

6.1 Uniform Probability Distribution "Continuous" (67)

- Discrete prob. distributions like 1) binomial prob. distribution
2) Poisson prob. distribution
- Continuous prob. distribution like 1) Uniform prob. distribution
2) Normal prob. distribution
3) Exponential prob. distribution

• Whenever the prob. is proportional to the length of the interval, the random variable is uniformly distributed.

• Uniform prob. density function

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

• The expected value and variance for the uniform continuous prob. distribution are:

$$E(x) = \frac{a+b}{2} \quad \text{and} \quad \text{Var}(x) = \frac{(b-a)^2}{12}$$

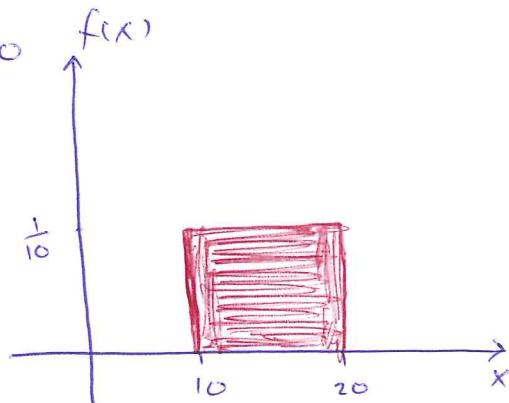
• The prob. of any single point is zero.

Example (Q2 page 229) The random variable X is known to be uniformly distributed between 10 and 20.

[a] Show the graph of the prob. density function.

$$f(x) = \begin{cases} \frac{1}{20-10} & \text{for } 10 \leq x \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$

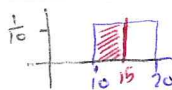
$$= \begin{cases} \frac{1}{10} & \text{for } 10 \leq x \leq 20 \\ 0 & \text{elsewhere} \end{cases}$$



[a'] Compute $P(X=15) = 0$ "No area"

[b] Compute $P(X < 15)$

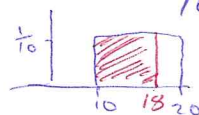
$$P(X < 15) = P(10 \leq X < 15) = \frac{1}{10} (15 - 10) = \frac{5}{10} = 0.5$$



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[c] Compute $P(12 \leq X \leq 18)$

$$= \frac{1}{10} (18 - 12) = \frac{6}{10} = 0.6$$



[d] Compute $E(X) = \frac{a+b}{2} = \frac{10+20}{2} = 15$

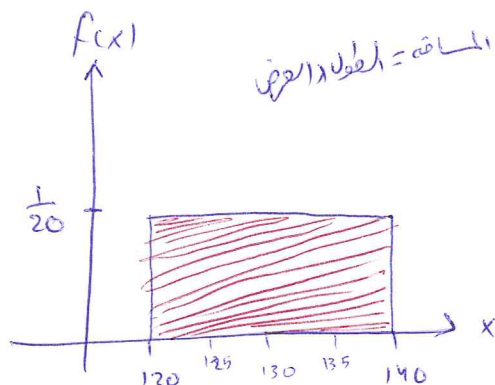
[e] Compute $Var(X) = \frac{(b-a)^2}{12} = \frac{(20-10)^2}{12} = \frac{10^2}{12} = \frac{100}{12} = 8.33$

Example: Consider the random variable X representing the flight time of an airplane traveling from Chicago to New York. Suppose X is known to be uniformly distributed between 120 min to 140 min.

[a] Show the graph of the prob. density function

$$f(x) = \begin{cases} \frac{1}{140-120} & \text{for } 120 \leq x \leq 140 \\ 0 & \text{elsewhere} \end{cases}$$

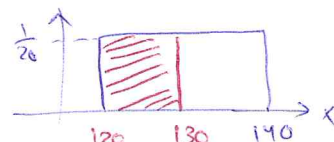
$$= \begin{cases} \frac{1}{20} & \text{for } 120 \leq x \leq 140 \\ 0 & \text{elsewhere} \end{cases}$$



[b] What is the prob. that the flight time is between 120 and 130 minutes?

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$$P(120 \leq X \leq 130) = \frac{1}{20} (130 - 120) = \frac{10}{20} = 0.5$$



[c] What is the prob. that the flight time is between 128 and 136 minutes?

$$P(128 \leq X \leq 136) = \frac{1}{20} (136 - 128) = \frac{8}{20} = 0.40$$

[d] $P(120 \leq X \leq 140) = \frac{1}{20} (140 - 120) = \frac{20}{20} = 1$

$f(x) \geq 0$ and $\sum f(x) = 1$
similar to discrete.

e) What is the expected time to arrive New York? (69)

$$E(x) = \frac{a+b}{2} = \frac{120+140}{2} = 130 \text{ minutes}$$

f) What is the variance in minutes?

$$\text{Var}(x) = \frac{(b-a)^2}{12} = \frac{(140-120)^2}{12} = \frac{(20)^2}{12} = \frac{400}{12} = 33.33 \text{ minute}$$

g) What is the prob. that x is exactly 135 minutes

$$p(x=135) = p(135 \leq x \leq 135) = \frac{1}{20} [135-135] = 0$$

Differences between

Continuous random variables x

discrete random variables x

1) Prob. of single value is zero

• Prob. of single value takes any value between 0 and 1

2) The random variable x takes values in interval $= [a, b]$

• The random variable x takes values $0, 1, 2, 3, \dots$

3) $p(a \leq x \leq b)$ = area under the graph between a and b
 $= p(a < x < b)$

• $p(a \leq x \leq b) \neq p(a < x < b)$