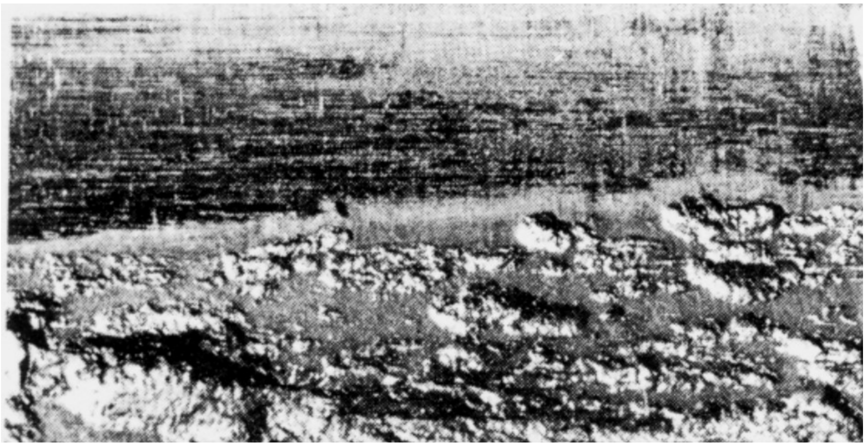
Introduction

Bending Failure

Pitting Failure



14.1 The Lewis Bending Equation

Table 14–1

Symbols, Their Names, and Locations

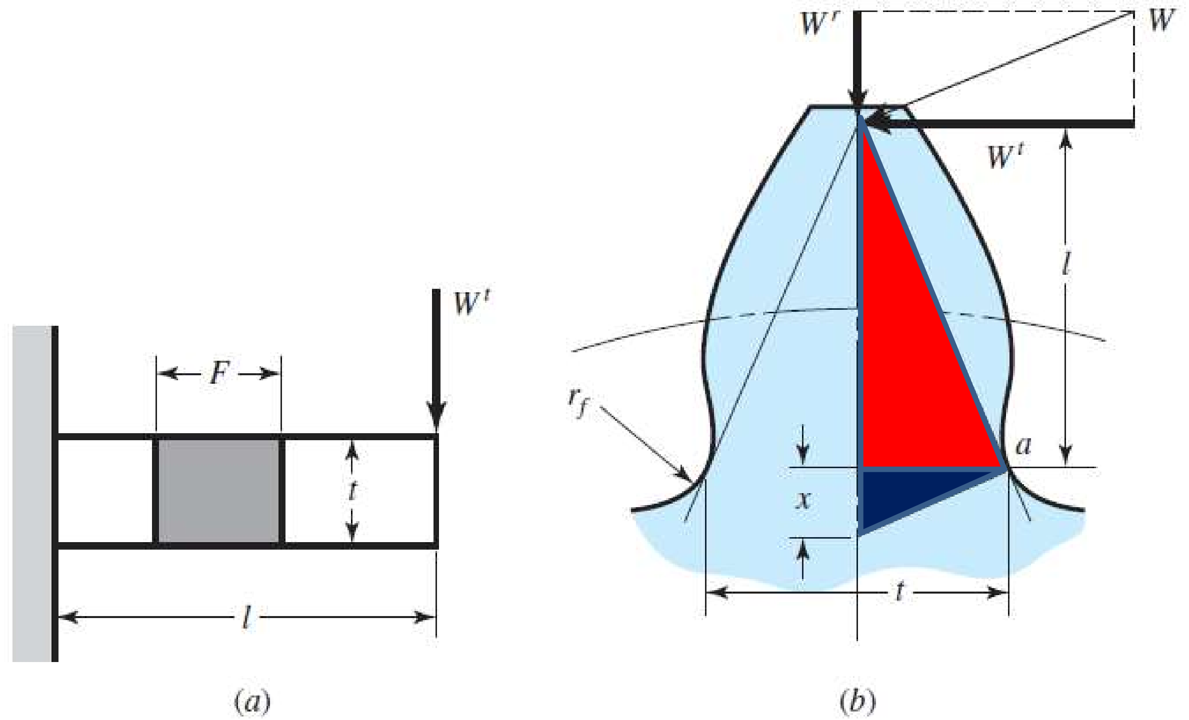
Symbol*	Name	Where Found
C_e	Mesh alignment correction factor	Eq. (14–35)
$C_f\,(Z_R)$	Surface condition factor	Eq. (14–16)
$C_H\,(Z_W)$	Hardness-ratio factor	Eq. (14–18)
C_{ma}	Mesh alignment factor	Eq. (14–34)
C_{mc}	Load correction factor	Eq. (14–31)
C_{mf}	Face load-distribution factor	Eq. (14–30)
$C_p\,(Z_E)$	Elastic coefficient	Eq. (14–13)
C_{pf}	Pinion proportion factor	Eq. (14–32)
C_{pm}	Pinion proportion modifier	Eq. (14–33)
d	Pitch diameter	Ex. (14–1)
d_P	Pitch diameter, pinion	Eq. (14–22)
d_G	Pitch diameter, gear	Eq. (14–22)
$F\,(b)$	Net face width of narrowest member	Eq. (14–15)
f_P	Pinion surface finish	Fig. 14–13
H	Power	Fig. 14–17
H_B	Brinell hardness	Ex. 14–3
H_{BG}	Brinell hardness of gear	Sec. 14–12

14.1 The Lewis Bending Equation

H_{BG}	Brinell hardness of gear	Sec. 14–12
H_{BP}	Brinell hardness of pinion	Sec. 14–12
hp	Horsepower	Ex. 14–1
h_t	Gear-tooth whole depth	Sec. 14–16
$I\ (Z_I)$	Geometry factor of pitting resistance	Eq. (14–16)
$J\ (Y_J)$	Geometry factor for bending strength	Eq. (14–15)
K_B	Rim-thickness factor	Eq. (14–40)
K_f	Fatigue stress-concentration factor	Eq. (14–9)
$K_m\ (K_H)$	Load-distribution factor	Eq. (14–30)
K_o	Overload factor	Eq. (14–15)
$K_R\ (Y_Z)$	Reliability factor	Eq. (14–17)
K_s	Size factor	Sec. 14–10
$K_T\ (Y_\theta)$	Temperature factor	Eq. (14–17)
K_v	Dynamic factor	Eq. (14–27)
m	Module	Eq. (14–15)
m_B	Backup ratio	Eq. (14–39)
m_F	Face-contact ratio	Eq. (14–19)
m_G	Gear ratio (never less than 1)	Eq. (14–22)
m_N	Load-sharing ratio	Eq. (14–21)

14.1 The Lewis Bending Equation

Figure 14-1



14.1 The Lewis Bending Equation

$$\sigma = \frac{W^t P}{FY} \qquad Y = \frac{2xP}{3}$$

Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

14.3 AGME Stress Equations – Bending Stress

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$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \tag{14–15}$$

where for U.S. customary units (SI units),

W^t is the tangential transmitted load, lbf (N)
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
P_d is the transverse diametral pitch
F (b) is the face width of the narrower member, in (mm)
K_m (K_H) is the load-distribution factor
K_B is the rim-thickness factor
J (Y_J) is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
(m_t) is the transverse metric module

14.5 Geometry Factor J (Y)

Bending-Strength Geometry Factor J (Y_J)

The AGMA factor J employs a modified value of the Lewis form factor, also denoted by Y; a *fatigue stress-concentration factor* K_f; and a tooth *load-sharing ratio* m_N. The resulting equation for J for spur and helical gears is

$$J = \frac{Y}{K_f m_N} \qquad (14\text{–}20)$$

It is important to note that the form factor Y in Eq. (14–20) is *not* the Lewis factor at all. The value of Y here is obtained from calculations within AGMA 908-B89, and is often based on the highest point of single-tooth contact.

The factor K_f in Eq. (14–20) is called a *stress-correction factor* by AGMA. It is based on a formula deduced from a photoelastic investigation of stress concentration in gear teeth over 50 years ago.

The load-sharing ratio m_N is equal to the face width divided by the minimum total length of the lines of contact. This factor depends on the transverse contact ratio m_p, the face-contact ratio m_F, the effects of any profile modifications, and the tooth deflection. For spur gears, $m_N = 1.0$. For helical gears having a face-contact ratio $m_F > 2.0$, a conservative approximation is given by the equation

$$m_N = \frac{p_N}{0.95Z} \qquad (14\text{–}21)$$

where p_N is the normal base pitch and Z is the length of the line of action in the transverse plane (distance L_{ab} in Fig. 13–15, p. 676).

14.5 Geometry Factor J (Y)

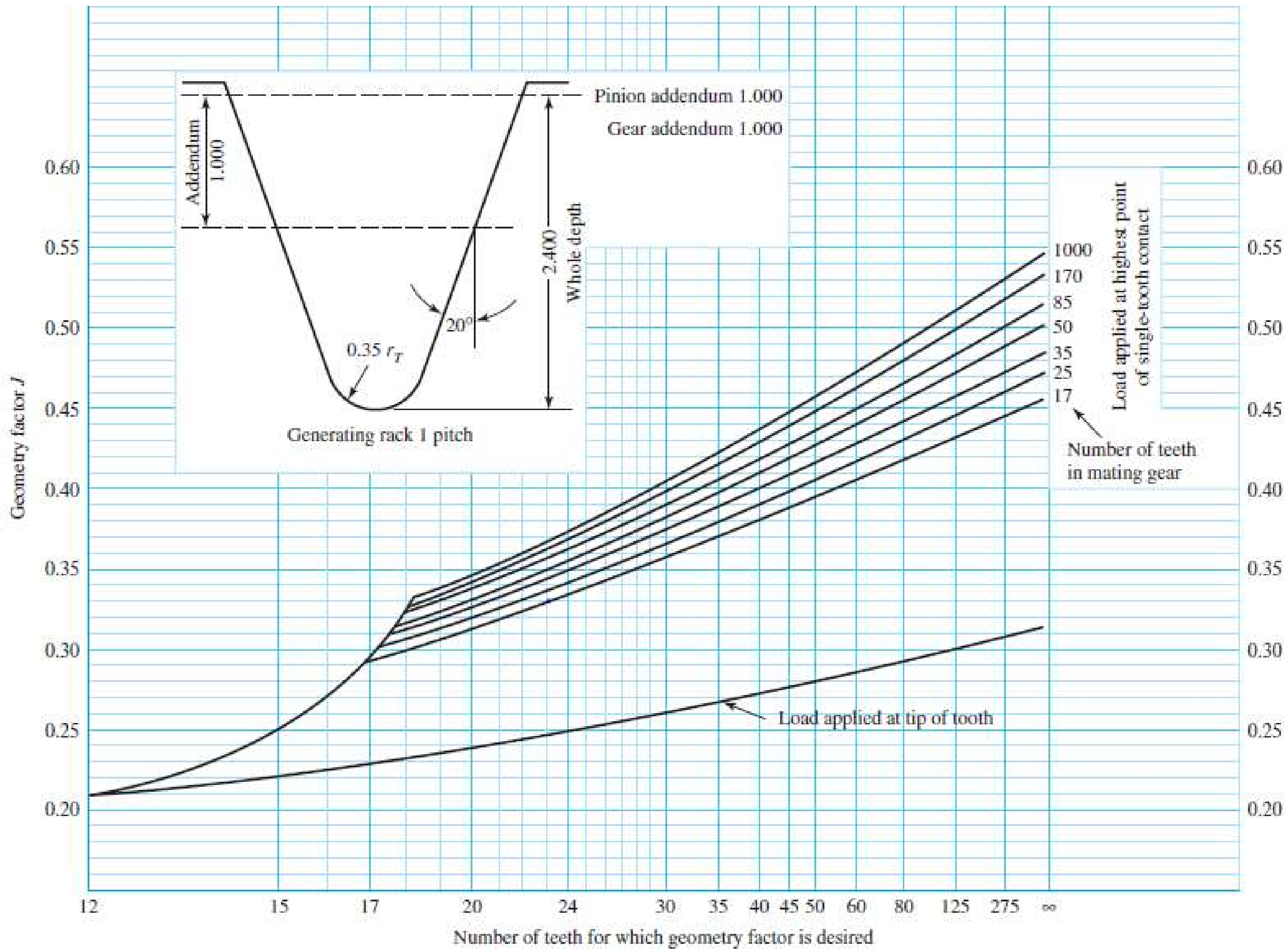


Figure 14–6

Spur-gear geometry factors *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

14.7 Dynamic Factor K_v

Dynamic factors are used to account for inaccuracies in the manufacture and meshing of gear teeth in action.

Transmission error is defined as the departure from uniform angular velocity of the gear pair. Some of the effects that produce transmission error are:

- Inaccuracies produced in the generation of the tooth profile; these include errors in tooth spacing, profile lead, and runout
- Vibration of the tooth during meshing due to the tooth stiffness
- Magnitude of the pitch-line velocity
- Dynamic unbalance of the rotating members
- Wear and permanent deformation of contacting portions of the teeth
- Gearshaft misalignment and the linear and angular deflection of the shaft
- Tooth friction

14.7 Dynamic Factor K_v

$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases}$$

where

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$

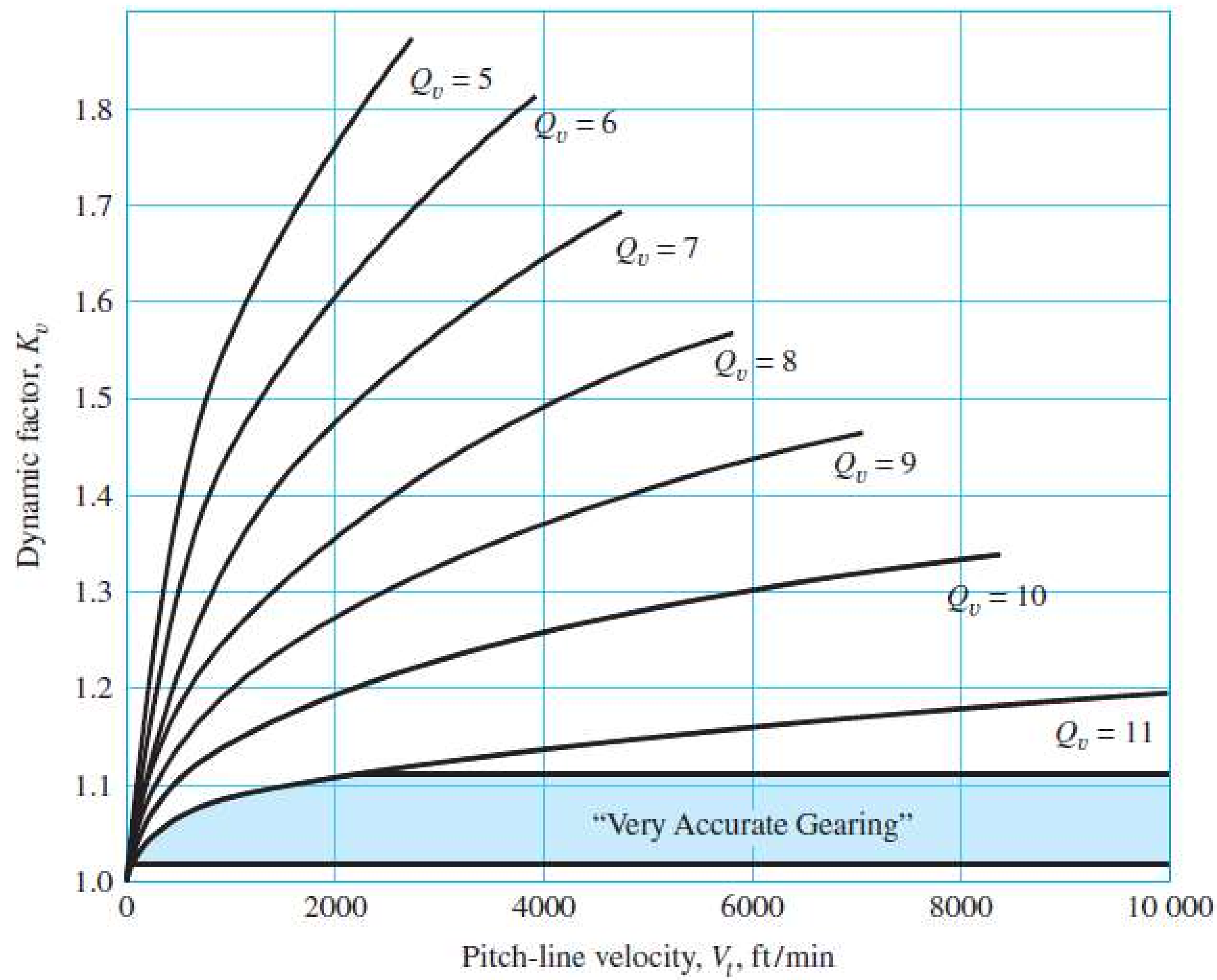
Qv (*quality numbers*) numbers define the tolerances for gears of various sizes manufactured to a specified accuracy.

- 3 to 7 will include most commercial quality gears.
- 8 to 12 are of precision quality.

14.7 Dynamic Factor K_v

Figure 14–9

Dynamic factor K_v. The equations to these curves are given by Eq. (14–27) and the end points by Eq. (14–29). *(ANSI/AGMA 2001-D04, Annex A)*



14.8 Overload Factor K_o

The overload factor K_o is intended to reflect the non-uniformity of driving and load torques drive (source of power).

Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

14.10 Size Factor K_s

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The size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

AGMA has identified and provided a symbol for size factor. Also, AGMA suggests $K_s = 1$, which makes K_s a placeholder in Eqs. (14–15) and (14–16) until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge. From Table 13–1, p. 688, $l = a + b = 2.25/P$. The tooth thickness t in Fig. 14–6 is given in Sec. 14–1, Eq. (*b*), as $t = \sqrt{4lx}$ where $x = 3Y/(2P)$ from Eq. (14–3). From Eq. (6–25), p. 297, the equivalent diameter d_e of a rectangular section in bending is $d_e = 0.808\sqrt{Ft}$. From Eq. (6–20), p. 296, $k_b = (d_e/0.3)^{-0.107}$. Noting that K_s is the reciprocal of k_b, we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \qquad (a)$$

14.1 The Lewis Bending Equation

$$\sigma = \frac{W^t P}{FY} \qquad Y = \frac{2xP}{3}$$

Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
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20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

14.3 AGME Stress Equations – Bending Stress

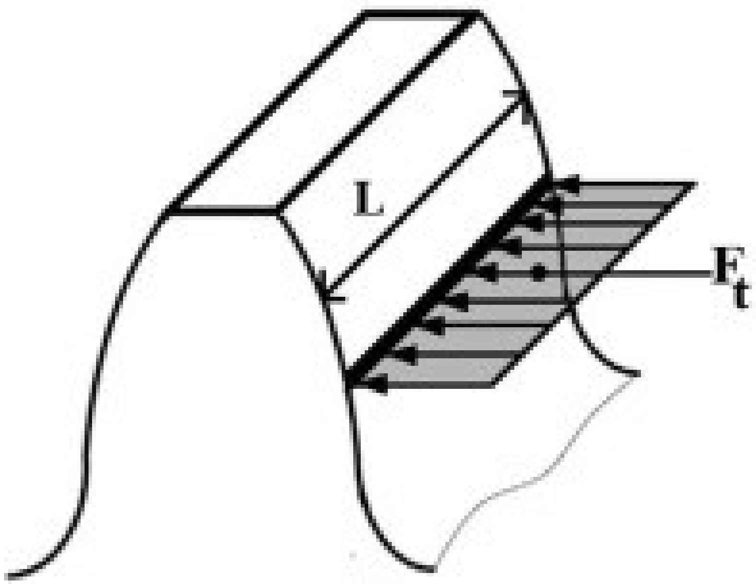
$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \qquad (14\text{–}15)$$

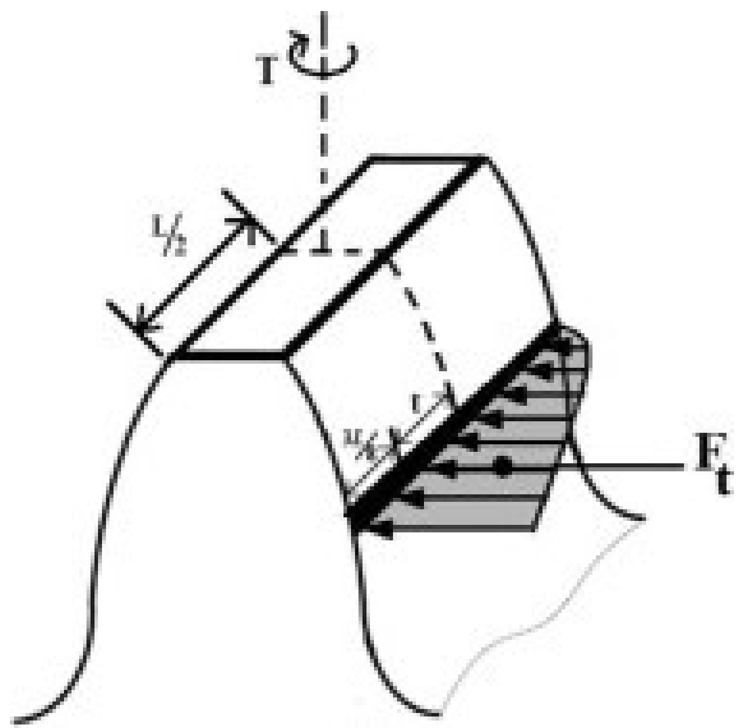
where for U.S. customary units (SI units),

W^t is the tangential transmitted load, lbf (N)
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
P_d is the transverse diametral pitch
F (b) is the face width of the narrower member, in (mm)
K_m (K_H) is the load-distribution factor
K_B is the rim-thickness factor
J (Y_J) is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
(m_t) is the transverse metric module

14.11 Load Distribution $K_M (K_H)$

The load-distribution factor modified the stress equations to reflect non-uniform distribution of load across the line of contact.





14.11 Load Distribution $K_M (K_H)$

The load-distribution factor modified the stress equations to reflect non-uniform distribution of load across the line of contact.

K_M is applied under the following conditions:

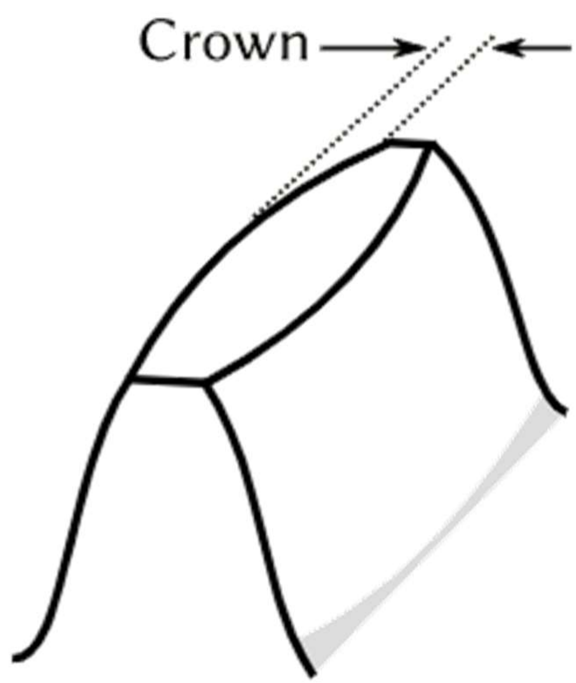
- Net face width to pinion pitch diameter ratio $F/d_p <= 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

14.11 Load Distribution K_M (K_H)

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$



14.11 Load Distribution K_M (K_H)

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d_P} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d_P} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d_P} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

Note that for values of $F/(10d_P) < 0.05$, $F/(10d_P) = 0.05$ is used.

14.11 Load Distribution $K_M (K_H)$

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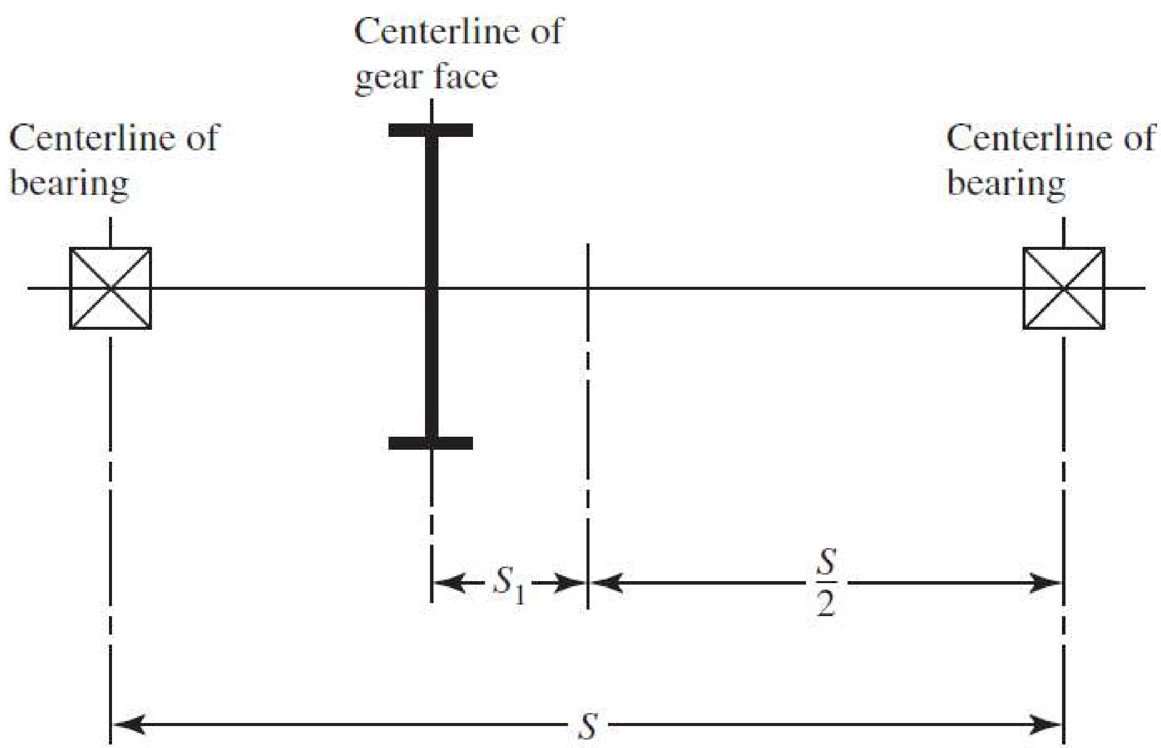
$$K_m = C_{mf} = 1 + C_{mc}(C_{pf} C_{pm} + C_{ma} C_e)$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases}$$

Figure 14–10

Definition of distances S and S_1 used in evaluating C_{pm}, Eq. (14–33). *(ANSI/AGMA 2001-D04.)*



14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{ma} = A + BF + CF^2$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

14.11 Load Distribution $K_M\,(K_H)$

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$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{ma} = A + BF + CF^2$$

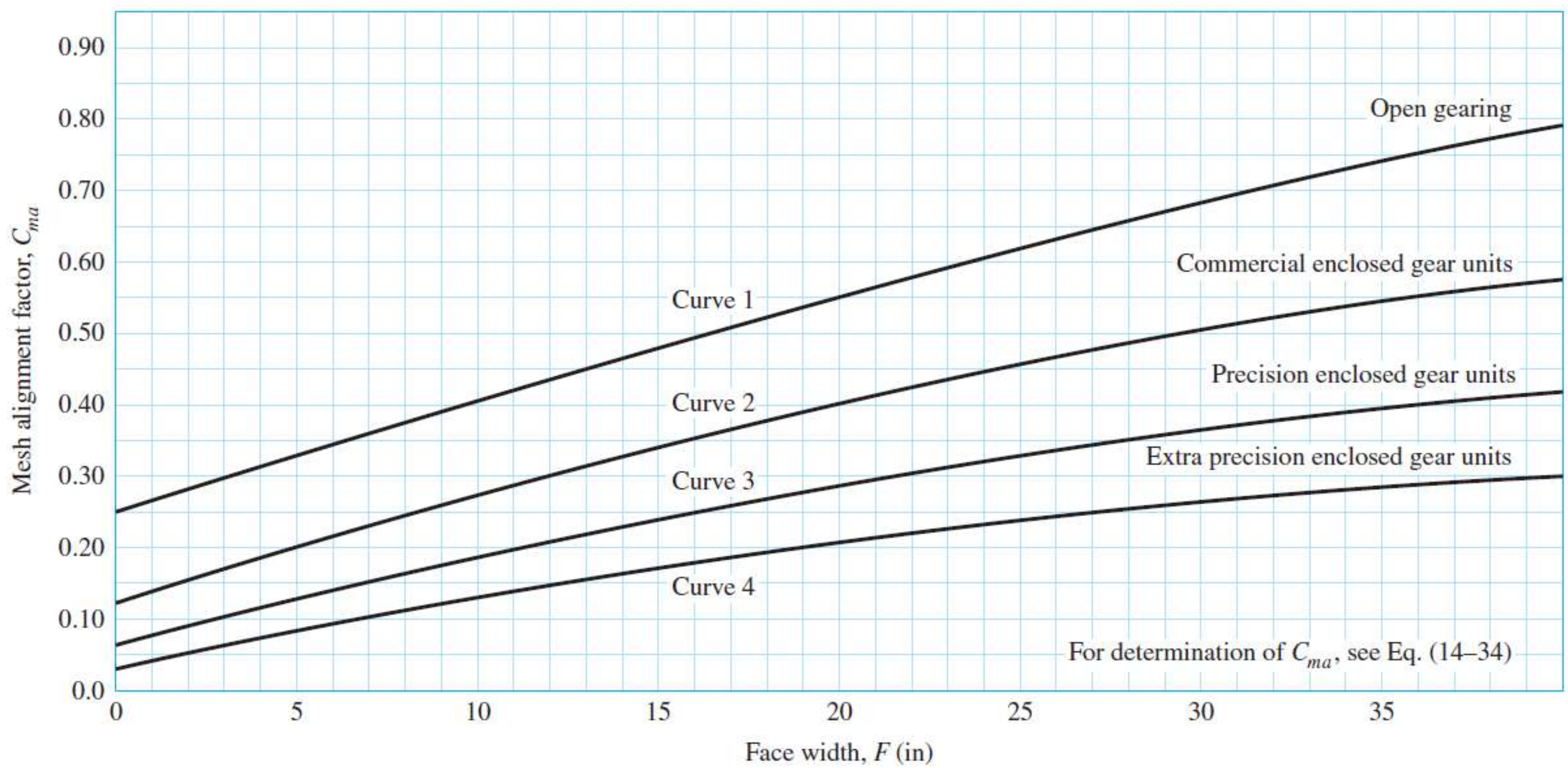


Figure 14–11

Mesh alignment factor C_{ma}. Curve-fit equations in Table 14–9. *(ANSI/AGMA 2001-D04.)*

14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

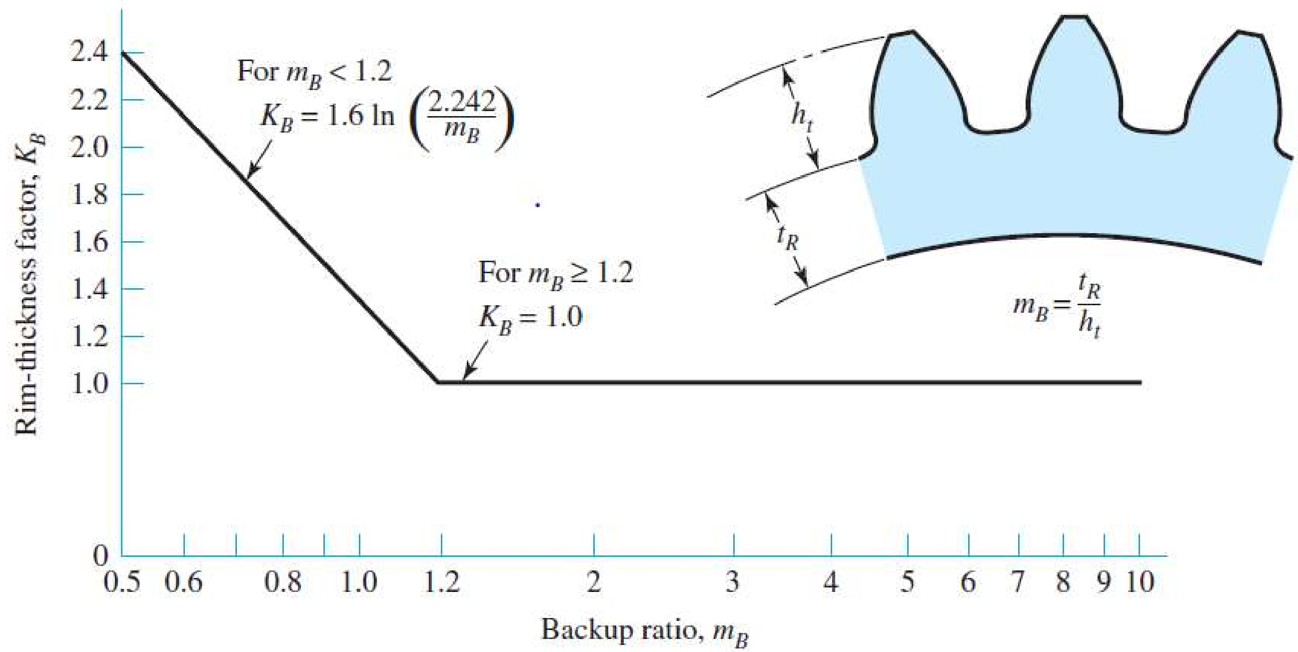
14.16 Rim-Thickness Factor K_B

Rim-Thickness factor (K_B) is a modifying factor adjusts the estimated bending stress for the thin-rimmed gear

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases}$$

Figure 14–16

Rim-thickness factor K_B. *(ANSI/AGMA 2001-D04.)*



14.4 AGMA Strength Equations

Instead of using the term *strength,* AGMA uses data termed ***allowable stress numbers (Gear Bending Strength)***

The equation for the allowable bending stress is

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}17)$$

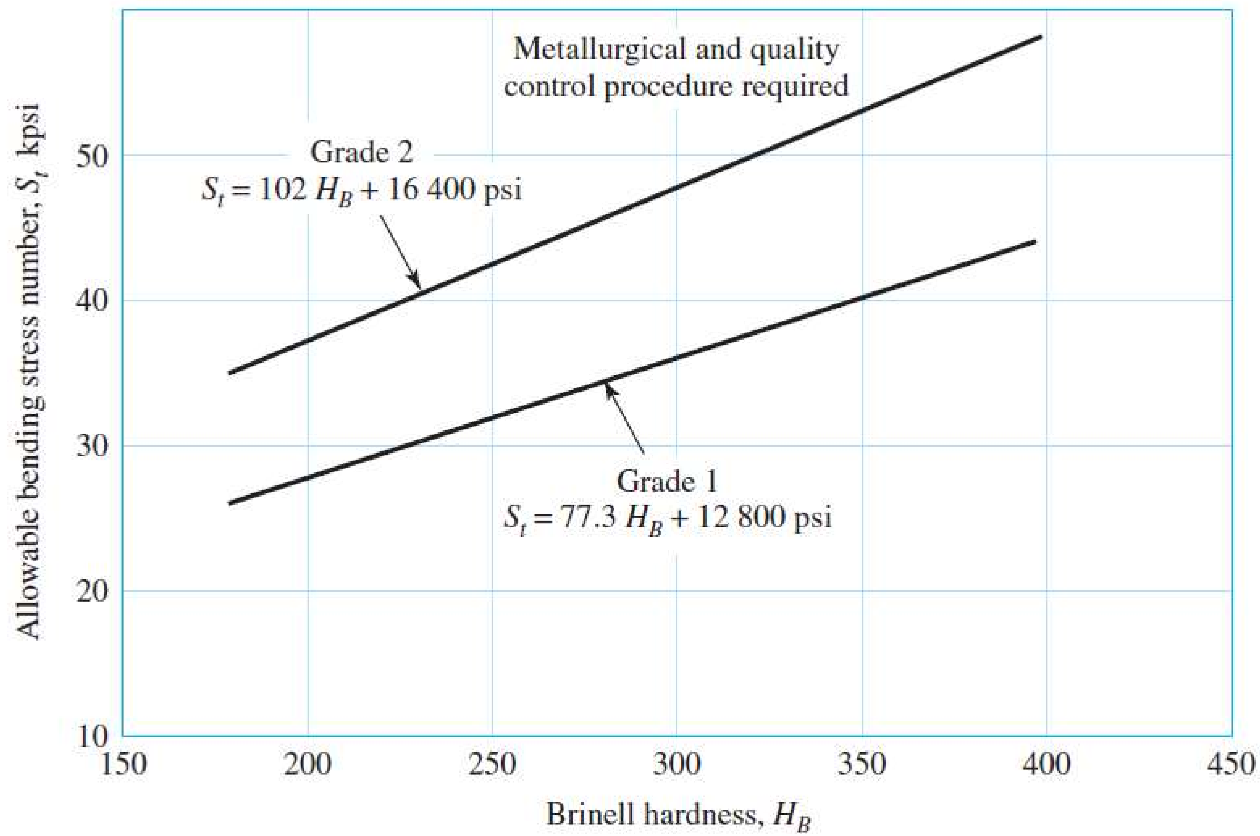
where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress-cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

14.4 AGMA Strength Equations

Figure 14–2

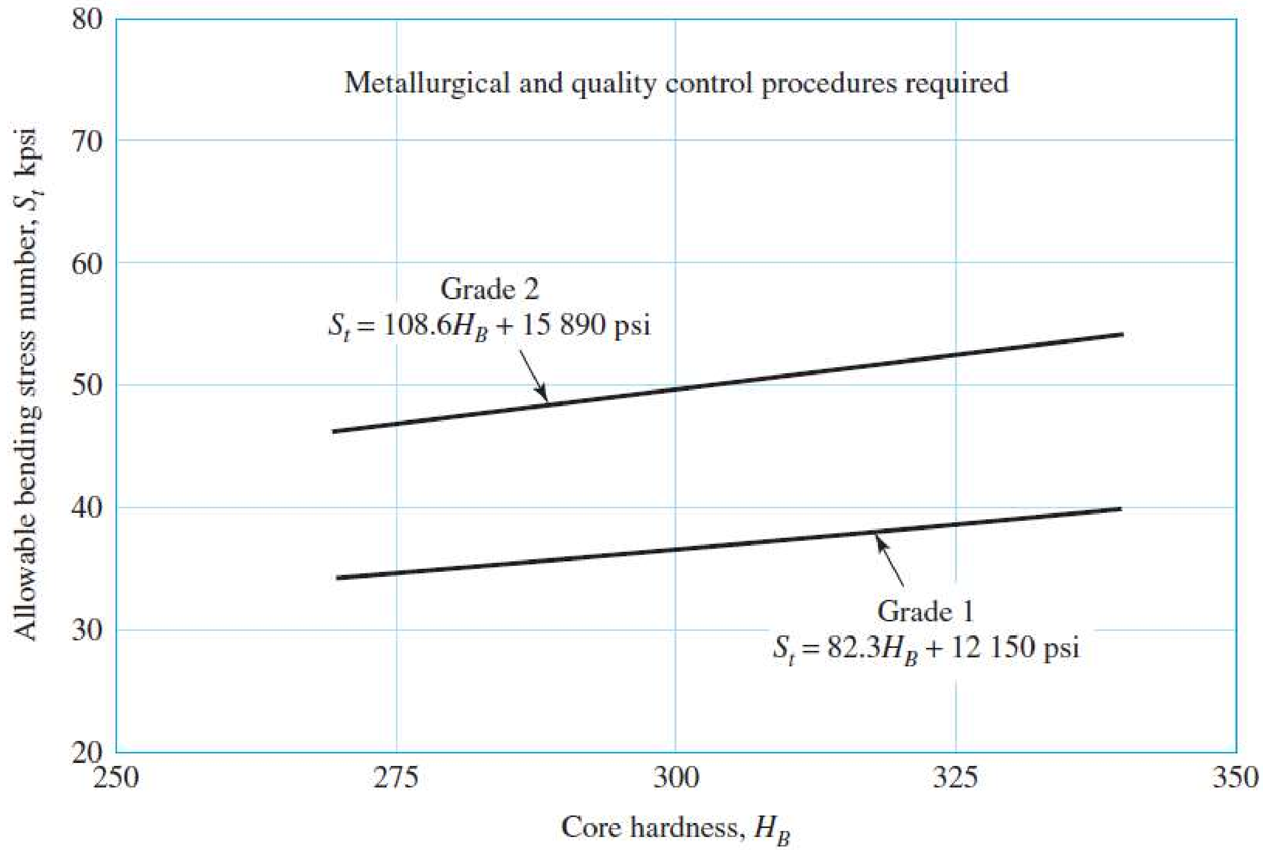
Allowable bending stress number for through-hardened steels, S_t. The SI equations are: $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



14.4 AGMA Strength Equations

Figure 14–3

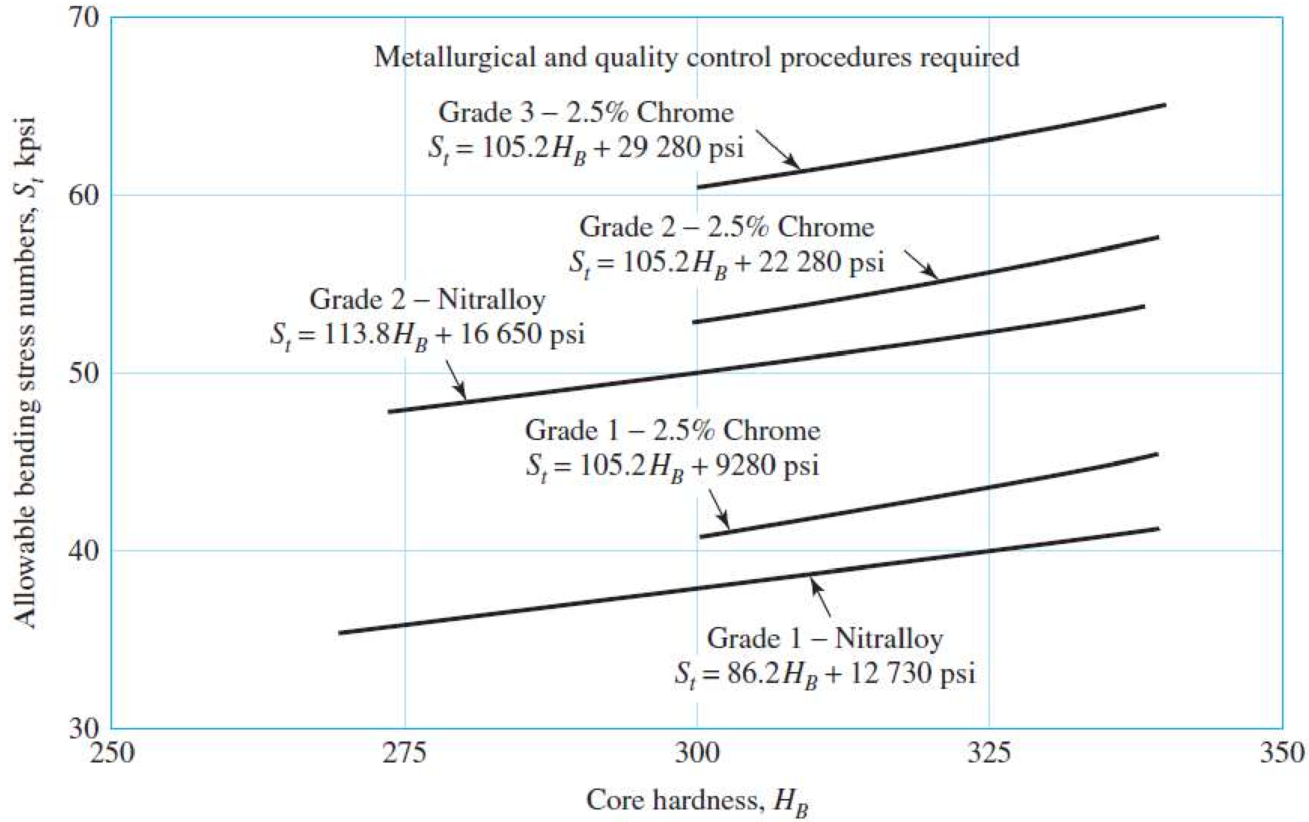
Allowable bending stress number for nitrided through-hardened steel gears (i.e., AISI 4140, 4340), S_t. The SI equations are: $S_t = 0.568H_B + 83.8$ MPa, grade 1, and $S_t = 0.749H_B + 110$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



14.4 AGMA Strength Equations

Figure 14–4

Allowable bending stress numbers for nitriding steel gears, S_t. The SI equations are:
$S_t = 0.594H_B + 87.76$ MPa
Nitralloy grade 1
$S_t = 0.784H_B + 114.81$ MPa
Nitralloy grade 2
$S_t = 0.7255H_B + 63.89$ MPa
2.5% chrome, grade 1
$S_t = 0.7255H_B + 153.63$ MPa
2.5% chrome, grade 2
$S_t = 0.7255H_B + 201.91$ MPa
2.5% chrome, grade 3
(Source: ANSI/AGMA 2001-D04, 2101-D04.)



14.4 AGMA Strength Equations

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Table 14–3

Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000	—
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]	75 000
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

14.4 AGMA Strength Equations

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Table 14–4

Repeatedly Applied Bending Strength S_t for Iron and Bronze Gears at 10^7 Cycles and 0.99 Reliability

Source: ANSI/AGMA 2001-D04.

Material	Material Designation[1]	Heat Treatment	Typical Minimum Surface Hardness[2]	Allowable Bending Stress Number, S_t,[3] psi
ASTM A48 gray cast iron	Class 20	As cast	—	5000
	Class 30	As cast	174 HB	8500
	Class 40	As cast	201 HB	13 000
ASTM A536 ductile (nodular) Iron	Grade 60–40–18	Annealed	140 HB	22 000–33 000
	Grade 80–55–06	Quenched and tempered	179 HB	22 000–33 000
	Grade 100–70–03	Quenched and tempered	229 HB	27 000–40 000
	Grade 120–90–02	Quenched and tempered	269 HB	31 000–44 000
Bronze		Sand cast	Minimum tensile strength 40 000 psi	5700
	ASTM B–148 Alloy 954	Heat treated	Minimum tensile strength 90 000 psi	23 600

14.4 AGMA Strength Equations

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Instead of using the term *strength,* AGMA uses data termed ***allowable stress numbers (Gear Bending Strength)***

The equation for the allowable bending stress is

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–17}$$

where for U.S. customary units (SI units),

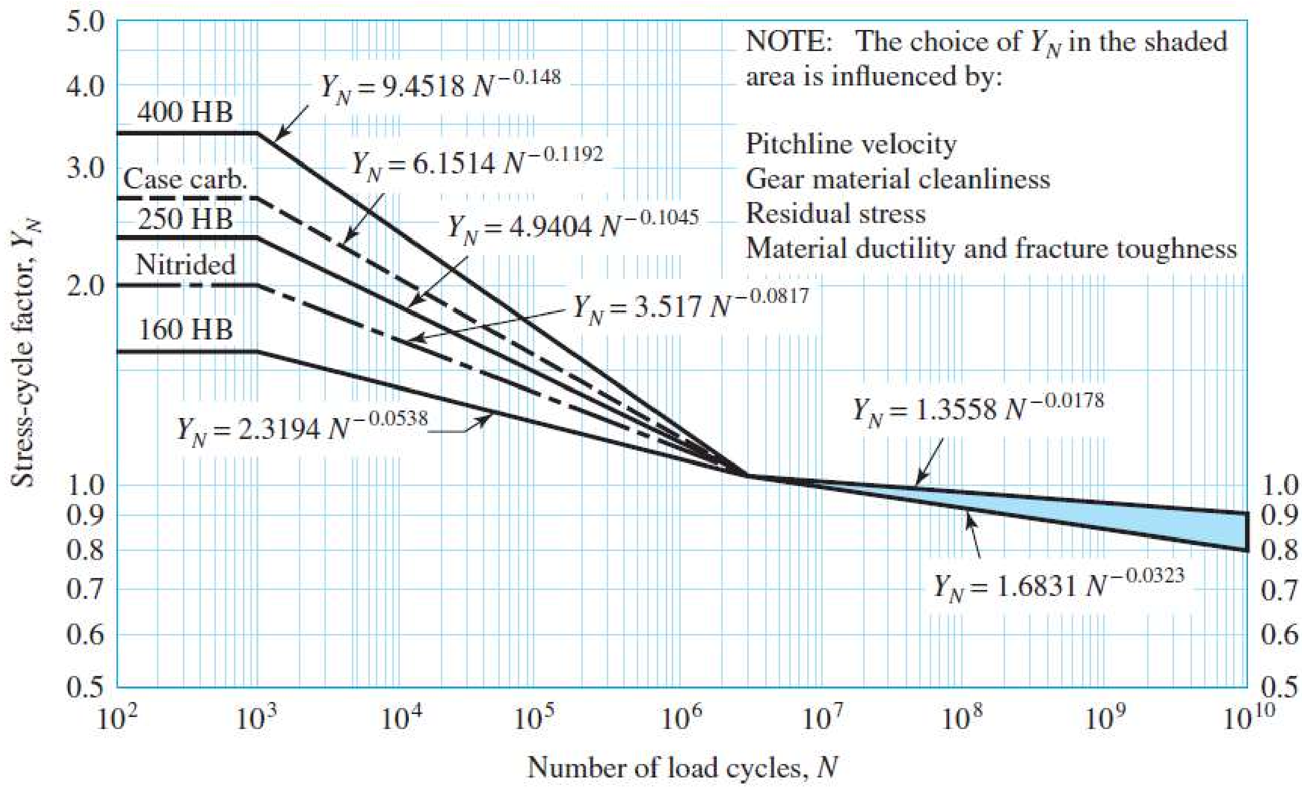
- S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
- Y_N is the stress-cycle factor for bending stress
- K_T (Y_θ) are the temperature factors
- K_R (Y_Z) are the reliability factors
- S_F is the AGMA factor of safety, a stress ratio

14.13 Stress-Cycle Factors Y_N

The purpose of the stress-cycle factors Y_N is to modify the gear strength for lives other than 10^7 cycles

Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*



14.14 Reliability Factors K_R

Table 14–10

Reliability Factors K_R (Y_Z)

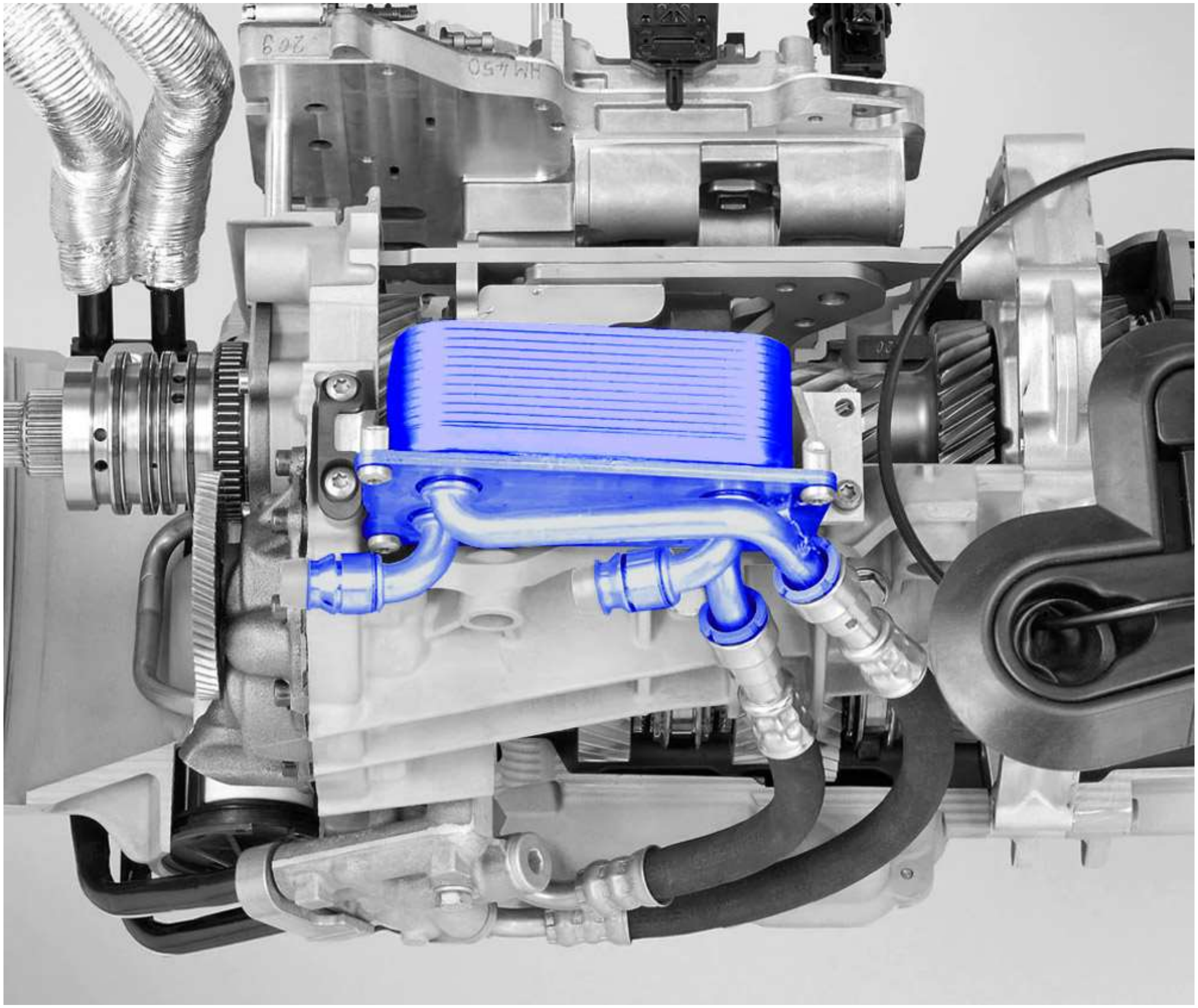
Source: ANSI/AGMA 2001-D04.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is:

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \le R \le 0.9999 \end{cases} \qquad (14\text{–}38)$$

14.15 Temperature Factors K_T



14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

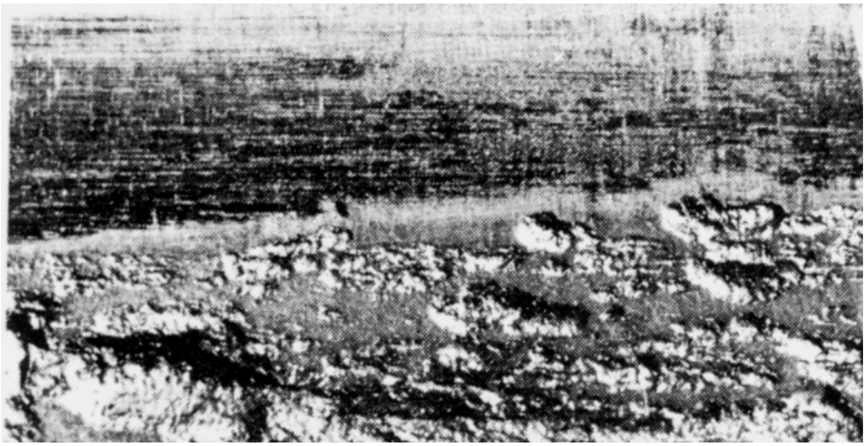
Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

Introduction

Bending Failure

Pitting Failure



14.3 AGME Stress Equations – Contact Stress (Hertzian Stress)

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \qquad (14\text{–}16)$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion*, in (mm)
I (Z_I) is the geometry factor for pitting resistance

The evaluation of all these factors is explained in the sections that follow. The development of Eq. (14–16) is clarified in the second part of Sec. 14–5.

14.3 Surface Durability

To obtain an expression for the surface-contact stress, we shall employ the Hertz theory. In Eq. (3–74), p. 138, it was shown that the contact stress between two cylinders may be computed from the equation

$$p_{\max} = \frac{2F}{\pi b l} \qquad (a)$$

where $p_{\max}$ = largest surface pressure

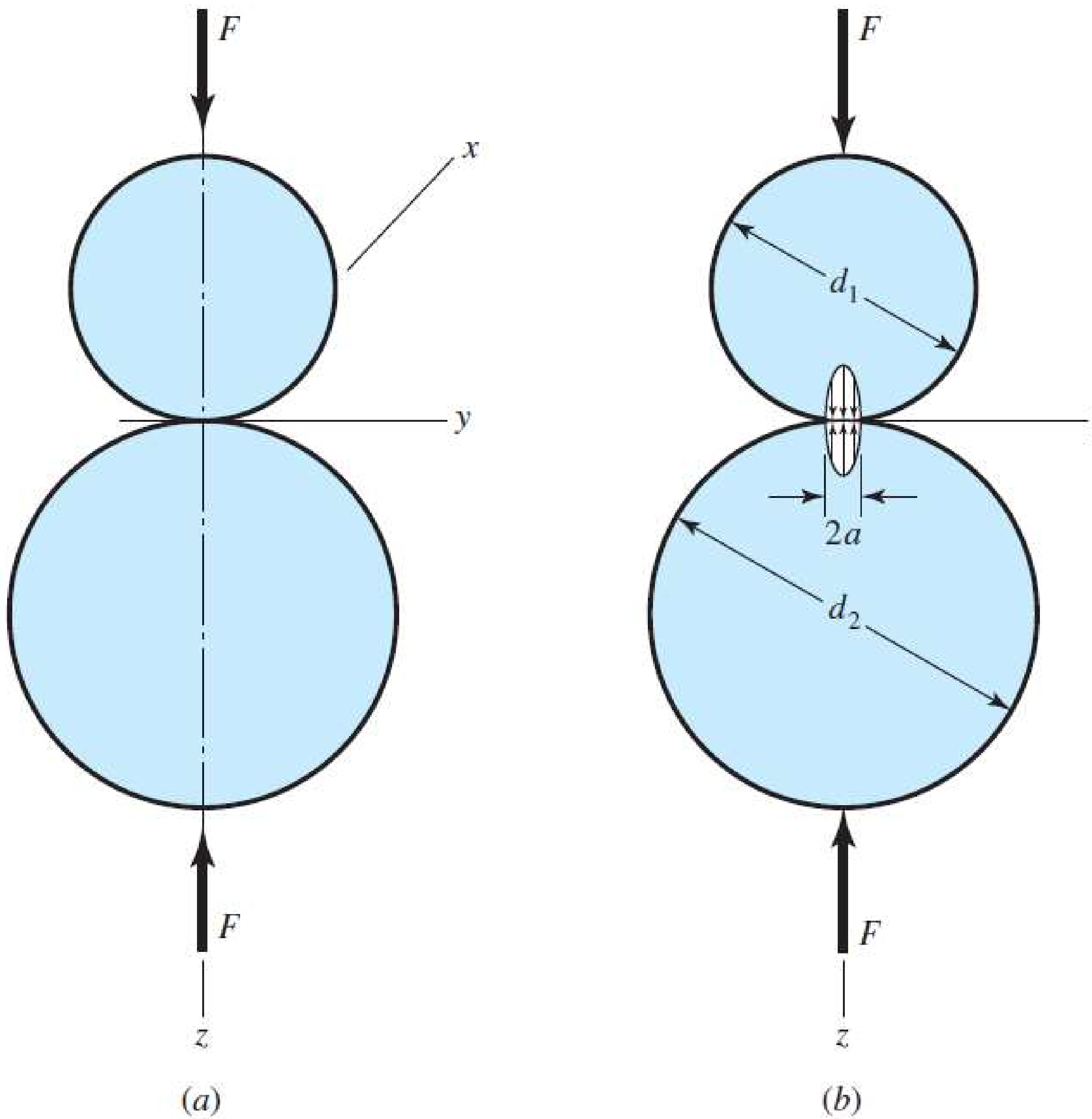
F = force pressing the two cylinders together

l = length of cylinders

14.3 Surface Durability

Figure 3–36

(*a*) Two spheres held in contact by force F; (*b*) contact stress has a hemispherical distribution across contact zone of diameter $2a$.



14.3 Surface Durability

To obtain an expression for the surface-contact stress, we shall employ the Hertz theory. In Eq. (3–74), p. 138, it was shown that the contact stress between two cylinders may be computed from the equation

$$p_{\max} = \frac{2F}{\pi b l} \tag{a}$$

where $p_{\max}$ = largest surface pressure
F = force pressing the two cylinders together
l = length of cylinders

and half-width b is obtained from Eq. (3–73), p. 138, given by

$$b = \left[\frac{2F}{\pi l}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}\right]^{1/2} \tag{14–10}$$

where ν_1, ν_2, E_1, and E_2 are the elastic constants and d_1 and d_2 are the diameters, respectively, of the two contacting cylinders.

14.3 Surface Durability

$$p_{max} = \frac{2F}{\pi b l}$$

+

$$b = \left[\frac{2F}{\pi l} \frac{(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2}{1/d_1 + 1/d_2} \right]^{1/2}$$

=

$$\sigma_C^2 = \frac{W^t}{\pi F \cos\phi} \frac{1/r_1 + 1/r_2}{(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2}$$

Where, AGMA defines an elastic coefficient C_p by the equation:

$$C_p = \left[\frac{1}{\pi \left(\dfrac{1 - \nu_P^2}{E_P} + \dfrac{1 - \nu_G^2}{E_G} \right)} \right]^{1/2}$$

14.6 The Elastic Coefficient C_p

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Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source: AGMA 218.01*

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

14.3 AGME Stress Equations – Contact Stress (Hertzian Stress)

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \tag{14–16}$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion*, in (mm)
I (Z_I) is the geometry factor for pitting resistance

The evaluation of all these factors is explained in the sections that follow. The development of Eq. (14–16) is clarified in the second part of Sec. 14–5.

14.5 Geometry Factor I

Pitting resistance geometry factor OR surface strength geometry factor:

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G+1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G-1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

Where m_N =1 for spur gear

$$p_N = p_n \cos\phi_n$$

$$Z = [(r_P + a)^2 - r_{bP}^2]^{1/2} + [(r_G + a)^2 - r_{bG}^2]^{1/2} - (r_P + r_G)\sin\phi_t \qquad (14\text{–}25)$$

14.3 AGME Stress Equations – Contact Stress (Hertzian Stress)

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \qquad (14\text{–}16)$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion*, in (mm)
I (Z_I) is the geometry factor for pitting resistance

The evaluation of all these factors is explained in the sections that follow. The development of Eq. (14–16) is clarified in the second part of Sec. 14–5.

14.9 Surface Condition Factor C_f

The surface condition factor C_f or Z_R is used only in the pitting resistance equation, Eq. (14–16). It depends on

- Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shotpeening
- Residual stress
- Plastic effects (work hardening)

Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA specifies a value of C_f greater than unity.

14.3 AGME Stress Equations – Contact Stress (Hertzian Stress)

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \tag{14–16}$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Eq. (14–15). For U.S. customary units (SI units), the additional terms are

C_p (Z_E) is an elastic coefficient, $\sqrt{\text{lbf/in}^2}$ ($\sqrt{\text{N/mm}^2}$)
C_f (Z_R) is the surface condition factor
d_P (d_{w1}) is the pitch diameter of the *pinion*, in (mm)
I (Z_I) is the geometry factor for pitting resistance

The evaluation of all these factors is explained in the sections that follow. The development of Eq. (14–16) is clarified in the second part of Sec. 14–5.

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.17 Safety Factor S_H – *in Wear (Surface)*

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$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

Contact (Pitting) Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

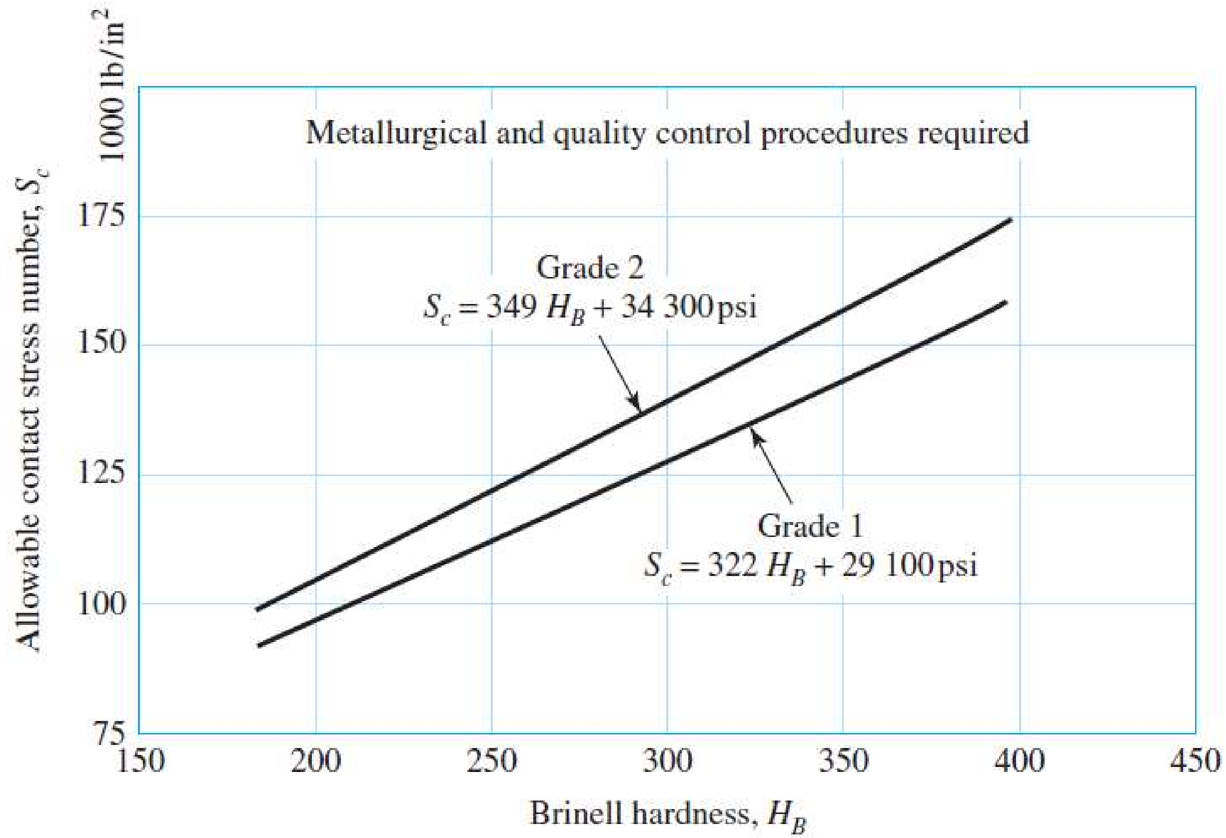
Allowable contact Stress

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
Z_N is the stress-cycle factor
C_H (Z_W) are the hardness ratio factors for pitting resistance
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_H is the AGMA factor of safety, a stress ratio

14.4 AGMA Strength Equations

Figure 14–5

Contact-fatigue strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are: $S_c = 2.22H_B + 200$ MPa, grade 1, and $S_c = 2.41H_B + 237$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



AGMA allowable stress numbers (strengths) for bending and contact stress are for

- Unidirectional loading
- 10 million stress cycles
- 99 percent reliability

14.4 AGMA Strength Equations

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Table 14–6

Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

Source: ANSI/AGMA 2001-D04.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

14.17 Safety Factor S_H – in Wear (Surface)

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$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

Contact (Pitting) Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Allowable contact Stress

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
Z_N is the stress-cycle factor
C_H (Z_W) are the hardness ratio factors for pitting resistance
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_H is the AGMA factor of safety, a stress ratio

14.12 Hardness Ratio Factor C_H (Z_W)

$$C_H = 1.0 + A'(m_G - 1.0) \qquad (14\text{–}36)$$

where

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3}) \qquad 1.2 \leq \frac{H_{BP}}{H_{BG}} \leq 1.7$$

The terms H_{BP} and H_{BG} are the Brinell hardness (10-mm ball at 3000-kg load) of the pinion and gear, respectively. The term m_G is the speed ratio and is given by Eq. (14–22). See Fig. 14–12 for a graph of Eq. (14–36). For

$$\frac{H_{BP}}{H_{BG}} < 1.2, \qquad A' = 0$$

$$\frac{H_{BP}}{H_{BG}} > 1.7, \qquad A' = 0.006\,98$$

14.17 Safety Factor S_H – *in Wear (Surface)*

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

Contact (Pitting) Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Allowable contact Stress

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
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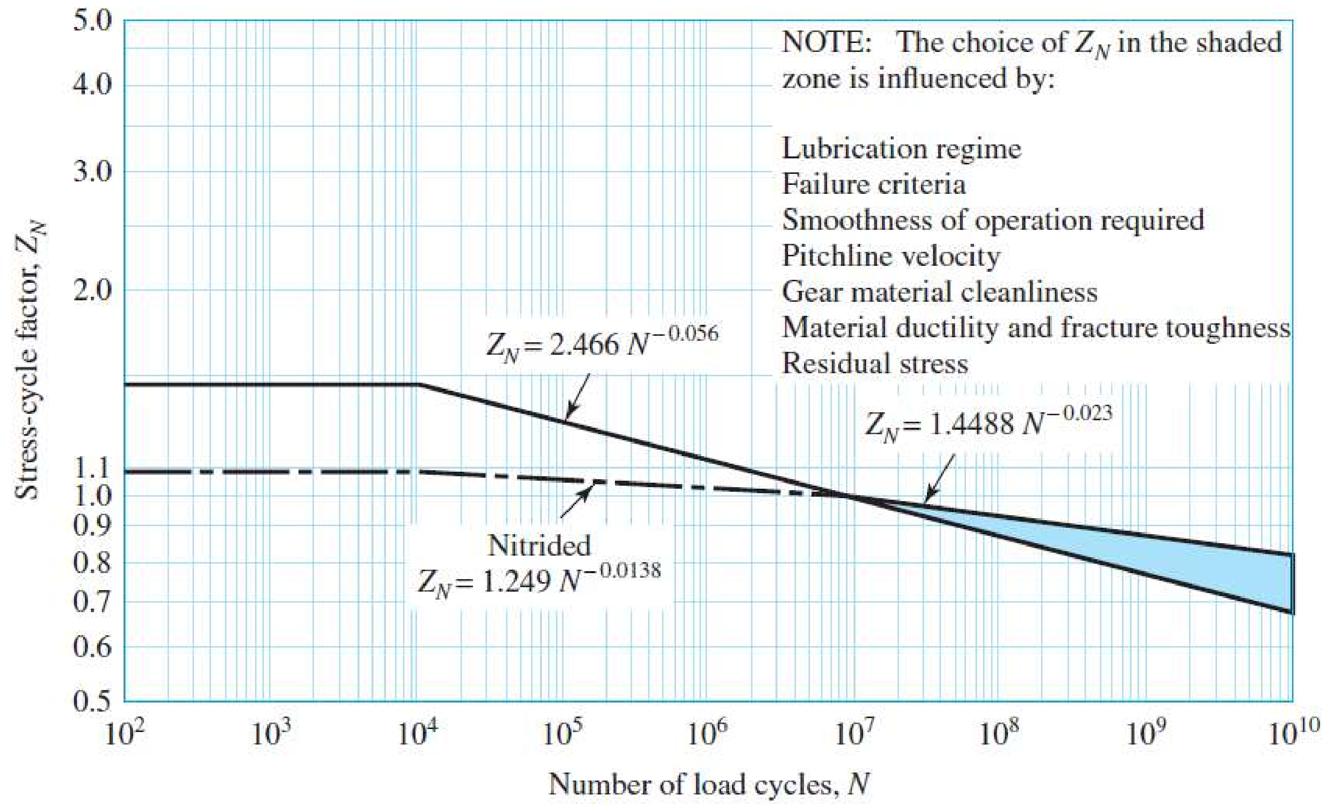
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_H is the AGMA factor of safety, a stress ratio

14.13 Stress-Cycle Factors Z_N

The purpose of the stress-cycle factors $\boldsymbol{Z_N}$ is to modify the gear strength for lives other than 10^7 cycles

Figure 14–15

Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*



14.17 Safety Factor S_H – *in Wear (Surface)*

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$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

Contact (Pitting) Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Allowable contact Stress

S_c is the allowable contact stress, lbf/in^2 (N/mm^2)
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S_H is the AGMA factor of safety, a stress ratio

14.17 Safety Factor S_H – *in Wear (Surface)*

$$\sigma_c = \begin{cases} C_p\sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F}\dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E\sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b}\dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

Contact (Pitting) Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

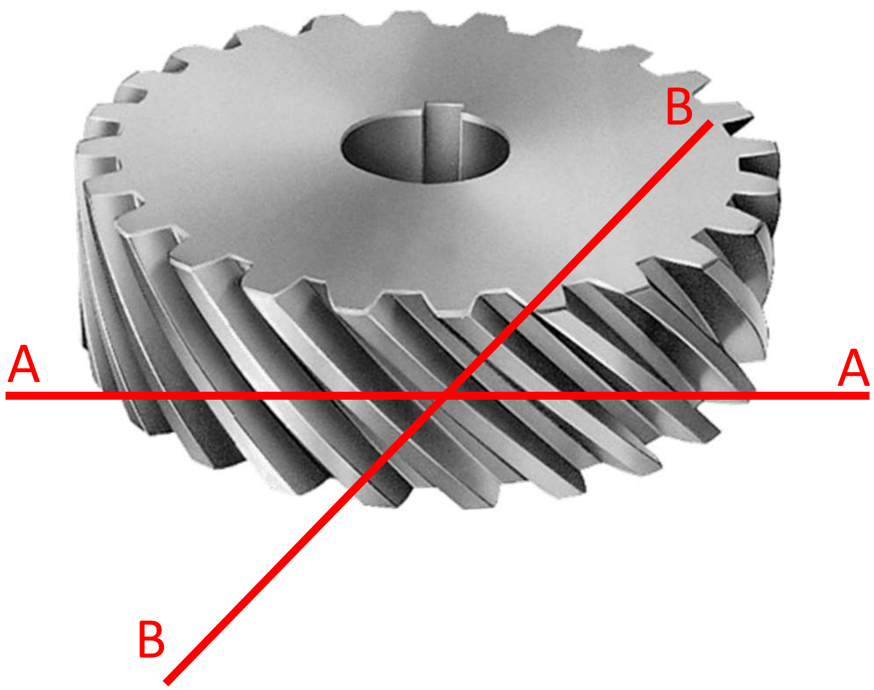
Allowable contact Stress

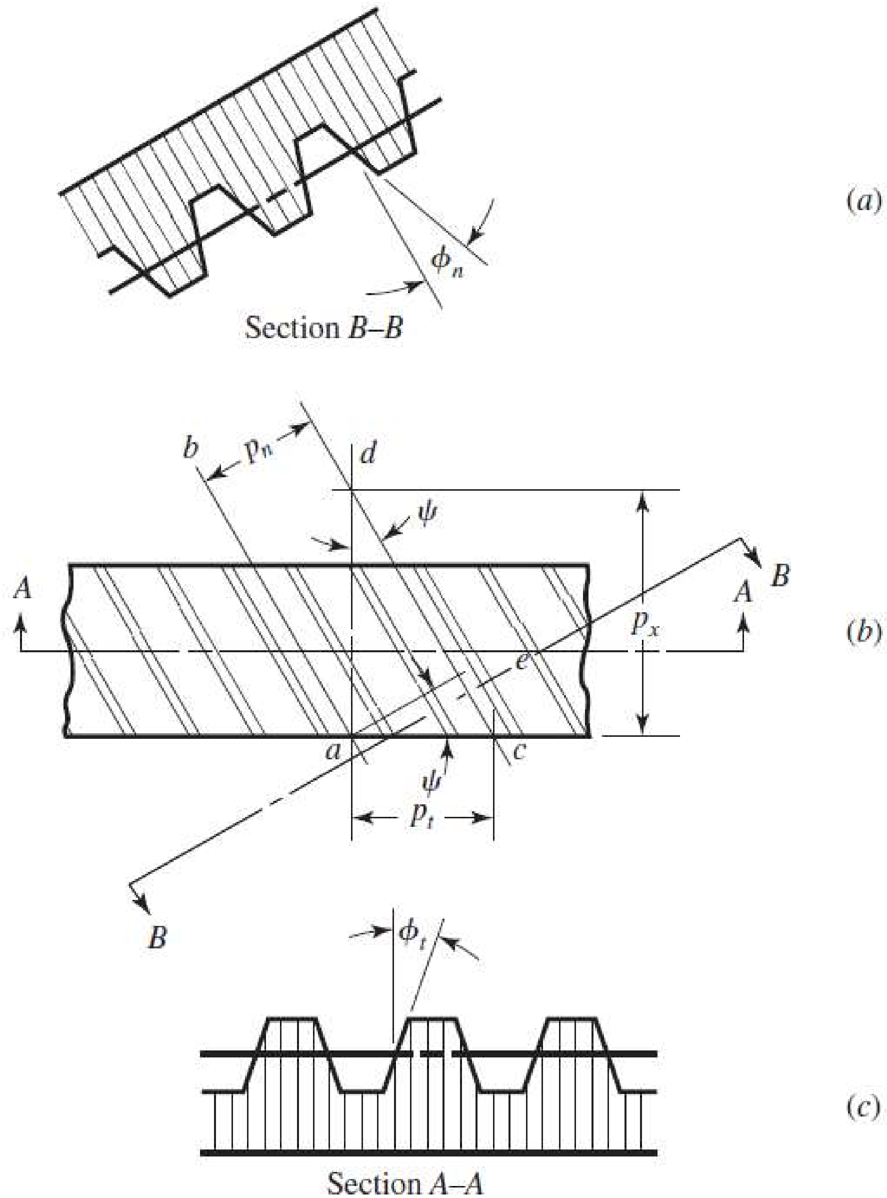
$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \qquad (14\text{–}42)$$

13.10 Parallel Helical Gears

Figure 13–22

Nomenclature of helical gears.

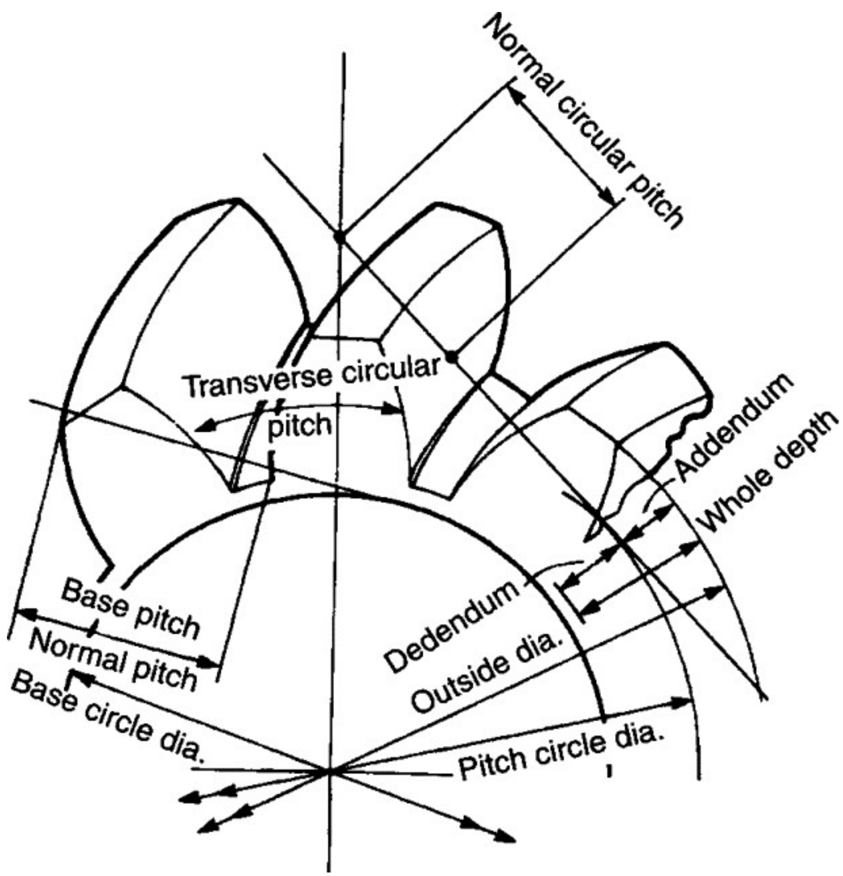


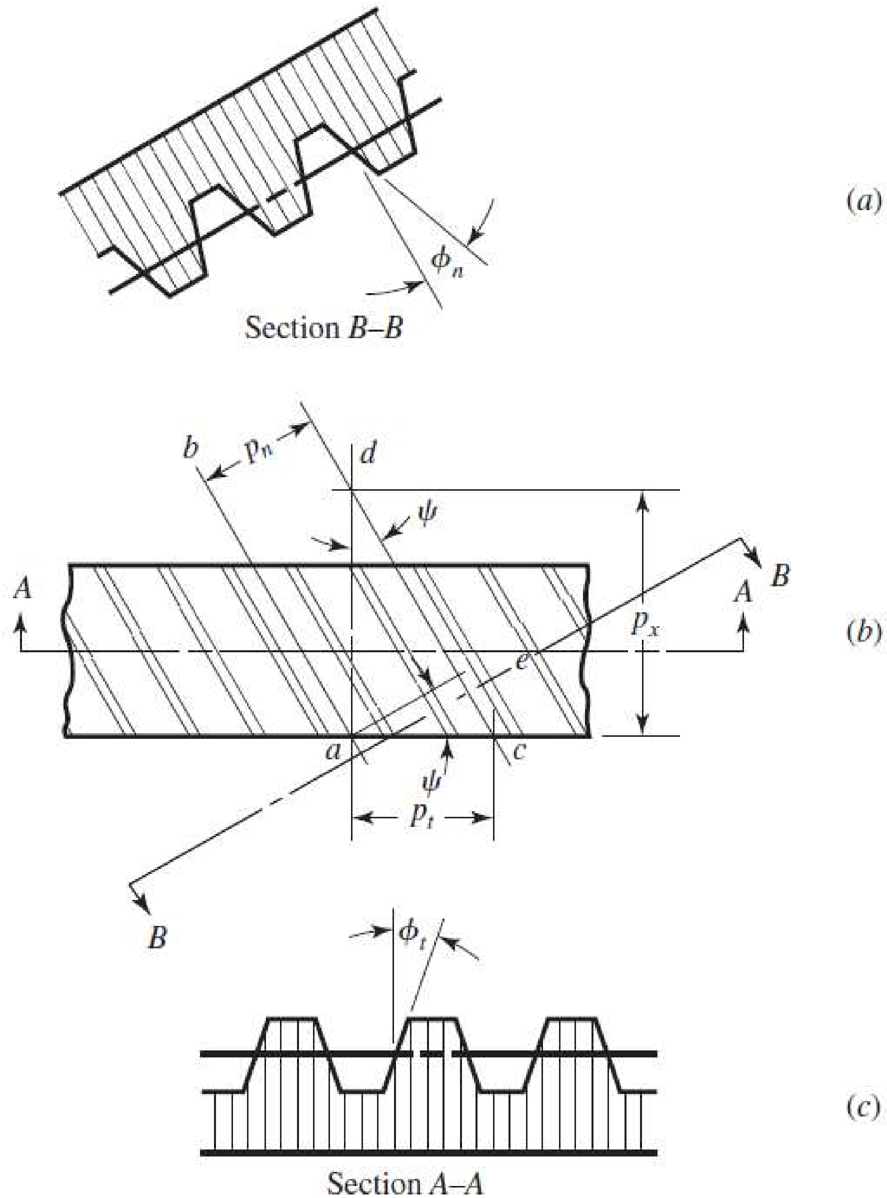


13.10 Parallel Helical Gears

Figure 13–22

Nomenclature of helical gears.





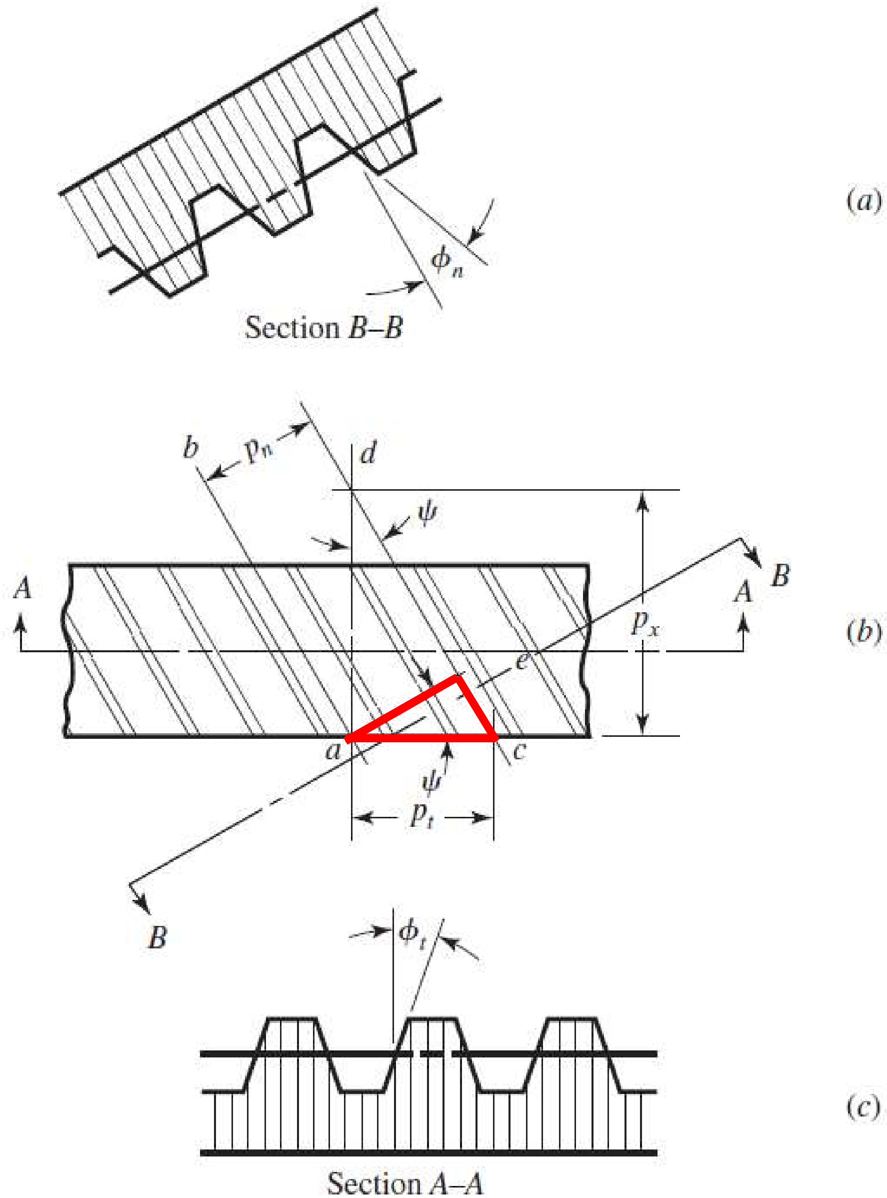
13.10 Parallel Helical Gears

Figure 13–22

Nomenclature of helical gears.

$$p_n = p_t \cos\psi$$

$$P_n = \frac{P_t}{\cos\psi}$$



13.10 Parallel Helical Gears

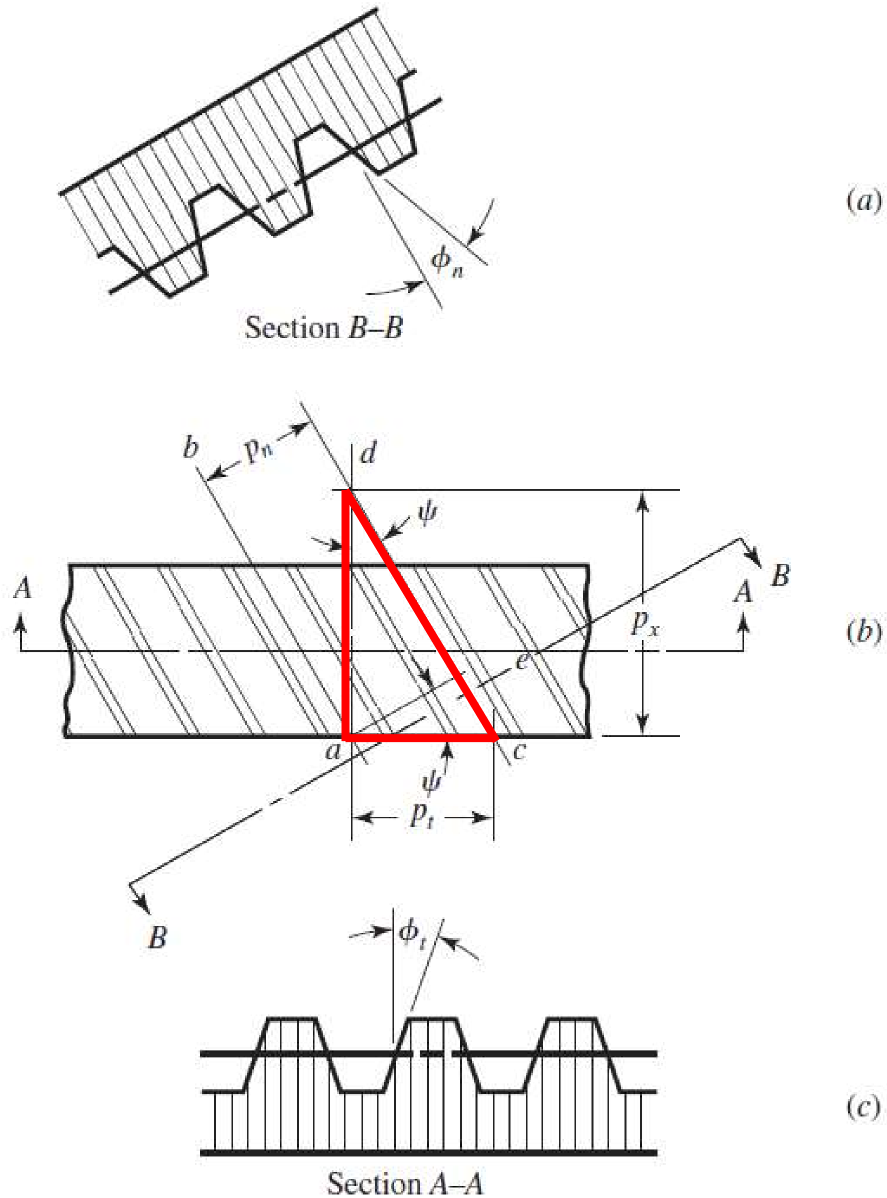
Figure 13–22

Nomenclature of helical gears.

$$p_n = p_t \cos\psi$$

$$p_x = \frac{p_t}{\tan\psi}$$

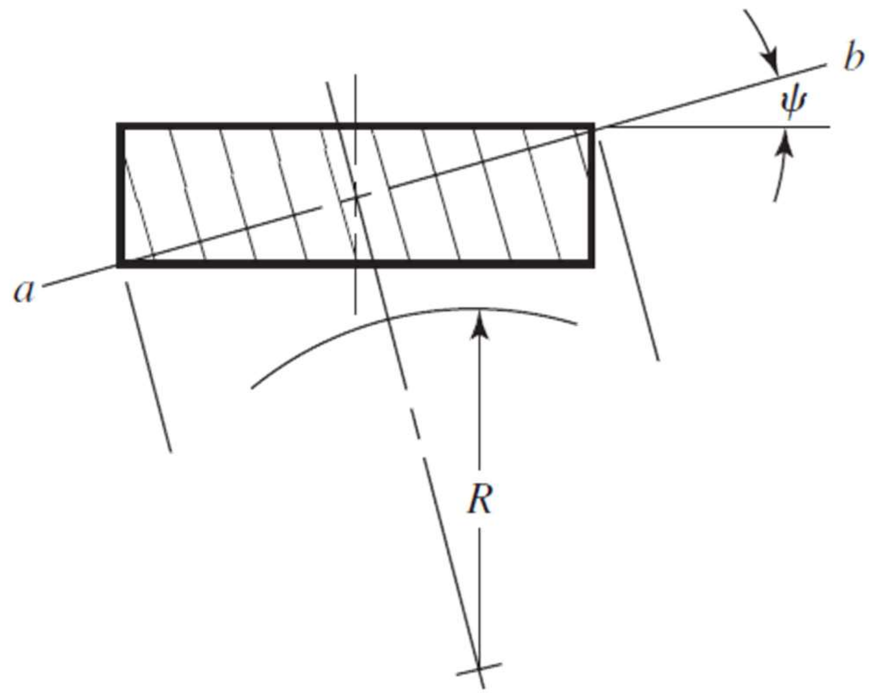
$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$



13.10 Parallel Helical Gears

Figure 13–23

A cylinder cut by an oblique plane.



$$N' = \frac{N}{\cos^3 \psi} \tag{13–20}$$

where N' is the virtual number of teeth and N is the actual number of teeth. It is necessary to know the virtual number of teeth in design for strength and also, sometimes, in cutting helical teeth. This apparently larger radius of curvature means that few teeth may be used on helical gears, because there will be less undercutting.

13.10 Parallel Helical Gears

Case 1: One-To-One Gear Teeth Ratio

The smallest number of teeth on a helical-spur pinion and gear without Interference:

$$N_P = \frac{2k \cos \psi}{3 \sin^2 \phi_t}\left(1 + \sqrt{1 + 3 \sin^2 \phi_t}\right) \qquad (13\text{–}21)$$

For example, if the normal pressure angle ϕ_n is 20°, the helix angle ψ is 30°, then ϕ_t is

$$\phi_t = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ$$

$$N_P = \frac{2(1) \cos 30^\circ}{3 \sin^2 22.80^\circ}\left(1 + \sqrt{1 + 3 \sin^2 22.80^\circ}\right) = 8.48 = 9 \text{ teeth}$$

13.10 Parallel Helical Gears

Case 2: Mating Gear has more teeth than the pinion

The smallest number of teeth on a helical-spur pinion without Interference:

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m + \sqrt{m^2 + (1+2m)\sin^2\phi_t}\right] \tag{13–22}$$

The largest gear with a specified pinion is given by

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t} \tag{13–23}$$

For example, for a nine-tooth pinion with a pressure angle ϕ_n of 20°, a helix angle ψ of 30°, and recalling that the tangential pressure angle ϕ_t is 22.80°,

$$N_G = \frac{9^2\sin^2 22.80° - 4(1)^2\cos^2 30°}{4(1)\cos 30° - 2(9)\sin^2 22.80°} = 12.02 = 12$$

13.10 Parallel Helical Gears

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Case 3: Mating Rack and Pinion:

The smallest number of teeth on a helical-spur pinion that can run with a rack without Interference:

$$N_P = \frac{2k \cos \psi}{\sin^2 \phi_t}$$

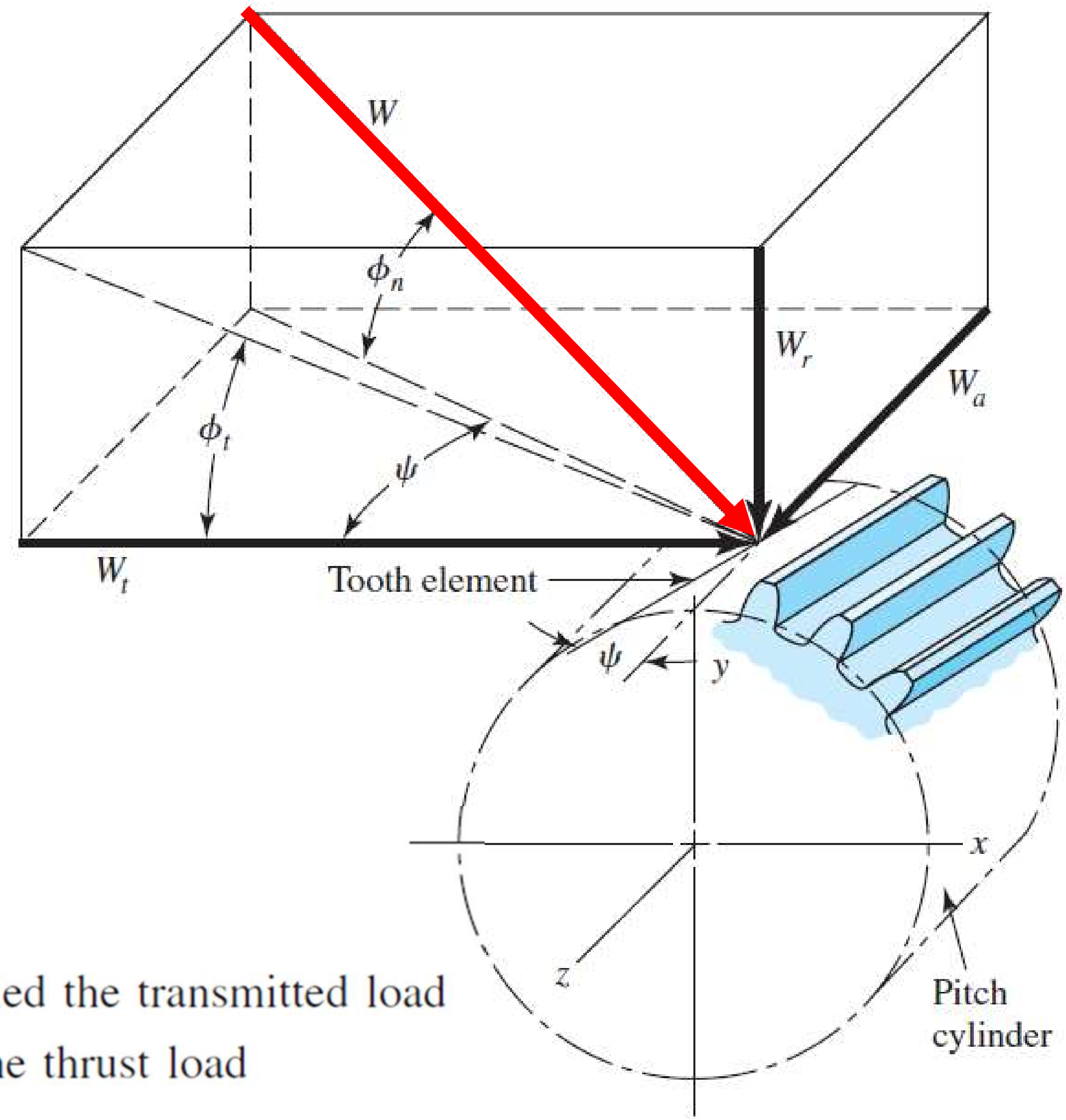
For a normal pressure angle ϕ_n of 20° and a helix angle ψ of 30°, and $\phi_t = 22.80°$,

$$N_P = \frac{2(1) \cos 30°}{\sin^2 22.80°} = 11.5 = 12 \text{ teeth}$$

13.16 Force Analysis – Helical Gearing

Figure 13–37

Tooth forces acting on a right-hand helical gear.



W = total force

W_r = radial component

W_t = tangential component, also called the transmitted load

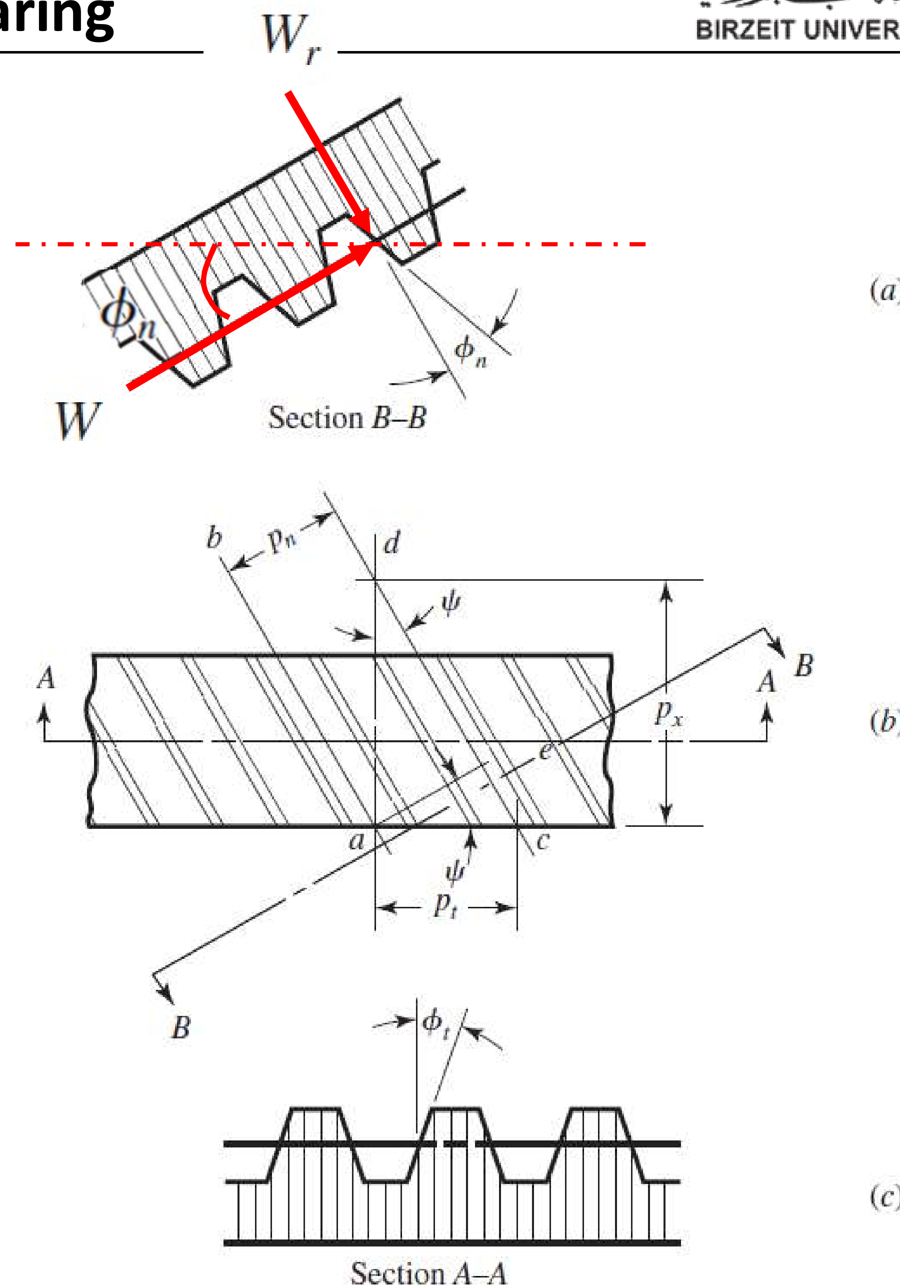
W_a = axial component, also called the thrust load

13.16 Force Analysis – Helical Gearing

Figure 13–22

Nomenclature of helical gears.

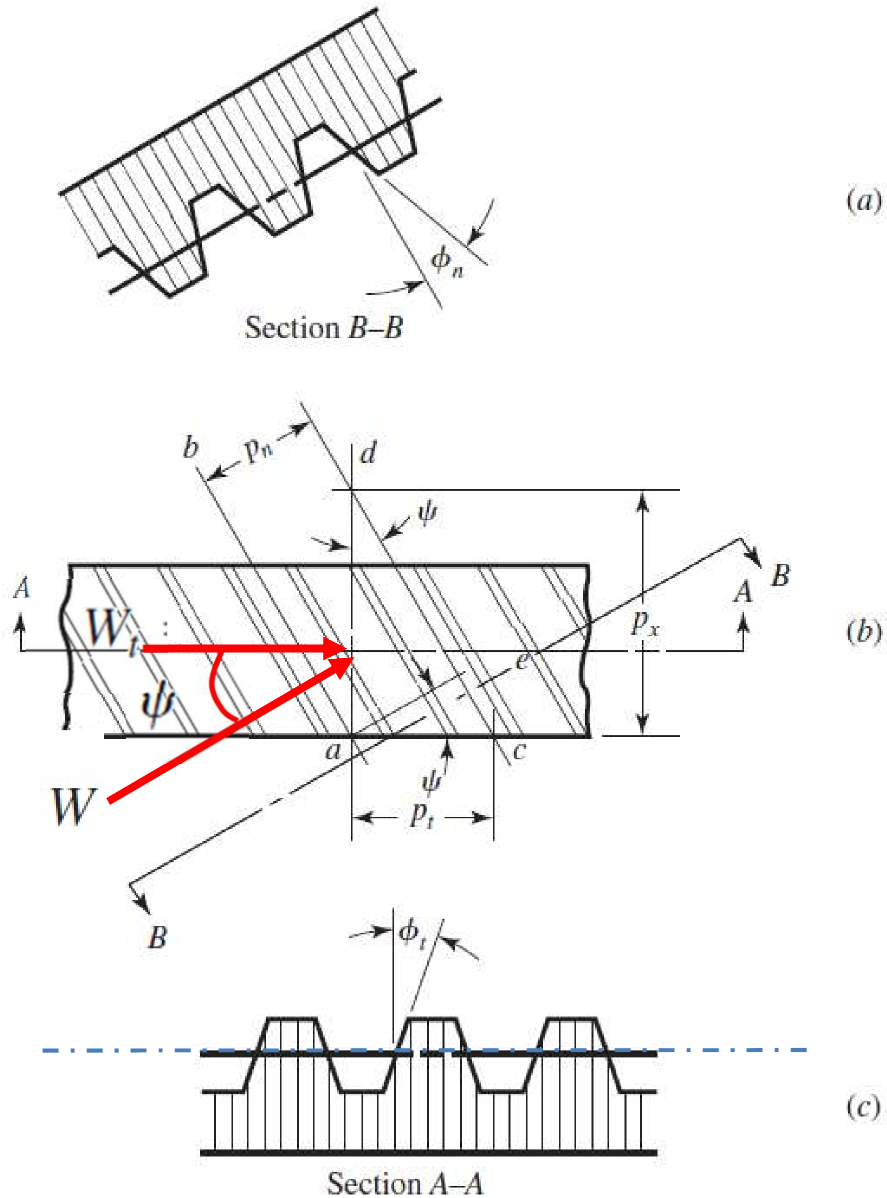
$$W_r = W \sin \phi_n$$



13.16 Force Analysis – Helical Gearing

Figure 13–22

Nomenclature of helical gears.



$$W_t = W \cos \psi$$

$$W_t = W \cos \phi_n \cos \psi$$

13.16 Force Analysis – Helical Gearing

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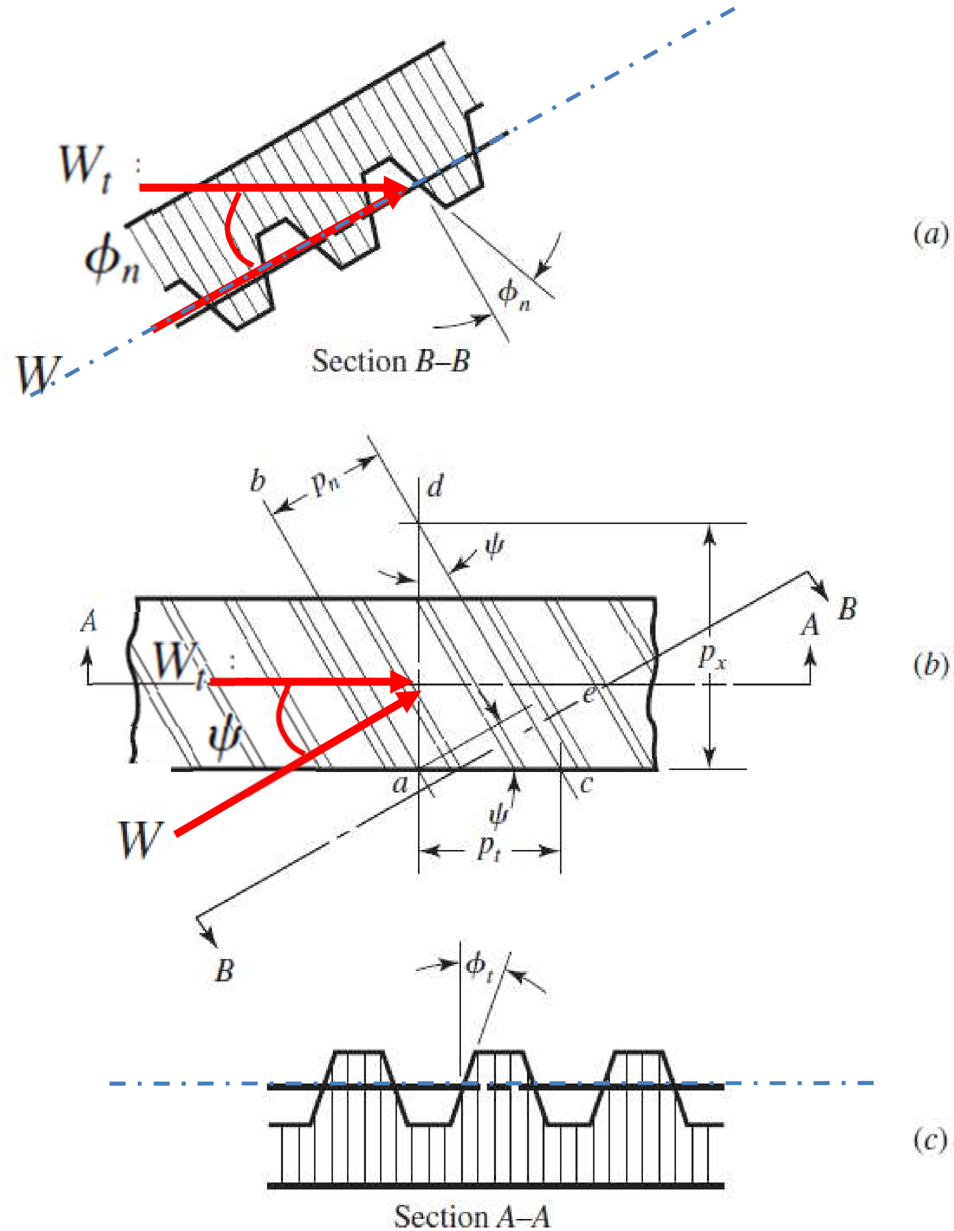
Figure 13–22

Nomenclature of helical gears.

$$W_t = W \cos \phi_n$$

$$W_t = W \cos \psi$$

$$W_t = W \cos \phi_n \cos \psi$$



13.16 Force Analysis – Helical Gearing

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Figure 13–37

Tooth forces acting on a right-hand helical gear.

$$W_r = W \sin \phi_n$$

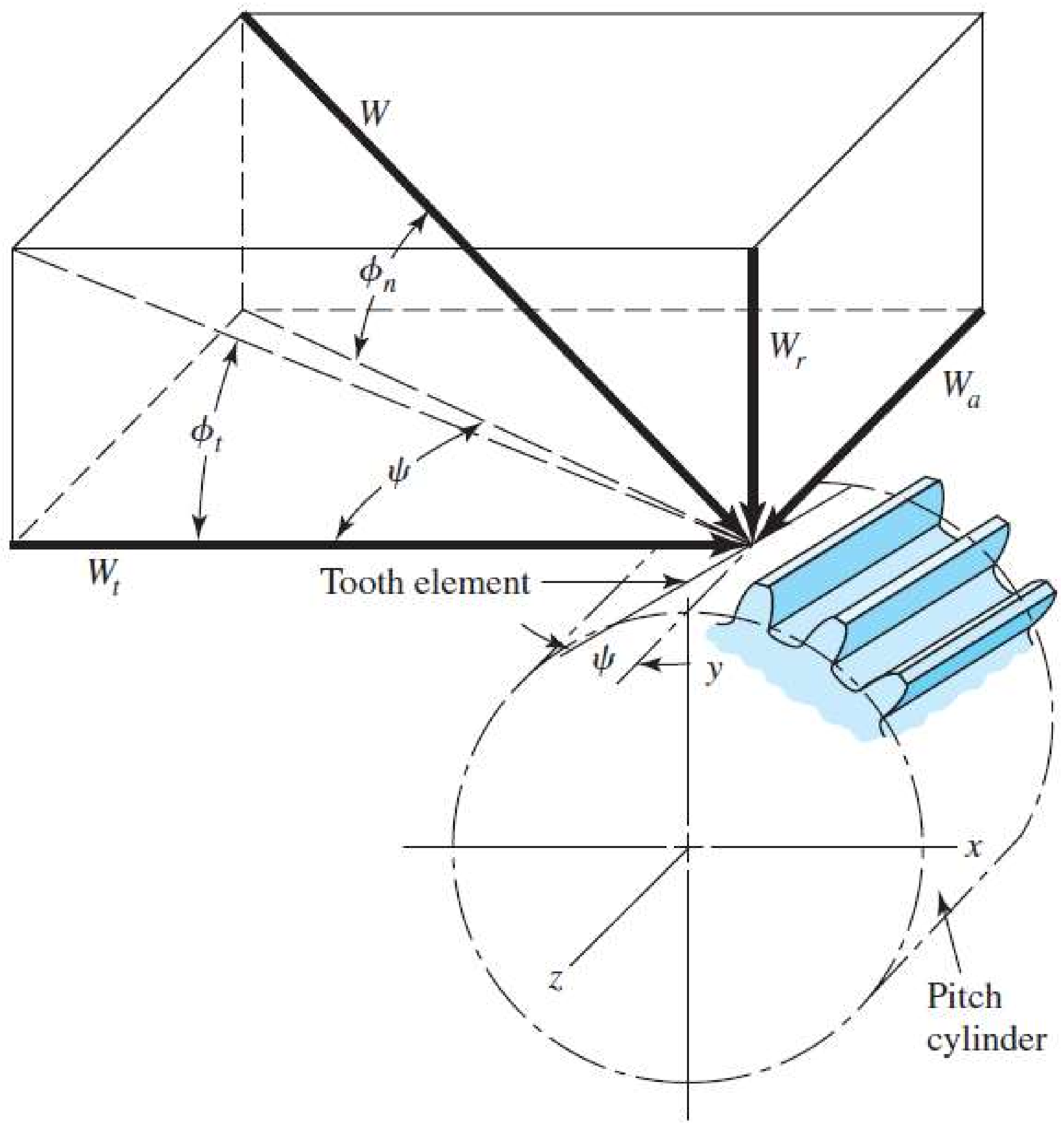
$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$

$$W_r = W_t \tan \phi_t$$

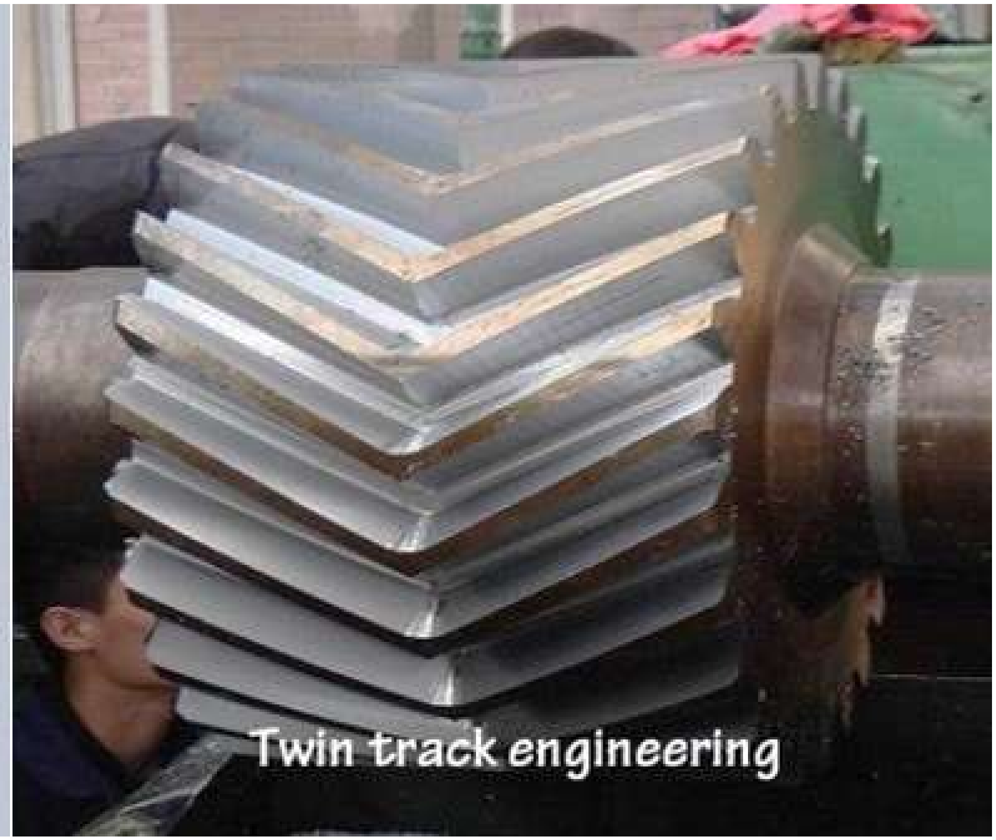
$$W_a = W_t \tan \psi$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$



13.10 Parallel Helical Gears

Double Helical Gear



13.10 Parallel Helical Gears – Example 13-9

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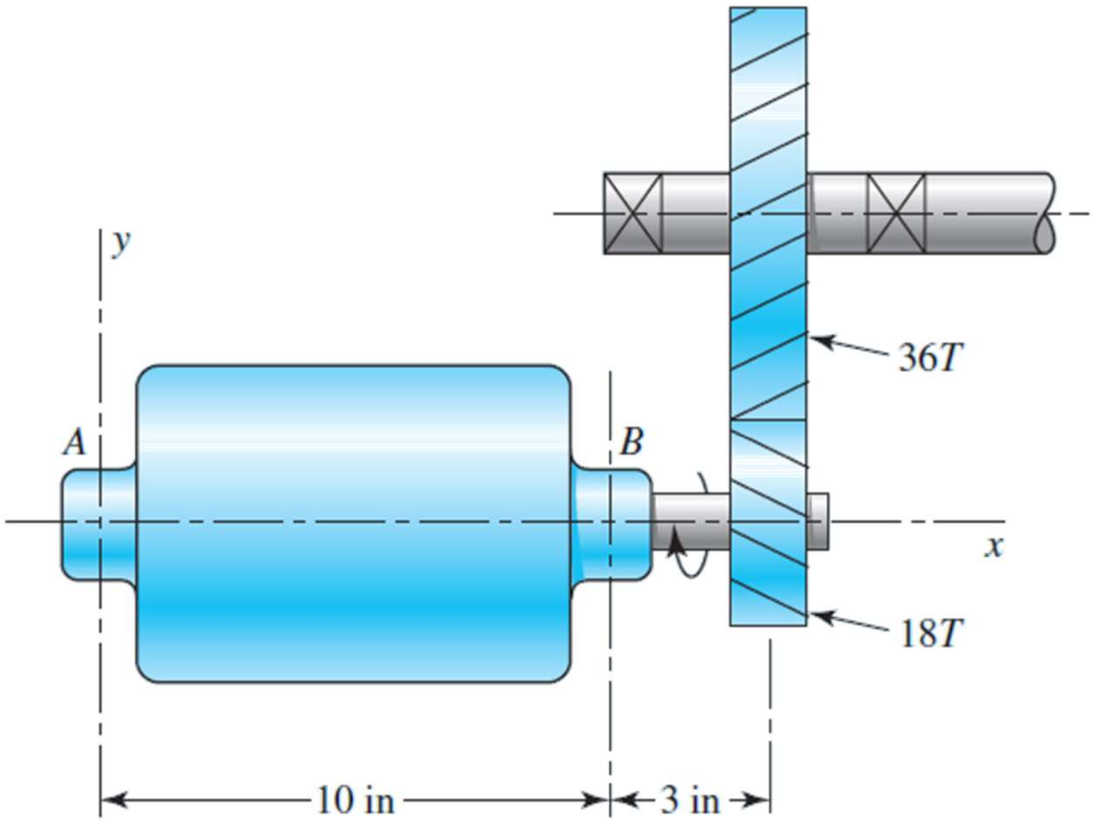
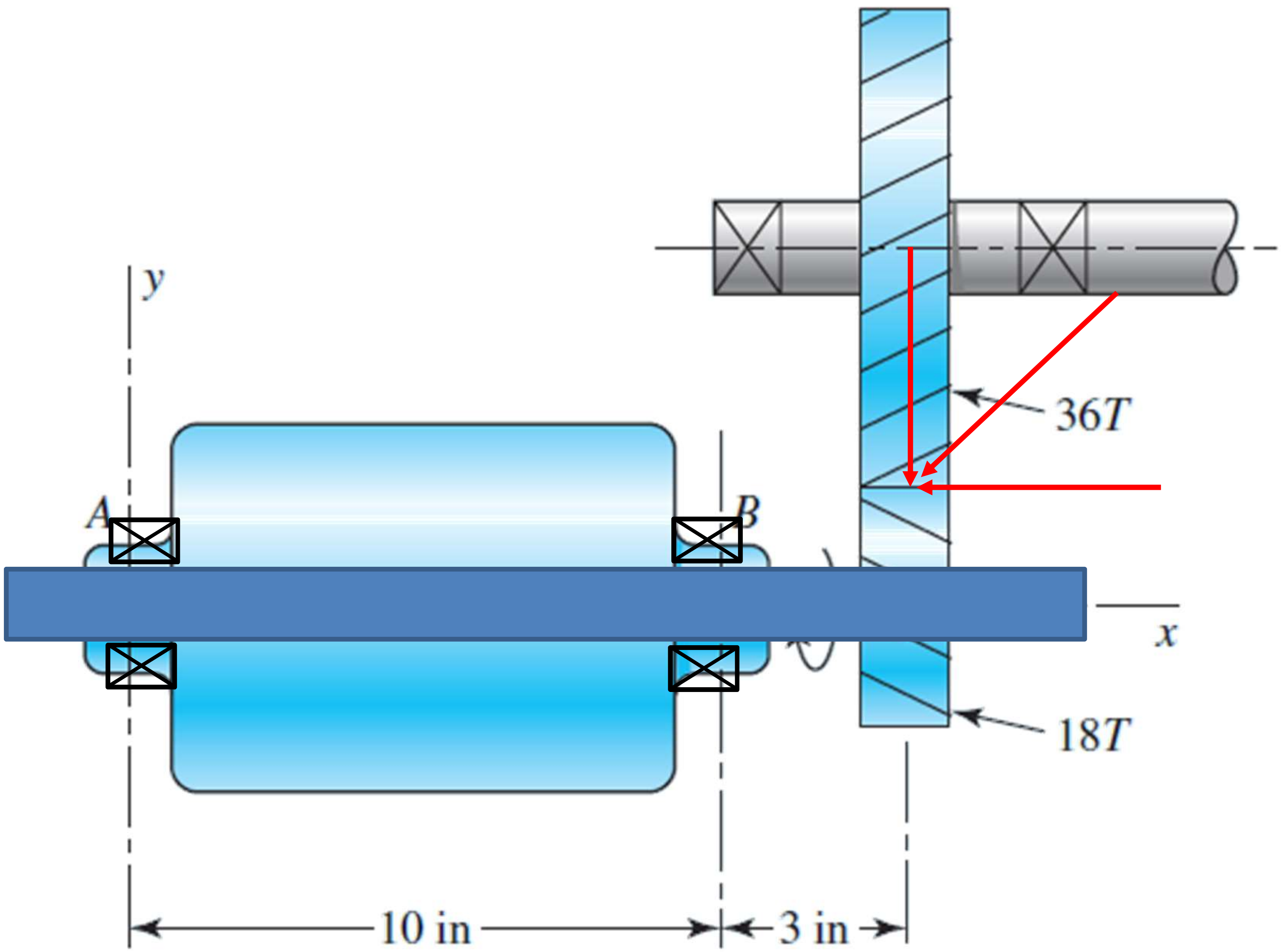
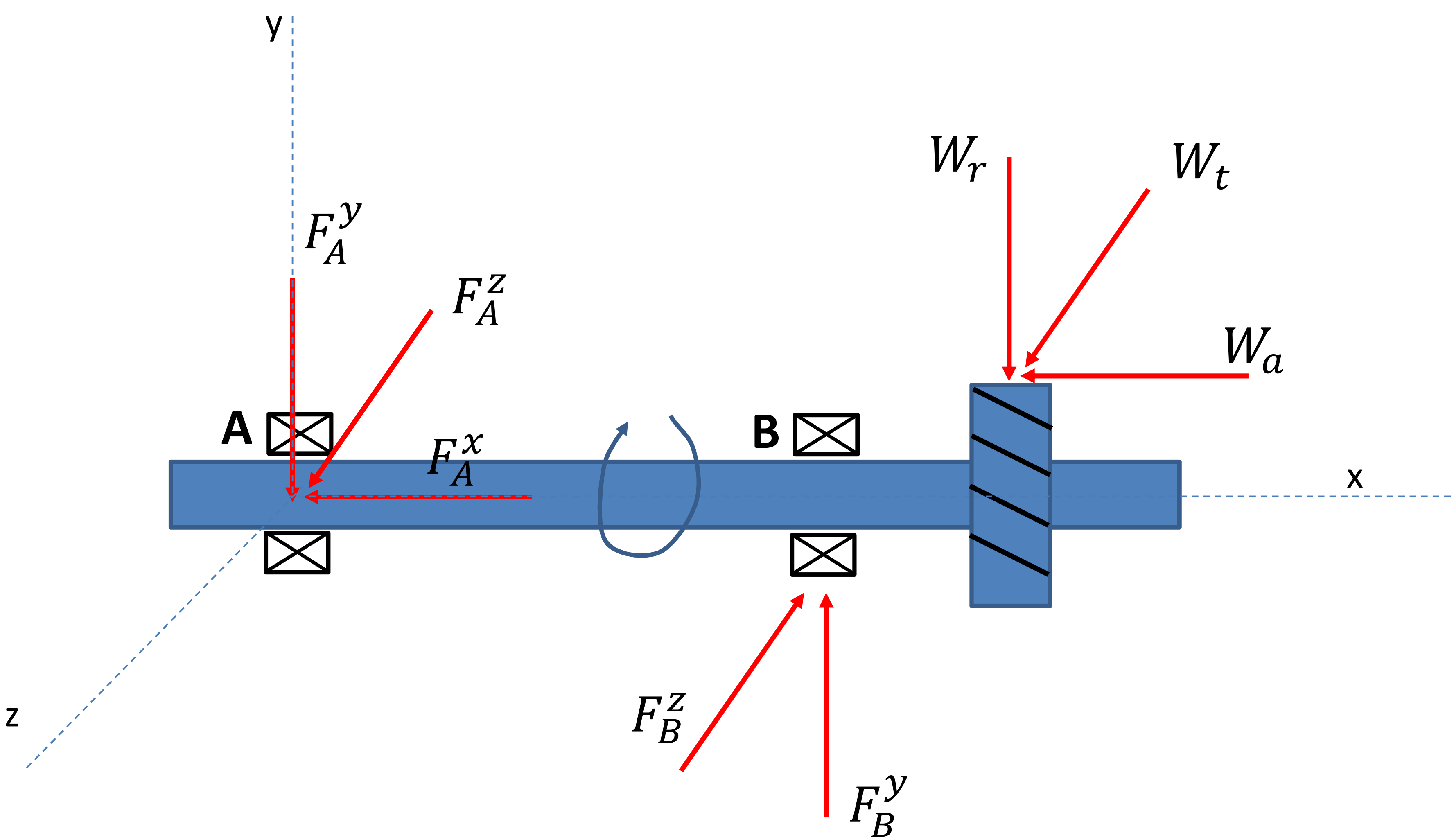
In Fig. 13–38 an electric motor transmits 1-hp at 1800 rev/min in the clockwise direction, as viewed from the positive *x* axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal diametral pitch of 12 teeth/in. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at *A* and *B*. The thrust should be taken out at *A*.



13.10 Parallel Helical Gears – Example 13-9



13.10 Parallel Helical Gears – Example 13-9



13.16 Force Analysis – Helical Gearing

Figure 13–37

Tooth forces acting on a right-hand helical gear.

$$W_r = W \sin \phi_n$$

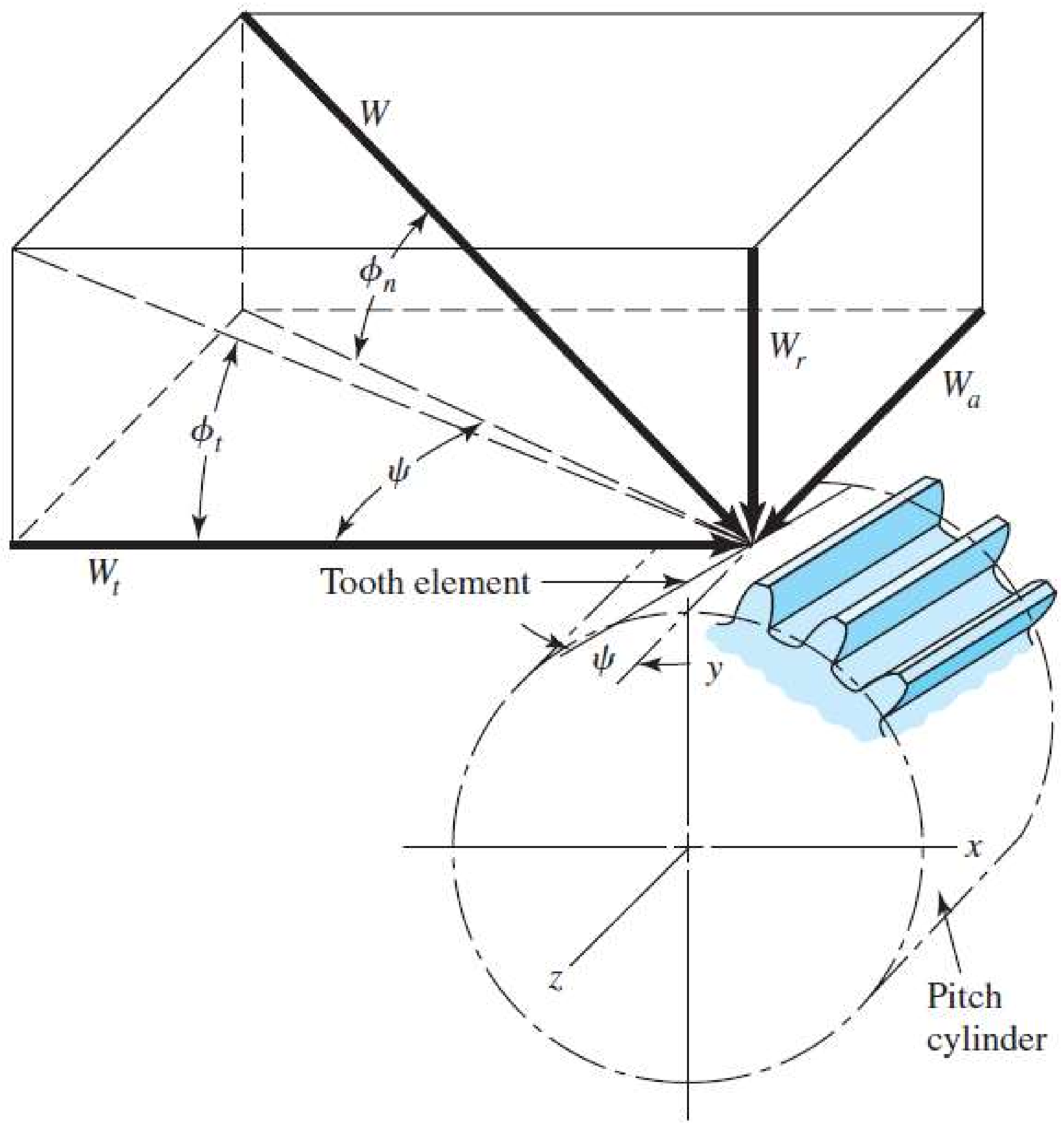
$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$

$$W_r = W_t \tan \phi_t$$

$$W_a = W_t \tan \psi$$

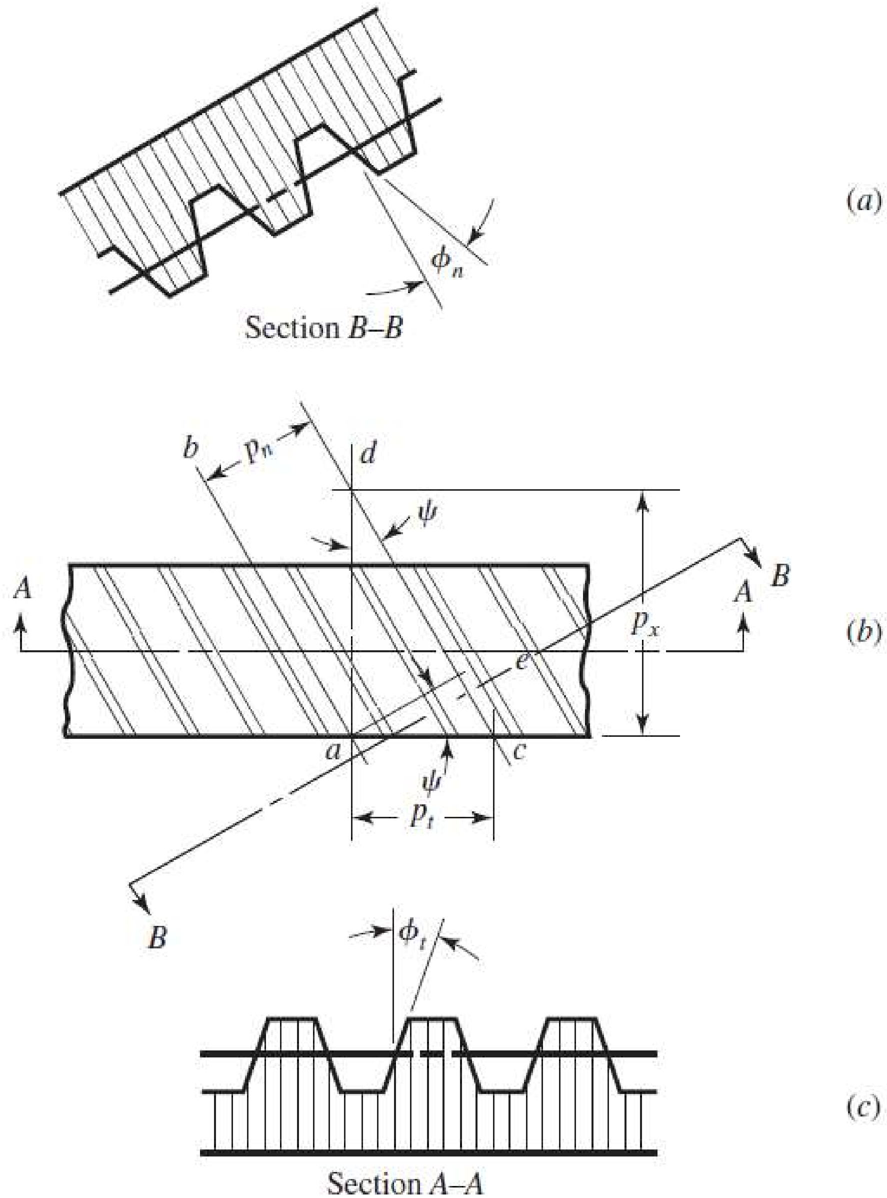
$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$



13.10 Parallel Helical Gears

Figure 13–22

Nomenclature of helical gears.

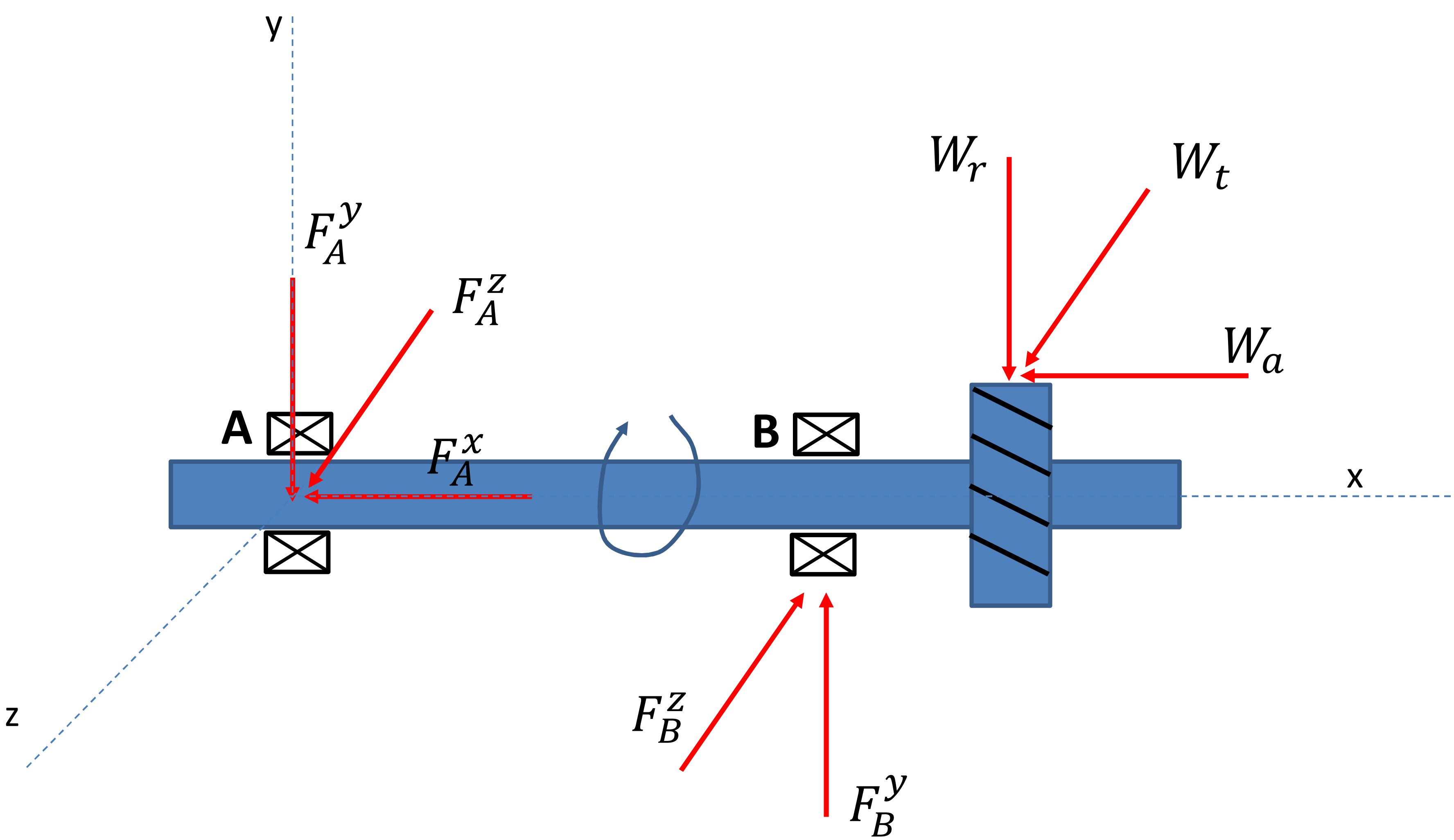


$$p_n = p_t \cos \psi$$

$$P_n = \frac{P_t}{\cos \psi}$$

$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t}$$

13.10 Parallel Helical Gears – Example 13-9



14.17 Safety Factor S_F – *in Bending*

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$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.5 Geometry Factor J (Y)

20° normal pressure angle and face-contact ratios of $m_F = 2$ or greater.

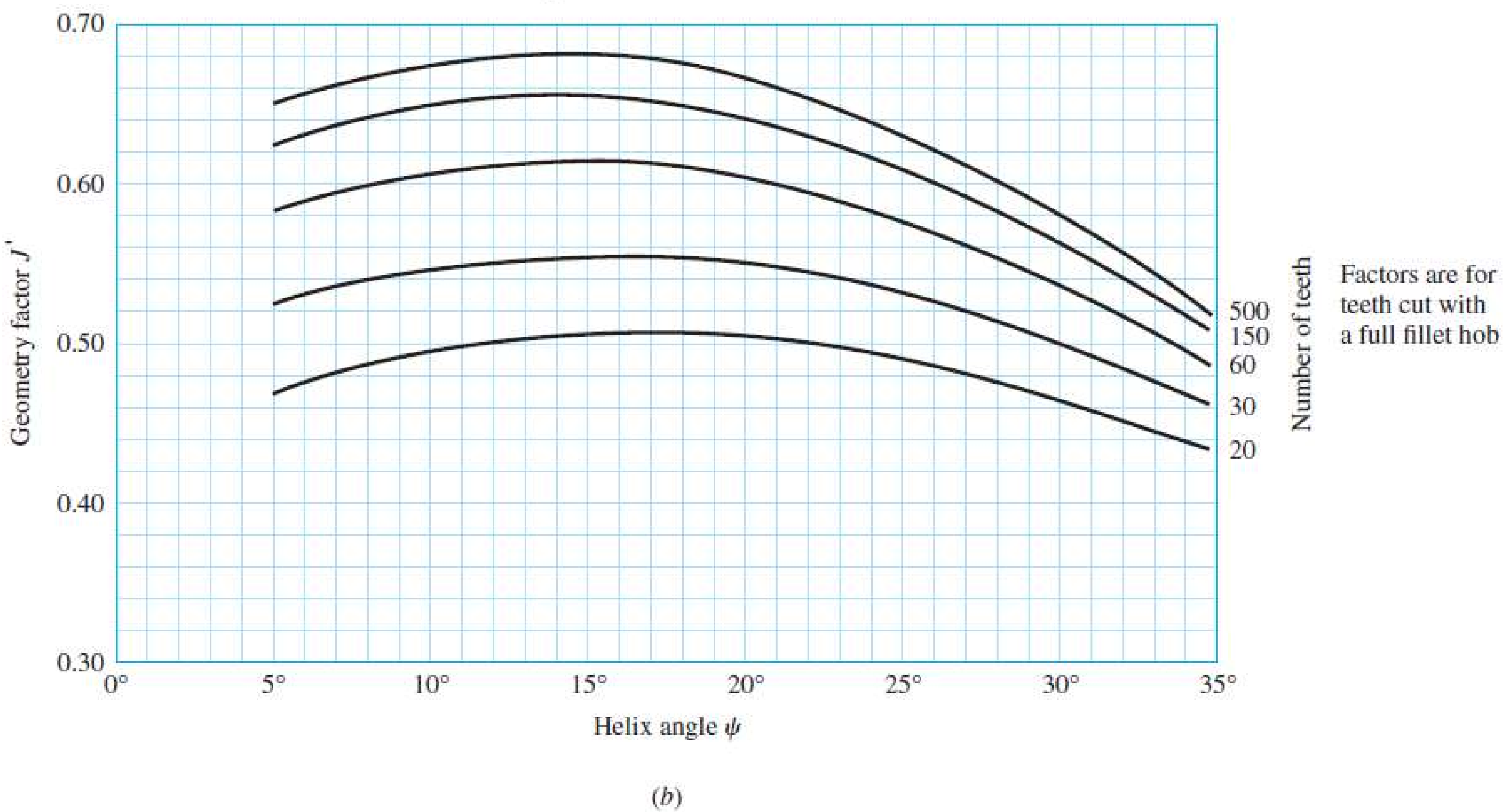


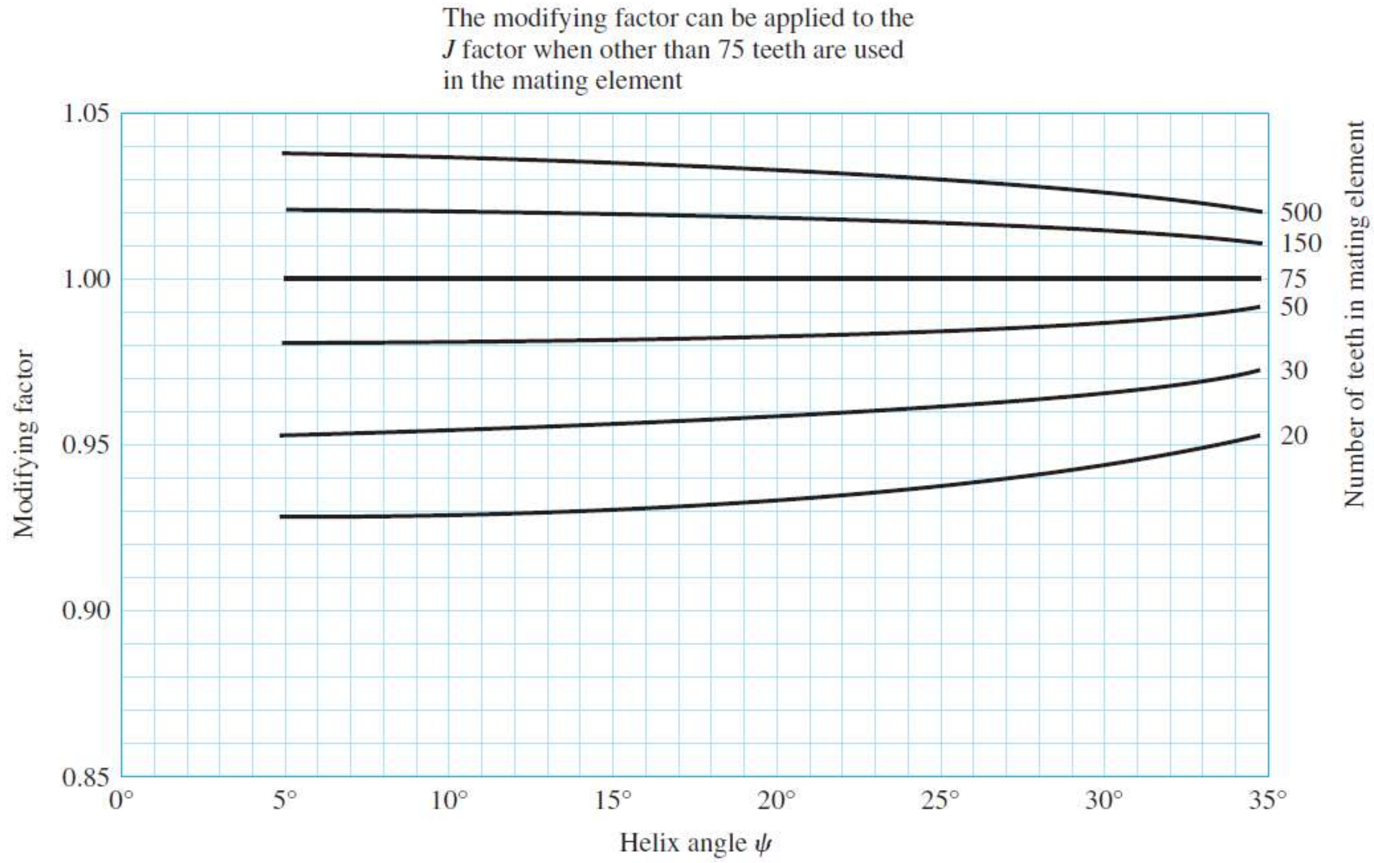
Figure 14–7

Helical-gear geometry factors J'. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

14.5 Geometry Factor J (Y)

Figure 14–8

J′-factor multipliers for use with Fig. 14–7 to find *J*.
Source: The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



14.17 Safety Factor S_H – *in Wear (Surface)*

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$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

Contact (Pitting) Stress

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Allowable contact Stress

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}} \qquad (14\text{–}42)$$

14.5 Geometry Factor I

Pitting resistance geometry factor OR surface strength geometry factor:

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G+1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G-1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

Where m_N =1 for spur gear

$$p_N = p_n \cos\phi_n \qquad\qquad m_N = \frac{p_N}{0.95Z}$$

$$Z = [(r_P + a)^2 - r_{bP}^2]^{1/2} + [(r_G + a)^2 - r_{bG}^2]^{1/2} - (r_P + r_G)\sin\phi_t \qquad (14\text{–}25)$$

14.18 Analysis

SPUR GEAR BENDING
Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

1 [or Eq. (a), Sec. 14–10]; p. 751

Eq. (14–30); p. 751

Eq. (14–40); p. 756

Fig. 14–6; p. 745

Eq. (14–27); p. 748

Table below

$_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 740, 741

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Fig. 14–14; p. 755

Table 14–10, Eq. (14–38); pp. 756, 755

1 if $T < 250°\mathrm{F}$

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

14.18 Analysis

SPUR GEAR WEAR
Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

- C_p: Eq. (14–13), Table 14–8; pp. 736, 749
- K_o: Table below
- K_v: Eq. (14–27); p. 748
- K_s: 1 [or Eq. (a), Sec. 14–10]; p. 751
- K_m: Eq. (14–30); p. 751
- C_f: 1
- I: Eq. (14–23); p. 747

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

- S_c: ${}_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 743, 744
- Z_N: Fig. 14–15; p. 755
- C_H: Section 14–12, gear only; pp. 753, 754
- K_T: 1 if $T < 250°\mathrm{F}$
- K_R: Table 14–10, Eq. (14–38); pp. 756, 755

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

- C_H: Gear only

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

14.18 Analysis – Example 14.4

A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4 hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, $J_P = 0.30$, $J_G = 0.40$, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558N^{-0.0178}$, $Z_N = 1.4488N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

(*a*) Find the factor of safety of the gears in bending.

(*b*) Find the factor of safety of the gears in wear.

(*c*) By examining the factors of safety, identify the threat to each gear and to the mesh.

14.18 Analysis – Example 14.5

A 17-tooth 20° normal pitch-angle helical pinion with a right-hand helix angle of 30° rotates at 1800 rev/min when transmitting 4 hp to a 52-tooth helical gear. The normal diametral pitch is 10 teeth/in, the face width is 1.5 in, and the set has a quality number of 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion and gear are made from a through-hardened steel with surface and core hardnesses of 240 Brinell on the pinion and surface and core hardnesses of 200 Brinell on the gear. The transmission is smooth, connecting an electric motor and a centrifugal pump. Assume a pinion life of 10^8 cycles and a reliability of 0.9 and use the upper curves in Figs. 14–14 and 14–15.

(*a*) Find the factors of safety of the gears in bending.

(*b*) Find the factors of safety of the gears in wear.

(*c*) By examining the factors of safety identify the threat to each gear and to the mesh.

14.17 Safety Factor S_F – *in Bending*

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$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.5 Geometry Factor J (Y)

20° normal pressure angle and face-contact ratios of $m_F = 2$ or greater.

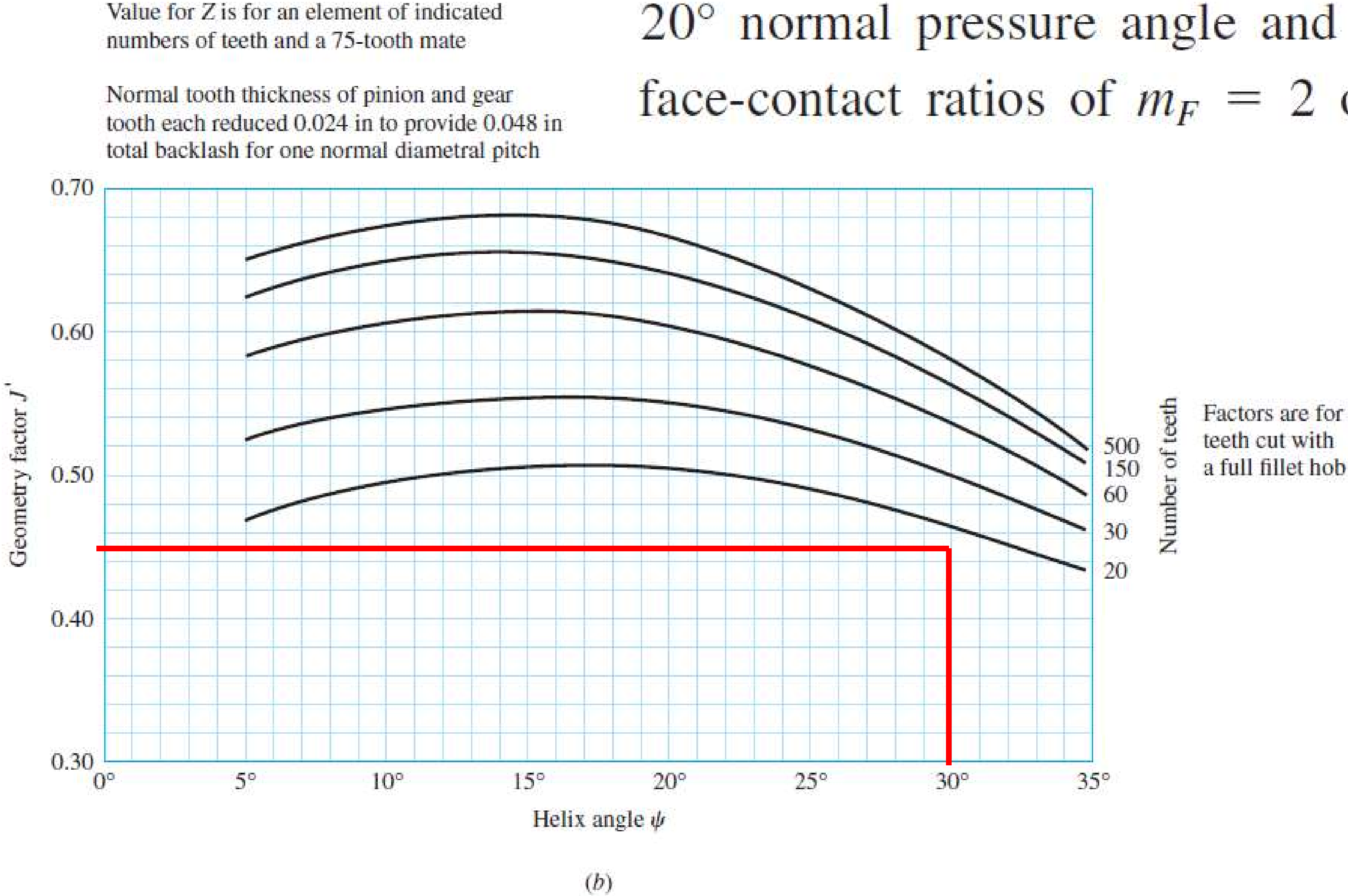

Figure 14–7

Helical-gear geometry factors J'. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

14.5 Geometry Factor J (Y)

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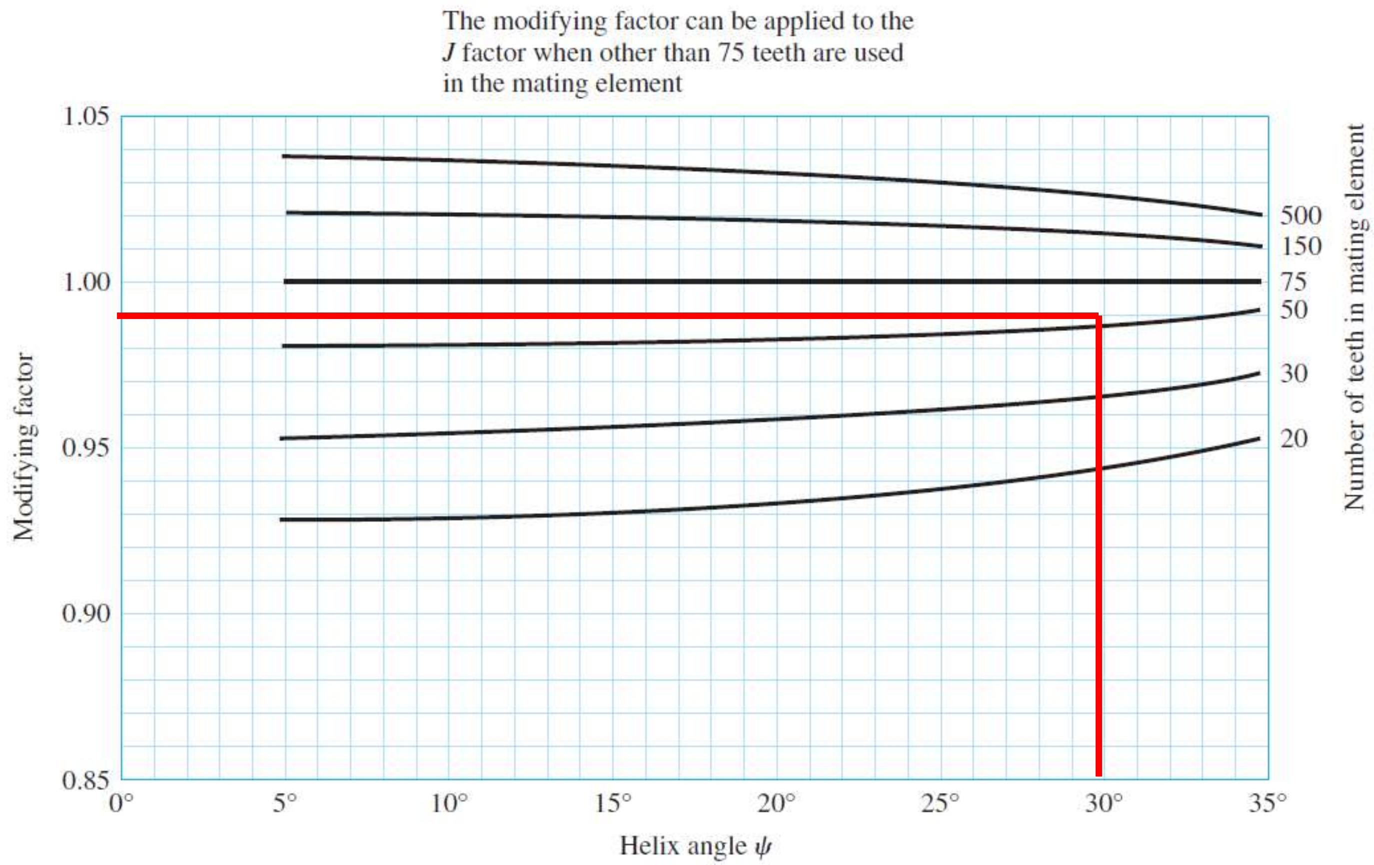
Figure 14–8

J′-factor multipliers for use with Fig. 14–7 to find *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.16 Rim-Thickness Factor K_B

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Rim-Thickness factor (K_B) is a modifying factor adjusts the estimated bending stress for the thin-rimmed gear

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases}$$

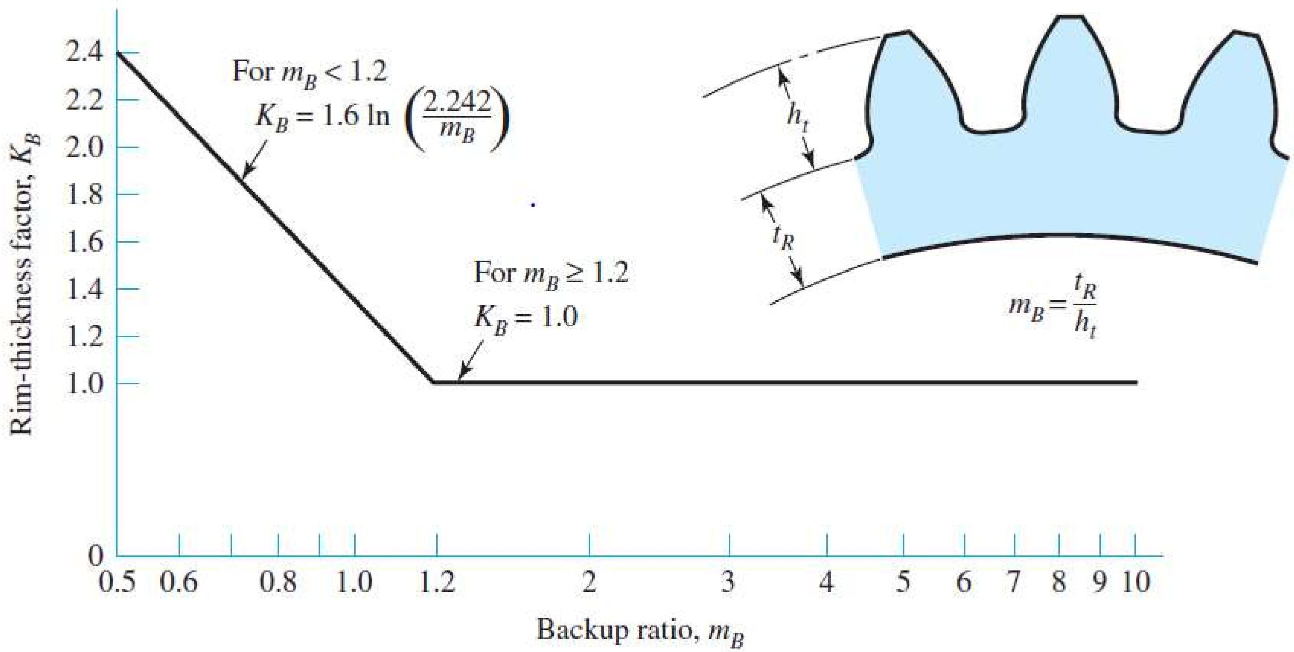


Figure 14–16

Rim-thickness factor K_B. *(ANSI/AGMA 2001-D04.)*

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.11 Load Distribution $K_M (K_H)$

The load-distribution factor modified the stress equations to reflect non-uniform distribution of load across the line of contact.

K_M is applied under the following conditions:

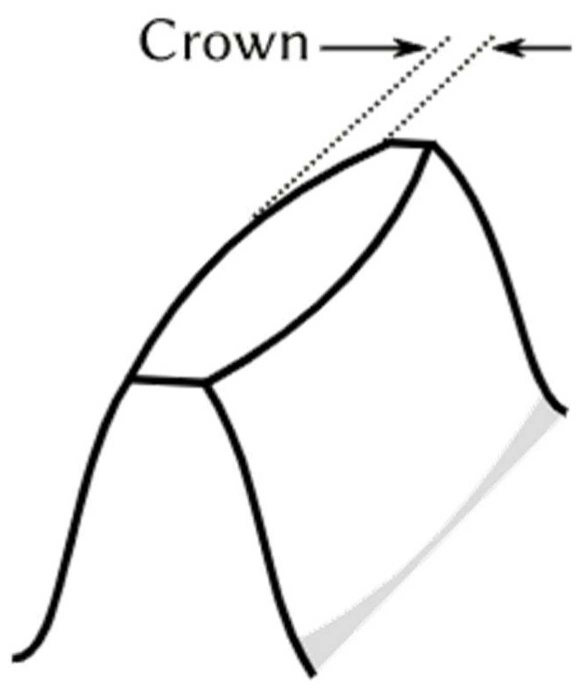
- Net face width to pinion pitch diameter ratio $F/d_p <= 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$



14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d_P} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d_P} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d_P} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

Note that for values of $F/(10d_P) < 0.05$, $F/(10d_P) = 0.05$ is used.

F = 1.5 in

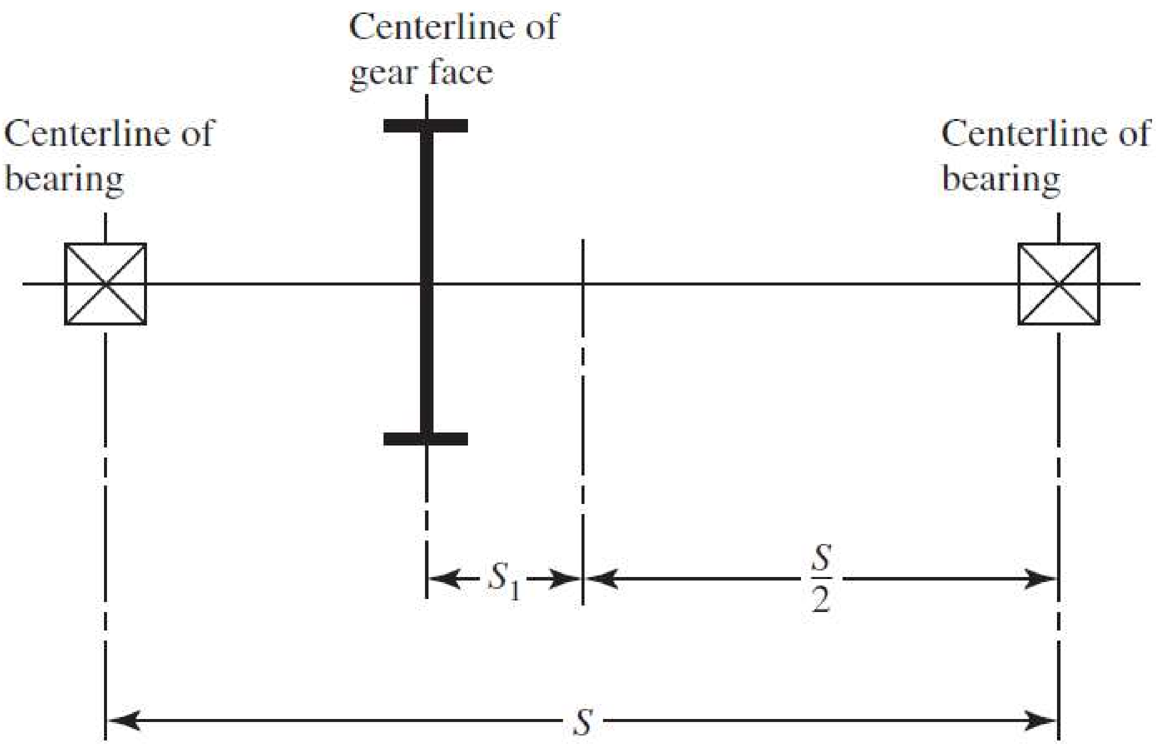
14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf} C_{pm} + C_{ma} C_e)$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases}$$

Figure 14–10

Definition of distances S and S_1 used in evaluating C_{pm}, Eq. (14–33). *(ANSI/AGMA 2001-D04.)*



14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{ma} = A + BF + CF^2$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{ma} = A + BF + CF^2$$

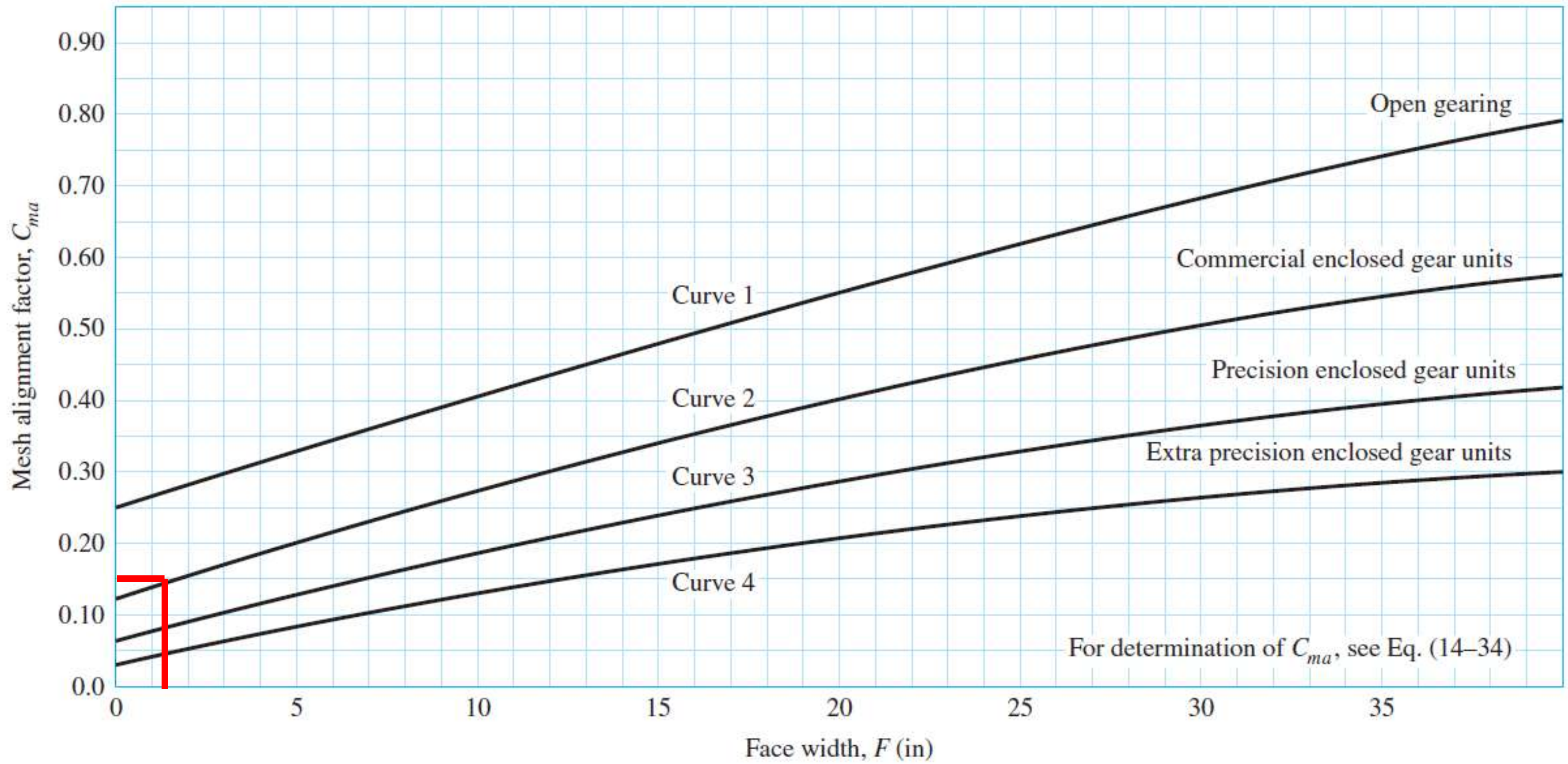


Figure 14–11

Mesh alignment factor C_{ma}. Curve-fit equations in Table 14–9. *(ANSI/AGMA 2001-D04.)*

14.11 Load Distribution $K_M (K_H)$

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$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

14.4 AGMA Strength Equations

Instead of using the term *strength,* AGMA uses data termed ***allowable stress numbers (Gear Bending Strength)***

The equation for the allowable bending stress is

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad (14\text{–}17)$$

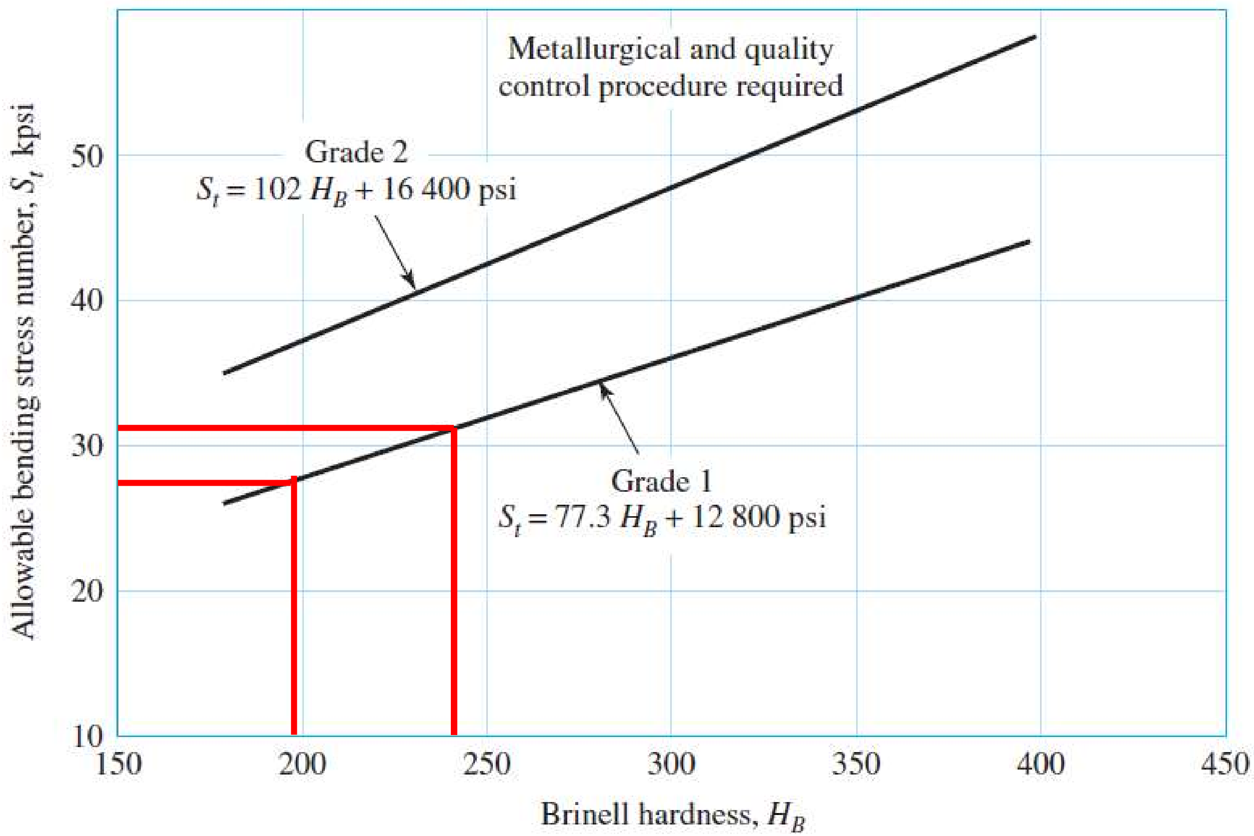
where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress-cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

14.4 AGMA Strength Equations

Figure 14–2

Allowable bending stress number for through-hardened steels, S_t. The SI equations are: $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



14.4 AGMA Strength Equations

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Instead of using the term *strength,* AGMA uses data termed ***allowable stress numbers (Gear Bending Strength)***

The equation for the allowable bending stress is

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–17}$$

where for U.S. customary units (SI units),

S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
Y_N is the stress-cycle factor for bending stress
K_T (Y_θ) are the temperature factors
K_R (Y_Z) are the reliability factors
S_F is the AGMA factor of safety, a stress ratio

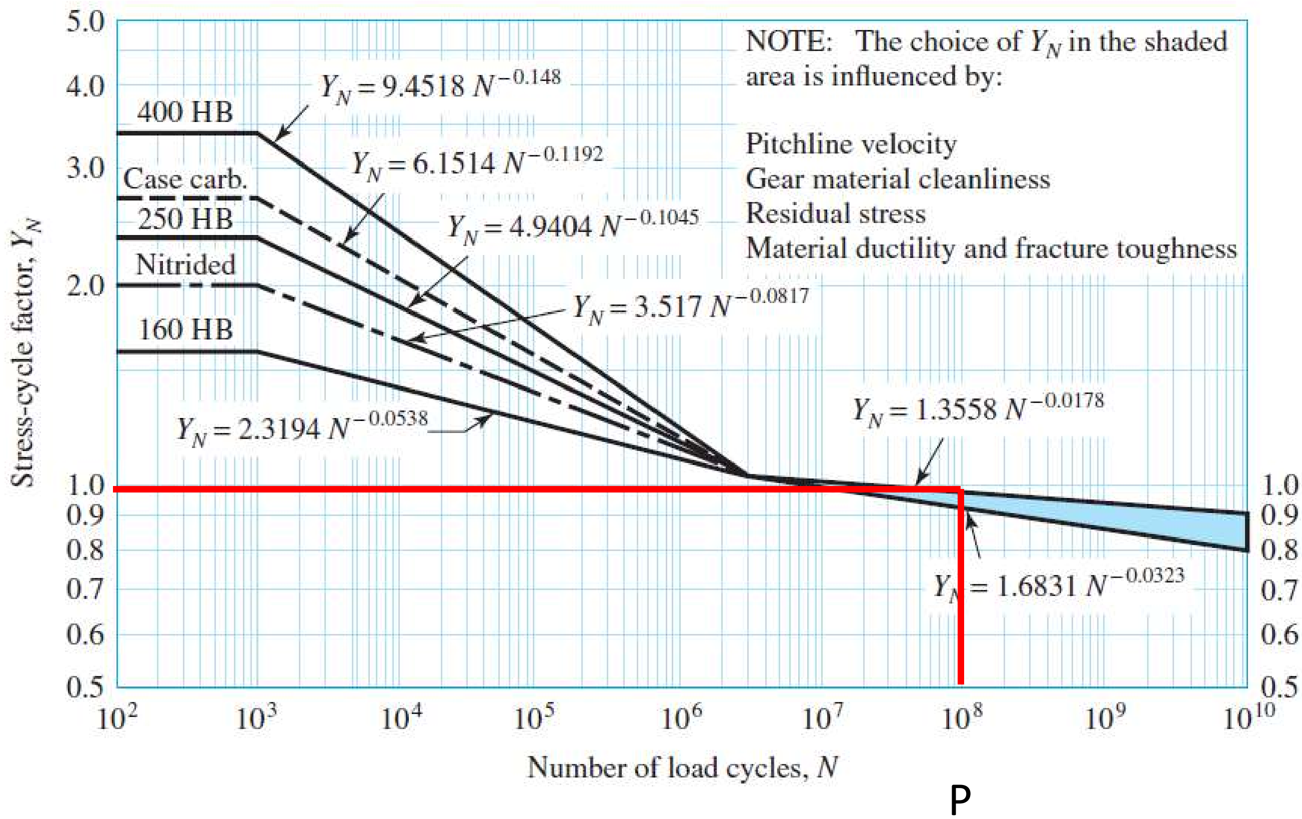
14.13 Stress-Cycle Factors Y_N

The purpose of the stress-cycle factors Y_N is to modify the gear strength for lives other than 10^7 cycles

Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

Gear Cycle = 10^8 \m$_g$



14.14 Reliability Factors K_R

Table 14–10

Reliability Factors K_R (Y_Z)

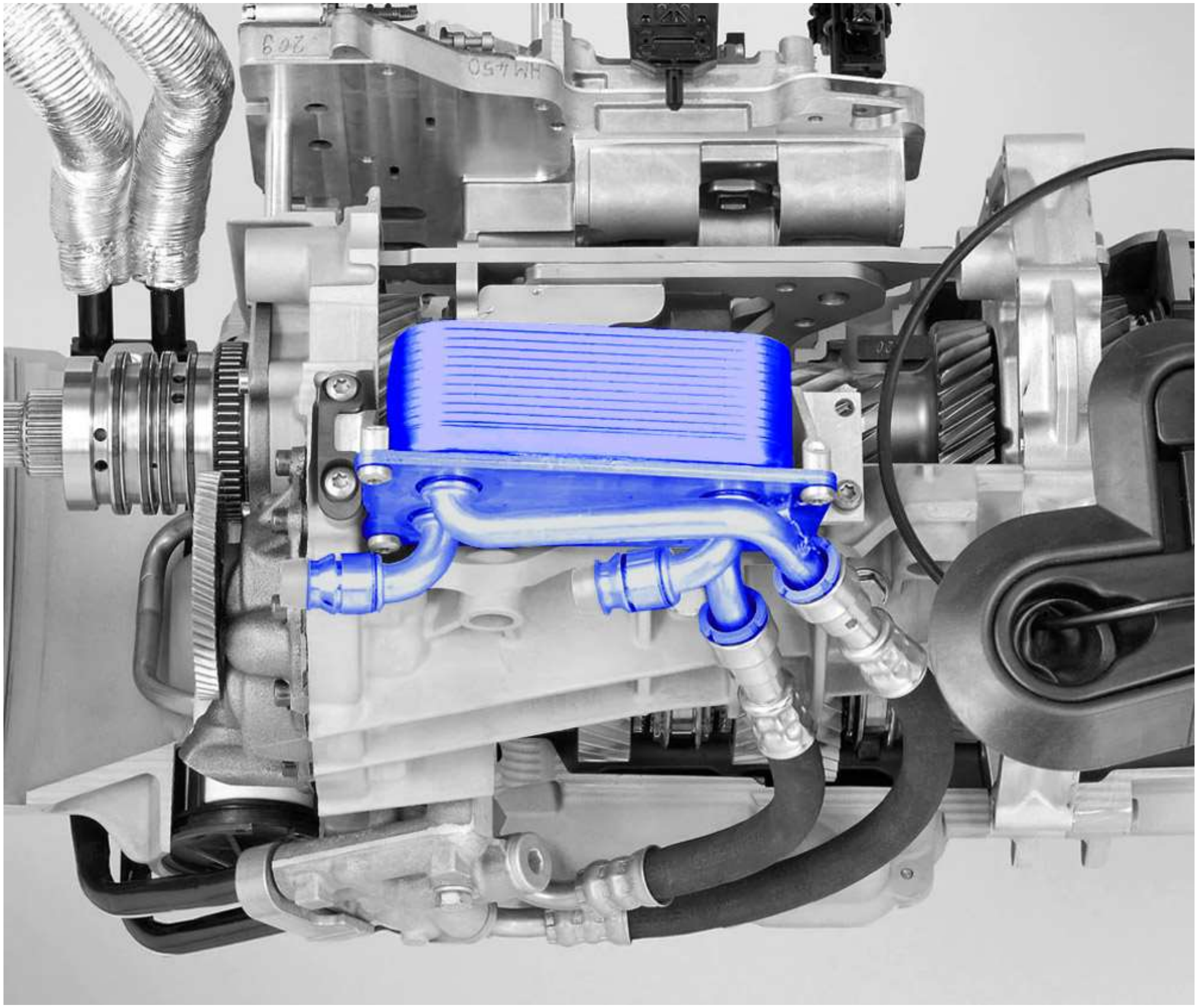
Source: ANSI/AGMA 2001-D04.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is:

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \le R \le 0.9999 \end{cases} \qquad (14\text{–}38)$$

14.15 Temperature Factors K_T



14.19 Design of a Gear Mesh

A useful decision set for spur gear and helical gears includes:

$$\left.\begin{array}{l} \bullet\ \text{Function: load, speed, reliability, life, } K_o \\ \bullet\ \text{Unquantifiable risk: design factor } n_d \\ \bullet\ \text{Tooth system: } \phi,\ \psi\text{, addendum, dedendum, root fillet radius} \\ \bullet\ \text{Gear ratio } m_G,\ N_p,\ N_G \\ \bullet\ \text{Quality number } Q_v \end{array}\right\}\ \text{a priori decisions}$$

$$\left.\begin{array}{l} \bullet\ \text{Diametral pitch } P_d \\ \bullet\ \text{Face width } F \\ \bullet\ \text{Pinion material, core hardness, case hardness} \\ \bullet\ \text{Gear material, core hardness, case hardness} \end{array}\right\}\ \text{design decisions}$$

14.19 Design of a Gear Mesh – Example 14-8

Design a 4:1 spur-gear reduction for a 100-hp, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Make the a priori decisions:

- Function: 100 hp, 1120 rev/min, $R = 0.95$, $N = 10^9$ cycles, $K_o = 1$
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20°$
- Tooth count: $N_P = 18$ teeth, $N_G = 72$ teeth (no interference, Sec. 13–7, p. 677)
- Quality number: $Q_v = 6$, use grade 1 material
- Assume $m_B \geq 1.2$ in Eq. (14–40), $K_B = 1$

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.5 Geometry Factor J (Y)

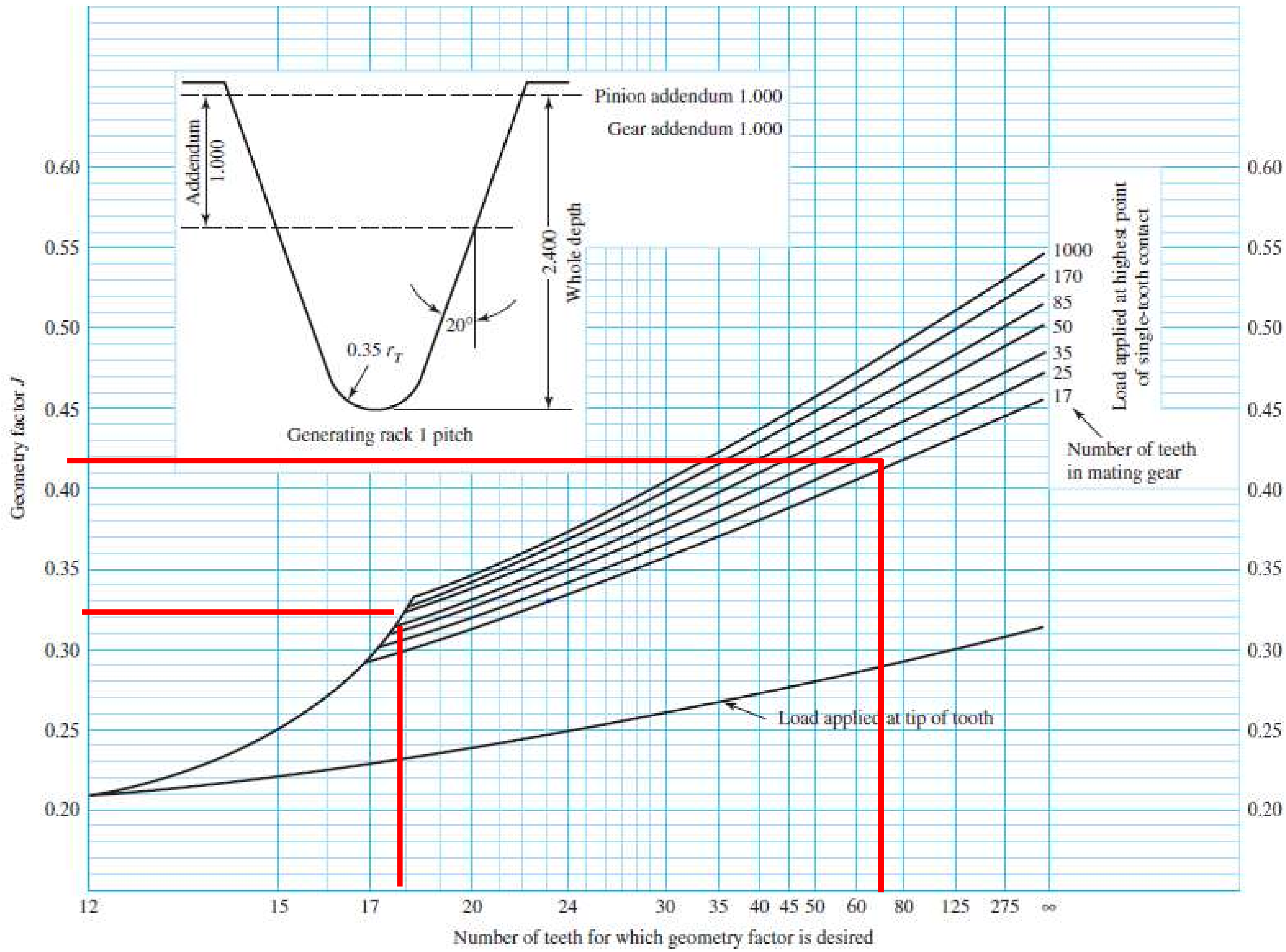


Figure 14–6

Spur-gear geometry factors *J*. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.19 Design of a Gear Mesh – Example 14-8

Design a 4:1 spur-gear reduction for a 100-hp, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Make the a priori decisions:

- Function: 100 hp, 1120 rev/min, $R = 0.95$, $N = 10^9$ cycles, $K_o = 1$
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20°$
- Tooth count: $N_P = 18$ teeth, $N_G = 72$ teeth (no interference, Sec. 13–7, p. 677)
- Quality number: $Q_v = 6$, use grade 1 material
- Assume $m_B \geq 1.2$ in Eq. (14–40), $K_B = 1$

Select a trial diametral pitch of $P_d = 4$ teeth/in.

$$W^t = \frac{33\,000H}{V} = \frac{33\,000(100)}{1319} = 2502 \text{ lbf} \qquad V = \frac{\pi d_P n_P}{12} = \frac{\pi(4.5)1120}{12} = 1319 \text{ ft/mir}$$

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.8 Overload Factor K_0

Design a 4:1 spur-gear reduction for a 100-hp, three-phase squirrel-cage induction motor running at 1120 rev/min. The load is smooth, providing a reliability of 0.95 at 10^9 revolutions of the pinion. Gearing space is meager. Use Nitralloy 135M, grade 1 material to keep the gear size small. The gears are heat-treated first then nitrided.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	$\boxed{1.00}$	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.7 Dynamic Factor K_v

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$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases}$$

Gearing space is meager

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$

Qv (*quality numbers*) define the tolerances for gears of various sizes manufactured to a specified accuracy.

- 3 to 7 will include most commercial quality gears.
- 8 to 12 are of precision quality.

$$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left(\frac{59.77 + \sqrt{1319}}{59.77}\right)^{0.8255} = 1.480$$

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14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.10 Size Factor K_s

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The size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

AGMA has identified and provided a symbol for size factor. Also, AGMA suggests $K_s = 1$, which makes K_s a placeholder in Eqs. (14–15) and (14–16) until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge. From Table 13–1, p. 688, $l = a + b = 2.25/P$. The tooth thickness t in Fig. 14–6 is given in Sec. 14–1, Eq. (b), as $t = \sqrt{4lx}$ where $x = 3Y/(2P)$ from Eq. (14–3). From Eq. (6–25), p. 297, the equivalent diameter d_e of a rectangular section in bending is $d_e = 0.808\sqrt{Ft}$. From Eq. (6–20), p. 296, $k_b = (d_e/0.3)^{-0.107}$. Noting that K_s is the reciprocal of k_b, we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \qquad (a)$$

14.1 The Lewis Bending Equation

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$$\sigma = \frac{W^t P}{FY} \qquad Y = \frac{2xP}{3}$$

Table 14–2

Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

14.10 Size Factor K_s

The size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

- Select a median face width for this pitch, $4\pi/P$

AGMA has identified and provided a symbol for size factor. Also, AGMA suggests $K_s = 1$, which makes K_s a placeholder in Eqs. (14–15) and (14–16) until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge. From Table 13–1, p. 688, $l = a + b = 2.25/P$. The tooth thickness t in Fig. 14–6 is given in Sec. 14–1, Eq. (*b*), as $t = \sqrt{4lx}$ where $x = 3Y/(2P)$ from Eq. (14–3). From Eq. (6–25), p. 297, the equivalent diameter d_e of a rectangular section in bending is $d_e = 0.808\sqrt{Ft}$. From Eq. (6–20), p. 296, $k_b = (d_e/0.3)^{-0.107}$. Noting that K_s is the reciprocal of k_b, we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \qquad (a)$$

14.17 Safety Factor S_F – in Bending

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$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.11 Load Distribution $K_M\,(K_H)$

The load-distribution factor modified the stress equations to reflect non-uniform distribution of load across the line of contact.

K_M is applied under the following conditions:

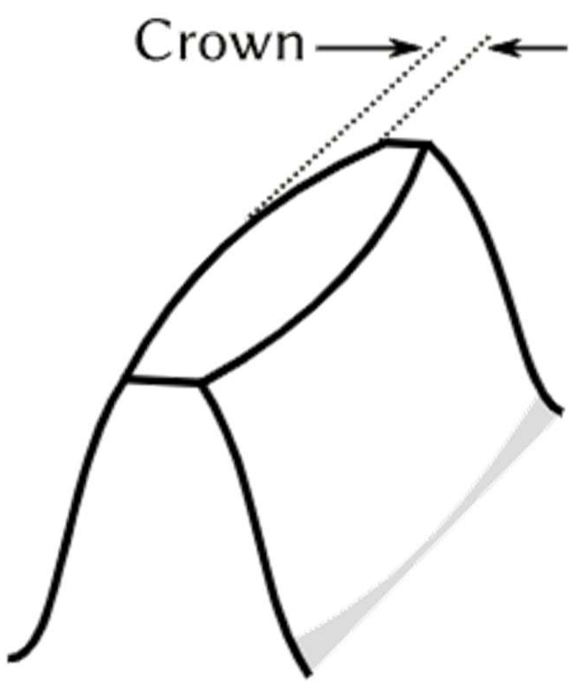
- Net face width to pinion pitch diameter ratio $F/d_p <= 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$



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14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d_P} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d_P} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d_P} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

Note that for values of $F/(10d_P) < 0.05$, $F/(10d_P) = 0.05$ is used.

$$3p \le F \le 5p$$

$$\boxed{F = 4p = 4\pi/P = \quad 4\pi/4 = 3.14 \text{ in.}}$$

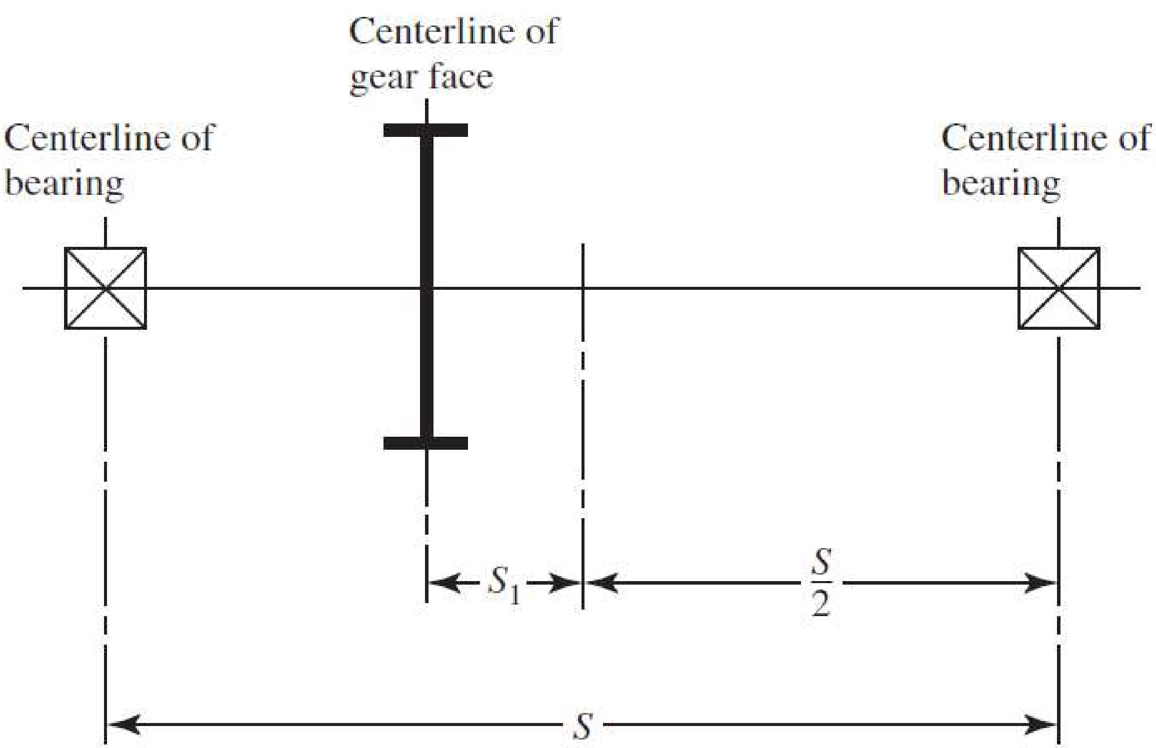
14.11 Load Distribution $K_M (K_H)$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf} C_{pm} + C_{ma} C_e)$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases}$$

Figure 14–10

Definition of distances S and S_1 used in evaluating C_{pm}, Eq. (14–33). *(ANSI/AGMA 2001-D04.)*



14.11 Load Distribution K_M (K_H)

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$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{ma} = A + BF + CF^2$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

14.11 Load Distribution $K_M (K_H)$

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$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{ma} = A + BF + CF^2$$

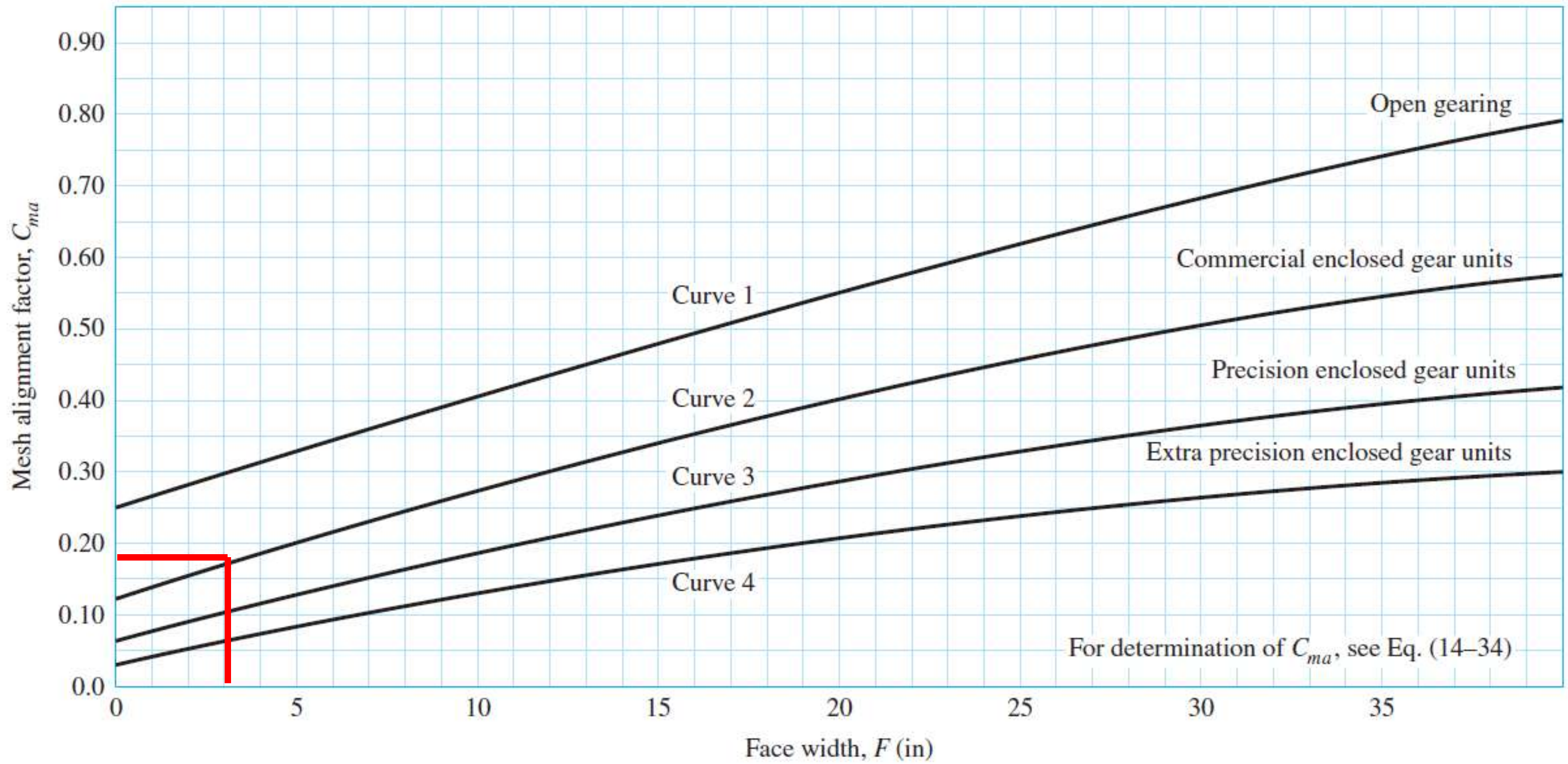

Figure 14–11

Mesh alignment factor C_{ma}. Curve-fit equations in Table 14–9. *(ANSI/AGMA 2001-D04.)*

14.11 Load Distribution $K_M (K_H)$

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$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

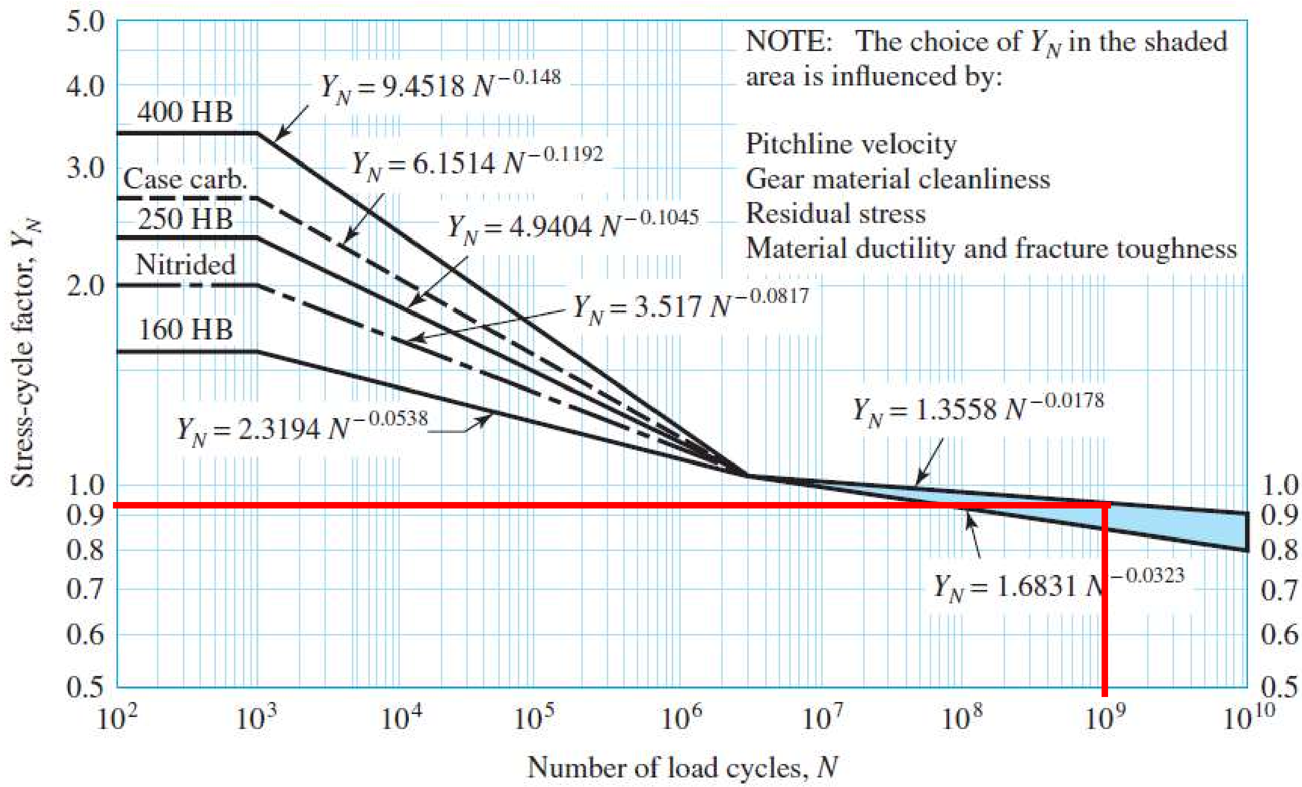
$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.13 Stress-Cycle Factors Y_N

The purpose of the stress-cycle factors Y_N is to modify the gear strength for lives other than 10^7 cycles

Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*



14.17 Safety Factor S_F – in Bending

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.14 Reliability Factors K_R

Table 14–10

Reliability Factors K_R (Y_Z)

Source: ANSI/AGMA 2001-D04.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

R = 0.95%

The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is:

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \le R \le 0.9999 \end{cases} \qquad (14\text{–}38)$$

14.17 Safety Factor S_F – in Bending

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.4 AGMA Strength Equations

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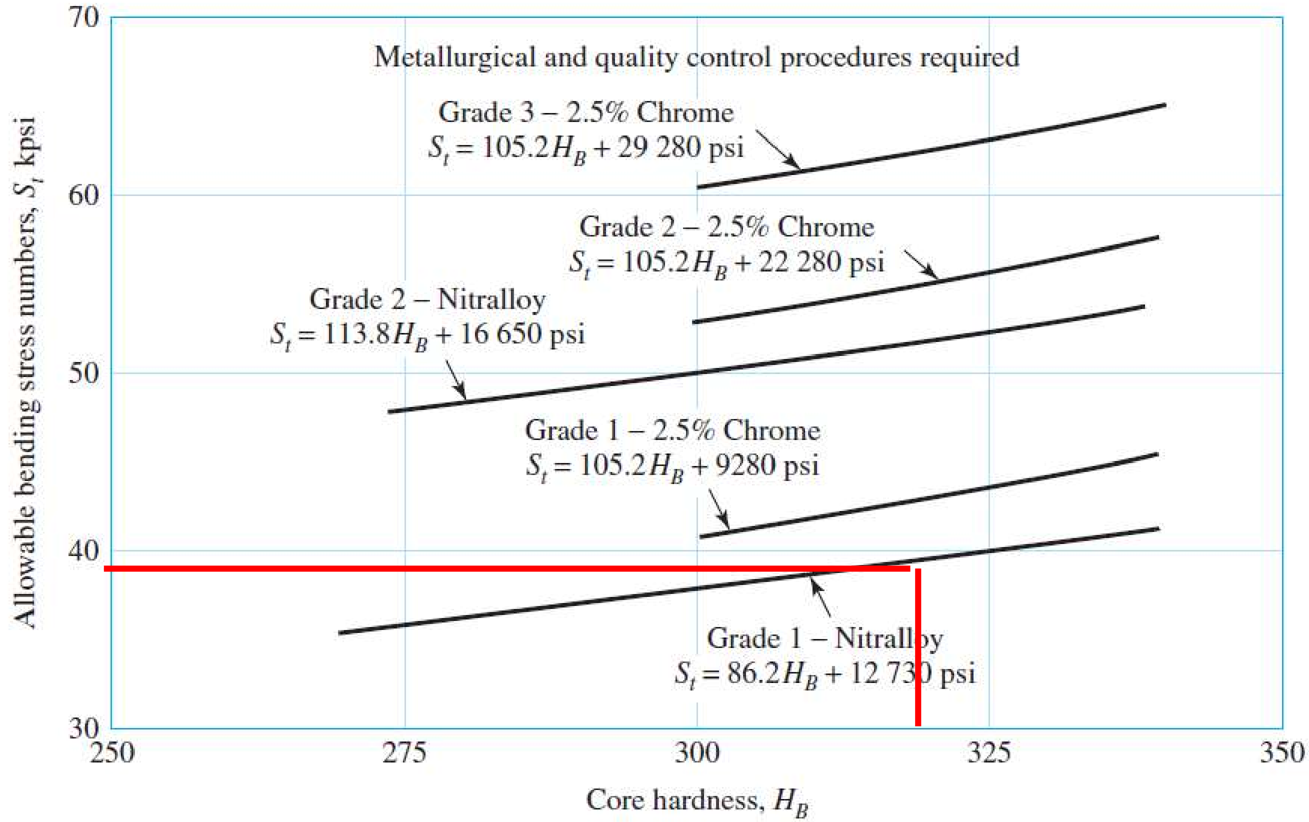
Figure 14–4

Allowable bending stress numbers for nitriding steel gears, S_t. The SI equations are: $S_t = 0.594H_B + 87.76$ MPa Nitralloy grade 1 $S_t = 0.784H_B + 114.81$ MPa Nitralloy grade 2 $S_t = 0.7255H_B + 63.89$ MPa 2.5% chrome, grade 1 $S_t = 0.7255H_B + 153.63$ MPa 2.5% chrome, grade 2 $S_t = 0.7255H_B + 201.91$ MPa 2.5% chrome, grade 3 *(Source: ANSI/AGMA 2001-D04, 2101-D04.)*



Choose a midrange
Hardness Pinion = 320

14.17 Safety Factor S_F – *in Bending*

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

Bending Stress

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Fully corrected Bending Strength

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}} \qquad (14\text{–}41)$$

14.19 Design of a Gear Mesh – Example 14-8

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Pinion Tooth Bending

$$(F)_{\text{bend}} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J_P} \frac{K_T K_R}{S_t Y_N} \tag{1}$$

Equating Eqs. (14–16) and (14–18), substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{\text{wear}}$ necessary to resist wear fatigue, we obtain

$$(F)_{\text{wear}} = \left(\frac{C_p K_T K_R}{S_c Z_N}\right)^2 n_d W^t K_o K_v K_s \frac{K_m C_f}{d_P I} \tag{2}$$

$$(F)_{\text{bend}} = 2(2502)(1)1.48(1.14)4 \frac{1.247(1)(1)0.885}{0.32(40\,310)0.938} = 3.08 \text{ in}$$

$$(F)_{\text{wear}} = \left(\frac{2300(1)(0.885)}{170\,000(0.900)}\right)^2 2(2502)1(1.48)1.14 \frac{1.247(1)}{4.5(0.1286)} = 3.22 \text{ in}$$

Make face width 3.5

14.10 Size Factor K_s

The size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

AGMA has identified and provided a symbol for size factor. Also, AGMA suggests $K_s = 1$, which makes K_s a placeholder in Eqs. (14–15) and (14–16) until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge. From Table 13–1, p. 688, $l = a + b = 2.25/P$. The tooth thickness t in Fig. 14–6 is given in Sec. 14–1, Eq. (*b*), as $t = \sqrt{4lx}$ where $x = 3Y/(2P)$ from Eq. (14–3). From Eq. (6–25), p. 297, the equivalent diameter d_e of a rectangular section in bending is $d_e = 0.808\sqrt{Ft}$. From Eq. (6–20), p. 296, $k_b = (d_e/0.3)^{-0.107}$. Noting that K_s is the reciprocal of k_b, we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \tag{a}$$

14.11 Load Distribution $K_M (K_H)$

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$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$$C_{ma} = A + BF + CF^2$$

Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-D04.

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

14.19 Design of a Gear Mesh – Example 14-8

$$(S_F)_P = \frac{40\,310(0.938)/[1(0.885)]}{19\,100} = 2.24$$

$$(S_H)_P = \frac{170\,000(0.900)/[1(0.885)]}{118\,000} = 1.465$$

By our definition of factor of safety, pinion bending is $(S_F)_P = 2.24$, and wear is $(S_H)_P^2 = (1.465)^2 = 2.15$.

$$(S_F)_G = \frac{40\,310(0.961)/[1(0.885)]}{14\,730} = 2.97$$

$$(S_H)_G = \frac{170\,000(0.929)/[1(0.885)]}{118\,000} = 1.51$$

So, for the gear in bending, $(S_F)_G = 2.97$, and wear $(S_H)_G^2 = (1.51)^2 = 2.29$.

14.19 Design of a Gear Mesh

A useful decision set for spur gear and helical gears includes:

- Function: load, speed, reliability, life, K_o
- Unquantifiable risk: design factor n_d
- Tooth system: ϕ, ψ, addendum, dedendum, root fillet radius
- Gear ratio m_G, N_p, N_G
- Quality number Q_v

a priori decisions

- Diametral pitch P_d
- Face width F
- Pinion material, core hardness, case hardness
- Gear material, core hardness, case hardness

design decisions