

15]

$$\int_0^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}} d\theta$$

8.7

1/

$$u = \theta^2 + 2\theta$$

$$du = 2(\theta+1)d\theta$$

$$\int \frac{1}{2\sqrt{u}} = \frac{1}{2} \int u^{-\frac{1}{2}} = \sqrt{u} = \sqrt{\theta^2+2\theta}$$

$$\lim_{c \rightarrow 0^+} \int_c^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}} d\theta = \lim_{c \rightarrow 0^+} \sqrt{\theta^2+2\theta} \Big|_c^1 = \lim_{c \rightarrow 0^+} \sqrt{3} - \sqrt{c^2+2c} = \sqrt{3} \quad \text{conv}$$

47]

$$\int_1^{\infty} \frac{dx}{x^3+1}$$

$$\frac{1}{x^3+1} \leq \frac{1}{x^3}$$

$$\int_1^{\infty} \frac{1}{x^3} \quad \text{conv} \quad p > 1$$

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$$\text{Hence, } \int_1^{\infty} \frac{dx}{x^3+1} \quad \text{conv by DCT}$$

63]

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{x^4+1}}$$

$$\frac{1}{\sqrt{x^4+1}} \leq \frac{1}{x^2}$$

$$= 2 \int_0^{\infty} \frac{dx}{\sqrt{x^4+1}} = 2 \underbrace{\int_0^1 \frac{dx}{\sqrt{x^4+1}}}_{\text{conv}} + 2 \underbrace{\int_1^{\infty} \frac{dx}{\sqrt{x^4+1}}}_{\text{conv by DCT}}$$

$$\text{Hence, } \int_{-\infty}^{\infty} \frac{dx}{\sqrt{x^4+1}} \quad \text{conv}$$

$$\int_0^4 \frac{dx}{\sqrt{4-x}}$$

$$\lim_{c \rightarrow 4^-} \int_0^c \frac{dx}{\sqrt{4-x}}$$

$$u = 4-x \quad \frac{du}{dx} = -1 \quad \int \frac{-du}{\sqrt{u}} = -2\sqrt{4-x}$$

$$\lim_{c \rightarrow 4^-} -2\sqrt{4-x} \Big|_0^c = \lim_{c \rightarrow 4^-} -2\sqrt{4-c} + 2\sqrt{4-0} = 4 \quad \text{conv}$$

$$\int_{-\infty}^2 \frac{2dx}{x^2+4}$$

$$\lim_{b \rightarrow -\infty} \int_b^2 \frac{2dx}{x^2+4}$$

$$\lim_{b \rightarrow -\infty} 2 \left[\frac{1}{2} \tan^{-1} \frac{x}{2} \right] \Big|_b^2 = \lim_{b \rightarrow -\infty} \left[\tan^{-1} 1 - \tan^{-1} \frac{b}{2} \right] = \frac{\pi}{4} - -\frac{\pi}{2} = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$$

$$\int_{-\infty}^0 \frac{x dx}{(x^2+4)^{3/2}}$$

$$u = x^2+4 \quad \frac{du}{dx} = 2x \quad \int \frac{du}{2u^{3/2}} = \frac{1}{2} \int u^{-3/2} du = \frac{1}{2} u^{-1/2} \cdot -2 = \frac{-1}{\sqrt{x^2+4}}$$

$$= \int_{-\infty}^0 \frac{x dx}{(x^2+4)^{3/2}} + \int_0^{\infty} \frac{x dx}{(x^2+4)^{3/2}}$$

$$= \lim_{b \rightarrow -\infty} \left[\frac{-1}{\sqrt{x^2+4}} \right] \Big|_b^0 + \lim_{b \rightarrow \infty} \left[\frac{-1}{\sqrt{x^2+4}} \right] \Big|_0^b = \lim_{b \rightarrow -\infty} \left[\frac{1}{\sqrt{b^2+4}} \right] + \lim_{b \rightarrow \infty} \left[\frac{-1}{\sqrt{b^2+4}} + \frac{1}{2} \right] = 0 \quad \text{conv}$$

$$\int_0^2 \frac{s+1}{\sqrt{4-s^2}} ds$$

$$u = 4-s^2 \quad \frac{du}{ds} = -2s \quad \int \frac{1}{\sqrt{u}} = \frac{1}{2} \int \frac{du}{\sqrt{u}} = \frac{1}{2} 2\sqrt{u} = \sqrt{4-s^2}$$

$$= \int_0^2 \frac{s ds}{\sqrt{4-s^2}} + \int_0^2 \frac{1}{\sqrt{4-s^2}} ds$$

$$\lim_{c \rightarrow 2^-} \left[-\sqrt{4-s^2} \right] \Big|_0^c + \lim_{c \rightarrow 2^-} \left[\sin^{-1} \frac{s}{2} \right] \Big|_0^c = \lim_{c \rightarrow 2^-} [0+2] + \lim_{c \rightarrow 2^-} [\sin^{-1} 1 - \sin^{-1} 0] = 2 + \frac{\pi}{2} \quad \text{conv}$$

25) $\int_0^1 x \ln x \, dx$

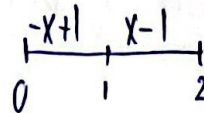
$u = \ln x \quad dv = x \, dx$
 $du = \frac{1}{x} \quad v = \frac{x^2}{2}$

$\lim_{c \rightarrow 0^+} \left[\frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx \right]$

$\lim_{c \rightarrow 0^+} \left[\frac{x^2}{2} \ln x - \frac{1}{4} x^2 \right]_c^1 = \lim_{c \rightarrow 0^+} \left[\left(\frac{1}{2} \ln 1 - \frac{1}{4} \right) - (0) \right] = -\frac{1}{4} \text{ conv}$

32) $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$

$x-1=0$
 $x=1$



$= \int_0^1 \frac{dx}{\sqrt{-x+1}} + \int_1^2 \frac{dx}{\sqrt{x-1}}$

$= \lim_{c \rightarrow 1^-} \left[-2\sqrt{-x+1} \right]_0^c + \lim_{c \rightarrow 1^+} \left[2\sqrt{x-1} \right]_c^2 = \lim_{c \rightarrow 1^-} [0 + 2] + \lim_{c \rightarrow 1^+} [2 + 0] = 4 \text{ conv}$

37) $\int_0^\pi \frac{\sin \theta \, d\theta}{\sqrt{\pi-\theta}}$

$\frac{\sin \theta}{\sqrt{\pi-\theta}} \leq \frac{1}{\sqrt{\pi-\theta}}$

$\int_0^\pi \frac{1}{\sqrt{\pi-\theta}} \, d\theta$

$\lim_{c \rightarrow \pi^-} \left[-2\sqrt{\pi-\theta} \right]_0^c = \lim_{c \rightarrow \pi^-} [0 + 2\sqrt{\pi}] = 2\sqrt{\pi}$

Hence, $\int_0^\pi \frac{\sin \theta \, d\theta}{\sqrt{\pi-\theta}}$ conv by DCT

41) $\int_0^\pi \frac{dt}{\sqrt{t} + \sin t}$

$\frac{1}{\sqrt{t} + \sin t} \leq \frac{1}{\sqrt{t}}$

$\int_0^\pi \frac{1}{\sqrt{t}} \, dt$

$\lim_{c \rightarrow 0^+} \int_c^\pi \frac{1}{\sqrt{t}} \, dt = \lim_{c \rightarrow 0^+} \left[2\sqrt{t} \right]_c^\pi = \lim_{c \rightarrow 0^+} [2\sqrt{\pi} - 0] = 2\sqrt{\pi}$

Hence, $\int_0^\pi \frac{dt}{\sqrt{t} + \sin t}$ conv by DCT

$$50] \int_0^{\infty} \frac{d\theta}{1+e^{\theta}}$$

$$\frac{1}{1+e^{\theta}} < \frac{1}{e^{\theta}}$$

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$$\int_0^{\infty} \frac{d\theta}{e^{\theta}}$$

$$\lim_{b \rightarrow \infty} \int_0^b e^{-\theta} d\theta = \lim_{b \rightarrow \infty} \left. -\frac{1}{e^{\theta}} \right|_0^b = \lim_{b \rightarrow \infty} [0 + 1] = 1$$

Hence, $\int_0^{\infty} \frac{d\theta}{1+e^{\theta}}$ conv by DCT

$$58] \int_2^{\infty} \frac{1}{\ln x} dx \quad \frac{1}{x} < \frac{1}{\ln x}$$

Hence, $\int_2^{\infty} \frac{1}{\ln x} dx$ div by DCT.

$$\int_2^{\infty} \frac{1}{x} dx = \lim_{b \rightarrow \infty} \ln b - \ln 2 = \infty$$

$$62] \int_1^{\infty} \frac{1}{e^x - 2^x} dx$$

$$\frac{1}{e^x} < \frac{1}{e^x - 2^x}$$

$$\lim_{b \rightarrow \infty} \left. \frac{1}{e^x} \right|_1^b = \lim_{b \rightarrow \infty} \left[-\frac{1}{e^b} + \frac{1}{e} \right] = \frac{1}{e} \text{ conv}$$

$$62] \int_1^{\infty} \frac{1}{e^x - 2^x} dx$$

$$\lim_{x \rightarrow \infty} \frac{1}{e^x - 2^x} \cdot e^x = \lim_{x \rightarrow \infty} \frac{1}{1 - \left(\frac{2}{e}\right)^x} = \frac{1}{1-0} = 1$$

Hence, $\int_1^{\infty} \frac{1}{e^x - 2^x} dx$ conv by LCT

$$1) \int_0^{\infty} \frac{dx}{x^2+1}$$

$$\lim_{b \rightarrow \infty} [\tan^{-1} b - \tan^{-1} 0] = \frac{\pi}{2} \quad \text{conv}$$

$$7) \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$\lim_{c \rightarrow 1} [\sin^{-1} c - \sin^{-1} 0] = \frac{\pi}{2} - 0 = \frac{\pi}{2} \quad \text{conv}$$

$$2) \int_{-\infty}^0 \theta e^{\theta} d\theta$$

$$= \theta e^{\theta} - e^{\theta}$$

$$\begin{aligned} u &= \theta & dv &= e^{\theta} d\theta \\ du &= d\theta & v &= e^{\theta} \end{aligned}$$

$$\lim_{b \rightarrow -\infty} [-1 - (be^b - e^b)] = -1$$

$$40) \int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$$

$$\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}} dx \end{aligned}$$

$$\lim_{c \rightarrow 0^+} \int_c^1 2e^{-u} du$$

$$\lim_{c \rightarrow 0^+} -2e^{-u} \Big|_c^1 = -2 \lim_{c \rightarrow 0^+} \left[\frac{1}{e} - 1 \right] = -\frac{2}{e} + 2$$

$$56) \int_{\pi}^{\infty} \frac{1 + \sin x}{x^2} dx$$

$$\frac{1 + \sin x}{x^2} \leq \frac{2}{x^2}$$

$$\int_{\pi}^{\infty} \frac{2}{x^2} = \lim_{b \rightarrow \infty} \left. -\frac{2}{x} \right|_{\pi}^b$$

$$= -2 \lim_{b \rightarrow \infty} \left[\frac{1}{b} - \frac{1}{\pi} \right] = \frac{2}{\pi} \text{ conv}$$

Hence, $\int_{\pi}^{\infty} \frac{1 + \sin x}{x^2}$ conv by DCT

$$65) a - \int_1^2 \frac{dx}{x (\ln x)^p}$$

$$b - \int_2^{\infty} \frac{dx}{x (\ln x)^p}$$

$$u = \ln x \\ du = \frac{dx}{x}$$

$$x=2 \rightarrow u = \ln 2 \\ x=1 \rightarrow u = 0$$

$$p=1 \Rightarrow \lim_{a \rightarrow 0^+} \ln |u| \Big|_a^{\ln 2}$$

$$= \lim_{a \rightarrow 0^+} (\ln \ln 2 - \ln a) = \infty \quad p \geq 1 \text{ div}$$

$$p \neq 1 \Rightarrow \lim_{a \rightarrow 0^+} \frac{u^{-p+1}}{-p+1} \Big|_a^{\ln 2} = \lim_{a \rightarrow 0^+} \left(\frac{(\ln 2)^{1-p}}{1-p} - \frac{a^{1-p}}{1-p} \right) \quad p < 1$$

$$= \frac{(\ln 2)^{1-p}}{1-p} \quad p < 1 \text{ conv}$$

65)

$$b - \int_2^{\infty} \frac{dx}{x (\ln x)^p}$$

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$$\lim_{b \rightarrow \infty} \ln |u| \Big|_{\ln 2}^b$$

$$u = \ln x$$

$$x = \infty \rightarrow u = \infty$$

$$x = 2 \rightarrow u = \ln 2$$

$$\lim_{b \rightarrow \infty} (\ln b - \ln \ln 2) = \infty \quad \text{div } p \leq 1$$

$$\lim_{b \rightarrow \infty} \frac{u^{-p+1}}{-p+1} \Big|_{\ln 2}^b = \lim_{b \rightarrow \infty} \frac{b^{-p+1}}{-p+1} - \frac{\ln 2^{-p+1}}{-p+1} = \frac{\ln 2^{1-p}}{p-1} \quad \text{conv } p > 1$$