{-3(9)}{13}$$

$$X_t = 4 - 2.1 = 1.9\,m$$

8-77

m = 2 kg moves along the x-axis, under the influence of F(x) conservative

U(x) Associated with $F_{cons.}$



At $x = 2\,m$, $V_x = -1.5\,m/s$

a) Find $\vec{F}$ at $x = 2.m$

$$F_x = -\frac{dU}{dx} = (-)\frac{U(4) - U(1)}{4-1} = (-)\left(\frac{^{-}17.5 - ^{-}2.8}{3}\right) = 4.9\,N$$

$$F_x = +4.9\,N, \quad \vec{F} = +4.9\,\hat{i}\,N$$

c) We have to find E_{mec} of the system we have information to find U at $x = 2\,m$

At $x = 2\,m$

$\rightarrow V = -1.5\,m/s$

$\rightarrow K = \frac{1}{2}mv^2 = \frac{1}{2}(2)(1.5)^2 = 2.25\,J$

$\rightarrow U = -7\,J$ from the curve

$\rightarrow E = 2.25 + ^{-}7 = -4.75\,J$

(8-77) Continuation

the Particle will move between 2 turning Points

At these 2 Point $\rightarrow K = 0$

$\rightarrow E = U = -4.75 \approx -5\,J$

from the curve these two points $\rightarrow x_{t_1} = 1.2\,m$

$\rightarrow x_{t_2} = 14\,m$

e) find the Particle speed at $x = 7m$

$E_{mec} = -4.75\,J$ (at any Point, from $x = 0 \rightarrow 15m$

At $x = 7m$

$\rightarrow E = -4.75\,J$

$\rightarrow U = -17.5\,J$

$\rightarrow K = E - U$

$= -4.75 - {}^{-}17.5 = 12.75\,J$

$\rightarrow K = \frac{1}{2} m v^2$

$V = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(12.75}{2}} =$

$V = 3.6\,m/s$