

## Chapter 8: (Potential energy and Conservation of energy).

→ In chapter 7:

$$F \text{ const} \rightarrow W = \vec{F} \cdot \vec{d} \quad x_P$$

$$F \text{ non-const} \rightarrow W = \int_{x_i}^{x_f} F(x) dx.$$

$$W_{\text{net}} = \Delta K.$$

### 8.1 → Potential energy (u).

$$\Delta u = u_2 - u_1.$$

$$\Delta u = -W \Rightarrow \left. \begin{array}{l} 1. \text{ Isolated system} \\ 2. \text{ Conservative forces.} \end{array} \right\} \text{ شروط.}$$

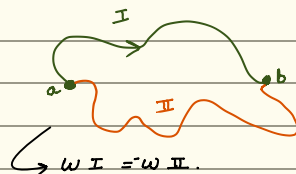
\* Force

- conservative.
- non-conservative.

#### \* Conservative Force :-

work does not depend on the path.

work net on close path = zero.



#### \* non-Conservative Force :-

work depends on the path. → k. Friction, Drag force.

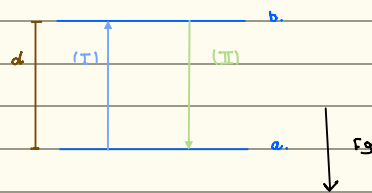
#### \* Gravitational Force :-

$$W I = -mgd$$

$$W II = mgd.$$

$$W_{\text{net}} = 0.$$

∴ conservative force.



$$W = \int_{x_i}^{x_f} F(x) dx$$

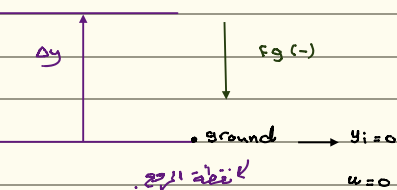
$$\Delta u = -W = - \int_{x_i}^{x_f} F(x) dx \rightarrow u_2 - u_1 = - \int_{x_i}^{x_f} F(x) dx.$$

#### Case I :- Gravitational Force.

$$\Delta u = - \int_{y_i}^{y_f} F(y) dy.$$

$$= - \int_{y_i}^{y_f} (-mg) dy$$

$$= mg \int_{y_i}^{y_f} dy$$



$$\Delta u = mg (y_f - y_i).$$

#### Case II :- Spring Force.

$$F_s = -kx \quad (\text{Hooke's law}).$$

$$W_s = \frac{1}{2} k (x_i^2 - x_f^2)$$

$$W_a = -W_s$$

→ From chapter 7

$$\Delta u = - \int_{x_i}^{x_f} F(x) dx$$

$$= - \int_{x_i}^{x_f} -kx dx$$

$$\Delta u = \frac{1}{2} k (x_f^2 - x_i^2) \rightarrow \Delta u = -W_s$$

## 8.2 → conservation of mechanical energy. (E)

$$\left. \begin{array}{l} E = K + U \\ \text{Isolated [no external force]} \\ \text{conservative forces.} \end{array} \right\} \begin{array}{l} E_{\text{mec}} = \text{conserved} \\ W = \Delta K \\ W = -\Delta U \end{array}$$

$$\Rightarrow \Delta K = -\Delta U$$

$$\therefore K_f - K_i = -(U_f - U_i)$$

$$K_f + U_f = K_i + U_i$$

$$E_f = E_i \rightarrow \Delta E = 0.$$

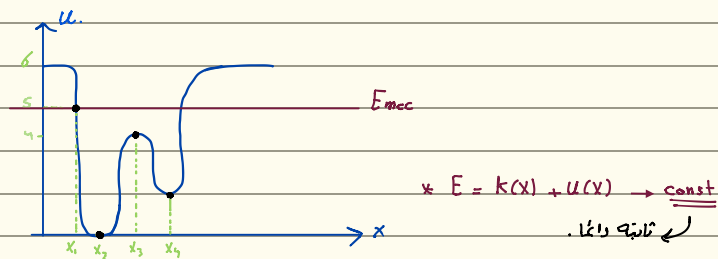
## 8.3 → Reading a potential energy curve.

$$\Delta U(x) = -W, \quad W = F(x) \cdot \Delta x$$

$$\therefore \Delta U(x) = -F(x) \cdot \Delta x$$

$$F(x) = -\frac{\Delta U}{\Delta x} \rightarrow$$

$$= -\frac{dU(x)}{dx} \rightarrow \text{معدل التغير مع } x$$



$$E_{\text{mec}} = 5 \text{ J.}$$

\* turning Point : النقطة التي عندها يكون الـ Potential energy الـ M.E.C. energy

$$\hookrightarrow E_{\text{mec}} = U(x), \quad K(x) = 0.$$

بالنسبة للموقع الذي فيه يكون الـ turning Point

$$U = 0 \leftarrow U(x) = E_{\text{mec}}$$

فيكون الـ turning Point هو المكان الذي فيه يكون الـ potential energy هو الـ E\_mec. والسرعة تكون صفر.

\* Equilibrium Point : نقطة الاتزان.

$$\vec{F}_{\text{net}} = 0, \quad F = -\frac{dU}{dx}$$

$$\Delta U = 0$$

$$\text{if } \vec{F} = 0, \text{ slope} = 0$$

\* بالنسبة للموقع الذي فيه يكون الـ Equilibrium Point

Equilibrium Point يكون الـ slope يكون يساوي صفر.

\* 1. Stable Eq. Point :

في النقطة التي عندها الجسم يتحرك حولها، يكون الـ

الجسم يمشي بمراد الموضع. نكتب  $(x_1, y_1)$

local min

2. un-Stable Eq. Point :

في النقطة التي عندها الجسم يتحرك بسرعة

local max

Ex:  $u(x,y) = x^2y + 2x$

$F_x = -\frac{du}{dx} \rightarrow -(2xy + 2)$  , y const

$F_y = -\frac{du}{dy} \rightarrow -(x^2 + 0)$  , x const

$\Rightarrow \vec{F} = -(2xy + 2)\hat{i} - (x^2)\hat{j}$

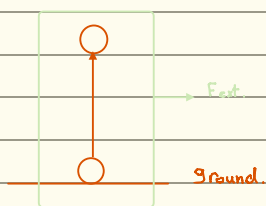
### 8.4 $\rightarrow$ work done on a system by an external force.

work: is energy transferred to or from the system when external force acts on that system.

\* non-Isolated system. ( F external).

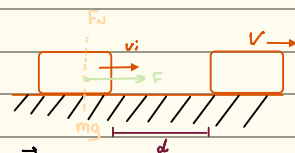
$\rightarrow$  no Friction. ( conservative force).  
 $\rightarrow$  Friction. ( non-conservative force).

case I: non-Isolated system / without friction force.



$W = \Delta E_{mec}$  ,  $\Delta E \neq 0$ .  
 $= \Delta K + \Delta U$

case II: non-Isolated system / with friction force.



\*  $\vec{F}_{net} = ma$

$F - f_k = m\vec{a}$  .... 1

\*  $v_f^2 = v_i^2 + 2ad$

$a = \frac{v_f^2 - v_i^2}{2d}$  .... 2

$\therefore Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + f_k d$

$\therefore Fd = \Delta K = f_k d$  ,  $U = 0$

$Fd = \Delta E_{mec} + f_k d$   $\rightarrow \Delta E_{th}$  thermal energy

$W = \Delta E_{mec} + \Delta E_{th}$

### 8.5 $\rightarrow$ conservation of energy.

$W = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$

$\hookrightarrow$  Isolated system: / Friction.

$W = 0$

$\Delta E_{mec} + \Delta E_{th} = 0$

$\Delta U + \Delta K + f_k d = 0$

\* Power = [watt]

$P_{inst} = \frac{dE}{dt}$

$P_{avg} = \frac{\Delta E}{\Delta t}$

# Chapter 9: (center mass, linear momentum).

النظام.  $\lambda$   $\rightarrow$  لم مركز الكتلة.

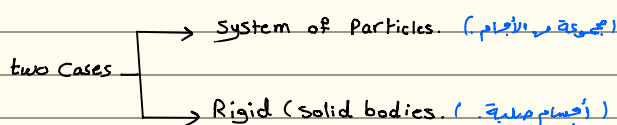
## 9.1 $\rightarrow$ center of mass

أعينة عن نقطة يتركز عندها الكتلة  $\rightarrow$

للجسم أو النظام.

عملية القوي المستقيم يتكره بهاءى النقطة.

is the point that moves as though, all of the system's mass concentrated there, and all external forces applied there. (Physical meaning.)



### Case I: system of Particles:-

كل جسم إلى كتلة وإحداثيات

كيف نطلع المركز للكتلة؟

1. نطلع الـ  $x_{com}$  خلال.

2. نطلع الـ  $y_{com}$  خلال.

3. نطلع الـ  $z_{com}$  خلال. (إحداثيات)

4. يكتب الجواب الذي نطلع معى على

شكل vectors



$$X_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$= \frac{1}{M} = \sum_{i=1}^n m_i x_i$$

total mass  $\leftarrow$

$$y_{com} = \frac{1}{M} = \sum_{i=1}^n m_i y_i$$

$$z_{com} = \frac{1}{M} = \sum_{i=1}^n m_i z_i$$

$$\vec{r}_{com} = x_{com} \hat{i} + y_{com} \hat{j} + z_{com} \hat{k}$$

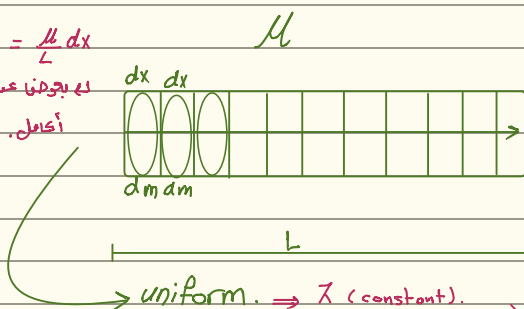
$$= \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$

### Case II: Solid bodies (Rigid).

$$X_{com} = \frac{1}{M} \int x (dm)$$

$$y_{com} = \frac{1}{M} \int y dm$$

$$z_{com} = \frac{1}{M} \int z dm$$



uniform.  $\Rightarrow \lambda$  (constant).

لم الجسم يكون منتظم اذا لا أقسم الـ  $\frac{dm}{dx}$   $\rightarrow$  قيمتها بالشكل المنتظم يتكون.  $\leftarrow$  ثابتة.

القطعة الأولى تساعد نفس فيه باقى

القطعة، ومكانه أقسم  $\frac{M}{L}$   $\rightarrow$  يعطى

نفس الجواب

$$\frac{dm}{dx} = \frac{M}{L} = \lambda \text{ (linear density)}$$

كثافة خطية.

لم قديم عنده كتلة فيه وحدة طول.

$\rightarrow$  غالباً بس صاير راج استند.

1. اذا كانت الجسم فيه بعد واحد (1D):  $dm = \lambda dx$

$$\frac{M}{L} = \text{uniform}$$

2. اذا كانت الجسم فيه بعدين (2D):  $dm = \sigma dA$

$$\frac{M}{A} = \text{surface density}$$

3. اذا كانت الجسم فيه سائدين (3D):  $dm = \rho dV$

$$\frac{M}{V} = \text{volume density}$$

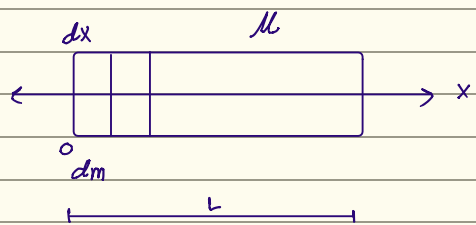


$$x_{com} = \frac{1}{M} \int x dm$$

$$dm = \lambda dx = \frac{M}{L} dx$$

$$\Rightarrow x_{com} = \frac{1}{M} \int_0^L x \left(\frac{M}{L}\right) dx$$

$$= \frac{1}{L} \int_0^L x dx \rightarrow \frac{x^2}{2L} \Big|_0^L = \frac{L}{2} \rightarrow \text{uniform rod} \rightarrow \text{الساكنة في المنتصف}$$

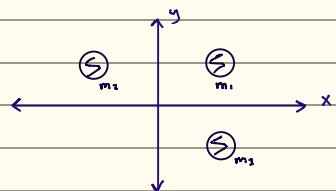


9.2 → Newton's 2<sup>nd</sup> Law for system of Particles :-

$$F_{net} = M \vec{a}_{com}$$

→ الساكنة في المنتصف

$$\vec{r}_{com} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{M}$$



$$M \vec{r}_{com} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n$$

→ مشتقة بالنسبة للزمن

$$M \vec{v}_{com} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n$$

→ مشتقة

$$M \vec{a}_{com} = \frac{m_1 \vec{a}_1}{F_1} + \frac{m_2 \vec{a}_2}{F_2} + \dots + \frac{m_n \vec{a}_n}{F_n}$$

$$\hookrightarrow F_{net} = M \vec{a}_{com} \rightarrow \text{يستعملو الساكنة}$$

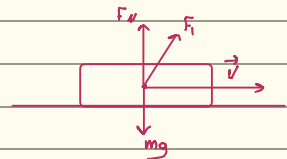
→ الساكنة ثابتة

9.3 → linear momentum ( $\vec{P}$ ) « زخم خطي »

$$\vec{P} = m\vec{v}$$

$\vec{P}$  in the same direction of  $\vec{v}$ .

$$\frac{d\vec{P}}{dt} = F_{net}$$



Newton's 2<sup>nd</sup> law

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}, \vec{P} = m\vec{v} \rightarrow \text{صار القانون الزخم الخطي}$$

→ ينطبق مع الحالة ثابتة (لا يتغير)

$$= \frac{d(mv)}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt}$$

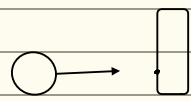
$$\hookrightarrow \text{if } m \equiv \text{constant} \rightarrow \frac{dm}{dt} = 0$$

$$\therefore F_{net} = m \frac{dv}{dt} = m\vec{a}$$

9.4 → Collision and impulse :-

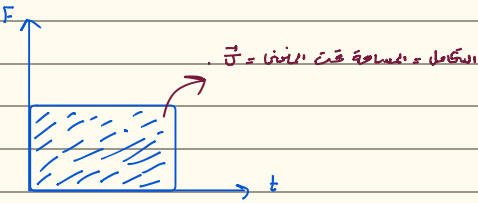
الزخم (Impulse)      الدفع (Impulse)

$$\text{Impulse } (\vec{J}) = F_{avg} \cdot \Delta t$$



$$\hookrightarrow \vec{F} \rightarrow \text{constant} \text{ « شدة »}$$

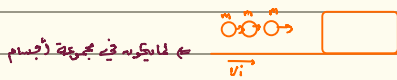
$$\vec{J} = \int_t^{t_f} F(t) dt \rightarrow \vec{F} \rightarrow \text{not constant} \text{ « depends on time »}$$



$$\begin{aligned} \vec{J} &= \Delta \vec{P} \\ &= m \Delta \vec{V} \\ &= m(\vec{V}_f - \vec{V}_i) \end{aligned}$$

$$\begin{aligned} \vec{J} &= F_{avg} \cdot \Delta t = \Delta \vec{P} = m(\vec{V}_f - \vec{V}_i) \quad \text{constant} \\ \vec{J} &= \int F dt = \Delta \vec{P} \quad \text{not constant} \end{aligned}$$

\* special case :-



في ميكانيكا فيزياء مجموعة الجسيمات

$$\vec{J} = -n \Delta \vec{P}$$

$n$ : # of particles.

$\Delta P$ : change in linear momentum of 1-Particle.

$$\vec{J} = -n \Delta \vec{P} = -n m \Delta V$$

$$F_{avg} \Delta t = -n m \Delta V$$

$$F_{avg} = -n m \frac{\Delta V}{\Delta t}$$

## 9.5 → Conservation of linear momentum.

\* From Newton's 2<sup>nd</sup> law.

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}, \quad \vec{P} \text{ is conserved.}$$

$$\text{if } \Delta \vec{P} = 0$$

$\vec{P}$  is conserved ( $\Delta \vec{P} = 0$ ), if the system is Isolated and close.

$$\Delta \vec{P} = 0$$

$$\vec{P}_f - \vec{P}_i = 0$$

$$\vec{P}_f = \vec{P}_i$$

\* Isolated system

$$\Delta \vec{P} = 0 \Rightarrow \vec{P}_f = \vec{P}_i$$

$$\begin{aligned} \vec{P}_x &= \vec{P}_{fx} \\ \vec{P}_y &= \vec{P}_{fy} \end{aligned}$$

## 9.6 → momentum and kinetic energy in collisions:-

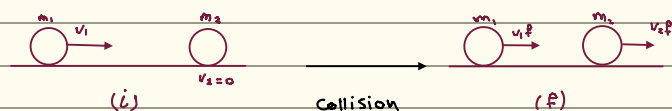
\* types of collisions:-

1. Elastic collision.  $\Delta \vec{P} = 0, \Delta K = 0$
2. Inelastic collision.  $\Delta \vec{P} = 0, \Delta K \neq 0$
3. completely inelastic collision.  $\Delta \vec{P} = 0, \Delta K \neq 0$

1. elastic collision:-

$$\vec{P}_f = \vec{P}_i$$

$$\vec{K}_f = \vec{K}_i$$



$$\begin{aligned} \text{if } m_1 &= m_2 \Rightarrow v_{1f} = 0 \\ v_{2f} &= \frac{2m_1}{m_1 + m_2} v_{1i} \end{aligned}$$

$$\begin{aligned} * m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \\ m_1 v_{1i} &= m_1 v_{1f} + m_2 v_{2f} \quad \dots 1 \end{aligned}$$

$$\therefore v_{2f} = v_{1i}$$

أي يعني الساكنة يتحرك بسرعة الأولى  
والمتحركة يتوقف.

$$* \frac{1}{2} m v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \dots 2$$

Find  $v_{2f}$  from equ 1, substitute in equ 2

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

سأصل صرولا بعد التصادم هو يساوي في جسم  
بتوقف قبل التصادم والجسم الآخر ساكنة.


\* target at rest ( $v_{2i} = 0$ )

$$\left. \begin{aligned} v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \\ v_{2f} &= \frac{2m_1}{m_1 + m_2} v_{1i} \end{aligned} \right\} \text{تساوي}$$

Case I :- ( $m_1 = m_2$ )

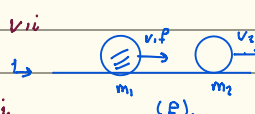
$$\begin{aligned} v_{1f} &= 0 \\ v_{2f} &= v_{1i} \end{aligned}$$

Case II :- massive target ( $m_2 \gg m_1$ ), neglect  $m_1$

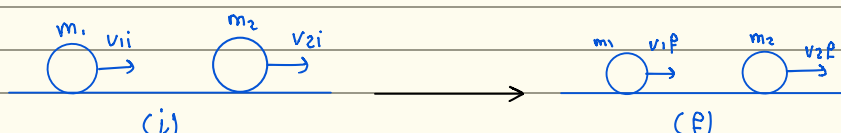
$$v_{1f} \approx -\frac{m_2}{m_1} v_{1i} = -v_{1i}$$


$$v_{2f} \approx \frac{2m_1}{m_2} v_{1i}$$

Case III :- massive Projectile ( $m_1 \gg m_2$ ).

$$\begin{aligned} v_{1f} &\approx \frac{m_1}{m_1} v_{1i} = v_{1i} \\ v_{2f} &\approx \frac{2m_1}{m_1} v_{1i} = 2v_{1i} \end{aligned}$$


\* moving target



$$\vec{P}_i = \vec{P}_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$K_i = K_f$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

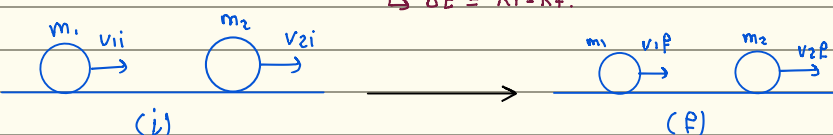
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

2. Inelastic collision :-

$$\vec{P}_i = \vec{P}_f$$

$$K_i \neq K_f \rightarrow \text{بسبب وجود احتكاك} \\ \rightarrow \Delta E = K_i - K_f$$



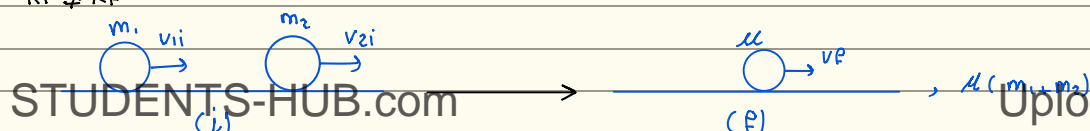
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \dots \perp$$

$$\Delta E = \left( \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \right) - \left( \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \right)$$

3. Completely inelastic collision :-

$$\vec{P}_i = \vec{P}_f$$

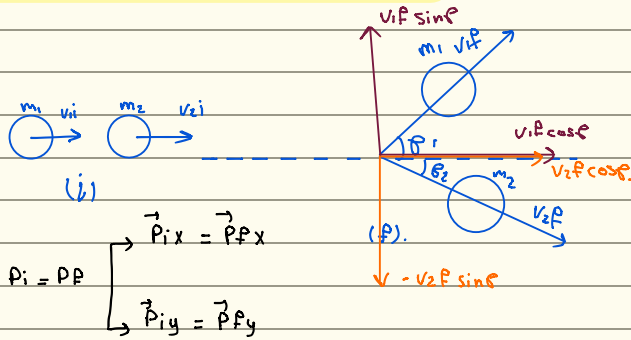
$$K_i \neq K_f$$



$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$DE = \left( \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \right) - \left( \frac{1}{2} (m_1 + m_2) v_f^2 \right)$$

### 9.8 → collisions in two dimensions.



$$P_x \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \sin \theta_2$$

$$P_y \rightarrow 0 = m_1 v_{1f} \sin \theta_1 + m_2 v_{2f} \cos \theta_2$$

$$\vec{P}_{net} = \sqrt{(P_x)^2 + (P_y)^2}$$

$$\tan \theta = \frac{P_y}{P_x}$$

### 9.9 → systems with varying mass : A Rocket.

• The first rocket equation:  $R v_{rel} = \mu a$  (قوة الدفع = Thrust)

$R$  is the mass rate of fuel consumption.  $= \frac{dm}{dt}$  (معدل استهلاك الوقود)

$v_{rel} \rightarrow$  سرعة نسبية.

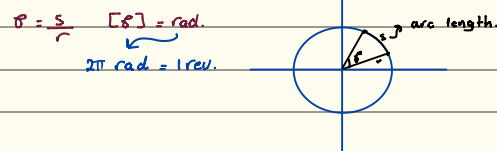
• The second rocket equation:

$$v_f - v_i = v_{rel} \ln \frac{M_i}{M_f}$$

سرعة الغاز بالنسبة للصاروخ

## Chapter 10 :- (Rotational variables).

### 1. Angular position ( $\theta$ ) :- موقع زاوي.



### 2. Angular displacement ( $\Delta\theta$ ) :- إزاحة زاوية.

$$\vec{\Delta\theta} = \vec{\theta_2} - \vec{\theta_1}, \quad [\Delta\theta] = \text{rad}.$$

Clock wise  $\Rightarrow \Delta\theta$  is negative. (-).

Counter clock wise  $\Rightarrow \Delta\theta$  is Positive. (+).

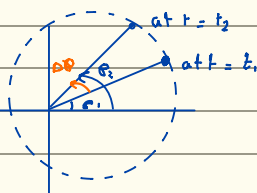
### 3. Angular velocity ( $\omega_{\text{avg}}$ ) :- سرعة زاوية.

$$\vec{\omega}_{\text{avg}} = \frac{\vec{\Delta\theta}}{\Delta t} \rightarrow \frac{\vec{\theta_2} - \vec{\theta_1}}{t_2 - t_1}, \quad [\omega] = \text{rad/sec}$$

لخفضة اليد اليمنى ← عند ايدي مع حركة الجير.

القيم الايجابية بكونه القيم « $\omega$ »

$\Rightarrow \text{angular speed} = |\text{angular velocity}|$



### 4. Angular acceleration ( $\alpha_{\text{avg}}$ ) :- تسارع زاوي.

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} \rightarrow \frac{\omega_2 - \omega_1}{t_2 - t_1}, \quad [\alpha] = \text{rad/sec}^2$$

$$* \theta(t) \xrightarrow{\frac{d\theta}{dt}} \omega(t) \xrightarrow{\frac{d\omega}{dt}} \alpha(t) \Rightarrow \text{ان اولى عند نفس وقتها}$$

$\theta(t) \rightarrow$  Instantaneous angular position. بشئته.

$\omega(t) \rightarrow$  Instantaneous angular velocity.

$\alpha(t) \rightarrow$  Instantaneous angular acceleration.

### \* Rotation motion at constant $\alpha$ :-

$$\omega_f = \omega_i + \alpha t.$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta.$$

$$\Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2.$$

translation Rotation

$$x, v, a \xleftarrow{?} \theta, \omega, \alpha$$

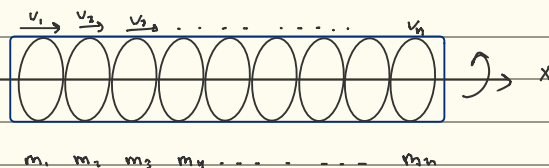
$$s = r\theta \quad / \quad \text{اشئته}$$

$$\frac{ds}{dt} = r \frac{d\theta}{dt} \Rightarrow v = r\omega$$

$$\frac{dv}{dt} = r \frac{d\omega}{dt} \Rightarrow a = r\alpha$$

### \* Kientic energy (K.E) in Rotation :-

$$K.E = \sum_{i=1}^n \frac{1}{2} m_i v_i^2$$



$$\hookrightarrow v_i = r_i \omega_i$$

$$\Rightarrow K.E = \sum_{i=1}^n \frac{1}{2} m_i (r_i \omega_i)^2$$

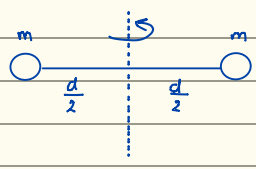
$$= \frac{1}{2} \sum_{i=1}^n (m_i r_i^2) \omega_i^2$$

$\hookrightarrow$  Inertia (I). ... ممانعة

Inertia (I) :-

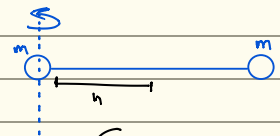
Case I :- \* (rotational axis through the center of mass.

$$I = \sum m_i r_i^2$$
$$= m \left(\frac{d}{2}\right)^2 + m \left(\frac{d}{2}\right)^2$$
$$= \frac{md^2}{2}$$



\* (rotational axis through a point from the center of mass.

$$I = I_{com} + Mh^2$$
  
total mass ← distance bt center of mass and rotational axis.  
center of mass. المسافة بين مركز الكتلة والمحور الدوراني



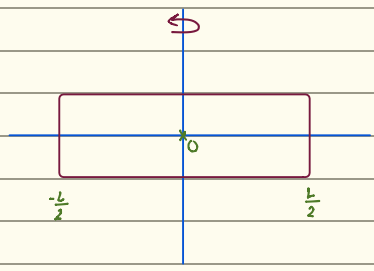
$$I = I_{com} + Mh^2$$
$$= \frac{md^2}{2} + 2m \left(\frac{d}{2}\right)^2$$
$$= md^2$$

or حسب كلا جسم خلال. في خلال اي نقطة الجذب  
كلا جسم يتحرك ويجزو عن محور الدوراني

$$\Rightarrow m(d) + m(d)^2 = \frac{md^2}{2}$$

Case II :- rigid body (solid).

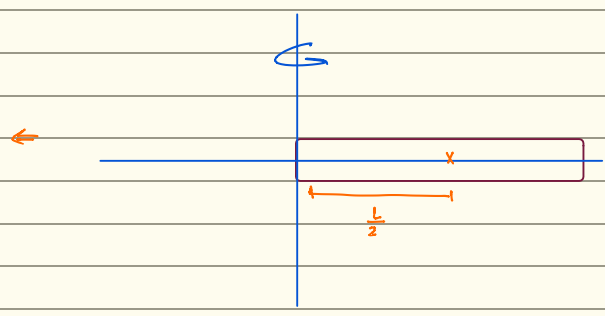
$$I = \int r^2 dm$$
  
$$\lambda = \frac{dm}{dx} = \frac{M}{L} \rightarrow dm = \frac{dx}{L} M$$
  
$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \lambda dx \rightarrow \frac{1}{12} M L^2$$



\*  $I_{rod} = \frac{1}{12} M L^2$  شرطها محور الدوراني  
في ال CoP

Parallel-axis theorem.

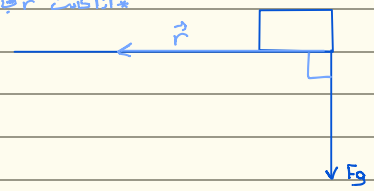
$$I = I_{com} + Mh^2$$
$$= \frac{1}{12} M L^2 + M \left(\frac{L}{2}\right)^2$$
$$= \frac{M L^2}{3}$$



Torque: ( $\vec{\tau}$ )  $\Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$   
$$= |\vec{r}| |\vec{F}| \sin \theta$$

\* انما كانت  $\vec{r}$  هي المسافة بين نقطة القوة والمحور الدوراني.

$$[\tau] = N.m$$



\* Newton's second law:-  $\Rightarrow \vec{F}_{net} = M\vec{a}$

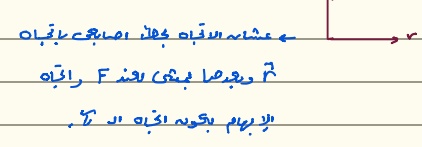
$$\vec{\tau}_{net} = I\alpha$$

$\vec{\tau}_{net} \rightarrow$  net torque

I  $\rightarrow$  inertia.

$\alpha \rightarrow$  angular acceleration.

\* 
$$\vec{\tau} = \vec{r} \times \vec{F}$$



the last part in chapter 10:-

\* 
$$W_{net} = \Delta K \rightarrow K = \frac{1}{2} I \omega^2$$

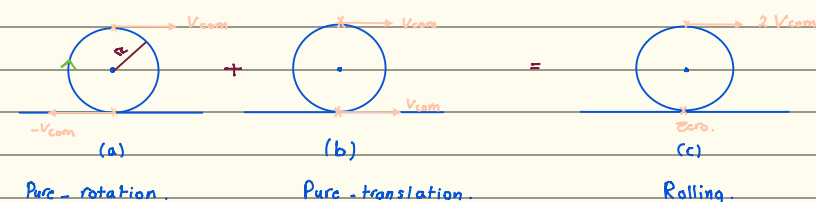
$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

$$W = \tau_{avg} \Delta \theta$$

\* Power  $\Rightarrow \vec{P} = \frac{dW}{dt} = \vec{r} \cdot \vec{\omega}$

## Chapter 11: Rolling, Torque, and Angular Momentum.

\* Rolling: Translation + Rotation.



$V_{cm} = R\omega$  angular velocity.

$$K = \frac{1}{2} I_{cm} \omega^2$$

$$I_P = I_{cm} + MR^2$$

$$K = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} MR^2 \omega^2$$

$\hookrightarrow v = R\omega$

$$K = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} MV^2$$

$K.E_{rolling} = K.E_{rotation} + K.E_{translation}$

\* 2<sup>nd</sup> law of Newton

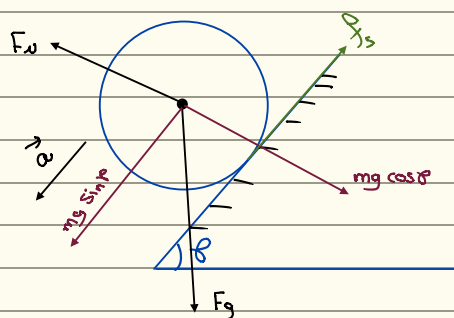
- translation

$$F_{net} = ma$$

$$F_y = 0$$

$$F_x = ma$$

$$mg \sin \theta - f_s = ma \quad \dots \quad \underline{\quad}$$



- rotation

$$\vec{\tau}_{net} = I\alpha$$

$\vec{\tau} = \vec{r} \times \vec{F}$ ;  $F_g, F_N \rightarrow$  have no torque

$$Rf_s = I\alpha \quad \dots \quad \underline{\quad}, \quad a = R\alpha$$

$$Rf_s = I\left(\frac{a}{R}\right)$$

$$* a_{cm} = \frac{-g \sin \theta}{1 + I_{cm}/MR^2}$$

$$f_s = \frac{I_{cm} \alpha}{R} = \frac{I_{cm} a_{cm}}{R^2}$$

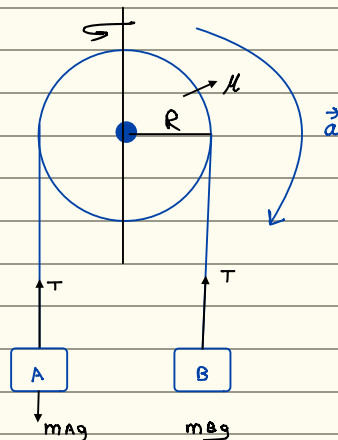
$$F_{net} = ma$$

$$T - m_A g = (m_A) a$$

$$m_B g - T = (m_B) a$$

\*  $\vec{\tau}_{net} = I\alpha$

$$TR = I\alpha, \quad a = R\alpha$$



Section 4.2: (Torque Revisited).

$$|\tau| = rF \sin \theta$$

$$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \text{الاعمال}$$

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \underline{\underline{11.5}}$$

$$= m \vec{r} \times \vec{v} \Rightarrow \text{Angular Momentum}$$