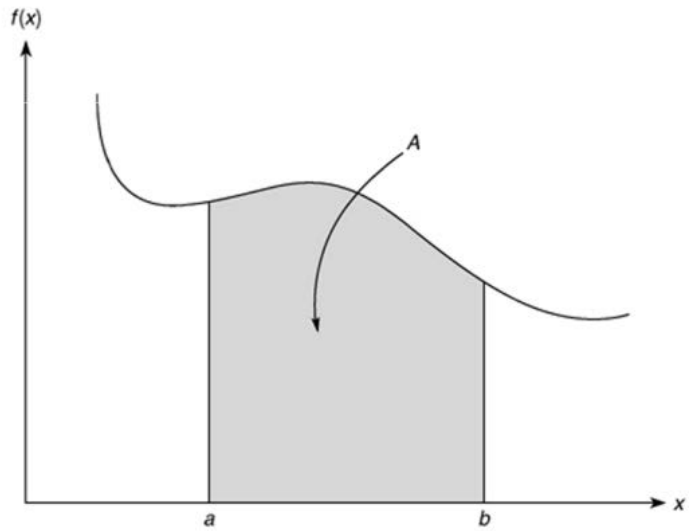
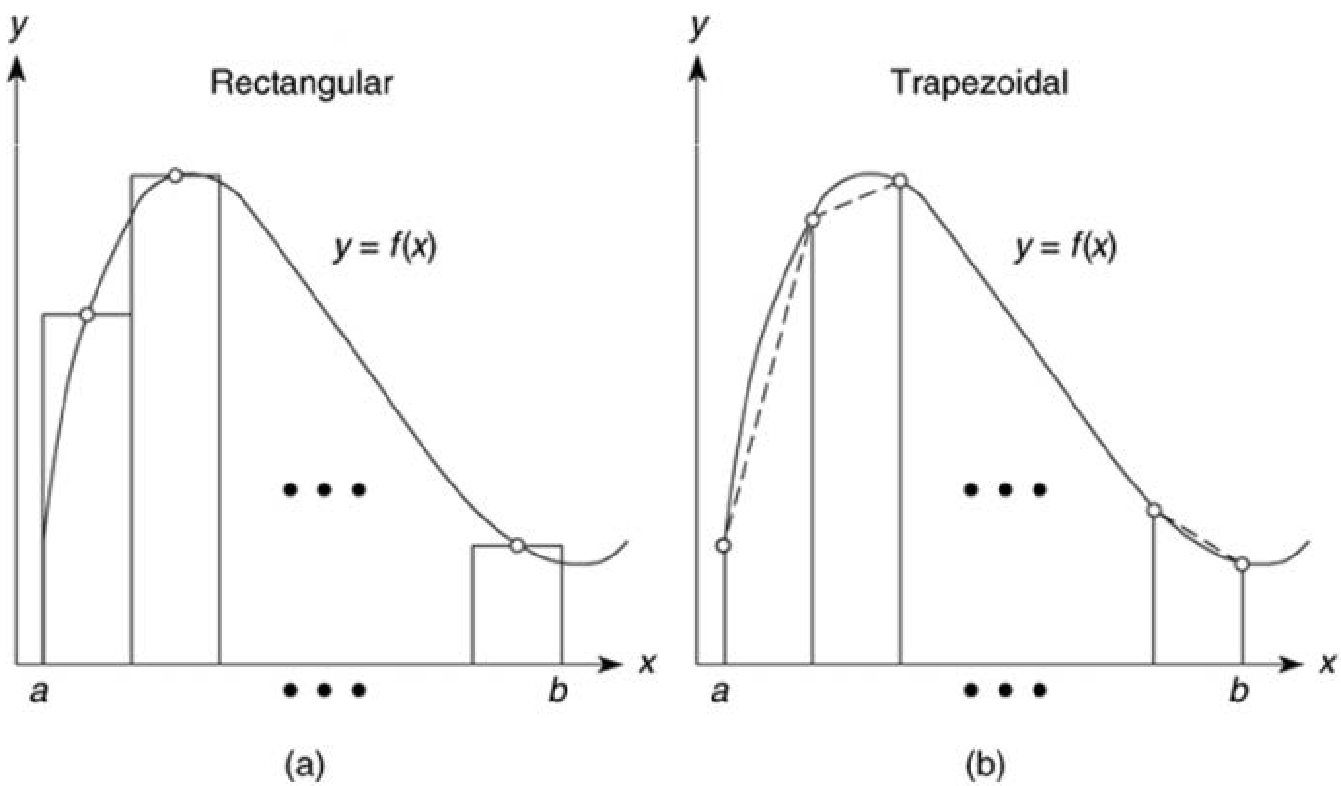
Lab 7: Numerical Calculus and Symbolic Processing

Numerical Calculus – Integration

- Integration of function f(x) in the interval $a \le x \le b$ can be represented as the area between f(x) and the x-axis.



- Numerical Integration can be approximated by partitioning the area into rectangles or trapezoids then summing up their areas.



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Function	Description
`trapz(x,y)`	Uses trapezoidal integration to compute the integral of **y** with respect to **x**, where the array **y** contains the function values at the points contained in the array **x**.
`quad('func',a,b,tol)`	Uses an adaptive Simpson's rule to compute the integral of the function **'func'** with **a** as the lower integration limit and **b** as the upper limit. The parameter **tol** is optional. **tol** indicates the specified error tolerance.
`integral(fun,xmin,xmax)`	Numerically integrates function **fun** from **xmin** to **xmax**. fun is usually a function handle.
`polyint(p,k)`	Returns the integral of the polynomial represented by the coefficients in **p** using a constant of integration **k**
`integral2(fun,xmin,xmax,ymin,ymax)`	Numerically evaluates double integral of the function **fun** from **xmin** to **xmax** and from **ymin** to **ymax**. fun is usually a function handle.
`Integral3(fun,xmin,xmax,ymin,ymax,zmin,zmax)`	Numerically evaluates triple integral of the function **fun** from **xmin** to **xmax,** from **ymin** to **ymax,** and from **zmin** to **zmax**. fun is usually a function handle.

Examples:

$$\int_0^{\pi} \sin(x)dx = -\cos(\pi) + \cos(0) = +1 + 1 = 2$$

```
>> x = [0:0.1:pi];
>> y = sin(x);
>> trapz(x,y)

ans =

    1.9975
```

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```
>> quad('sin(x)',0,pi)

ans =

    2.0000
```

$$\int_0^1 e^{-x^2}(\ln x)^2\,dx$$

```
>> fun = @(x) exp(-x.^2).*(log(x)).^2;
>> integral(fun,0,1)

ans =

    1.9331

>> quad(fun,0,1)

ans =

    1.9331
```

$$f(x,y) = \frac{1}{\sqrt{x+y}\ \ (1+x+y)^2}$$

```
>> fun = @(x,y) 1./(sqrt(x+y).*(1+x+y).^2);
>> integral2(fun,0,1,0,1)

ans =

    0.3695
```

$$f(x,y,z) = y\sin(x) + z\cos(x)$$

```
>> fun = @(x,y,z) y.*sin(x)+z.*cos(x);
>> integral3(fun,0,pi,0,1,-1,1)

ans =

    2.0000
```

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- Polynomial Integration

$$f(x) = 3x^4 - 4x^2 + 10x - 25 \qquad\qquad k = 3$$

```
>> p = [3 0 -4 10 -25];
>> polyint(p,3)

ans =

    0.6000         0   -1.3333    5.0000  -25.0000    3.0000
```

Which corresponds to $\frac{3}{5}x^5 - \frac{4}{3}x^3 + \frac{10}{2}x^2 - 25x + 3$

Numerical Calculus – Differentiation

- The derivative of function can be interpreted graphically as the slope function.

Function	Description
`diff(x)`	Returns a vector containing the differences between adjacent elements in the vector **x**
`polyder(p)`	Returns a vector containing the coefficients of the derivative of the polynomial represented by the vector **p**
`polyder(p1,p2)`	Returns a vector containing the coefficients of the polynomial that is the derivative of the product of the polynomials represented by **p1** and **p2**.
`[num,den]=polyder(p2,p1)`	Returns the vectors num and den containing the coefficients of the numerator and denominator polynomials of the derivative of the quotient p2/p1, where **p1** and **p2** are polynomials.

Examples:

```
>> x = [1 1 2 3 5 8 13 21];
>> diff(x)

ans =

     0     1     1     2     3     5     8
```

- Ans has one less element that the original vector
- This is not the derivative

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How to use the diff function to perform derivations?

```
>> h = 0.001;       % step size
>> X = -pi:h:pi;    % domain
>> f = sin(X);      % range
>> Y = diff(f)/h;   % first derivative
>> Z = diff(Y)/h;   % second derivative
>> plot(X(:,1:length(Y)),Y,'r',X,f,'b', X(:,1:length(Z)),Z,'k')
```

$$f(x) = x^2 + 2x + 1$$
$$g(x) = x^3 + 5x^2 - x + 8$$

Find the derivative of $f(x).g(x)$ and $f(x)/g(x)$

```
>> p1 = [1 2 1];
>> p2 = [1 5 -1 8];
>> polyder(p1,p2)

ans =

     5    28    30    22    15
```

$$5x^4 + 28x^3 + 30x^2 + 22x + 15$$

```
>> [num,den]=polyder(p1,p2)

num =

    -1    -4   -14     6    17
```

$$\frac{-x^4 - 4x^3 - 14x^2 + 6x + 17}{x^6 + 10x^5 + 23x^4 + 6x^3 + 81x^2 - 16x + 64}$$

```
den =

     1    10    23     6    81   -16    64
```

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Symbolic Processing

- Refers to the way that computers perform operations on mathematical expressions the way that humans do it with pencil and paper.
- Matlab contains many functions to define symbols, symbolic expressions, simplification, and solution to symbolic expressions.

Symbolic Processing – Defining Symbols

- To define a symbol in Matlab use the **syms** functions. We can create symbolic constants or symbolic variables.
- When mathematical operations are used with symbols, the result is symbolic.
- We can use symbols to define symbolic expressions. We can use the operators + - / ^ and the built-in functions.
- We can use the vector and matrix notation with symbols.

```
>> syms x y ;
>> r = sqrt(x^2+y^2)
r = (x^2+y^2)^(1/2)
```

- If we later assign **x** and **y** to 3 and 5, typing **r** will not result in the evaluation of the expression, because **r** is saved as a symbolic variable.

```
>> x = 3;
>> y = 5;
>> r

r =

(x^2 + y^2)^(1/2)

>> eval(r)

ans =

    5.8310
```

Manipulating Symbolic Expressions

- With symbolic expressions in Matlab, we can expand, collect, simplify, and create new expressions from old expressions using the mathematical operators.
- The function **collect(E)** collects coefficients of like powers in the expression **E**. If there is more than one variable, you can use the optional form **collect(E,v)**, which collects all the coefficients with the same power of **v**.

```
>>syms x y
>>E = (x-5)^2+(y-3)^2;
>>collect(E)             % will collect x powers
```

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```
ans =
x^2-10*x+25+(y-3)^2

>>collect(E,y)           % will collect y powers
ans =
y^2-6*y+(x-5)^2+9
```

- The **expand** and **simplify** functions.

```
>>syms x y

>>expand((x+y)^2) % applies algebra rules
ans =
x^2+2*x*y+y^2


>>expand(sin(x+y)) % applies trig identities
ans =
sin(x)*cos(y)+cos(x)*sin(y)


>>simplify(6*((sin(x))^2+(cos(x))^2))      % applies another trig identity
ans = 6
```

- The **factor** function finds the factors of the expression.

```
>>syms x y
>>factor(x^2-1)
ans = (x-1)*(x+1)
```

- The function **subs(E,old,new)** substitutes **new** for **old** in the expression **E**, where **old** can be a symbolic variable or expression and **new** can be a symbolic variable, expression, or matrix, or a numeric value or matrix.

```
>>syms x y
>>E = x^2+6*x+7;
>>F = subs(E,x,y)
F =
y^2+6*y+7
```

- The function **poly2sym(p)** converts the coefficient vector **p** to a symbolic polynomial.
- The function **sym2poly(E)** converts the expression **E** to a polynomial coefficient vector.

```
>> ploy2sym([2,4,5])
ans = 2*x^2 + 4*x + 5
>> ploy2sym([2,4,5],y)
ans = 2*y^2 + 4*y + 5
```

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```
>> syms x ;
>> sym2poly(9*x^2+4)
ans =
9 0 4
```

- The function **[num,den] = numden(E)** returns two expressions **num** and **den** for the numerator and denominator for expression **E**.

```
>> syms x
>> E1 = x^2 + 5 ;
>> E2 = 1 / (x+6) ;
>> [num,den] = numden(E1+E2)
num = x^3 + 6*x^2 + 5*x + 31
den = x + 6
>> pretty(num) %this function produces a mathematical friendly expression
 3      2
x  + 6 x  + 5 x + 31
```

Evaluating Symbolic Expressions

- Use the **subs** function to evaluate an expression numerically. Use **subs(E,old,new)** to replace **old** with a numeric value new in the expression **E**. The result is of class double.

```
% evaluate symbolic expressions
>>syms x
>>E = x^2+6*x+7;
>>G = subs(E,x,2)
G =
23
```

Plotting Symbolic Expressions

- The Matlab function **ezplot(E)** generates a plot of a symbolic expression **E**, which is a function of one variable. The default range of the independent variable is the interval **[−2 π, 2 π].**
- The optional form **ezplot(E,[xmin xmax])** generates a plot over the range from **xmin** to **xmax**.
- The **ezplot** function accepts **E** as string also.
- The **ezplot** function does not accept the specification of line styles and data markers.

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The `solve` Function

- It is used to find the solution of an equation.

```
>> syms x
>> eq1 = x+5
>> solve(eq1)
ans = -5
```

```
>> solve(exp(2*x)+3*exp(x)==54)
ans =
[log(9) + pi*1i]
[log(6) ]
```

- We can solve an equation in terms of symbols

```
>> syms b c
>> solve (b^2+8*c+2*b==0,b)             % solve for b
ans = [-1+(1-8*c)^(1/2)]
     [-1-(1-8*c)^(1/2)]
```

- We can solve more than one equation

```
>> eq1 = 6*x+2*y==14;
>> eq2 = 3*x+7*y==31;
>> solve(eq1,eq2)
result = x: [1x1 sym] % the answer is a structure
y: [1x1 sym]
>> x = result.x
x = 1
>> y = result.y
y = 4
```

Example:

```
>>  syms w x y z ;
>>  M = w - x^2 + log(y) + 5*z ;
>>  N = subs(M,[y,z],[5 1])       % evaluate M with y = 5 and z = 1

N =

- x^2 + w + log(5) + 5

% solve in terms of w i.e. solve for x
>> G = solve(N,x) % the result is symbolic array
G = (w+log(5)+5)^(1/2)
-(w+log(5)+5)^(1/2)
```

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```
>> % evaluate the first root if w = 6
>> subs(G(1),w,6)
ans =
3.5510
```

- Find the intersection of $y = x^2$ and $y = 2x$.

```
>> syms x ;
>> y1 = x^2 ;
>> y2 = 2*x ;
>> solve(y1-y2)
ans
=
0
2
```

- If x+6y=a and 2x-3y=9, then find a solution for x and y in terms of the parameter a

```
>> syms a x y ;
>> [x,y] = solve(x+6*y == a,2*x-3*y == 9)
x = 1/5*a+18/5
y = 2/15*a-3/5
```

Symbolic Differentiation and Integration

- We can use the **diff** function to perform symbolic differentiation by passing the expression directly, as a string, or as a symbolic expression.

```
>> diff(sin(x*y)) % differentiation with respect to
x
ans = cos(x*y)*y


>> syms x y ; % differentiation with respect to y

>> diff(sin(x*y),y)
ans = x*cos(x*y)

>> diff(x*sin(x*y),y,2) % Computes the second derivative
ans = -x^3*sin(x*y)
```

- We can use the **int** function to perform symbolic integration by passing the expression directly, as a string, or as a symbolic expression.

```
>> syms x y
>> int(2*x)                                   % x is defined as a symbol
ans = x^2
```

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```
>> int(sin(x*y),y)              % integration with respect to y
ans = -cos(x*y)/x
```

```
>> int(sin(x*y),y,0,1)          % integrates with respect to y over [0,1]
ans = -(-1+cos(x))/x
```

```
>> int(sin(x*y),x,0,1)            %integrates with respect to x over [0,1]
ans = -(-1+cos(y))/y
```

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