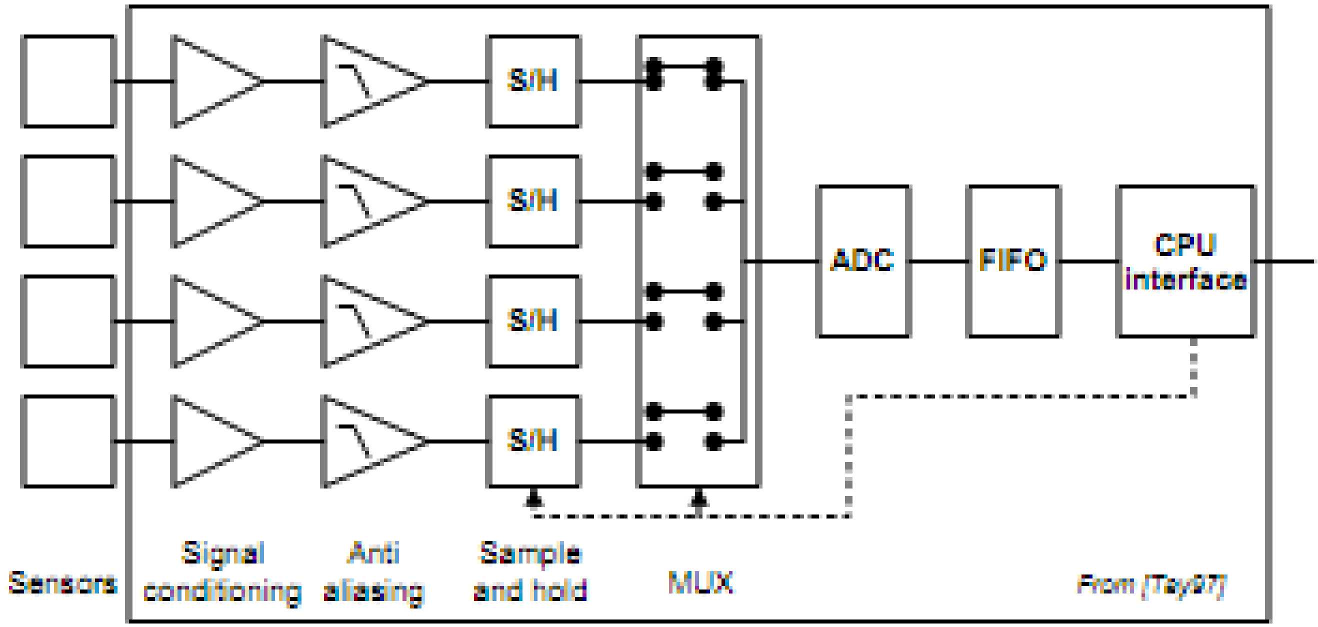
# Data Acquisition I

- Architecture of DAQ systems
- Signal conditioning
- Aliasing

# Architecture of data acquisition systems

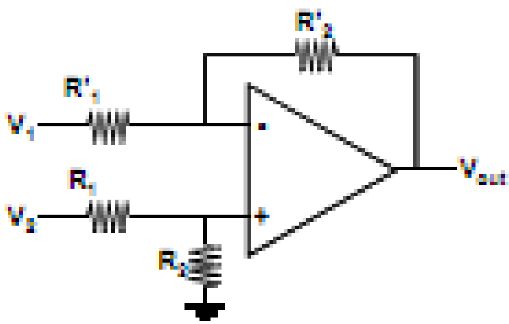



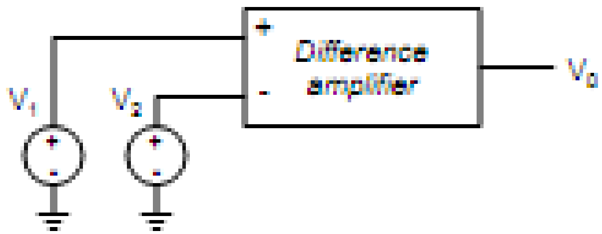
# Signal conditioning

- Instrumentation amplifiers
- Filters
- Integrators/differentiators

# Instrumentation amplifiers

- Consider the difference amplifier we saw in the previous lecture

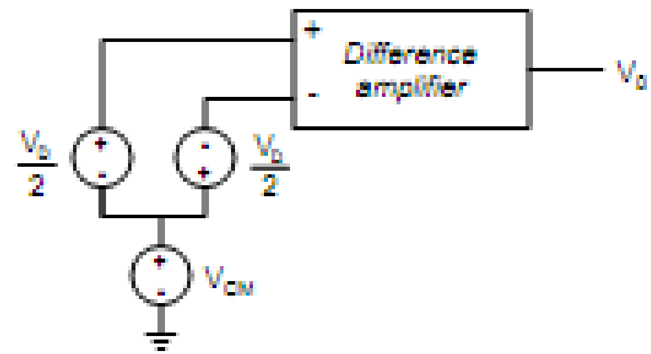



- We define COMMON-MODE and DIFFERENCE-MODE voltage as

$$V_{CM} = \frac{V_2 + V_1}{2}$$

$$V_D = V_2 - V_1$$



# Instrumentation amplifiers

- **As a result of a mismatch in the resistors ($R'_k \neq R_k$), the differential inputs may not have the same gain**

$$V_0 = G(V_2 - V_1) \overset{R'_k \neq R_k}{=} G_2V_2 - G_1V_1 = G_2\left(-\frac{V_D}{2} + V_{CM}\right) - G_1\left(\frac{V_D}{2} + V_{CM}\right) =$$

$$= -V_D\left(\frac{G_2 + G_1}{2}\right) + V_{CM}(G_2 - G_1) = -V_D G_D + V_{Cm} G_{CM}$$

- **We define COMMON-MODE REJECTION RATIO as**

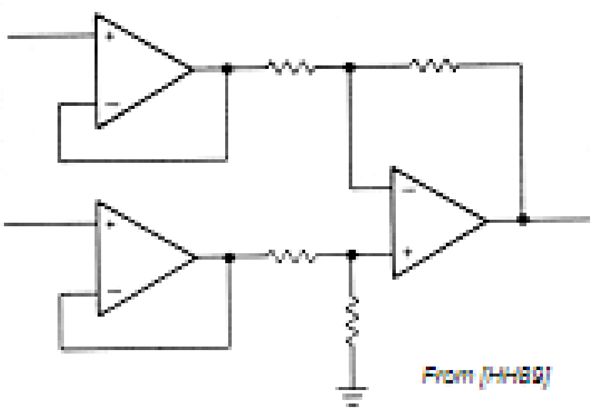
$$CMRR = 20\log_{10}\left(\frac{G_D}{G_{CM}}\right) = 20\log_{10}\left(\frac{G_2 + G_1}{2(G_2 - G_1)}\right)$$

  - CMRR is, in practice, a function of frequency, and its magnitude decreases with increasing frequency

- **An additional shortcoming of the difference amplifier is its LOW INPUT IMPEDANCE**

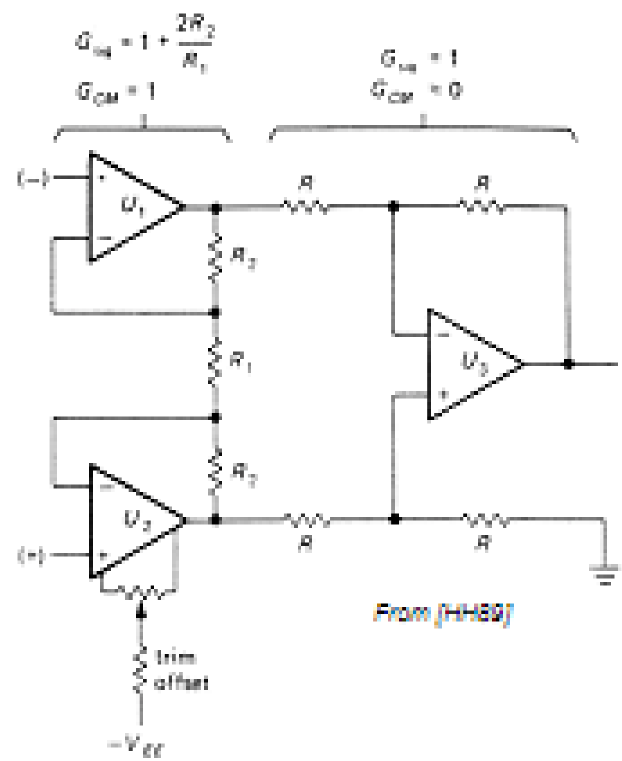
# Instrumentation amplifiers

- The term INSTRUMENTATION AMPLIFIER is used to denote a difference amplifier with
  - High gain (INA2126)
  - Single-ended output
  - High input impedance
  - High CMRR
- High input impedance may be achieved by buffering the differential inputs
  - This solution, however, requires high CMRR both in the followers and in the final op-amp
    - Otherwise, since the input buffers have unity gain, all the CM rejection must come in the output op-amp, requiring precise resistor matching

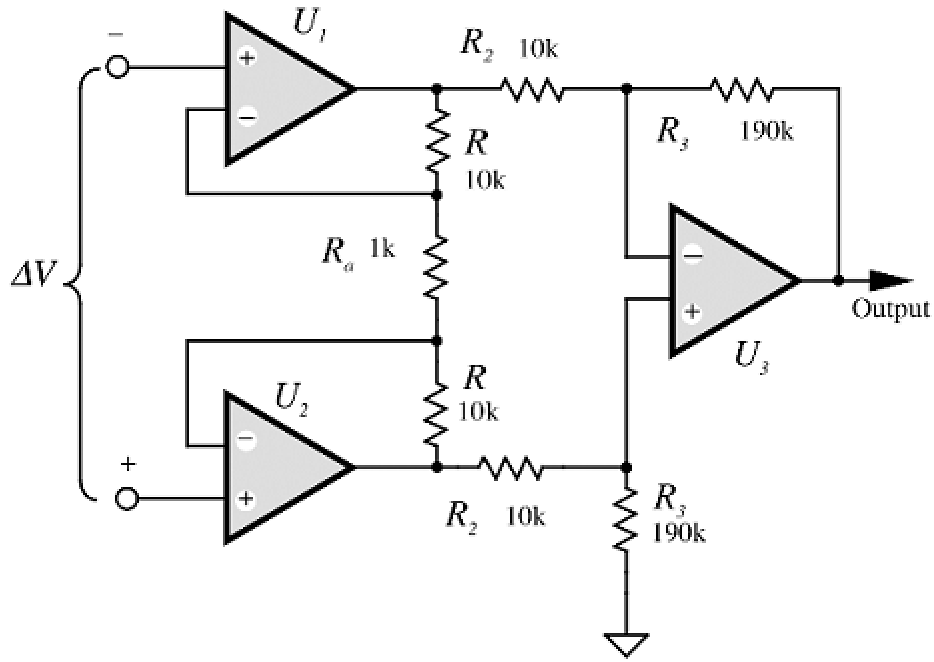


# Common mode rejection ratio

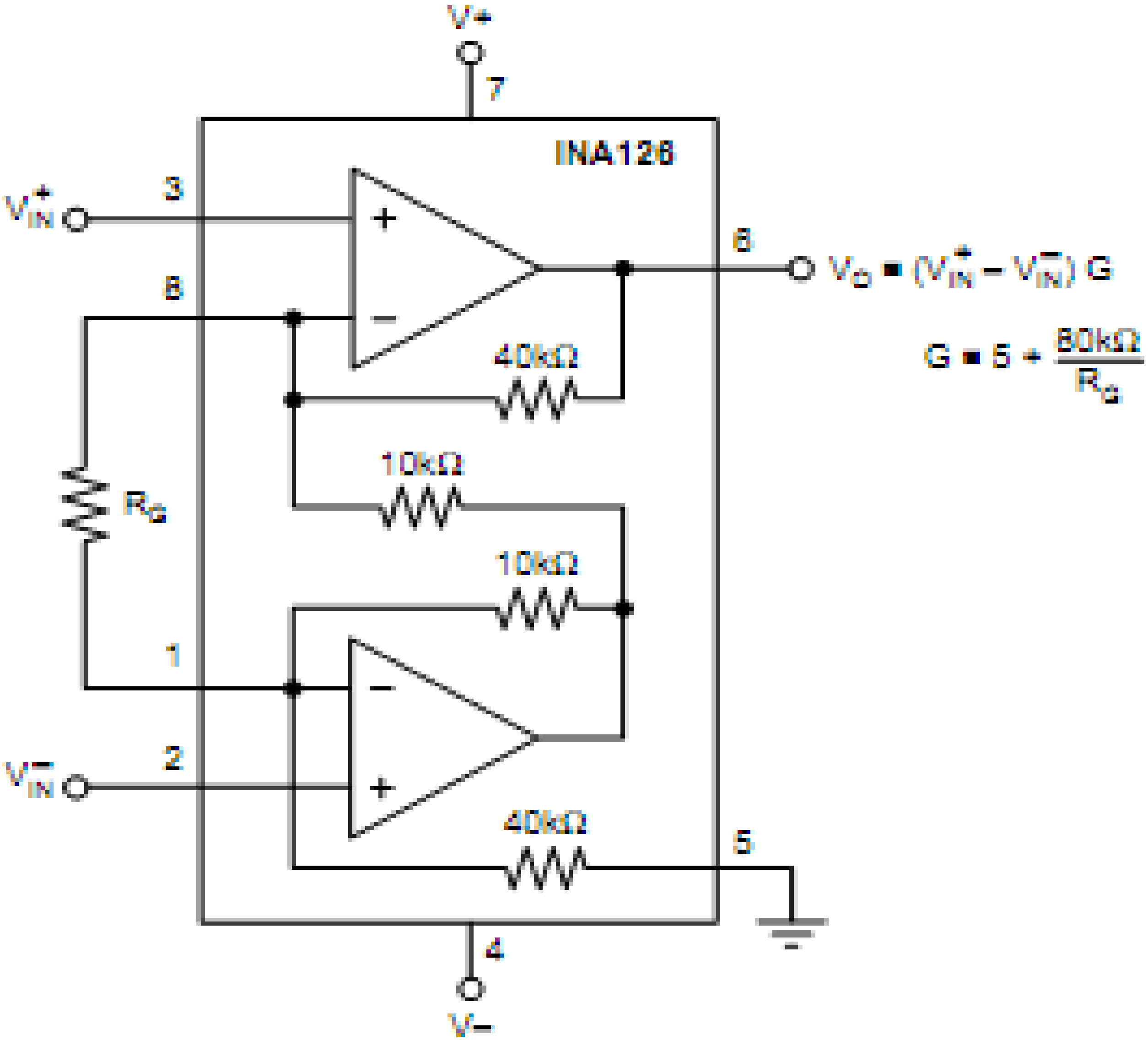
- A better solutions is the “standard” instrumentation amplifier shown below
  - Input stage provides high GD and unity GCM
    - Close resistor (R2) matching is NOT critical
  - As a result, the output op-amp (U3) does not require exceptional CMRR and resistor matching in U3is not critical
  - Offset trimming can be done at one of the input op-amps



# INA...



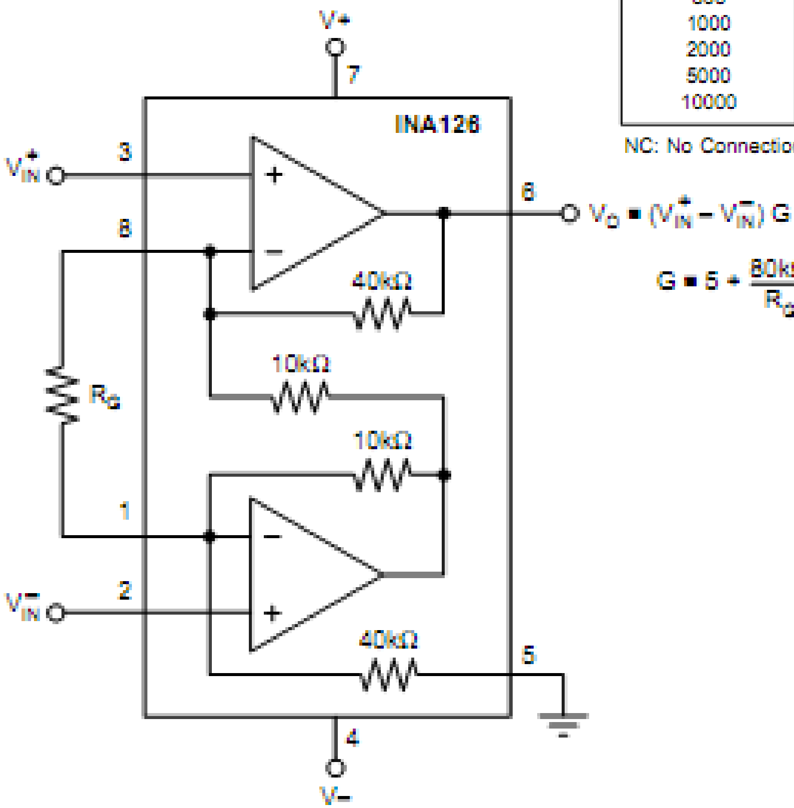

$$A = \left(1 + \frac{2R}{R_a}\right)\frac{R_3}{R_2}$$


V+
7
INA128
3
$V_{IN}^{+}$
8
6
$V_O = (V_{IN}^{+} - V_{IN}^{-})\,G$
$G = 5 + \frac{80k\Omega}{R_G}$
40kΩ
10kΩ
$R_G$
10kΩ
1
2
$V_{IN}^{-}$
40kΩ
5
4
V−

# INA126.....

| DESIRED GAIN (V/V) | $R_G$ ($\Omega$) | NEAREST 1% $R_G$ VALUE |
|---|---|---|
| 5 | NC | NC |
| 10 | 16k | 15.8k |
| 20 | 5333 | 5360 |
| 50 | 1779 | 1780 |
| 100 | 842 | 845 |
| 200 | 410 | 412 |
| 500 | 162 | 162 |
| 1000 | 80.4 | 80.6 |
| 2000 | 40.1 | 40.2 |
| 5000 | 16.0 | 15.8 |
| 10000 | 8.0 | 7.87 |

NC: No Connection.

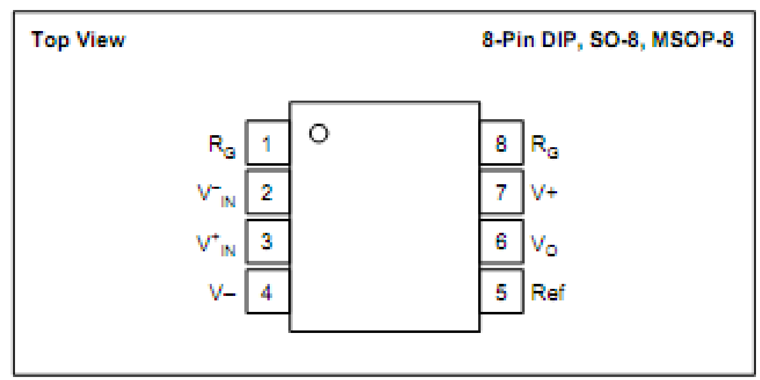



$$G = 5 + \frac{80k\Omega}{R_G}$$

## APPLICATIONS

- INDUSTRIAL SENSOR AMPLIFIER: Bridge, RTD, Thermocouple
- PHYSIOLOGICAL AMPLIFIER: ECG, EEG, EMG
- MULTI-CHANNEL DATA ACQUISITION
- PORTABLE, BATTERY OPERATED SYSTEMS

## PIN CONFIGURATION (Single)

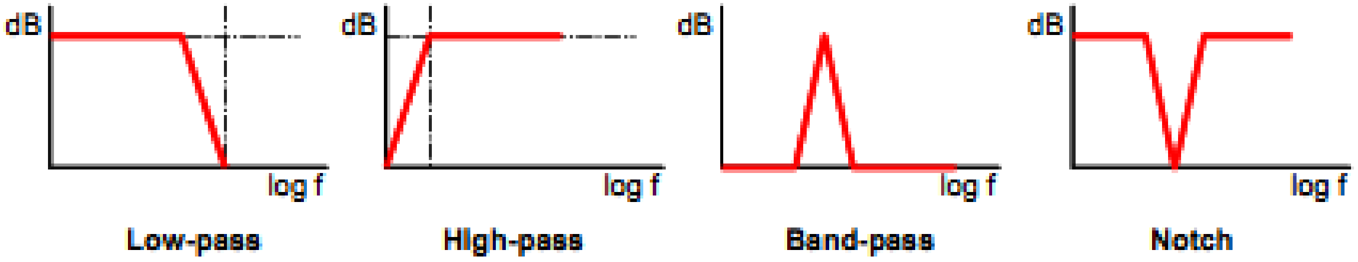


# Filters

- Filters are used to remove unwanted bandwidths from a signal
- Filter classification according to implementation
  - Active filters include RC networks and op-amps
    - Suitable for low frequency, small signal
    - Active filters are preferred since avoid the bulk and non-linearity of inductors and can have gains greater than 0dB
    - However, active filters require a power supply
  - Passive filters consist of RCL networks
    - Simple, more suitable for frequencies above audio range, where active filters are limited by the op-map bandwidth
- Digital filters
  - DSP is beyond the scope of this course

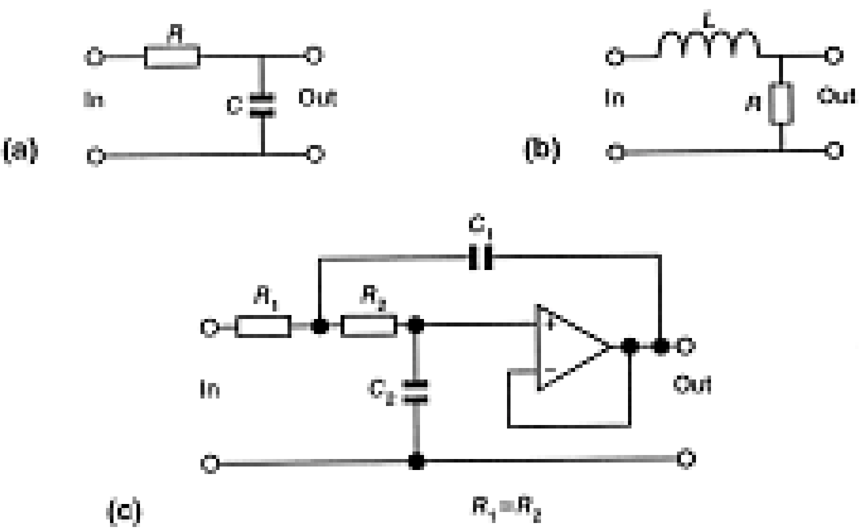
# Filters

- Filter classification according to frequency response
  - Low-pass filter
  - n High-pass filter
  - Band-pass filter
  - Band-stop (Notch)



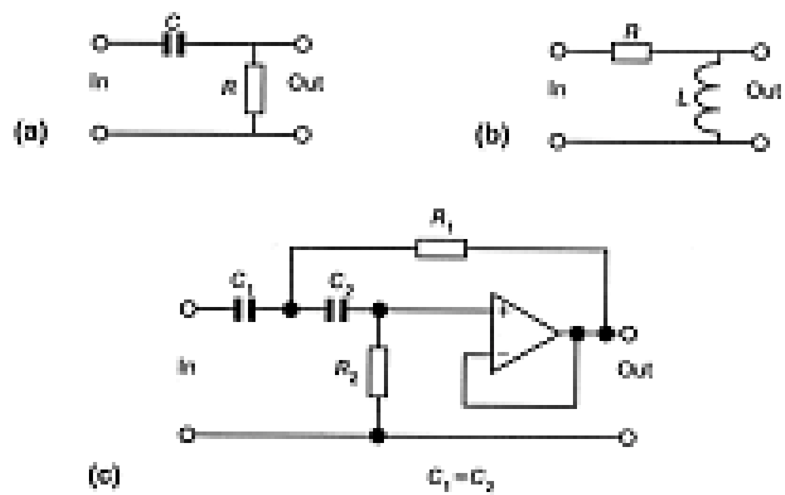
# Low- and high-pass filters

- Low pass filters




*From [Ram96]*

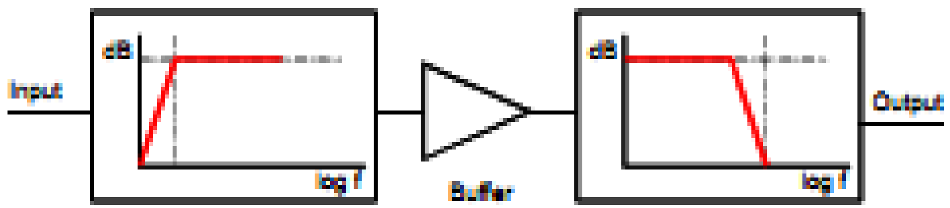
- High pass filters



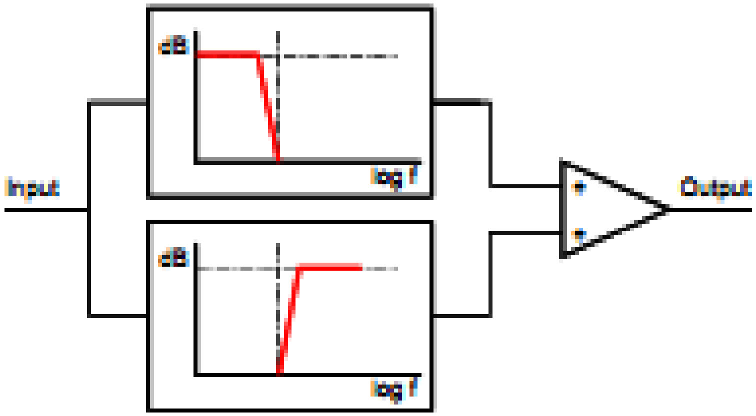

*From [Ram96]*

# Band-pass and band-stop filters

- **Band-pass**
  - High-pass and low pass in series
    - High-pass should usually precede
      - Corner frequency of low-pass must then be higher
    - If these are passive filters they should be buffered in between

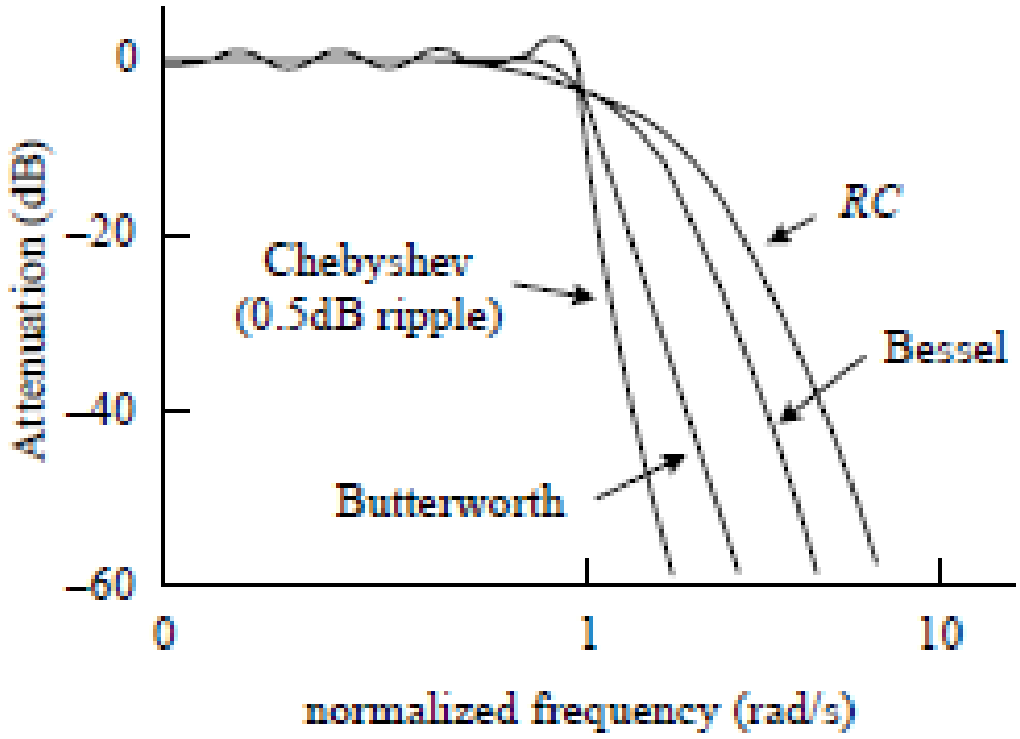



- **Band-stop**
  - High-pass and low-pass in parallel followed by a summer
    - Corner frequency of high-pass must be higher

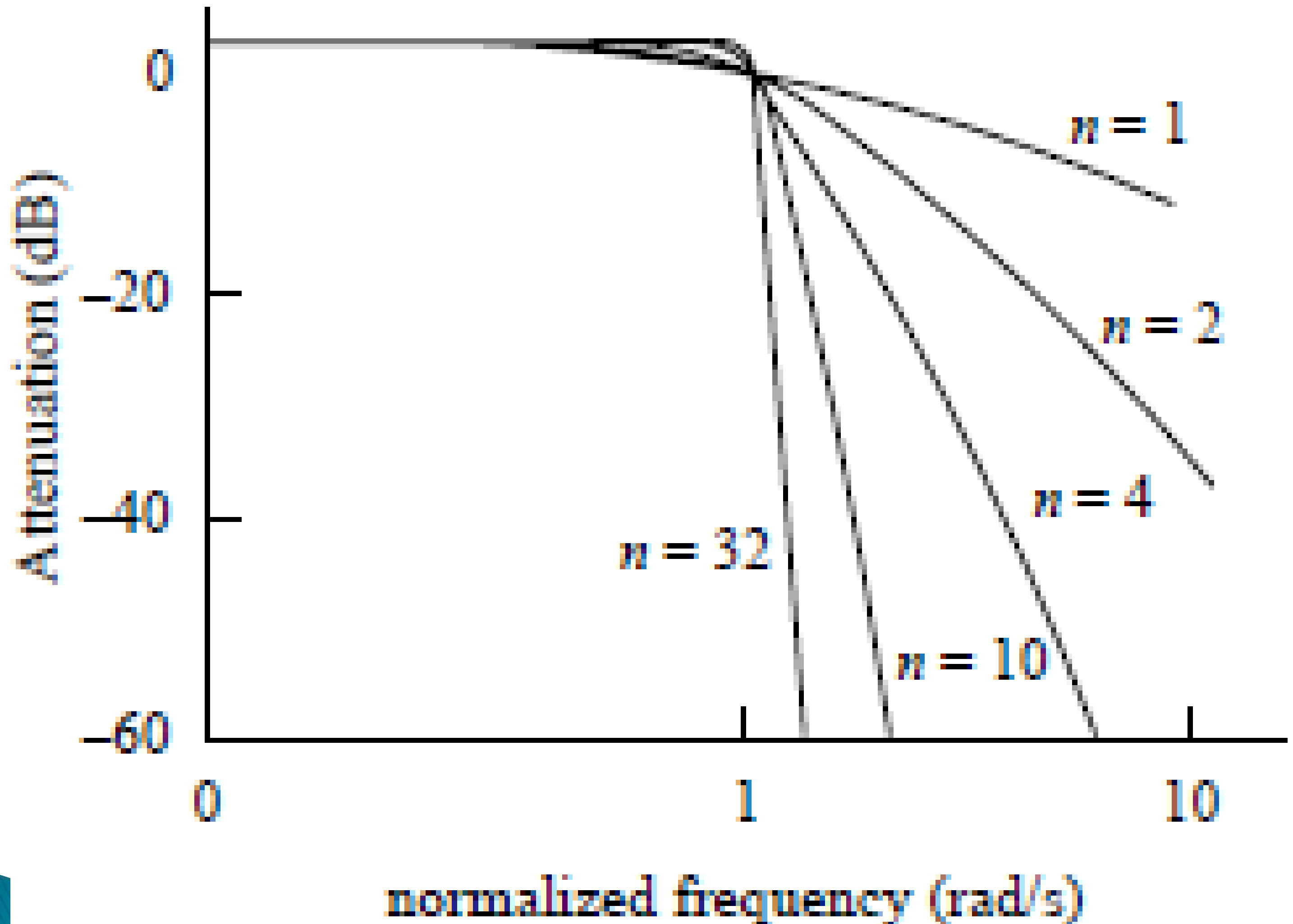


# Types of Filters

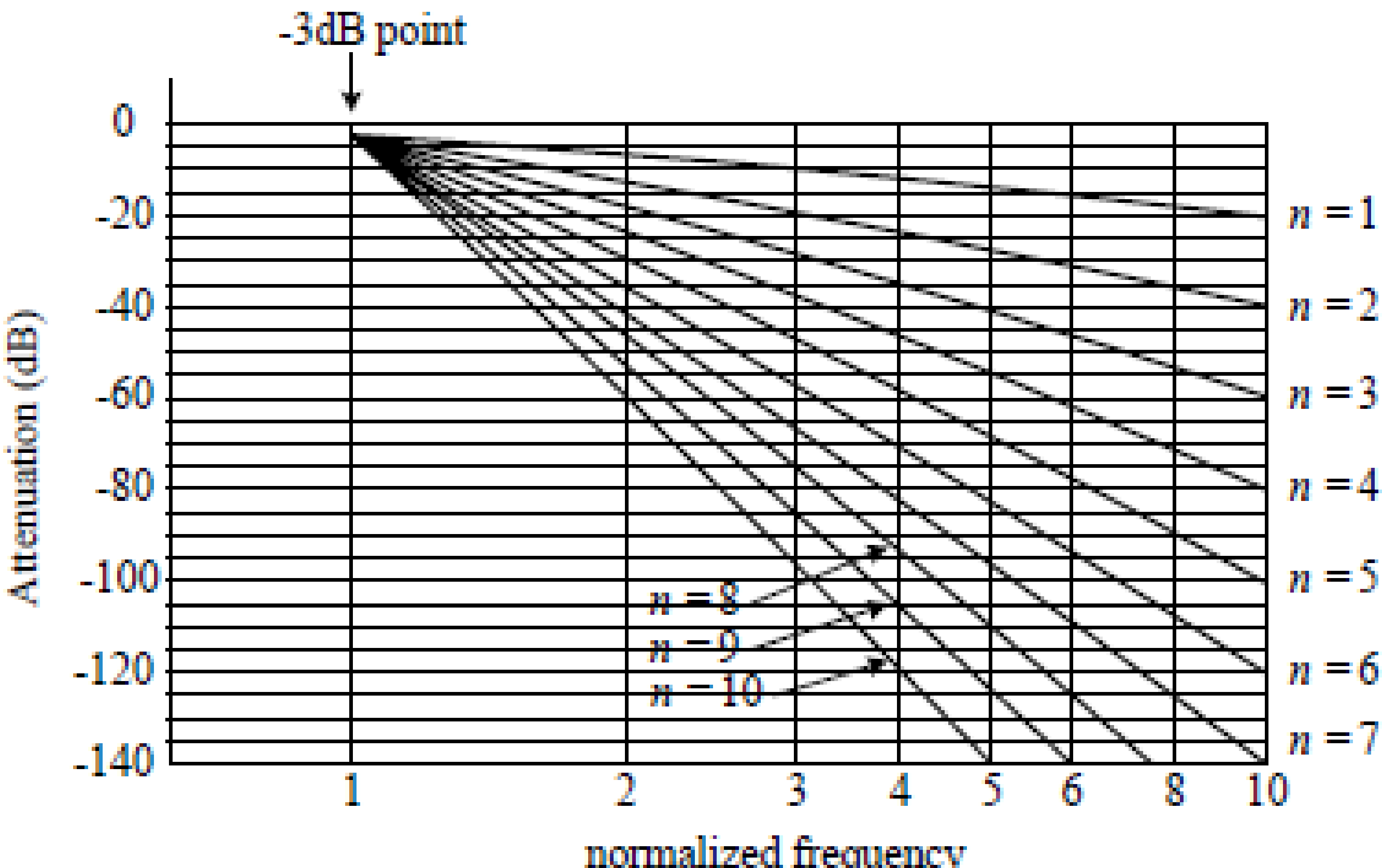
$$T(S) = \frac{V_{out}}{V_{in}} = \frac{N_m S^m + N_{m-1} S^{m-1} + \cdots + N_1 S + N_0}{D_n S^n + D_{n-1} S^{n-1} + \cdots + D_1 S + D_0}$$



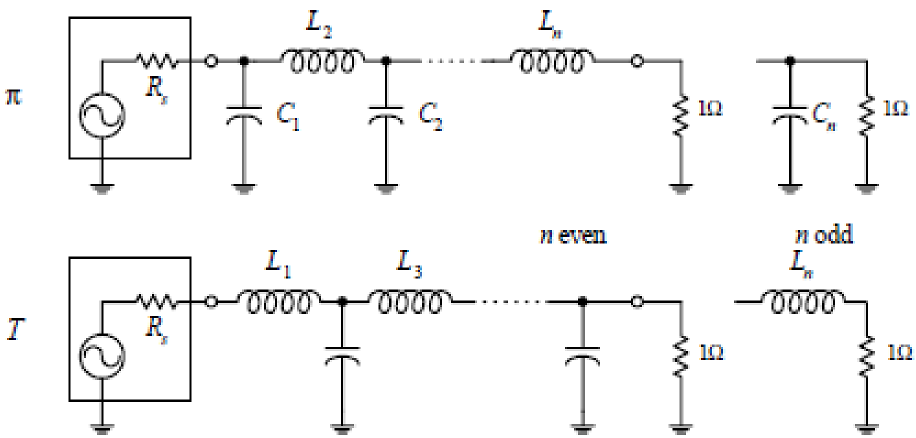
# Normalized low-pass Butterworth filter reponse curves



# Attenuation curves for Butterworth LPF



# LC Low Pass Filter network



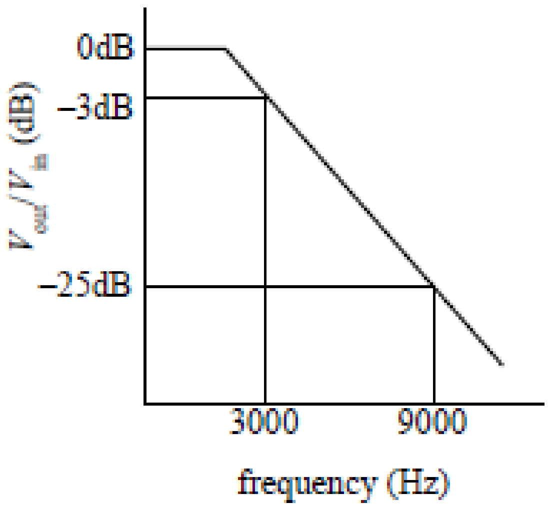

| π {T} n | $R_s$ {$1/R_s$} | $C_1$ {$L_1$} | $L_2$ {$C_2$} | $C_3$ {$L_3$} | $L_4$ {$C_4$} | $C_5$ {$L_5$} | $L_6$ {$C_6$} | $C_7$ {$L_7$} |
|---|---|---|---|---|---|---|---|---|
| 2 | 1.000 | 1.4142 | 1.4142 | | | | | |
| 3 | 1.000 | 1.0000 | 2.0000 | 1.0000 | | | | |
| 4 | 1.000 | 0.7654 | 1.8478 | 1.8478 | 0.7654 | | | |
| 5 | 1.000 | 0.6180 | 1.6180 | 2.0000 | 1.6180 | 0.6180 | | |
| 6 | 1.000 | 0.5176 | 1.4142 | 1.9319 | 1.9319 | 1.4142 | 0.5176 | |
| 7 | 1.000 | 0.4450 | 1.2470 | 1.8019 | 2.0000 | 1.8019 | 1.2470 | 0.4450 |

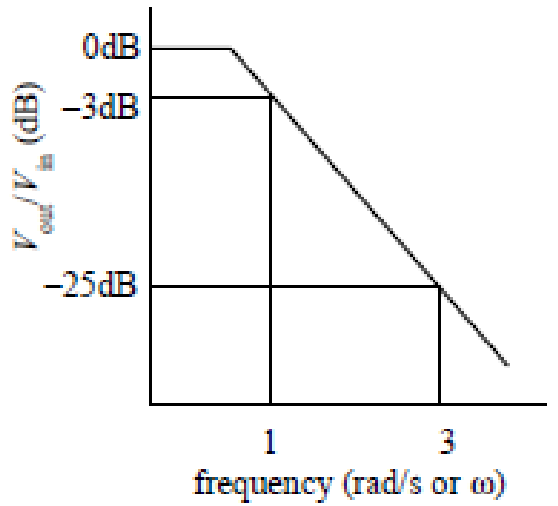
*Note:* Values of $L_n$ and $C_n$ are for a 1-Ω load and −3-dB frequency of 1 rad/s and have units of H and F. These values must be scaled down. See text.

# Example

Suppose that you want to design a low-pass filter that has a $f_{3dB} = 3000$ Hz (attenuation is –3 dB at 3000 Hz) and an attenuation of –25 dB at a frequency of 9000 Hz—which will be called the *stop frequency* $f_s$. Also, let's assume that both the signal-source impedance $R_s$ and the load impedance $R_L$ are equal to 50 Ω. How do you design the filter?

## Step 1: Normalization



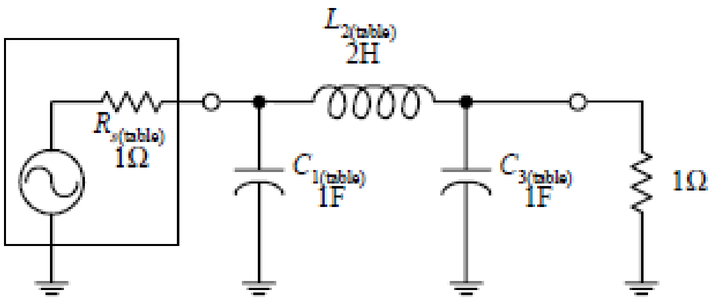




## Step 2&3: Pick Response Curve and determine number of Poles

n=3 from the curve for Butterworth

## Step 4: Create Normalized Filter

π

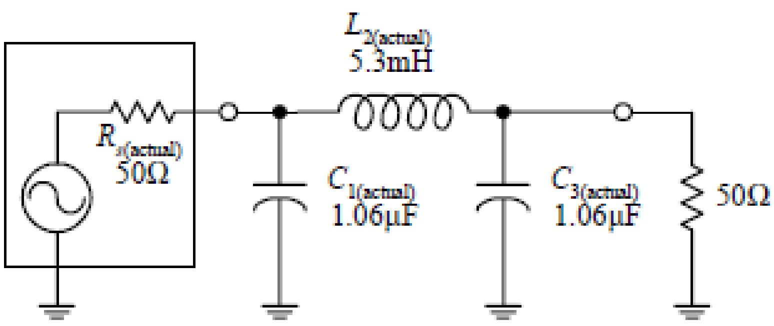



## Step 5: Frequency and Impedance Scaling

$$L_{2(actual)} = \frac{R_L L_{2(table)}}{2\pi f_{3dB}} = \frac{(50\Omega)(2\ \text{H})}{2\pi(3000\ \text{Hz})} = 5.3\ \text{mH}$$

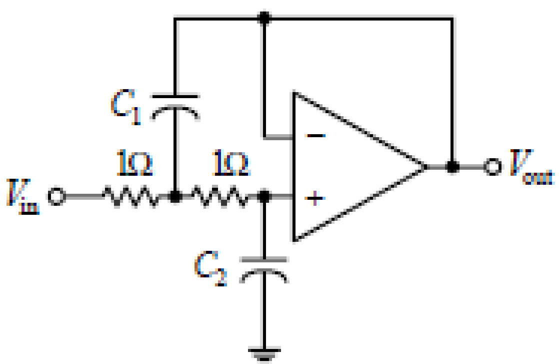
$$C_{1(actual)} = \frac{C_{1(table)}}{2\pi f_{3dB} R_L} = \frac{1\ \text{F}}{2\pi(3000\ \text{Hz})(50\ \Omega)} = 1.06\ \mu\text{F}$$

$$C_{3(actual)} = \frac{C_{3(table)}}{2\pi f_{3dB} R_L} = \frac{1\ \text{F}}{2\pi(3000\ \text{Hz})(50\ \Omega)} = 1.06\mu\text{F}$$

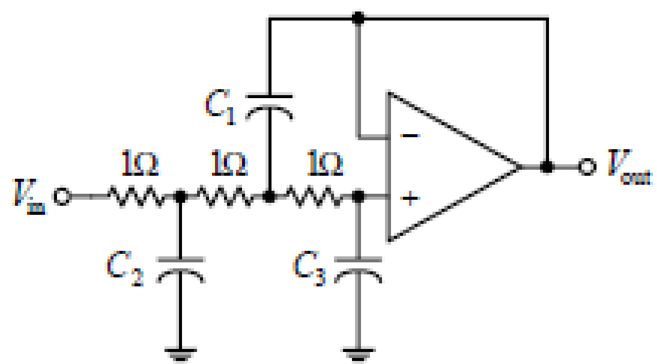


# Active Filter Design

Basic two-pole section




Basic three-pole section



# Normalized Curves

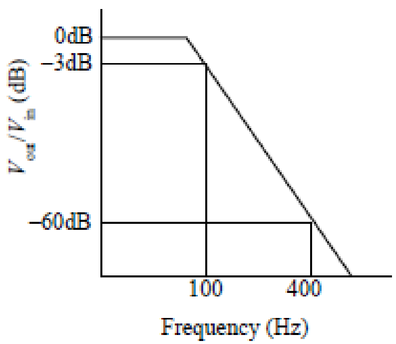
**TABLE 8.2 Butterworth Normalized Active Low-Pass Filter Values**

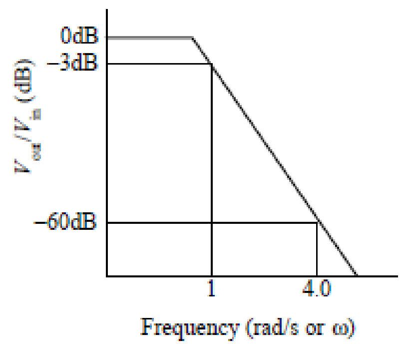
| ORDER $n$ | NUMBER OF SECTIONS | SECTIONS | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|---|---|
| 2 | 1 | 2-pole | 1.414 | 0.7071 | |
| 3 | 1 | 3-pole | 3.546 | 1.392 | 0.2024 |
| 4 | 2 | 2-pole | 1.082 | 0.9241 | |
| | | 2-pole | 2.613 | 0.3825 | |
| 5 | 2 | 3-pole | 1.753 | 1.354 | 0.4214 |
| | | 2-pole | 3.235 | 0.3090 | |
| 6 | 3 | 2-pole | 1.035 | 0.9660 | |
| | | 2-pole | 1.414 | 0.7071 | |
| | | 2-pole | 3.863 | 0.2588 | |
| 7 | 3 | 3-pole | 1.531 | 1.336 | 0.4885 |
| | | 2-pole | 1.604 | 0.6235 | |
| | | 2-pole | 4.493 | 0.2225 | |
| 8 | 4 | 2-pole | 1.020 | 0.9809 | |
| | | 2-pole | 1.202 | 0.8313 | |
| | | 2-pole | 2.000 | 0.5557 | |
| | | 2-pole | 5.758 | 0.1950 | |

# Exqample

Suppose that you wish to design an active low-pass filter that has a 3-dB point at 100 Hz and at least 60 dB worth of attenuation at 400 Hz—which we'll call the *stop frequency* $f_s$.

# Solution

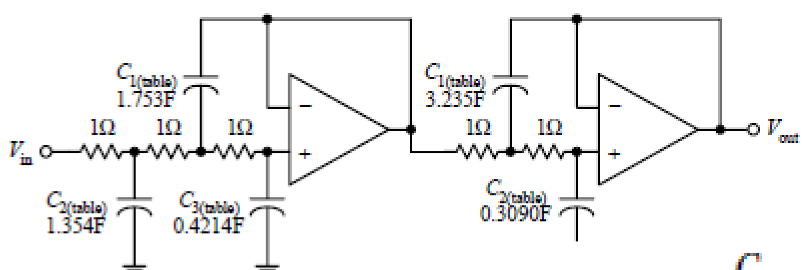



$$A_s = \frac{f_s}{f_{3dB}} = \frac{400\text{ Hz}}{100\text{ Hz}} = 4$$

n=5 from the curves

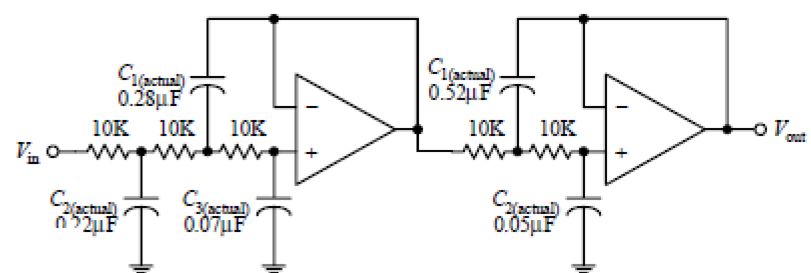
Normalized low-pass filter




$$C_{(actual)} = \frac{C_{(table)}}{Z \cdot 2\pi f_{3dB}}$$

$$R_{(actual)} = ZR_{(table)}$$

Final low-pass filter



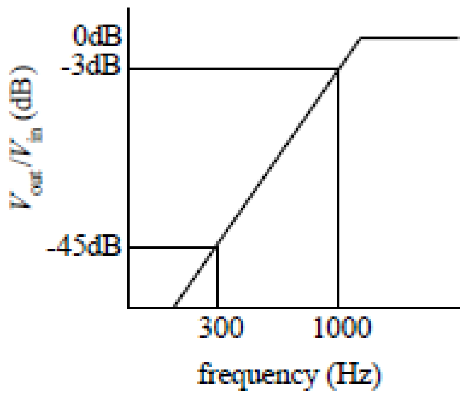
# High Pass Filter -Passive-

Suppose that you want to design a high-pass filter that has an *f3dB = 1000 Hz and an*
attenuation of at least −45 dB at 300 Hz—which we call the *stop frequency fs. Assume*
that the filter is hooked up to a source and load that both have impedances of 50 Ω
and that a Butterworth response is desired

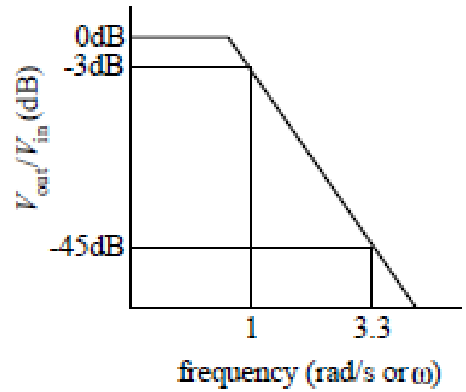
How do you design the filter?

# Solution

Frequency Response Curve



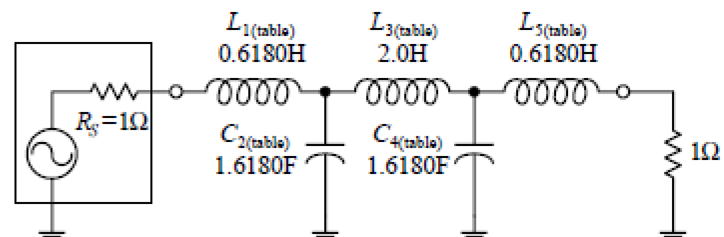

Normalized translation to low-pass filter



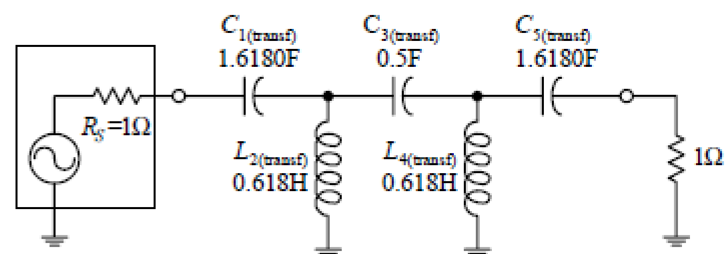

$$A_s = \frac{f_{3dB}}{f_s} = \frac{1000\text{ Hz}}{300\text{ Hz}} = 3.3$$

n=5 from the curves

Start with a "*T*" low-pass filter...




Transform low-pass filter into a high-pass filter...




To convert the low-pass into a high-pass filter, replace the inductors with capacitors that have value of $1/L$, and replace the capacitors with inductors that have values of $1/C$. In other words, do the following:

$$L_{2(transf)} = 1/C_{2(table)} = 1/1.6180 = 0.6180\text{ H}$$
$$L_{4(transf)} = 1/C_{4(table)} = 1/1.6180 = 0.6180\text{ H}$$

$$C_{1(transf)} = 1/L_{1(table)} = 1/0.6180 = 1.6180\text{ F}$$
$$C_{3(transf)} = 1/L_{3(table)} = 1/2.0 = 0.5\text{ F}$$
$$C_{5(transf)} = 1/L_{5(table)} = 1/0.6180 = 1.6180\text{ F}$$

# Now Scaling

Next, frequency and impedance scale to get the actual component values:

$$C_{1(\text{actual})} = \frac{C_{1(\text{trans})}}{2\pi f_{3\text{dB}} R_L} = \frac{1.618\ \text{H}}{2\pi(1000\ \text{Hz})(50\ \Omega)} = 5.1\ \mu\text{F}$$

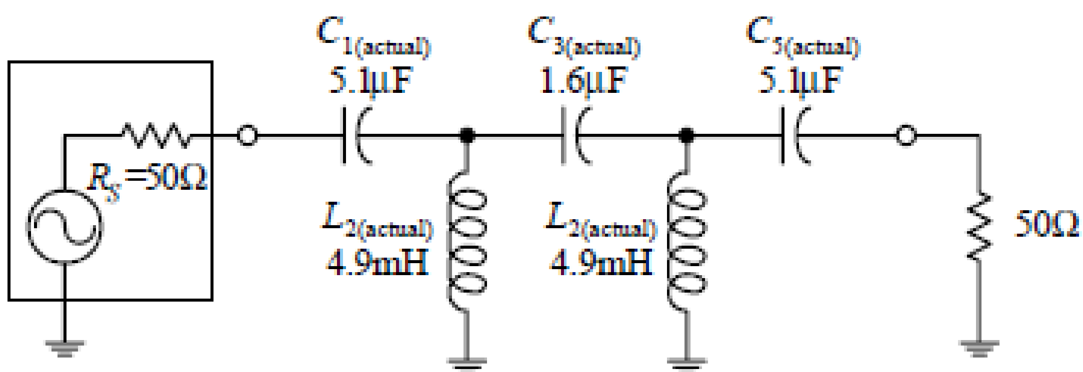
$$L_{2(\text{actual})} = \frac{L_{2(\text{trans})} R_L}{2\pi f_{3\text{dB}}} = \frac{(0.6180\ \text{F})(50\ \Omega)}{2\pi(1000\ \text{Hz})} = 4.9\ \text{mH}$$

$$C_{3(\text{actual})} = \frac{C_{3(\text{trans})}}{2\pi f_{3\text{dB}} R_L} = \frac{0.5\ \text{H}}{2\pi(1000\ \text{Hz})(50\ \Omega)} = 1.6\ \mu\text{F}$$

$$L_{4(\text{actual})} = \frac{L_{4(\text{trans})} R_L}{2\pi f_{3\text{dB}}} = \frac{(0.6180\ \text{F})(50\ \Omega)}{2\pi(1000\ \text{Hz})} = 4.9\ \text{mH}$$

$$C_{5(\text{actual})} = \frac{C_{5(\text{trans})}}{2\pi f_{3\text{dB}} R_L} = \frac{1.618\ \text{H}}{2\pi(1000\ \text{Hz})(50\ \Omega)} = 5.1\ \mu\text{F}$$

Impedance and frequency scale high-pass filter to get final circuit



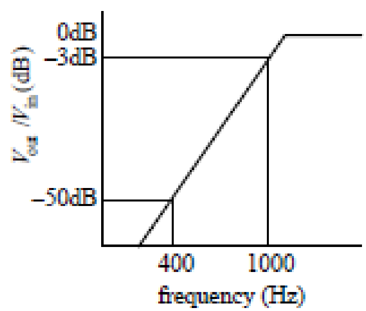
# High Pass Filter-Active-

suppose

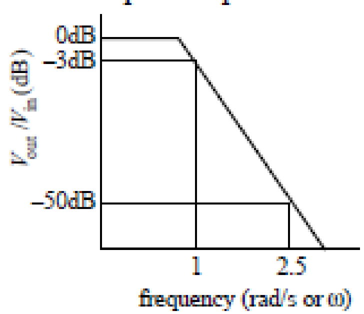
that you want to design a high-pass filter with a –3-dB frequency of 1000 Hz and 50 dB worth of attenuation at 300 Hz. What do you do?

n=5 from the curves

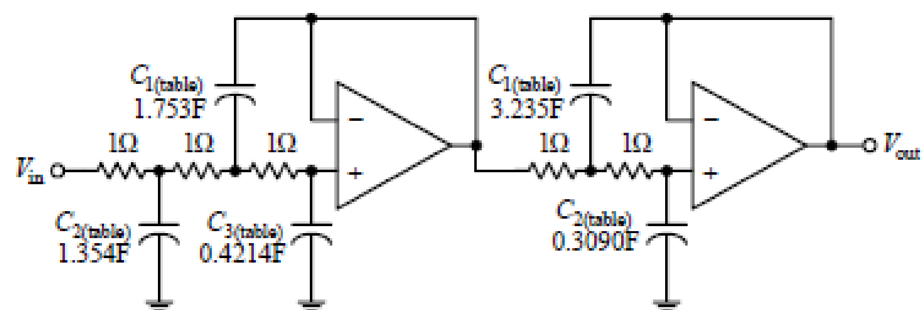
High-pass frequency response




Translation to normalized low-pass response



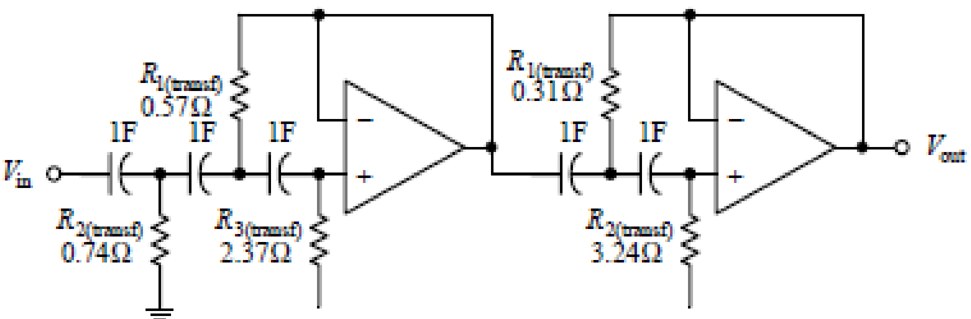

Normalized low-pass filter




Next, the normalized low-pass filter must be converted into a normalized high-pass filter. To make the conversion, exchange resistors for capacitors that have values of $1/R$ F, and exchange capacitors with resistors that have values of $1/C$ Ω.

# Follow.....

Normalized high-pass filter (transformed low-pass filter)

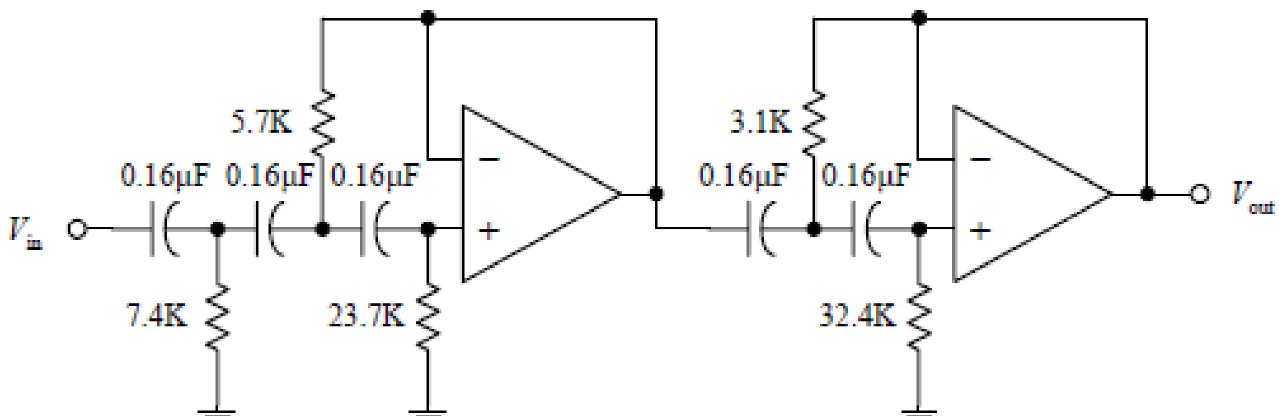



$$C_{(\text{actual})} \frac{C_{(\text{transf})}}{Z \cdot 2\pi f_{3\text{dB}}}$$

$$R_{(\text{actual})} = ZR_{(\text{transf})}$$

let $Z = 10{,}000\ \Omega$.

Final high-pass filter



# Bandpass Filter Design-Wide Band-Passive-

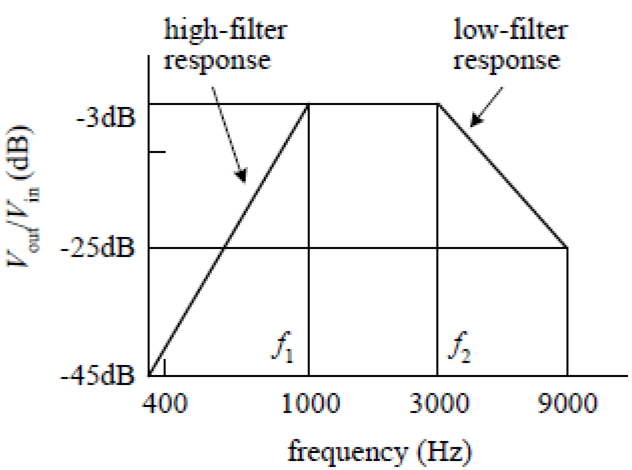
When is the filter wide band??

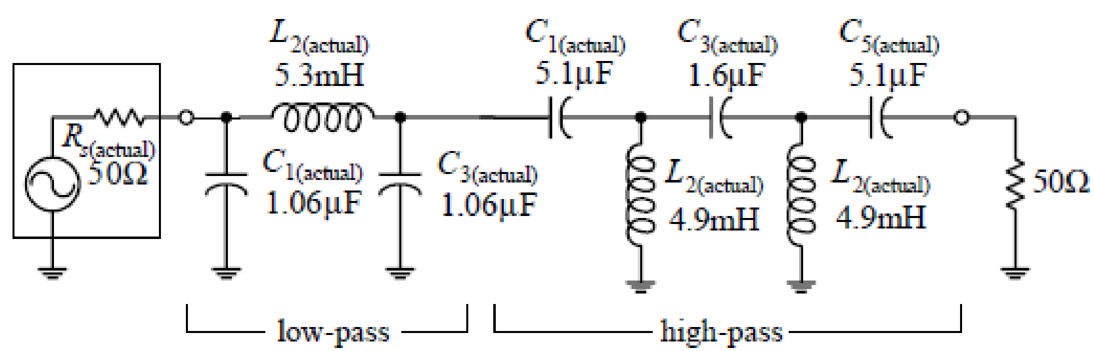
If $f_2/f_1$ is greater than 1.5,

Suppose that you want to design a bandpass filter that has –3-dB points at $f_1 = 1000$ Hz and $f_2 = 3000$ Hz and at least –45 dB at 300 Hz and more than –25 dB at 9000 Hz. Also, again assume that the source and load impedances are both 50 Ω and a Butterworth design is desired.

| | |
|---|---|
| Low-pass | –3 dB at 3000 Hz |
| | –25 dB at 9000 Hz |
| High-pass | –3 dB at 1000 Hz |
| | –45 dB at 300 Hz |

Bandpass Response Curve




Just Cascade

# Bandpass Filter Design–Wide Band–Active–

Suppose that you want to design a bandpass filter that has –3-dB points at $f_1 = 1000$ Hz and $f_2 = 3000$ Hz and at least –30 dB at 300 and 10,000 Hz. What do you do?

$$\frac{f_2}{f_1} = \frac{3000\text{ Hz}}{1000\text{ Hz}} = 3$$

Low-pass: –3 dB at 3000 Hz

–30 dB at 10,000 Hz

High-pass: –3 dB at 1000 Hz

–30 dB at 300 Hz

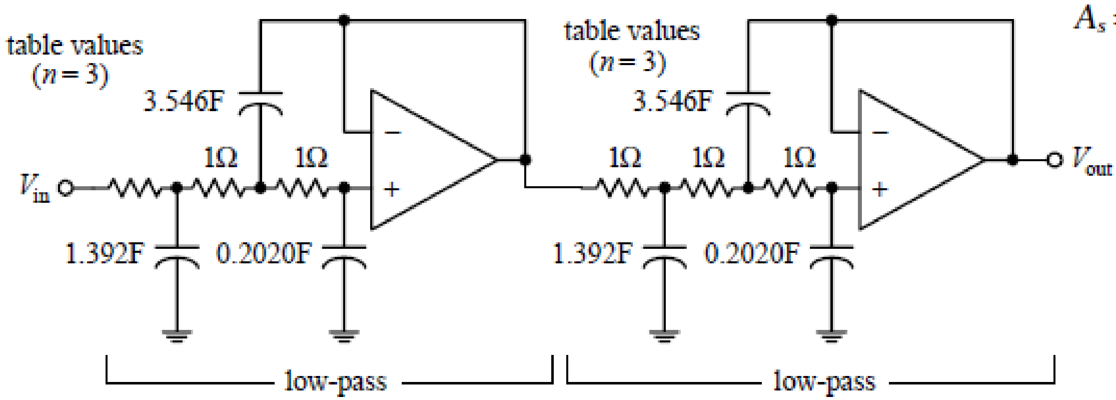
The steepness factor for the low-pass filter is

$$A_s = \frac{f_s}{f_{3\text{dB}}} = \frac{10{,}000\text{ Hz}}{3000\text{ Hz}} = 3.3$$

while the steepness factor for the high-pass filter is

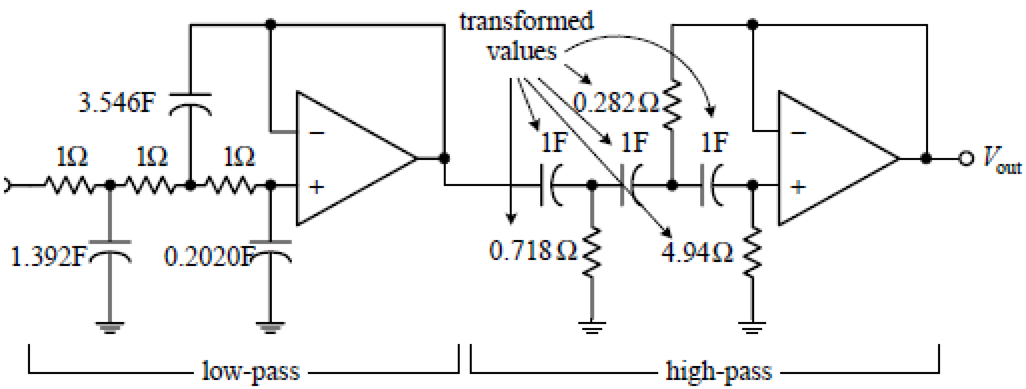
$$A_s = \frac{f_{3\text{dB}}}{f_s} = \frac{1000\text{ Hz}}{300\text{ Hz}} = 3.3$$

Normalized low-pass/low-pass inital setup

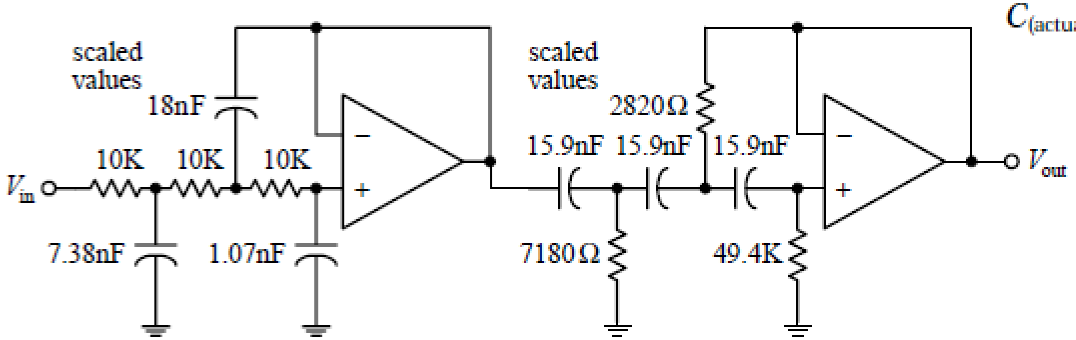


# Follow…

Normalized and transformed bandbass filter




Final bandpass filter




Low-pass section:

$$C_{(\text{actual})} = \frac{C_{\text{table}}}{Z \cdot 2\pi f_{3\text{dB}}} = \frac{C_{\text{table}}}{Z \cdot 2\pi(3000\ \text{Hz})}$$

High-pass section:

$$C_{(\text{actual})} = \frac{C_{\text{table}}}{Z \cdot 2\pi f_{2\text{dB}}} = \frac{C_{\text{table}}}{Z \cdot 2\pi(1000\ \text{Hz})}$$

# Narrow Bandwidth BPF–Passive (optional)

Suppose that you want to design a bandpass filter with –3-dB points at $f_1 = 900$ Hz and $f_2 = 1100$ Hz and at least –20 dB worth of attenuation at 800 and 1200 Hz. Assume that both the source and load impedances are 50 Ω and that a Butterworth design is desired.

Since $f_2/f_1 = 1.2$, which is less than 1.5, a narrow-band filter is needed.

geometric center frequency $$f_0 = \sqrt{f_1 f_2} = \sqrt{(900\text{ Hz})(1100)} = 995\text{ Hz}$$

Next, compute the two pair of geometrically related stop-band frequencies by using

$$f_a f_b = f_0^2$$

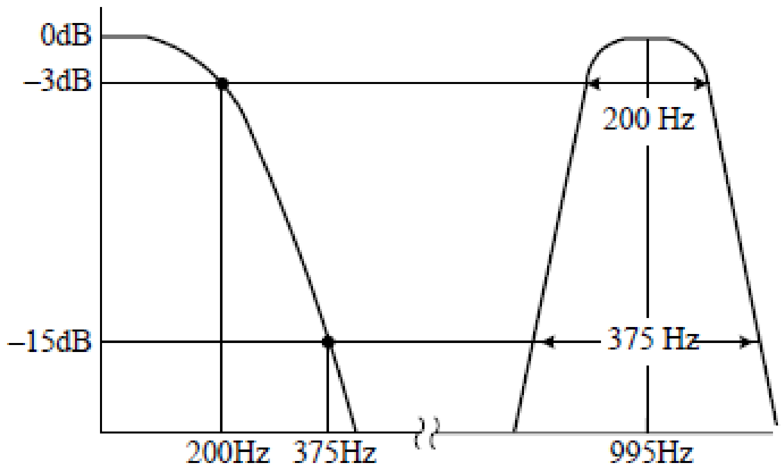
$$f_a = 800\text{ Hz} \qquad f_b = \frac{f_0^2}{f_a} = \frac{(995\text{ Hz})^2}{800\text{ Hz}} = 1237\text{ Hz} \qquad f_b - f_a = 437\text{ Hz}$$

$$f_b = 1200\text{ Hz} \qquad f_a = \frac{f_0^2}{f_b} = \frac{(995\text{ Hz})^2}{1200\text{ Hz}} = 825\text{ Hz} \qquad f_b - f_a = 375\text{ Hz}$$

Choose the pair having the least separation, which represents more severe requirements–375 Hz

# Follow.....

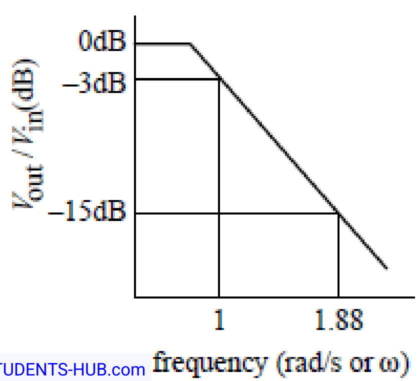
Low-pass bandpass relationship



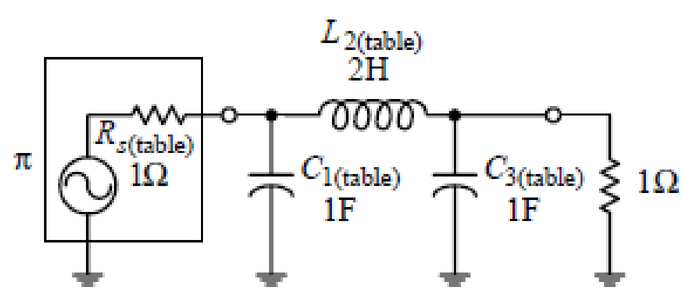

$$A_s = \frac{\text{stop-band bandwidth}}{\text{3-dB bandwidth}} = \frac{375\ \text{Hz}}{200\ \text{Hz}} = 1.88$$

From the curve n=3

Normalized low-pass response




Normalized low-pass filter



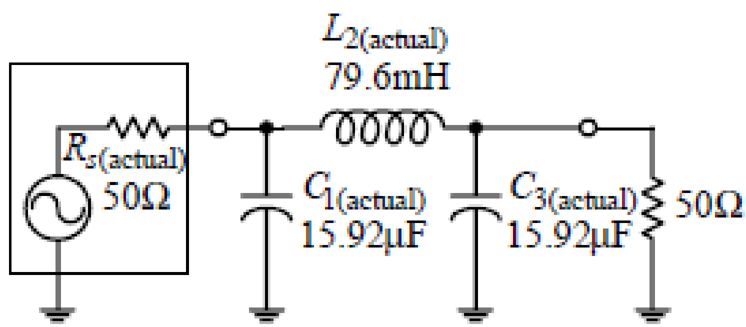
# Follow……

$$C_{1(\text{actual})} = \frac{C_{1(\text{table})}}{2\pi(\Delta f_{BW})R_L} = \frac{1\ \text{F}}{2\pi(200\ \text{Hz})(50\ \Omega)} = 15.92\ \mu\text{F}$$

$$C_{3(\text{actual})} = \frac{C_{3(\text{table})}}{2\pi(\Delta f_{BW})R_L} = \frac{1\ \text{F}}{2\pi(200\ \text{Hz})(50\ \Omega)} = 15.92\ \mu\text{F}$$

$$L_{2(\text{actual})} = \frac{L_{2(\text{table})}R_L}{2\pi(\Delta f_{BW})} = \frac{(2\ \text{H})(50\ \Omega)}{2\pi(200\ \text{Hz})} = 79.6\ \text{mH}$$

Impedance and frequency scaled low-pass filter




The important part comes now. Each circuit branch of the low-pass filter must be resonated to $f_0$ by adding a series capacitor to each inductor and a parallel inductor to each capacitor. The $LC$ resonant equation is used to determine the additional component values:

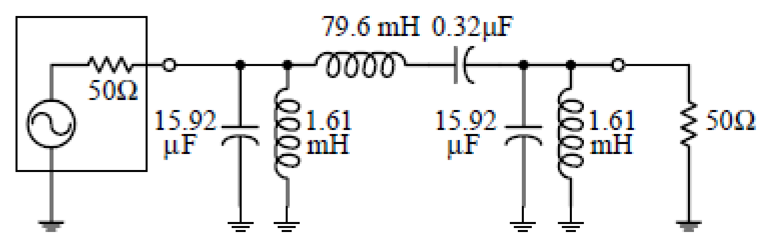
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$L_{(\text{parallel with } C1)} = \frac{1}{(2\pi f_0)^2 C_{1(\text{actual})}} = \frac{1}{(2\pi \cdot 995\ \text{Hz})^2(15.92\ \mu\text{F})} = 1.61\ \text{mH}$$

$$L_{(\text{parallel with } C3)} = \frac{1}{(2\pi f_0)^2 C_{3(\text{actual})}} = \frac{1}{(2\pi \cdot 995\ \text{Hz})^2(15.92\ \mu\text{F})} = 1.61\ \text{mH}$$

$$C_{(\text{series with } L2)} = \frac{1}{(2\pi f_0)^2 L_{2(\text{actual})}} = \frac{1}{(2\pi \cdot 995\ \text{Hz})^2(79.6\ \text{mH})} = 0.32\ \mu\text{F}$$

Final bandpass filter



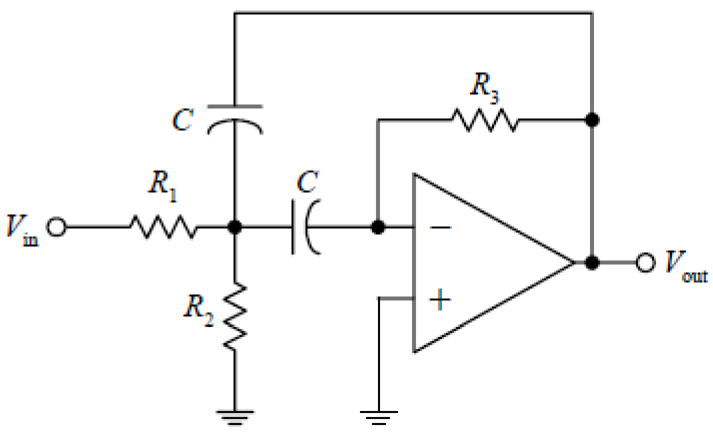
# BPF–Narrow Band–Active Design

Suppose that you want to design a bandpass filter that has a center frequency $f_0$ = 2000 Hz and a –3-dB bandwidth $\Delta f_{BW} = f_2 - f_1 = 40$ Hz. How do you design the filter?

Since $f_2/f_1 = 2040\text{ Hz}/1960\text{ Hz} = 1.04$,

No Cascading

Narrow-band filter circuit




$$Q = \frac{f_0}{f_2 - f_1} = \frac{2000\text{ Hz}}{40\text{ Hz}} = 50$$

$$R_1 = \frac{Q}{2\pi f_0 C} \qquad R_2 = \frac{R_1}{2Q^2 - 1} \qquad R_3 = 2R_1$$

$$R_1 = \frac{50}{2\pi(2000\text{ Hz})(0.01\ \mu\text{F})} = 79.6\text{ k}\Omega$$

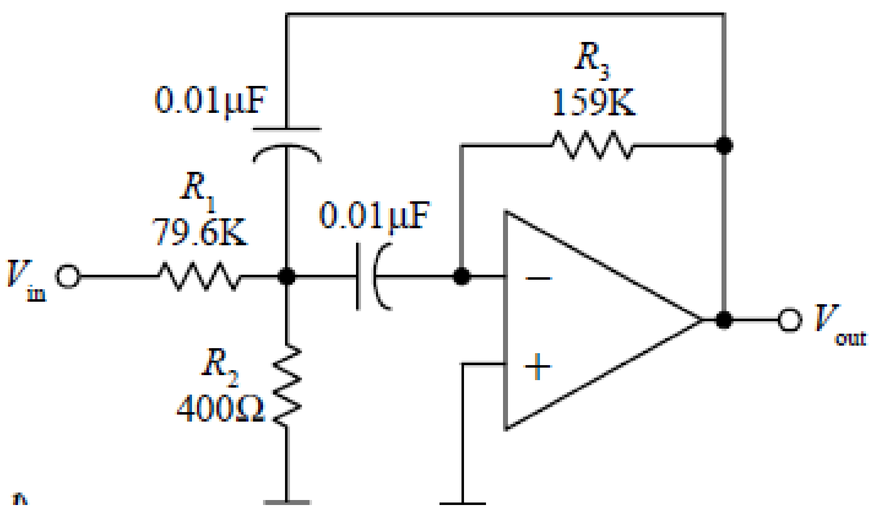
$$R_2 = \frac{79.6\text{ k}\Omega}{2(50)^2 - 1} = 400\ \Omega$$

$$R_3 = 2(79.6\text{ k}\Omega) = 159\text{ k}\Omega$$

Choose convenient value of C, let it 0.01 µF

# Final Design …

Final filter circuit



# Passive Notch Filter (optional)

**EXAMPLE**

Suppose that you want to design a notch filter with –3-dB points at $f_1 = 800$ Hz and $f_2 = 1200$ Hz and at least –20 dB at 900 and 1100 Hz. Let's assume that both the source and load impedances are 600 Ω and that a Butterworth design is desired.

First, you find the geometric center frequency:

$$f_0 = \sqrt{f_1 f_2} = \sqrt{(800\ \text{Hz})(1200\ \text{Hz})} = 980\ \text{Hz}$$

Next, compute the two pairs of geometrically related stop-band frequencies:

$$f_a = 900\ \text{Hz} \qquad f_b = \frac{f_0^2}{f_a} = \frac{(980\ \text{Hz})^2}{900\ \text{Hz}} = 1067\ \text{Hz}$$

$$f_b - f_a = 1067\ \text{Hz} - 900\ \text{Hz} = 167\ \text{Hz}$$

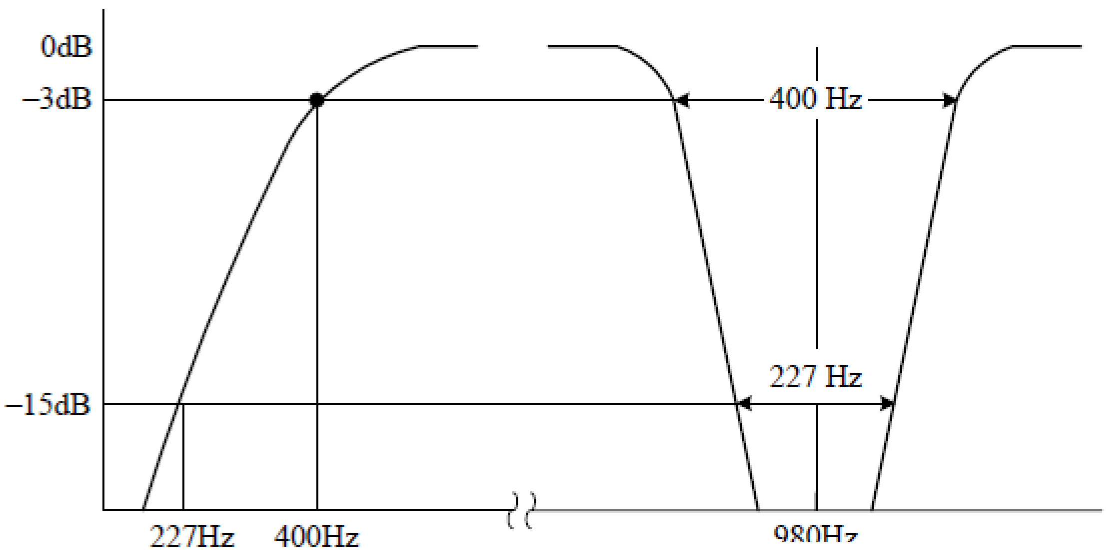
$$f_b = 1100\ \text{Hz} \qquad f_a = \frac{f_0^2}{f_b} = \frac{(980\ \text{Hz})^2}{1100\ \text{Hz}} = 873\ \text{Hz}$$

$$f_b - f_a = 1100\ \text{Hz} - 873\ \text{Hz} = 227\ \text{Hz}$$

Choose the pair of frequencies that gives the more severe requirement—227 Hz.

# Follow....

High-pass bandpass relationship



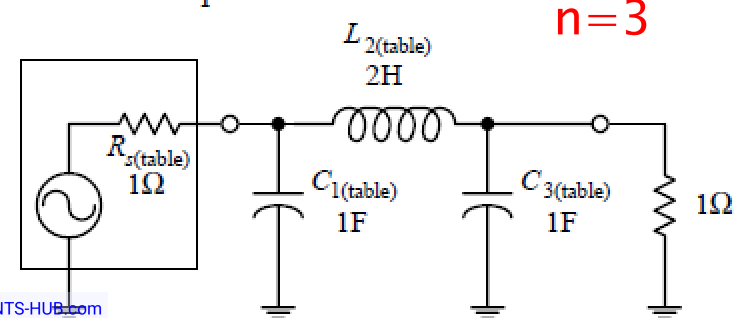

$$A_s = \frac{\text{3-dB bandwidth}}{\text{stop-band bandwidth}} = \frac{400\ \text{Hz}}{227\ \text{Hz}} = 1.7$$

$$L_{1(\text{transf})} = 1/C_{1(\text{table})} = 1/1 = 1\ \text{H}$$
$$L_{3(\text{transf})} = 1/C_{3(\text{table})} = 1/1 = 1\ \text{H}$$
$$C_{2(\text{transf})} = 1/L_{2(\text{table})} = 1/2 = 0.5\ \text{F}$$

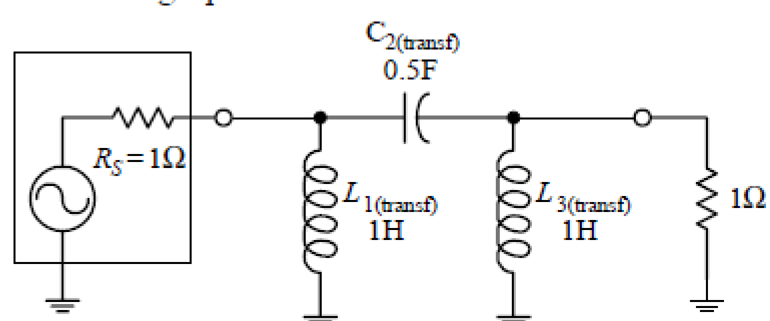
Normalized low-pass filter

n=3

π

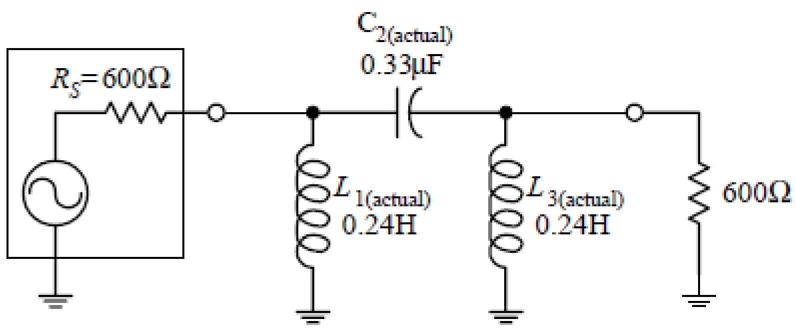



Normalized high-pass filter



# Follow…

Actual high-pass filter

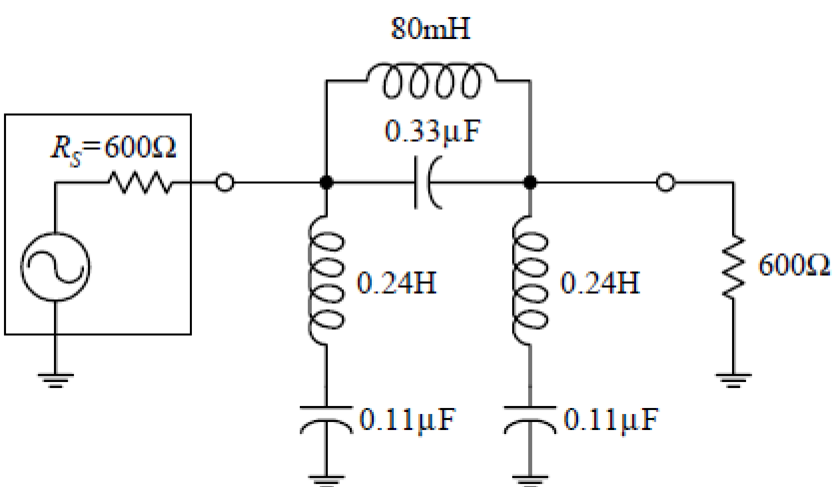



$$L_{1(actual)} = \frac{R_L L_{1(transf)}}{2\pi(\Delta f_{BW})} = \frac{(600\ \Omega)(1\ \text{H})}{2\pi(400\ \text{Hz})} = 0.24\ \text{H}$$

$$L_{3(actual)} = \frac{R_L L_{3(transf)}}{2\pi(\Delta f_{BW})} = \frac{(600\ \Omega)(1\ \text{H})}{2\pi(400\ \text{Hz})} = 0.24\ \text{H}$$

$$C_{2(actual)} = \frac{C_{1(transf)}}{2\pi(\Delta f_{BW}) R_L} = \frac{(0.5\ \text{F})}{2\pi(400\ \text{Hz})(600\ \Omega)} = 0.33\ \mu\text{F}$$

Final bandpass filter



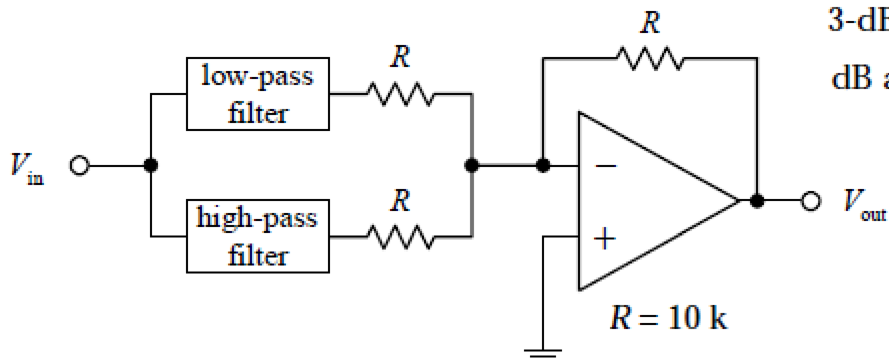

$$C_{(\text{series with } L1)} = \frac{1}{(2\pi f_0)^2 L_{1(actual)}} = \frac{1}{(2\pi \cdot 400\ \text{Hz})^2 (0.24\ \text{H})} = 0.11\ \mu\text{F}$$

$$C_{(\text{series with } L3)} = \frac{1}{(2\pi f_0)^2 L_{3(actual)}} = \frac{1}{(2\pi \cdot 400\ \text{Hz})^2 (0.24\ \text{H})} = 0.11\ \mu\text{F}$$

$$L_{(\text{parallel with } L1)} = \frac{1}{(2\pi f_0)^2 C_{2(actual)}} = \frac{1}{(2\pi \cdot 400\ \text{Hz})^2 (0.33\ \mu\text{F})} = 80\ \text{mH}$$

# Active Notch Filter –Wide–Band

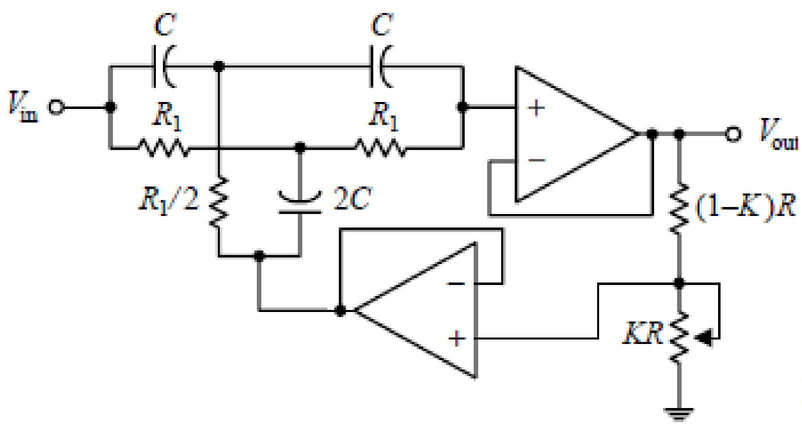
Basic wide-band notch filter




For example, if you need a notch filter to have –3-dB points at 500 and 5000 Hz and at least –15 dB at 1000 and 2500 Hz,

# Narrow-Band Notch Filter

Improved notch filter




Suppose that you want to make a "notch" at $f_0 = 2000$ Hz and desire a –3-dB bandwidth of $\Delta f_{BW} = 100$ Hz. To get this desired response, do the following: First determine the $Q$:

$$Q = \frac{\text{"notch" frequency}}{\text{–3-dB bandwidth}} = \frac{f_0}{\Delta f_{BW}} = \frac{2000\text{ Hz}}{100\text{ Hz}} = 20$$

$$R_1 = \frac{1}{2\pi f_0 C} \qquad \text{and} \qquad K = \frac{4Q-1}{4Q}$$

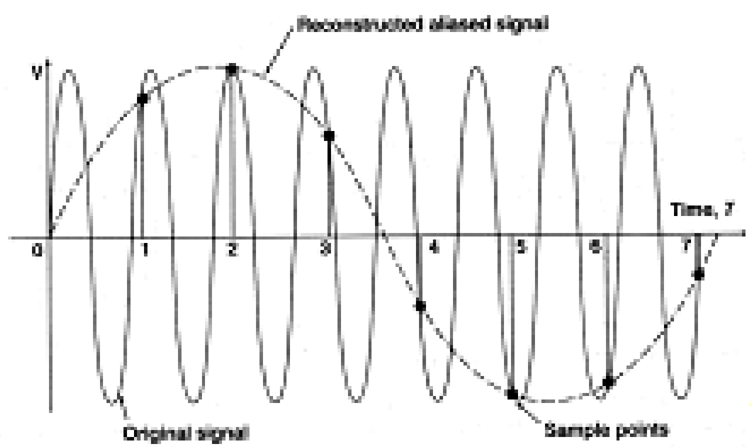
Now arbitrarily choose $R$ and $C$; say, let $R = 10$ k and $C = 0.01$ μF. Next, solve for $R_1$ and $K$:

$$R_1 = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi(2000\text{ Hz})(0.01\ \mu\text{F})} = 7961\ \Omega$$

$$K = \frac{4Q-1}{4Q} = \frac{4(20)-1}{4(20)} = 0.9875$$

# Anti-aliasing

- The sampling theorem
  - A continuous signal can be represented completely by, and reconstructed from, a set of instantaneous measurements or samples of its voltage which are made at equally-spaced times. The interval $T(=1/f_S)$ between such samples must be less than one-half the period of the highest-frequency component $f_{MAX}$ in the signal
  - In other words: you must sample at least twice the rate of the maximum frequency in your signal to prevent aliasing ($F_S \geq 2F_{MAX}$)
  - The sampling rate $F_S=2F_{MAX}$ is called the Nyquist rate



# Anti-aliasing

- The effects of aliasing can also be observed on the frequency spectrum of the signal
- In the figures below
  - $F_1$ appears correctly since $F_1 \leq F_S/2$
  - $F_2$, $F_3$ and $F_4$ have aliases at 30, 40 and 10Hz, respectively
  - You can compute these aliased frequencies by <u>folding</u> the spectrum around $F_S/2$ or with the expression

$$\text{Alias frequency } \hat{F} = \min\left|kF_S - F\right|_{\forall k \text{ integer}}$$

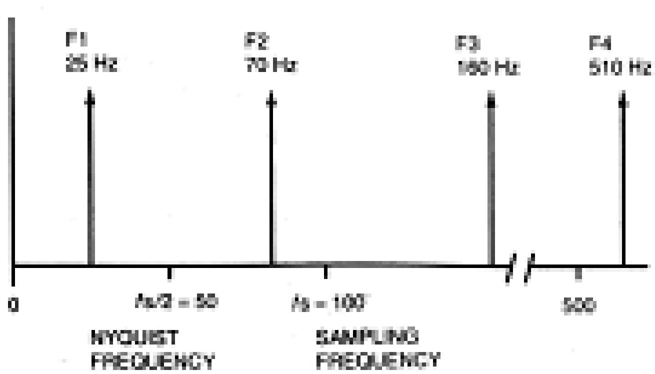



FIGURE 117.5 Spectral of signal with multiple frequencies.

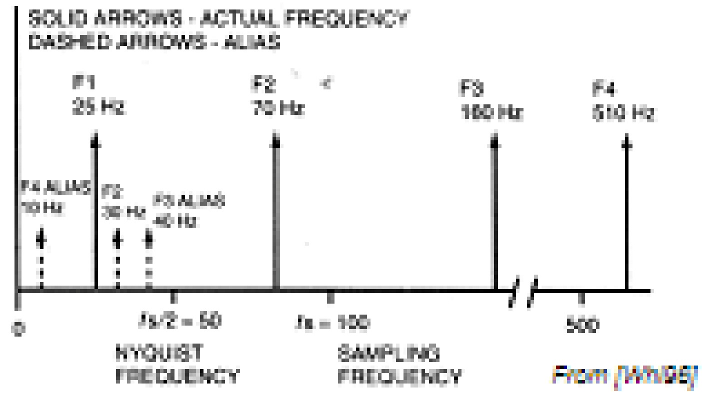
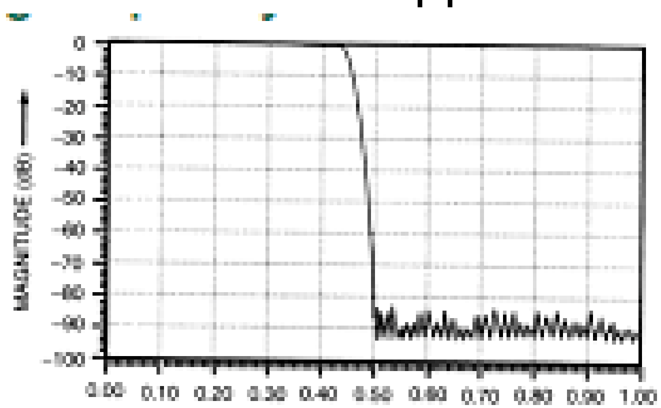



FIGURE 117.6 Spectral of signal with multiple frequencies after sampled at $fs = 100$ Hz.

# Anti-aliasing filters

- An anti-aliasing filter is a low-pass filter designed to filter out frequencies higher than the sampling frequency
  - An anti-aliasing filter should have
    - Steep cut-off and
    - Flat response in the frequency band
- Typical filters are:
  - Butterworth: flattest response in the frequency band but phase shifts well below the break frequency
  - Bessel: phase shift proportional to frequency, so the signal is not distorted by the filter
    - Recommended for anti-aliasing if it is important to preserve the waveform
  - Chebyshev: steepest cut-off but it has ripples in the band-pass




From [Whi96]