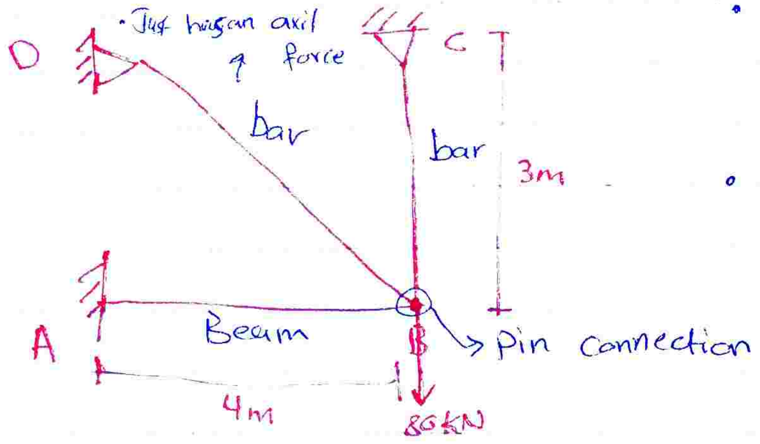
• Force Method: Combined system.

• Find member forces F_{BD}, F_{BC}.

• Draw B.M diagram AB.



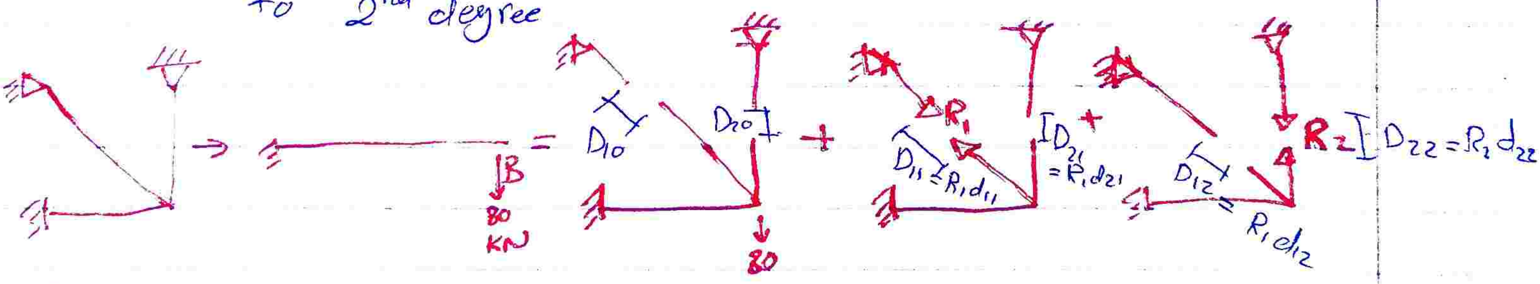
$R_1 = F_{BD}$

$R_2 = F_{BC}$

\# unknowns   \# eq

5   >   3

Statically Ind. to 2^{nd} degree



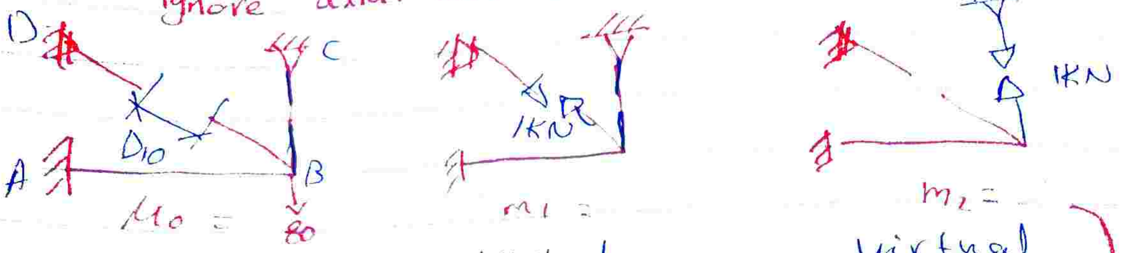
• Relative disp. between cut edges member BD.

$$0 = D_{10} + R_1 d_{11} + R_2 d_{12}$$

• Relative disp. between cut edges member BC

$$0 = D_{20} + R_1 d_{21} + R_2 d_{22}$$

→ In the calculation of the deformations ignore axial and shear deformations in the beam.



M_o = Real load System

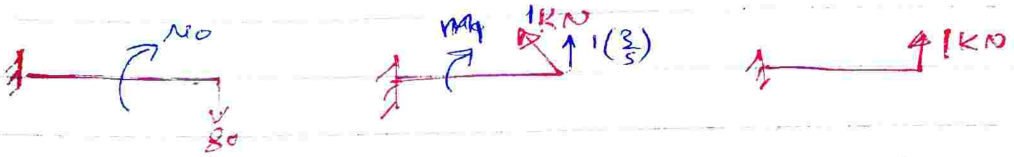
m_1 = Virtual D_{10}

m_2 = Virtual D_{20}

$$D_{10} = \int \frac{M m}{EI} dx + \sum n \frac{NL}{EA}$$

$(N_o)BD = 0$ $\quad (n_1)BD = 1$ $\quad (n_2)BD = 0$

$(N_o)BC = 0$ $\quad (n_1)BC = 0$ $\quad (n_2)BC = 1$



$M_o = -80X$ $\quad m_1 = 0.6X$ $\quad m_2 = X$

$0 \leq X \leq 4$ $\quad 0 \leq X \leq 4$ $\quad 0 \leq X \leq 4$

$$\rightarrow D_{10} = \int \frac{m_1 M_o}{EI} dx + \sum n \frac{N_o L}{EA}$$

$$= \int_0^4 \frac{(0.6x)(-80X)}{EI} dx + \left[\frac{(1)(0)(5)}{EA} + \frac{(0)(0)(3)}{EA} \right]$$

$$= \frac{-1024}{EI}$$

$$\rightarrow D_{20} = \int_0^4 \frac{(X)(-80)X}{EI} dx + \left[\frac{(0)(0)(5)}{EA} + \frac{(1)(0)(3)}{EA} \right] = \frac{-1706.67}{EI}$$

$$\rightarrow d_{11} = \int_0^4 \frac{(0.6X)^2}{EI} dx + \left[\frac{(1)^2(5)}{EA} + \frac{(0)^2(3)}{EA} \right] = \frac{7.68}{EI} + \frac{5}{EA}$$

$$\rightarrow d_{22} = \int \frac{(X)^2}{EI} dx + \left[\frac{0(5)^2}{EA} + \frac{(1)^2(3)}{EA} \right] = \frac{21.33}{EI} + \frac{3}{EA}$$

$\rightarrow d_{21} = \int \frac{m_2 m_1}{EI} dx = \int \frac{(0.6x)(x)}{EI} = \frac{12.8}{EI}$

$\rightarrow d_{12} = \frac{12.8}{EI}$

$E = 200 GPa \qquad I_b = 200 \times 10^6 mm^4 \qquad A = 100^2$

- $EI = (200 \times 10^6 \ KN/m^2)(200 \times 10^6 mm^4) \frac{10^{-12} m^4}{mm^4} =$

$= 4 \times 10^4 \ KN \cdot m^2$

- $EA = (200 \times 10^6 \ KN/m^2)(100 \ mm^2) \times \frac{10^{-6} m^2}{mm^2}$

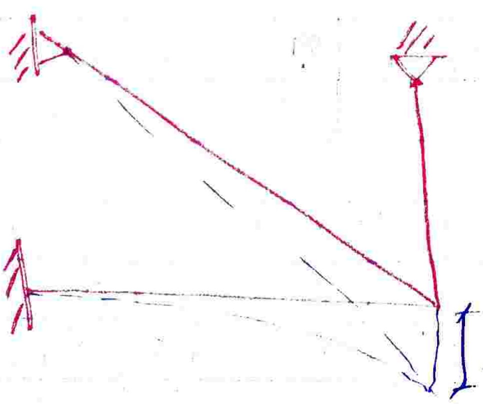
$= 2 \times 10^4 \ KN \cdot m$

$\rightarrow 0 = -\frac{1024}{EI} + R_1 \left[\frac{7.68}{EI} + \frac{5}{EA} \right] + R_2 \left[\frac{12.8}{EI} \right]$

$\rightarrow 0 = -\frac{1706.67}{EI} + R_1 \left[\frac{12.8}{EI} \right] + R_2 \left[\frac{21.33}{EI} + \frac{3}{EA} \right]$

$R_1 = F_{BD} = 19.2 \ KN \ (T)$

$R_2 = F_{BC} = 53.4 \ KN \ (T)$



$\Delta B = \Delta_{BC} = \frac{N_{BC} l}{EA}$

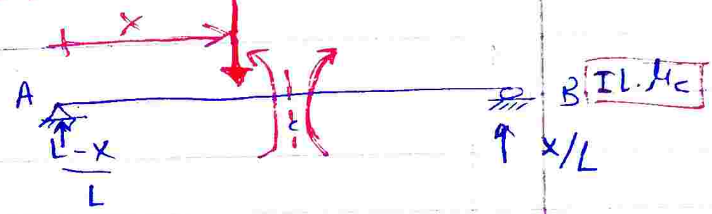
Influence line and Envelopes

- **Influence line:-** is a diagram shows the relation of an objective function (reaction force, Shear, bending moment) with the position of loading.

- Why do we construct Influence lines?
  - live loads are variable in position (gravity loading).
  - To decide the critical load cases when considering the live load, and develop load cases to get the required design values (V, R, M)

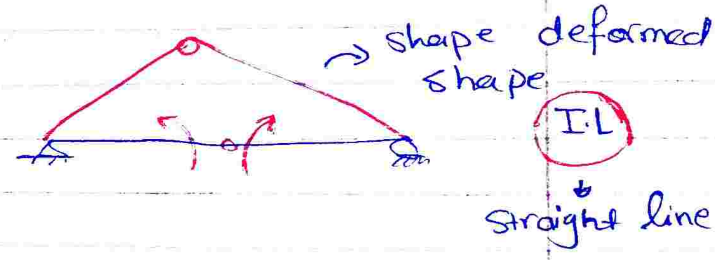
- Influence line of det. Systems? Can be determine By 3 ways:-

1- Point By Point, $x = 1m \Rightarrow M_c = ?$
$x = 2m \Rightarrow M_c = ?$
....



2- Equations $\Rightarrow M_c(x)$

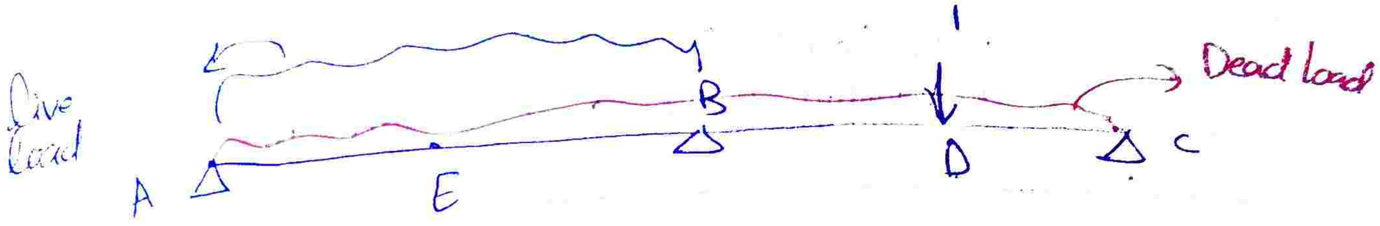
3- Qualitative approach

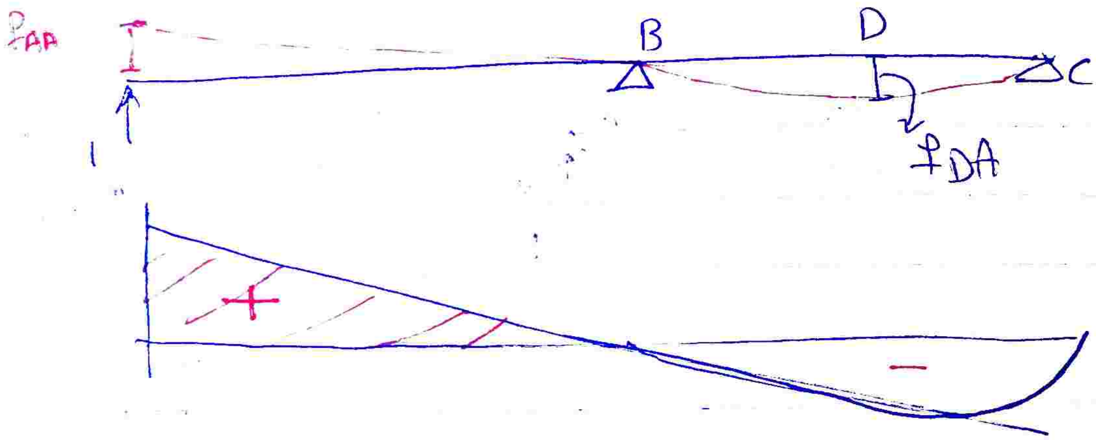


Qualitative approach to build influence lines - Muller Breslau principle.

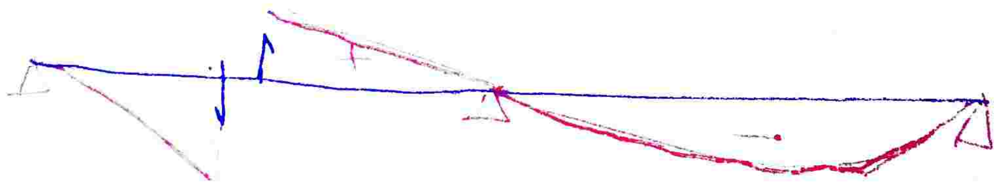
- Induce a unit deformation, the deformed shape due to unit deformation is the shape of the influenced.

- Load case for max Ay. (Reaction force)

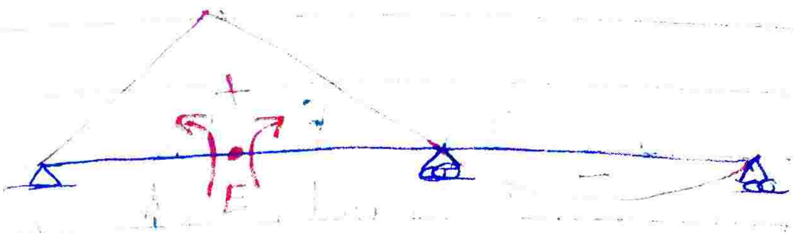




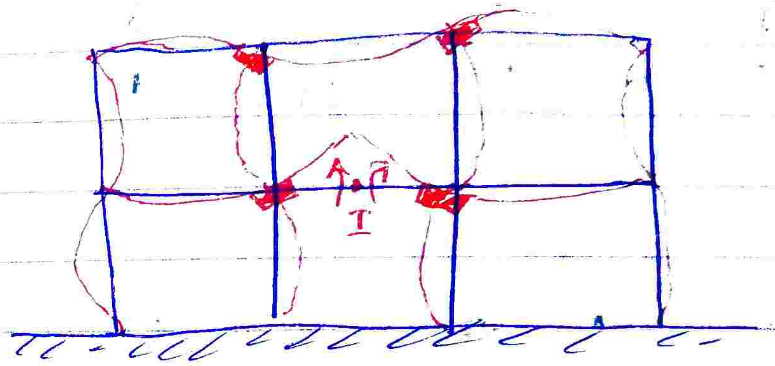
- IL for Shear VE.



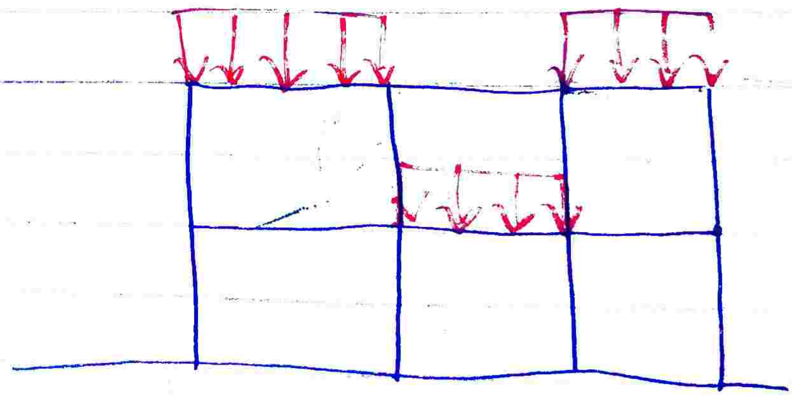
• IL (M)



⇒ I.L M_I



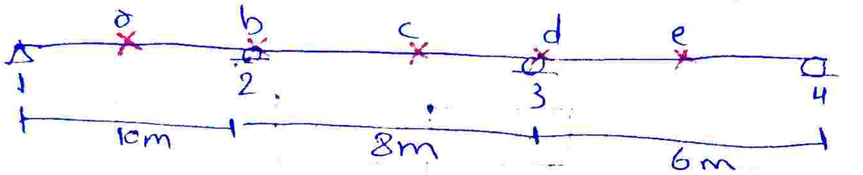
⇒ To get max M_I



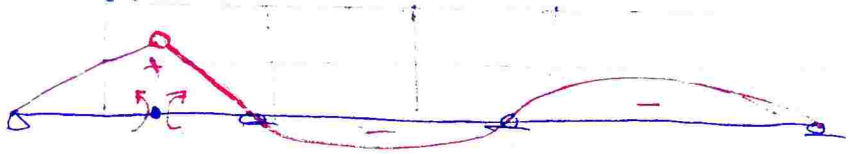
• Example: three span beam.

• Draw qualitative IL for the Bending Moment of the beam, and develop the load Cases.

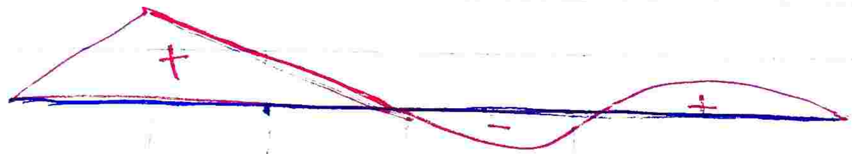
• Choose midspans & Interior supports.



① ⓐ

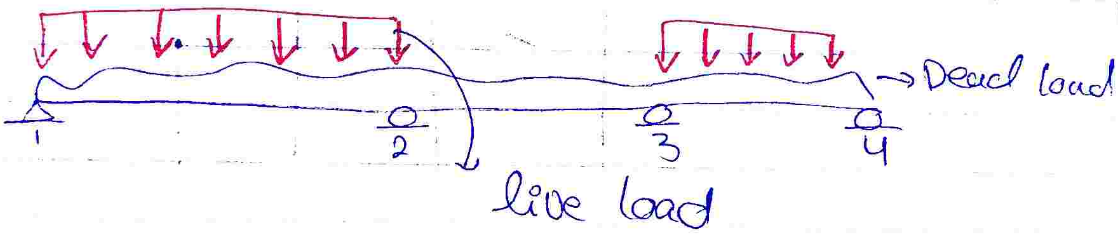


max M_a

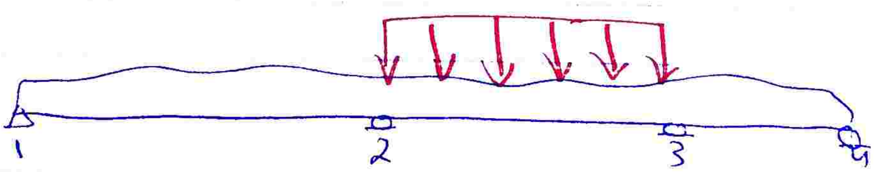


IL, M_a
- load Cases

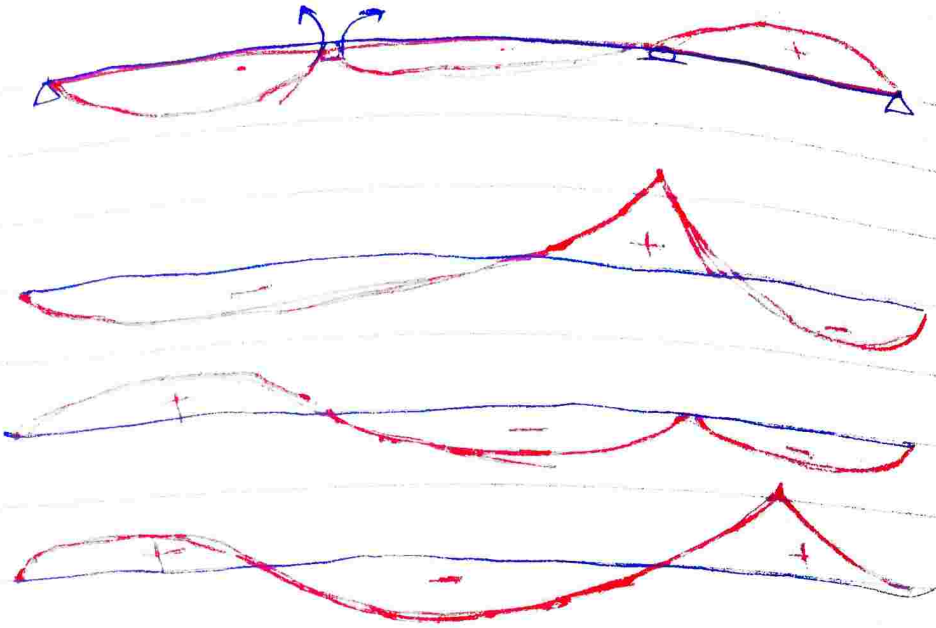
• load Case (LC1) for max M_a



• Load Case (LC2) for min M_a



② ⓑ I.L M_A

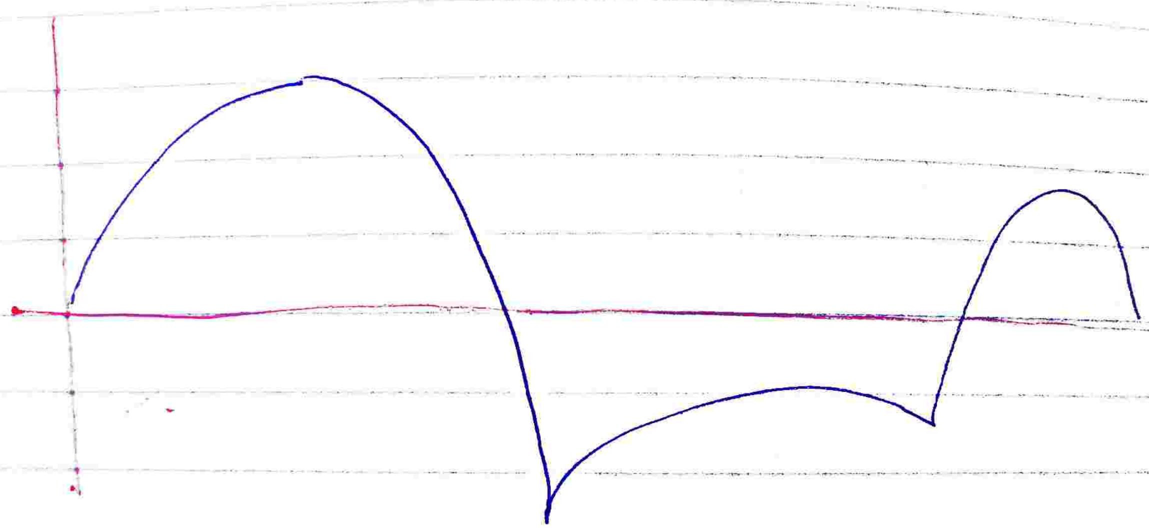


I.L M_B

I.L M_D

I.L M_E

B.M.L.C

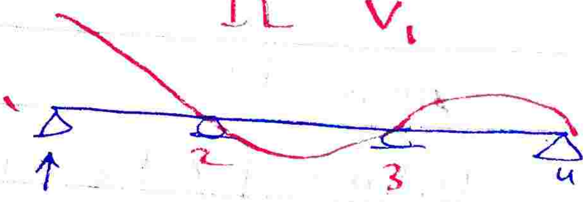


- Draw Bending Moment Diagras for all Cases

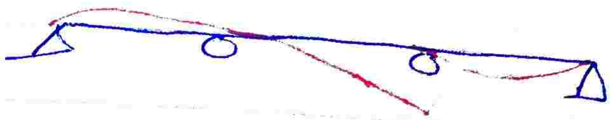
– then we take the max. Value from all cases.

- Influnce line for shearing forces of the Beam.

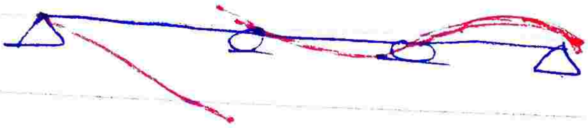
IL V_1



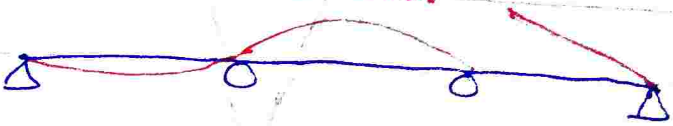
IL $(V_3)_L$



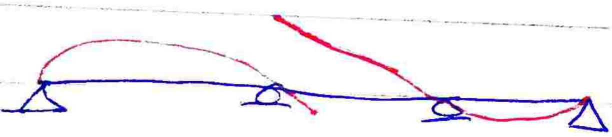
IL $(V_2)_L$



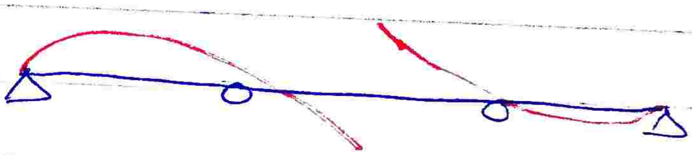
IL $(V_3)_R$



IL $(V_2)_R$



IL $(V_c)_R$



And so ...