(2.4) exrecises

1) a. False, b. True, c. False, d. True, e. True, f. True, g. FAlse, h. FALse
I. FAlse, J. True, k. True, L. FALse

2) a. True, b. FAlse, c. FAlse, d. True, e. True, f. True, g. True, h. True, i. True,
J. FALse, k. True

3) a. $\lim_{x\to 2^+} f(x) = \cancel{2}+1=2$, $\lim_{x\to 2^-} f(x) = 3-2=1$

b. DNE, Because $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

c. $\lim_{x\to 4^-} f(x) = \frac{4}{2}+1=3$, $\lim_{x\to 4^+} f(x) = 3$

D. it exsists $\lim_{x\to 4} f(x) = 3$

4) a. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x}{2} = \frac{2}{2}=1$, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 3-x = 3-2=1$, $f(2)=2$

b. it exists $\lim_{x\to 2} f(x) = 1$

c. $\lim_{x\to -1^-} f(x) = 3-(-1)=4$, $\lim_{x\to -1^+} f(x) = 3-(-1)=4$

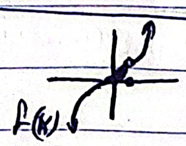
d. it exists $\Rightarrow \lim_{x\to -1} f(x) = 4$

5) a. it exist, because there is single number ($L=0$) as x approaches 0^+
b. it exists, $\lim_{x\to 0^-} f(x) = 0$

c. it does not exists because $\lim_{x\to 0^+} f(x)$ doesnot exist

6) a. it does not exist, because there is no single number L as x approaches 0^+
b. no, because $x \geq 0$
c. yes, $\therefore c$ is 0

7) a. $\lim_{x\to 1^-} f(x) = 1$, $\lim_{x\to 1^+} f(x) = 1$



b. it exists $\lim_{x\to 1} f(x) = 1$

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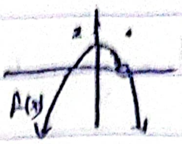
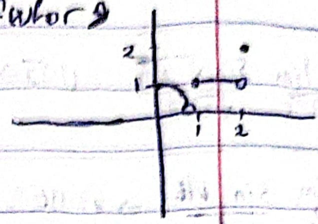
8) a. $\lim_{x\to 1^+} f(x) = 0$, $\lim_{x\to 1^-} f(x) = 0$



b. $\lim_{x\to 1} f(x) = 0$, (it exists)

9 and 10

f(x) for 9



a 9.

D: [0,2]

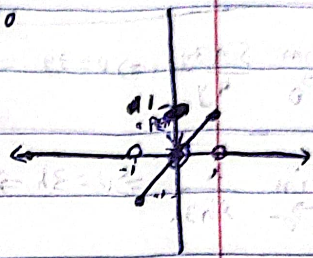
R: (0,2]

a 10.

D: [-1,1]

R: [-1,1)

f(x) for 10



b 9.

exists $\forall c \in (0,2) - \{1\}$

b 10.

exists $\forall c \in (-\infty,-1) \cup (-1,1)$

c 9.

at x=2

c 10.

no c

d 9.

at x=0

d 10.

no c

11) $\lim_{x\to 0.5^-} \sqrt{\frac{x+2}{x+1}} = \sqrt{\frac{2.5}{1.5}} = \sqrt{\frac{5}{3}}$

12) $\lim_{x\to 1^+} \sqrt{\frac{x-1}{x+2}} = \sqrt{\frac{0}{3}} = 0$

13) $\lim_{x\to 2^+} \left(\frac{x}{x+1}\right)\left(\frac{2x+5}{x^2+x}\right) = \left(\frac{2}{3}\right)\left(\frac{9}{6}\right) =$

14) $\lim_{x\to 1^-} \left(\frac{1}{x+1}\right)\left(\frac{x+6}{x}\right)\left(\frac{3-x}{7}\right) = \left(\frac{1}{2}\right)(7)\left(\frac{2}{7}\right) = 1$

15) $\lim_{h\to 0^+} \frac{\sqrt{h^2+4h+5} - \sqrt{5}}{h} \cdot \frac{(\sqrt{h^2+4h+5} - \sqrt{5})}{h(\sqrt{h^2+4h+5} - \sqrt{5})}$

$= \lim_{h\to 0^+} \frac{h^2+4h+5-5}{h(\sqrt{h^2+4h+5} - \sqrt{5})} = \lim_{h\to 0^+} \frac{\cancel{h}(h+4)}{\cancel{h}(\sqrt{h^2+4h+5})} = \frac{4}{\sqrt{5}}$

16) $\lim_{h\to 0^-} \frac{\sqrt{6} - \sqrt{5h^2+11h+6}}{h} \cdot \frac{(\sqrt{6} + \sqrt{5h^2+11h+6})}{(\sqrt{6} + \sqrt{5h^2+11h+6})}$

$\lim_{h\to 0^-} \frac{6 - 5h^2 - 11h - 6}{h \cdot (\sqrt{6} + \sqrt{5h^2+11h+6})} = \lim_{h\to 0^-} \frac{(5h-11)\cancel{h}}{\cancel{h}(\sqrt{6} + \sqrt{5h^2+11h+6})} = \frac{-11}{2\sqrt{6}}$

17) a. $\lim_{x\to -2^+} \frac{(x+3)|x+2|}{x+2} = \lim_{x\to -2^+} \frac{(x+3)\cancel{(x+2)}}{\cancel{(x+2)}} =$

17) b. $\lim_{x\to -2^-} \frac{(x+3)|x+2|}{x+2} = \lim_{x\to -2^-} \frac{-(x+3)\cdot\cancel{(x+2)}}{\cancel{(x+2)}} = -1$

18) a. $\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|} = \lim_{x\to 1^+} \frac{\sqrt{2x}\cancel{(x-1)}}{\cancel{(x-1)}} = \sqrt{2}$

18) b. $\lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|} = \lim_{x\to 1^-} \frac{\sqrt{2x}\cancel{(x-1)}}{-\cancel{(x-1)}} = -\sqrt{2}$

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19) a. $\lim_{\theta\to 3^+} \frac{\lfloor\theta\rfloor}{\theta} = \frac{3}{3} = 1$, $\lim_{\theta\to 3^-} \frac{\lfloor\theta\rfloor}{\theta} = \frac{2}{3}$

20) a. $\lim_{t\to 4^+} (t - \lfloor t\rfloor) = 4 - 4 = 0$, $\lim_{t\to 4^-} (t - \lfloor t\rfloor) = 4 - 3 = 1$

21) $\lim_{\theta\to 0} \frac{\sin\sqrt{2}\theta}{\sqrt{2}\theta}$ $U=\sqrt{2}\theta \Rightarrow \lim_{\frac{U}{\sqrt{2}}\to 0} \frac{\sin U}{U} = \lim_{U\to 0} \frac{\sin U}{U} = 1$

22) $\lim_{t\to 0} \frac{\sin kt}{t} \Rightarrow v = kt \Rightarrow \lim_{\frac{v}{k}\to 0} \frac{\sin v}{\frac{v}{k}} = k \lim_{v\to 0} \frac{\sin v}{v} = k$

23) $\lim_{y\to 0} \frac{\sin 3y}{4y} \Rightarrow v = 3y \Rightarrow \lim_{\frac{v}{3}\to 0} \frac{\sin v}{\frac{4}{3}v} = \lim_{v\to 0} \frac{3}{4} \times \frac{\sin v}{v} = \frac{3}{4}$

24) $\lim_{h\to 0^-} \frac{h}{\sin 3h} \Rightarrow v = 3h \Rightarrow \lim_{\frac{v}{3}\to 0^-} \frac{v}{3\sin v} = \frac{1}{3} \lim_{v\to 0} \frac{1}{\frac{\sin v}{v}} = \frac{1}{3} \frac{\lim_{v\to 0} 1}{\lim_{v\to 0} \frac{\sin v}{v}} = \frac{1}{3} \times \frac{1}{1} = \frac{1}{3}$

25) $\lim_{x\to 0} \frac{\tan 2x}{x} \Rightarrow v = 2x \Rightarrow \lim_{\frac{v}{2}\to 0} \frac{2\sin v}{v\cos v} = 2\lim_{v\to 0} \frac{\sin v}{v} \cdot \lim_{v\to 0} \frac{1}{\cos v} = 2 \cdot 1 \cdot 1 = 2$

26) $\lim_{t\to 0} \frac{2t}{\tan t} = \lim_{t\to 0} \frac{2t\cos t}{\sin t} = 2\lim_{t\to 0} \frac{1}{\frac{\sin t}{t}} \cdot \lim_{t\to 0} \cos t = 2 \cdot 1 \cdot 1 = 2$

27) $\lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x} = \lim_{x\to 0} \frac{x}{\sin x\cos x} + \lim_{x\to 0} \frac{x\cos x}{\sin x \cos x} = \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}} \cdot \frac{1}{\cos x} + \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}} = 1 \cdot 1 + 1 = 2$

~~28) $\lim_{x\to 0} 6x^2 \cot x \csc 2x = \lim_{x\to 0} 6x^2 \frac{\cos x}{\sin x} \cdot \frac{1}{\sin 2x} = 6\left[\lim_{x\to 0} \frac{1}{\frac{\sin x}{x}}\right]^2 \cdot \lim_{x\to 0} \cos x = 6 \cdot (1)^2 \cdot 1 = 6$~~

28) $\lim_{x\to 0} 6x^2 \cot x \csc 2x = \lim_{x\to 0} \frac{6x^2 \cos x}{\sin x \cdot 2\sin x \cos x} = \frac{6}{2}\left[\lim_{x\to 0} \frac{1}{\frac{\sin x}{x}}\right]^2 = 3$

29) $\lim_{x\to 0} \frac{x \csc 2x}{\cos 5x} = \lim_{x\to 0} \frac{x}{\sin 2x \cos 5x} = \lim_{x\to 0} \frac{x}{2\sin x \cos x \cos 5x} = \frac{1}{2} \lim_{x\to 0} \frac{1}{\frac{\sin x}{x}} \cdot \lim_{x\to 0} \frac{1}{\cos x \cos 5x} = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

30) $\lim_{x\to 0} \frac{x^2 - x + \sin x}{2x} = \frac{1}{2}\left(\lim_{x\to 0} \frac{x^2}{x} - \lim_{x\to 0} \frac{x}{x} + \lim_{x\to 0} \frac{\sin x}{x}\right) = \frac{1}{2}(0 - 1 + 1) = 0$

31) $\lim_{\theta\to 0} \frac{1-\cos\theta}{\sin 2\theta} = \lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin\theta\cos\theta} \cdot \frac{(1+\cos\theta)}{(1+\cos\theta)} = \lim_{\theta\to 0} \frac{\sin^2\theta}{2\sin\theta\cos\theta(1+\cos\theta)} = \frac{0}{2\times 1(2)} = 0$

32) $\lim_{x\to 0} \frac{x - x\cos x}{\sin^2 3x} =$

33) $\lim_{t\to 0} \frac{\sin(1-\cos t)}{(1-\cos t)} \Rightarrow v = 1-\cos t \Rightarrow \lim_{v\to 0} \frac{\sin v}{v} = 1$

as $t\to 0 \Rightarrow v \to 1-1 = 0$

34) $\lim_{h\to 0} \frac{\sin(\sin h)}{\sin h} \Rightarrow v = \sin h \Rightarrow \lim_{v\to 0} \frac{\sin v}{v} = 1$

$h\to 0 \Rightarrow v\to 0$

35) $\lim_{\theta\to 0} \frac{\sin\theta}{\sin 2\theta} = \lim_{\theta\to 0} \frac{\cancel{\sin\theta}}{2\cancel{\sin\theta}\cos\theta} = \frac{1}{2}$

36) using $\lim_{x\to 0} \frac{\sin 5x}{\sin 4x} = \lim_{x\to 0} \frac{5x \cdot \frac{\sin 5x}{5x}}{4x \cdot \frac{\sin 4x}{4x}}$

$= \lim_{x\to 0} \frac{5x}{4x} \cdot \lim_{x\to 0} \frac{\sin 5x}{5x} \cdot \lim_{x\to 0} \frac{4x}{\sin 4x} = \frac{5}{4} \lim_{v\to 0} \frac{\sin v}{v} \cdot \lim_{v\to 0} \frac{\frac{4}{5}v}{\sin\frac{4}{5}v}$

$v = 5x$

$= \frac{5}{4} \cdot 1 \cdot \lim_{v\to 0} \frac{\frac{4}{5}v}{\sin\frac{4}{5}v} = \frac{5}{4} \cdot \lim\left[\frac{1}{\frac{\sin h}{h}}\right] = \frac{5}{4} \cdot 1 = \frac{5}{4}$

$h = \frac{4}{5}v$

37) $\lim_{\theta\to 0} \theta\cos\theta = 0 \cdot 1 = 0$

38) $\lim_{\theta\to 0} \sin\theta \cot 2\theta = \lim_{\theta\to 0} \frac{\sin\theta \cdot \cos 2\theta}{\sin 2\theta} = \lim_{\theta\to 0} \frac{\cancel{\sin\theta}\cos 2\theta}{2\cancel{\sin\theta}\cos\theta} = \frac{1}{2}$

39) $\lim_{x\to 0} \frac{\tan 3x}{\sin 8x} = \lim_{x\to 0} \frac{3x \frac{\sin 3x}{3x}}{8x \frac{\sin 8x}{8x} \cdot \cos 3x} = \lim_{x\to 0} \frac{3x}{8x} \cdot \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \lim_{x\to 0} \frac{8x}{\sin 8x} \cdot \lim_{x\to 0} \frac{1}{\cos 3x}$

as 36

$\lim = \frac{3}{8} \cdot 1 \cdot 1 \cdot 1 = \frac{3}{8}$

40) $\lim_{y\to 0} \frac{\sin 3y \cot 5y}{y \cot 4y} = \lim_{y\to 0} \frac{\sin 3y}{y} \cdot \lim_{y\to 0} \frac{\cos 5y \cdot \sin 4y}{\sin 5y \cos 4y} =$

solving for $\lim_{y\to 0} \frac{\sin 3y}{y} \Rightarrow v = 3y$, $y\to 0 \Rightarrow v\to 0$ $\lim_{v\to 0} \frac{\sin v}{\frac{1}{3}v} = 3\times 1 = 3$

solving for $\lim_{y\to 0} \frac{\sin 4y}{\sin 5y} = \lim_{y\to 0} \frac{4y \frac{\sin 4y}{4y}}{5y \frac{\sin 5y}{5y}} = \lim_{y\to 0} \frac{4y}{5y} \frac{\lim_{v\to 0} \frac{\sin v}{v}}{\lim_{h\to 0} \frac{\sin h}{h}} = \frac{4}{5} \cdot \frac{1}{1} = \frac{4}{5}$

$v = 4y$

$h = 5y$

$\Rightarrow 3 \cdot \frac{4}{5} \cdot \lim_{y\to 0} \frac{\cos 5y}{\cos 4y} = 3 \cdot \frac{4}{5} \cdot 1 = \frac{12}{5}$

(2.4) exercises

41) $\lim_{\theta \to 0} \frac{\tan\theta}{\theta^2 \cot 3\theta} = \lim_{\theta \to 0} \frac{\sin\theta \cdot \sin 3\theta}{\theta^2 \cos\theta \cos 3\theta} = \lim_{\theta \to 0} \frac{1}{\cos\theta \cos 3\theta} \cdot \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \frac{\sin 3\theta}{\theta}$

$= 1 \cdot 1 \cdot \lim_{v \to 0} \frac{\sin v}{\frac{v}{3}} = 1 \cdot \lim_{v \to 0} \frac{3 \sin v}{v} = 1 \cdot 3 \cdot 1 = 3$

$v = 3\theta$

42) $\lim_{\theta \to 0} \frac{\theta \cot 4\theta}{\sin^2\theta \cot^2 2\theta} = \lim_{\theta \to 0} \frac{\theta \cos 4\theta \sin^2 2\theta}{\sin 4\theta \sin^2\theta \cos^2 2\theta} = \lim_{\theta \to 0} \frac{\cos 4\theta}{\cos^2 2\theta} \cdot \lim_{\theta \to 0} \frac{\theta \sin^2 2\theta}{\sin 4\theta \sin^2\theta}$

$= 1 \cdot \lim_{\theta \to 0} \frac{\theta \cancel{\cdot 4 \sin^2\theta} \cos^2\theta}{\sin 4\theta \cancel{\sin^2\theta}} = \lim_{\theta \to 0} \cos^2\theta \cdot \lim_{\theta \to 0} \frac{1}{\frac{\sin 4\theta}{4\theta}} = 1 \cdot 1 = 1$

$v = 4\theta$