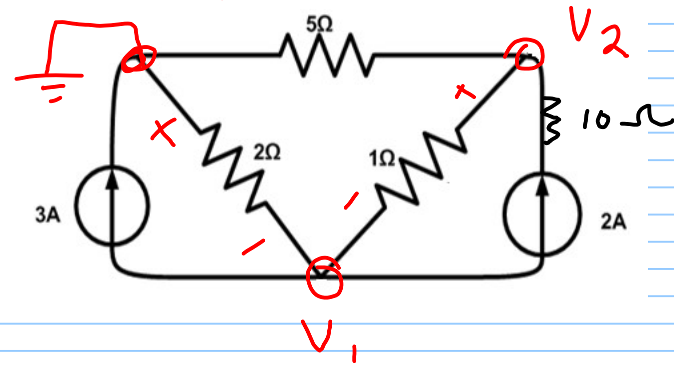
Ch 4

$\boxed{\text{Nodal method}}$

$V_2 - V_1$ 5) 2.5
-2.5 - ε



@ node V_1

All currents are out }
OR " " " IN }

$\Sigma I_{out} = 0$

OR

$\Sigma I_{IN} = 0$

I_{out} $$3 + \frac{V_1}{2} + \frac{V_1 - V_2}{1} + 2 = 0$$

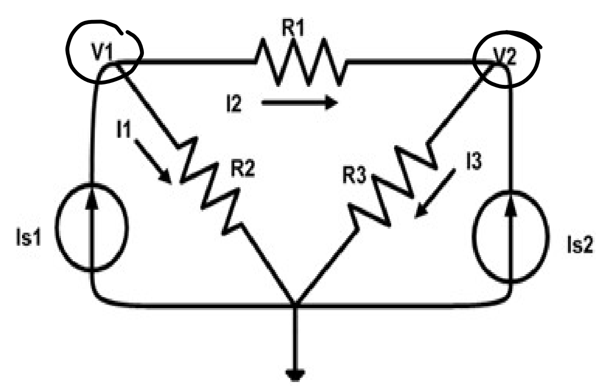
$$(1.5)\, V_1 - V_2 = -5 \quad —① $$

@ Node V_2

I_{OUT} $$-2 + \frac{V_2 - V_1}{1} + \frac{V_2}{5} = 0$$

$$-V_1 + 1.2\, V_2 = 2 \quad —②$$

$$\boxed{\begin{aligned} V_1 &= -5\ V \\ V_2 &= -2.5\ V \end{aligned}}$$



@ node V_1

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 - \left(\frac{1}{R_1}\right)V_2 = I_{s_1}$$

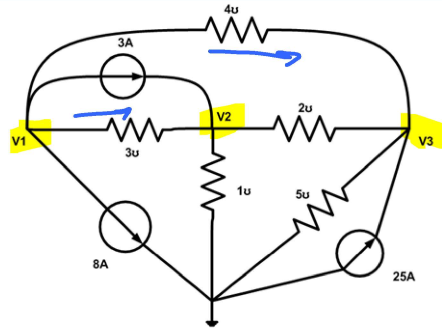
@ node V_2

$$-\left(\frac{1}{R_1}\right)V_1 + \left(\frac{1}{R_1} + \frac{1}{R_3}\right)V_2 = I_{s_2}$$

@ node 1

$$\sum I_{out} = \sum I_{in}$$

$$7V_1 - 3V_2 - 4V_3 = -11 \quad — (1)$$



$$3(V_1 - V_2) + 4(V_1 - V_3) + 3 + 8 = 0$$

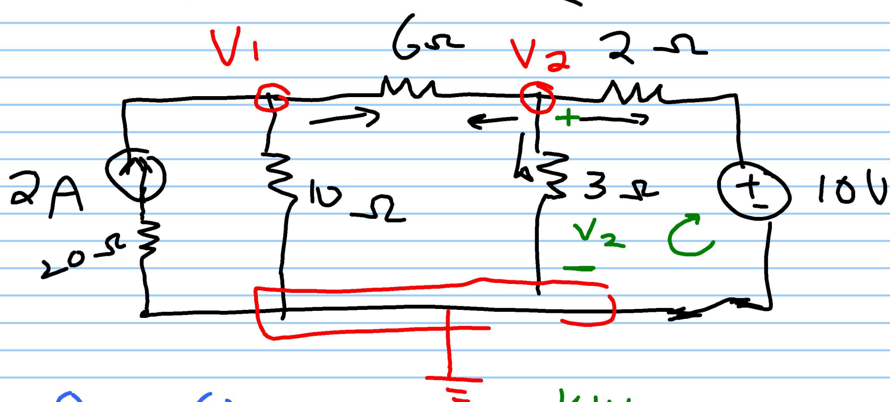
@ node V_2

$$-(3)V_1 + (3+2+1)V_2 - (2)V_3 = 3 \quad — (2)$$

@ node V_3

$$-4V_1 - 2V_2 + (5+2+4)V_3 = 25 \quad — (3)$$

Ex



$$\sum I_{out} = zero$$

node 1

$$-2 + \frac{V_1}{10} + \frac{V_1 - V_2}{6} = 0 \quad — (1)$$

رتب

KVL

$$-V_2 + 2I_x + 10 = 0$$

node 2

$$\sum I_{out} = 0$$

$$I_x = \frac{V_2 - 10}{2}$$

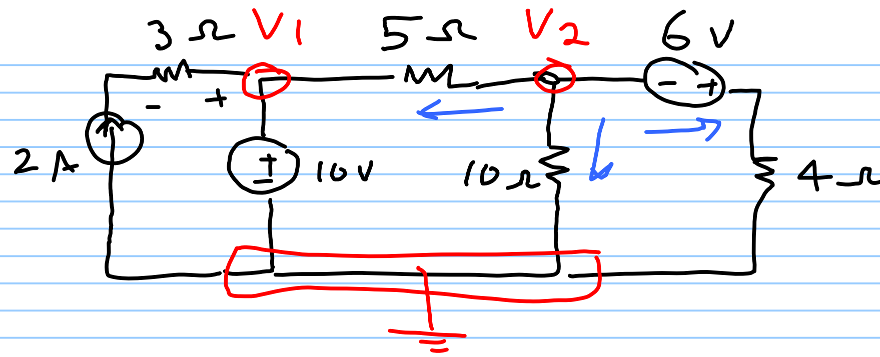
$$\frac{V_2 - V_1}{6} + \frac{V_2}{3} + \frac{V_2 - 10}{2} = 0 \quad — (2)$$

رتب

$V_1 =$ ___

$V_2 =$ ___ ✓

Ex



A V

$(-2) + (V_1 - 10)$ X

$V_1 = 10\,V$

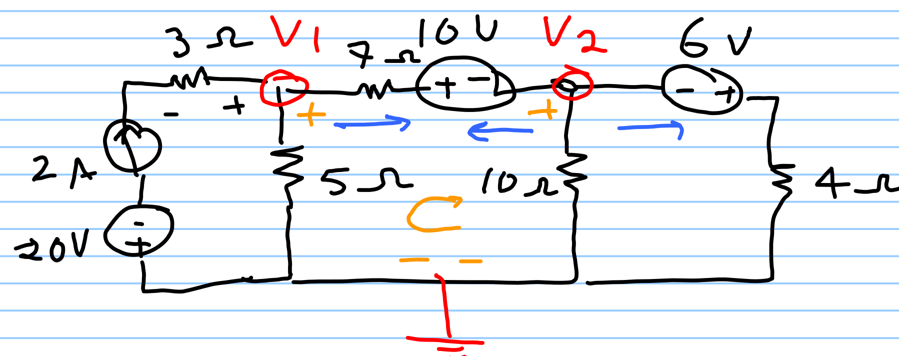
@ node V2

$$\frac{V_2 - \cancel{V_1}^{\,10}}{5} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0$$

$$0.55\,V_2 = 2 - 1.5$$

$$\boxed{V_2 = \frac{0.5}{0.55} = 0.909 \text{ Volt}}$$

Ex



$\Sigma I_{out} = \text{zero}$

@ node V1

$$-2 + \frac{V_1}{5} + \frac{V_1 - 10 - V_2}{7} = 0 \quad —(1)$$

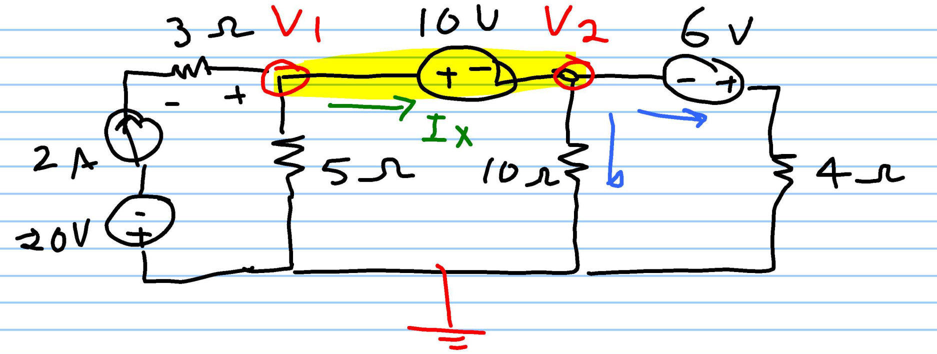
$-V_1 + 7I_x + 10 + V_2 = 0$

@ node V2

$$\frac{V_2 - V_1 + 10}{7} + \frac{V_2}{10} + \frac{V_2 + 6}{4} = 0 \quad —(2)$$

Special case

EX



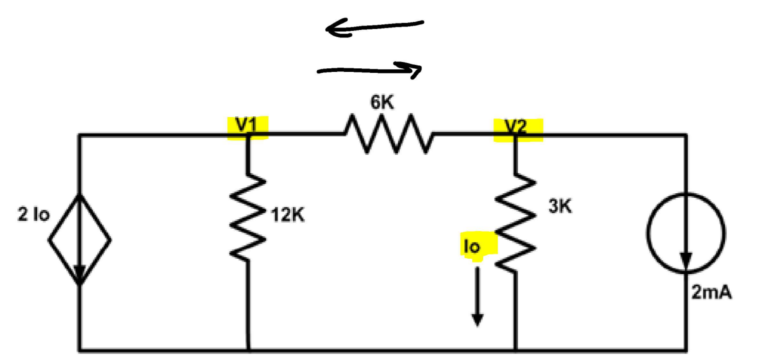
@node V_1

$$-2 + \frac{V_1}{5} + \underline{\underline{I_x}} = 0$$

$$-2 + \frac{V_1}{5} + \frac{V_2}{10} + \frac{V_2+6}{4} = 0 \quad — (1) \text{ KVL}$$

$$V_1 - V_2 = 10 \quad — (2) \text{ KVL @ supernode}$$

EX



$$I_o = \frac{V_2}{3k} \quad — (1)$$

@ node V_1

$$2 I_o + \frac{V_1}{12k} + \frac{V_1 - V_2}{6k} = 0$$

$$2\left(\frac{V_2}{3K}\right) + \frac{V_1}{12k} + \frac{V_1 - V_2}{6k} = 0 \quad — (1)$$

@ node V_2

$$\frac{V_2 - V_1}{6k} + \frac{V_2}{3K} + 2mA = 0 \quad — (2)$$

$$V_1 = -\frac{24}{5}\,V \;,\quad V_2 = \frac{12}{5}\,V$$

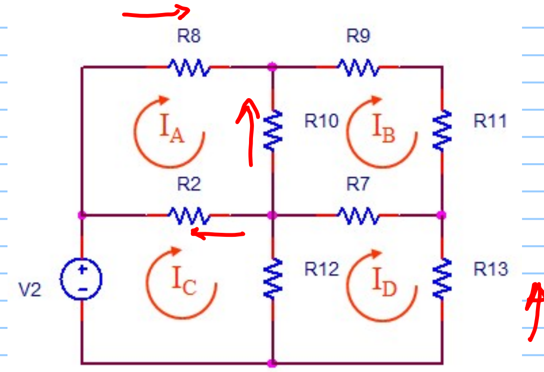
Mesh method

$I_{R8} = I_A$

$I_{R2} = I_A - I_C$

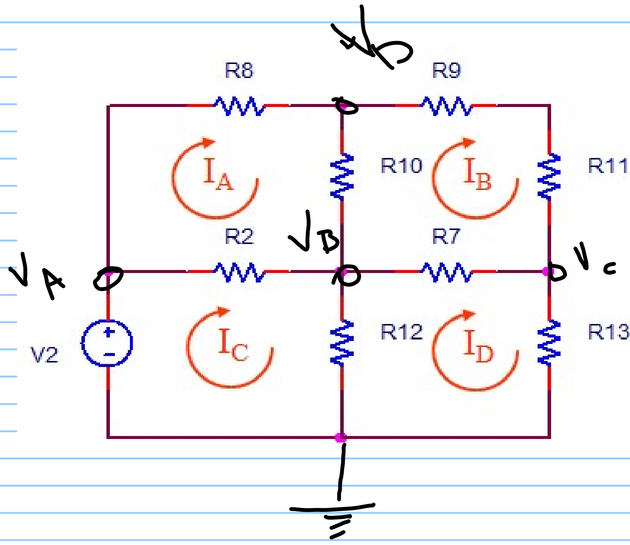
$I_{R13} = -I_D$

$I_{R10} = I_B - I_A$



4 equations

4 unknowns I_A, I_B, I_C & I_D



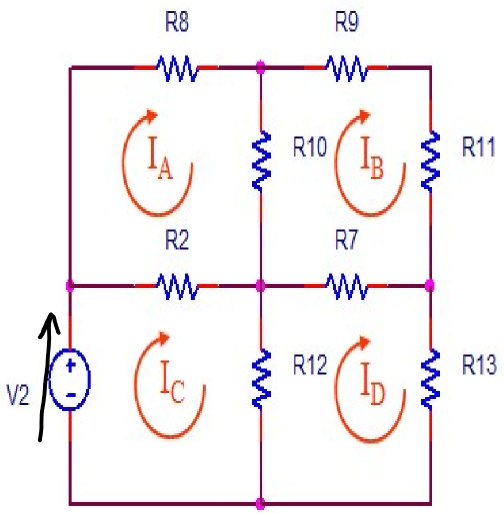
$V_A = V_2$

Nodal method

3 eqs

3 unknowns

V_B, C & D



Mesh KVL

$$R_8 I_A + R_{10}(I_A - I_B) + R_2(I_A - I_C) = 0$$

$$(R_8 + R_{10} + R_2) I_A - R_{10} I_B - R_2 I_C = 0 \quad (1)$$

$$-(R_{10}) I_A + (R_{10} + R_9 + R_{11} + R_7) I_B$$

$$-0 I_C - R_7 I_D = 0 \quad (2)$$

$$-V_2 + R_2(I_C - I_A) + R_{12}(I_C - I_D) = 0$$

$$(R_2 + R_{12}) I_C - R_2 I_A - R_{12} I_D = V_2 \quad (3)$$

$(R_{12}+R_7+R_{13})I_D - 0I_A - R_7 I_B - R_{12} I_C = 0$ — ④

EX

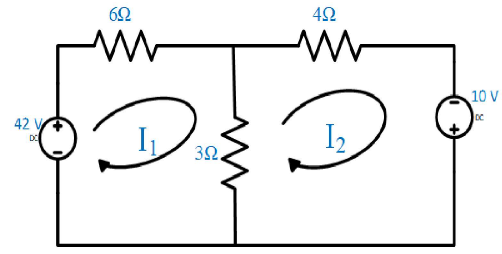


Figure3:example 1 of mesh analysis

$-42 + 6I_1 + 3(I_1 - I_2) = 0$

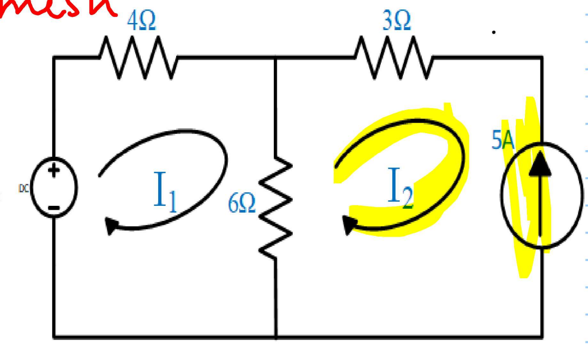
$9I_1 - 3I_2 = 42$ — ①

$-3I_1 + 7I_2 = 10$ — ②

6A, 4A

special case 1 | current source exists only in one mesh

EX



Figur6:mesh with current sourcee.

$I_2 = -5A$ ✓

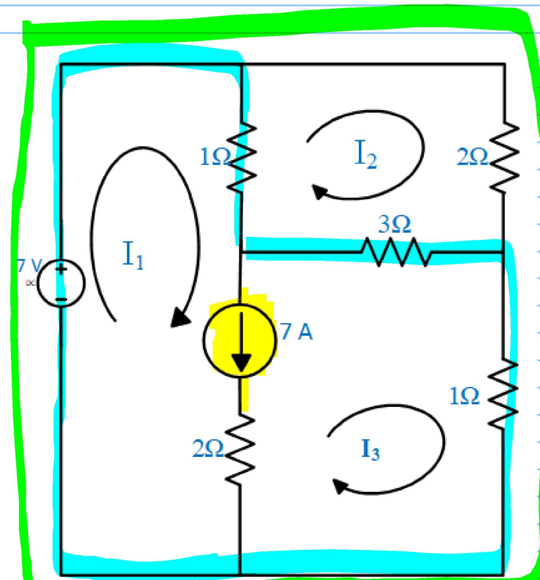
$-10 + 4I_1 + 6(I_1 - 5) = 0$

+

$I_1 = -2A$ ✓

Case 2:

Current source exists between two meshes, a **Super mesh** is obtained



Figur 7mesh with current sources

→ KVL for mesh(2)

$$(1+2+3)I_2 - (1)I_1 - (3)I_3 = 0 \quad — (1)$$

→ constraint equation

$$I_1 - I_3 = 7 \quad — (2)$$

→ super mesh equation

$$-7 + (1)(I_1 - I_2) + (3)(I_3 - I_2) + (1)I_3 = 0$$

$$I_1 - 4I_2 + 4I_3 = 7 \quad — (3)$$

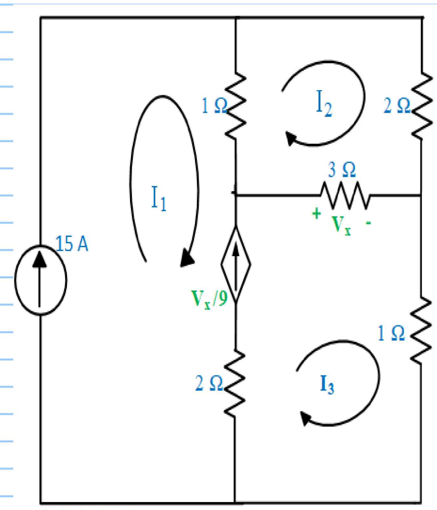
$I_3 = -7A$ X

OR

*

$$-7 + 2I_2 + I_3 = 0 \quad — (3)$$

EX) Mesh Analysis with dependent sources



Figur9mesh with dependant current source.

→ $V_x = (3)(I_3 - I_2)$

$\boxed{I_1 = 15A}$ ✓

→ constraint equation

$$\frac{V_x}{9} = I_3 - I_1$$

$$\frac{3}{9}(I_3 - I_2) = I_3 - 15$$

$$\frac{3}{9}I_2 + \frac{2}{3}I_3 = 15$$

$$\boxed{I_2 + 2I_3 = 45} \quad — (1)$$

$$(1)(I_2 - 15) + (2)I_2 + (3)(I_2 - I_3) = 0$$

$$\boxed{6I_2 - 3I_3 = 15} \quad — (2)$$

$I_2 = 11A$ ✓

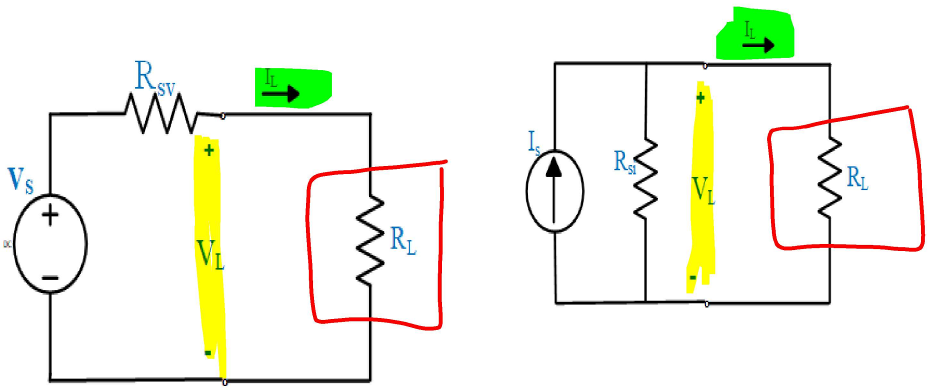
$I_3 = 17A$ ✓

Node or mesh: How to choose?

- Use the one with fewer equations.
- Use the method you like best.

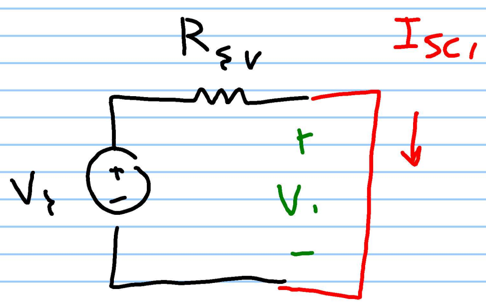
slides

Source Transformation

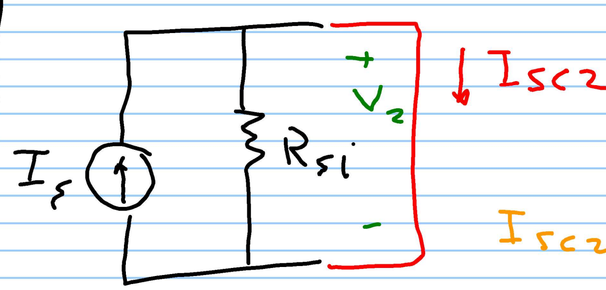


Two sources are equivalent, if each produces identical current and identical voltage in any load which is placed across its terminal.

let $R_L = \text{Zero}$ (short circuit S.C.)



$$I_{SC1} = \frac{V_S}{R_{SV}}$$



$$I_{SC2} = \frac{R_{SI}}{R_{SI} + 0} I_S$$

$$I_{SC2} = I_S$$

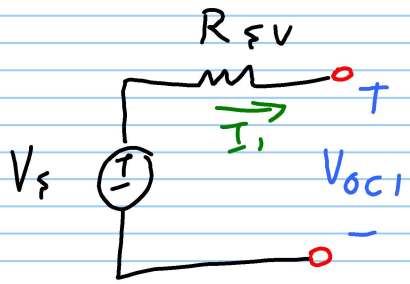
$$\boxed{V_1 = V_2 = \text{Zero}}$$

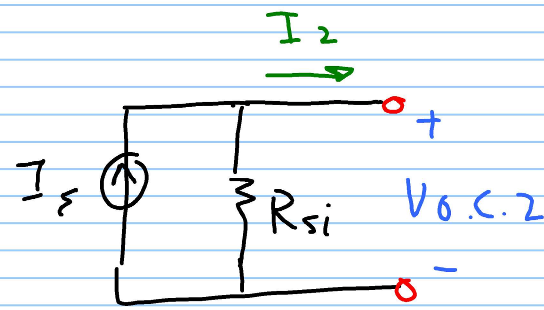
For $I_{SC1} = I_{SC2}$

$$\boxed{I_S = \frac{V_S}{R_{SV}}} \text{ --- } \oplus$$

→ let $R_L = \infty$ (open circuit)

$\boxed{I_1 = I_2 = Zero}$





$V_{oc1} = V_s$

$V_{oc2} = I_s R_{si}$

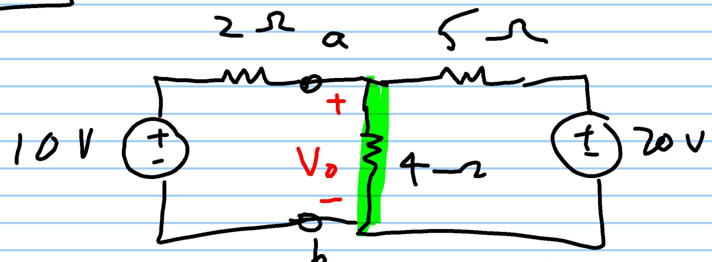
For $V_{oc1} = V_{oc2}$

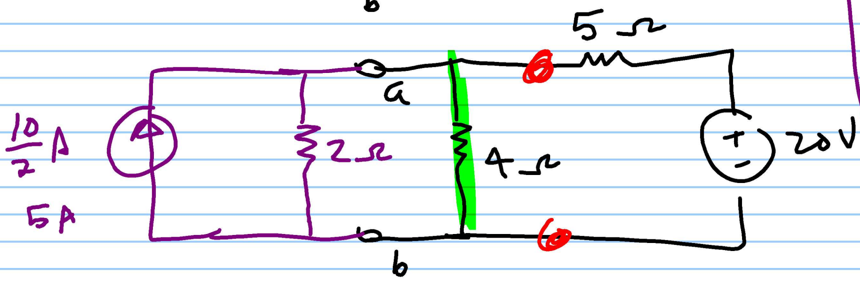
$V_s = I_s R_{si}$

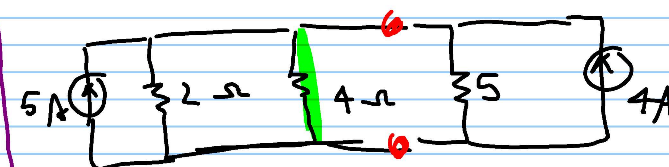
OR $\boxed{I_s = \frac{V_s}{R_{si}}}$ ** $\boxed{I_s = \frac{V_s}{R_{sv}}}$

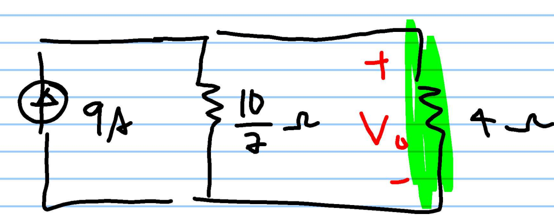
or $\boxed{R_{si} = R_{sv}}$

EX | find V_o



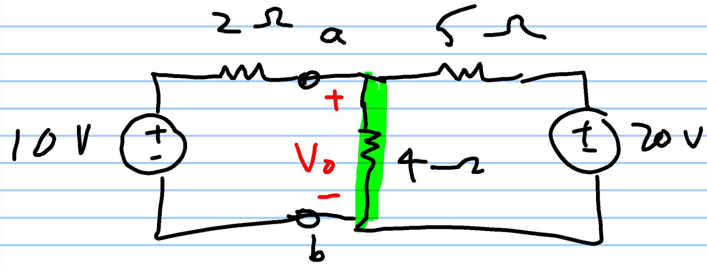






$\left(\frac{10/7}{10/7+4}\cdot 9\right)(4) = V_o$

$V_o = 9.47$ Volt.



OR

Nodal

$$\frac{V_0 - 10}{2} + \frac{V_0}{4} + \frac{V_0 - 20}{5} = 0$$

$$V_0\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{5}\right) = \frac{10}{2} + \frac{20}{5}$$

$$V_0(0.95) = 9$$

$$V_0 = 9.473 \text{ Volt.}$$

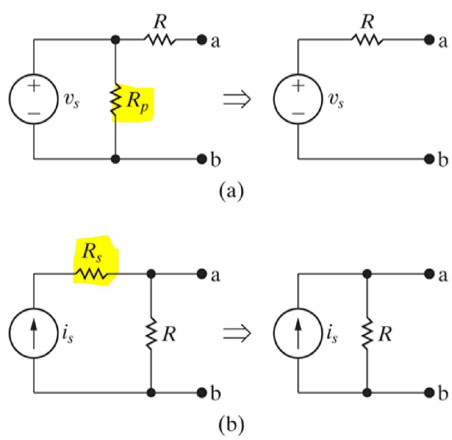


Figure: 04-39a,b

Thevenin &

Norton eq. circuit

- The two circuits depicted in Fig. 4.39(a) are equivalent with respect to terminals a,b because they produce the same voltage and current in any resistor *RL* inserted between nodes a,b.
- The same can be said for the circuits in Fig. 4.39(b).

Example: Find Vo using source transformation

neglect

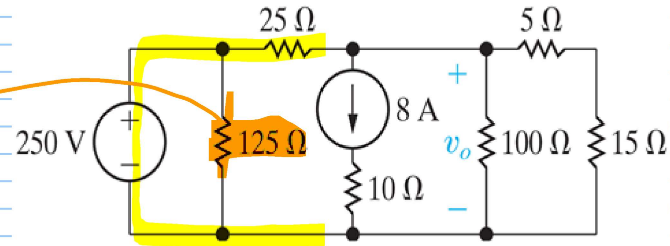
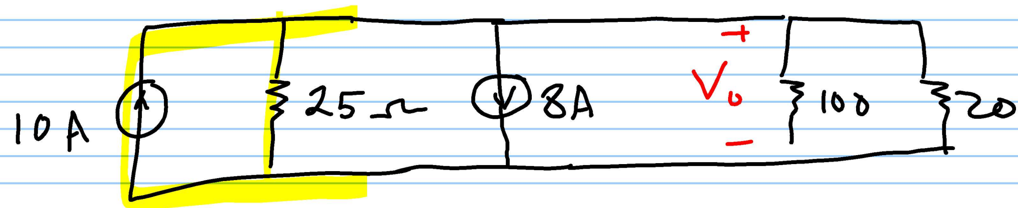
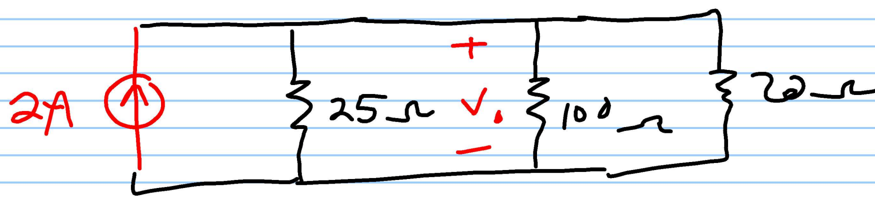
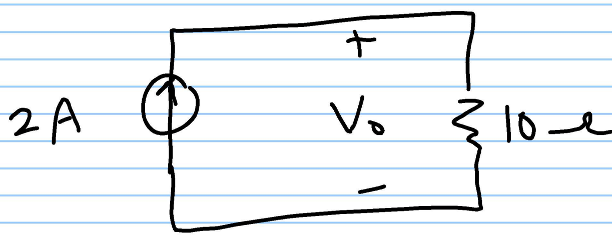


Figure: 04-40Ex4.9







$V_o = (2A)(10\,\Omega)$

$= 20$ Volt.

Dependent sources

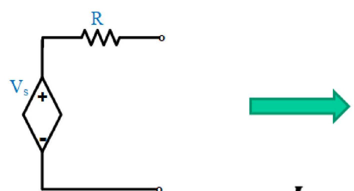


Figure 16 a

$$I_s = \frac{V_s}{R}$$

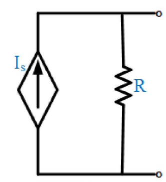


Figure 16 b

The control variable must be outside the transformation.

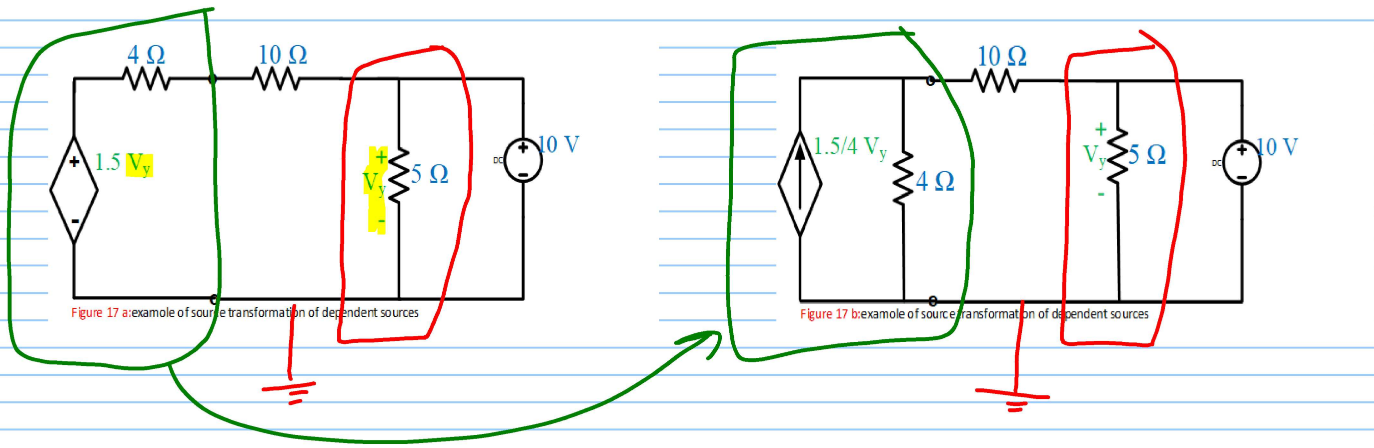
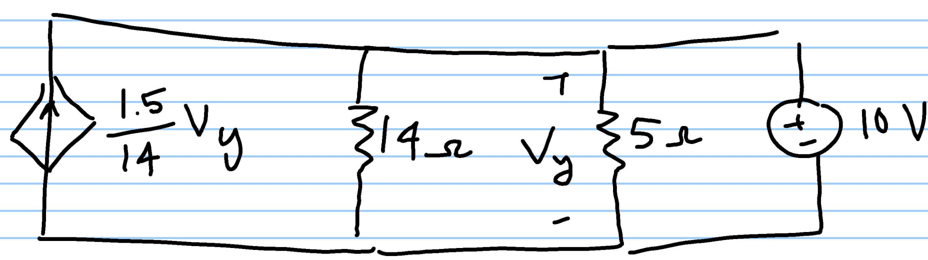


Figure 17 a:examole of source transformation of dependent sources

Figure 17 b:examole of source transformation of dependent sources

OR



The Superposition Theorem

In a linear network, the voltage across or the current through any element may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, i.e. with:

2 1) All other independent voltage sources replaced by short circuits.

3 2) All other independent current sources replaced by open circuits.

1 ➤ Dependent sources are left intact because they are controlled by circuit variables.

Steps to apply superposition principle

- Turn off all independent sources except one source. Find the output (voltage or current) due to that source using nodal, mesh…. .
- Repeat step 1 for each of the other independent sources.
- Find the total contribution by adding algebraically all contributions due to each independent sources.

EX)

find (I) using superposition

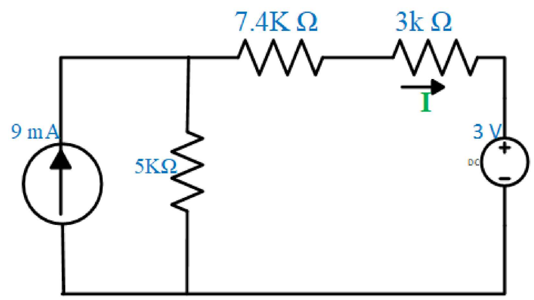
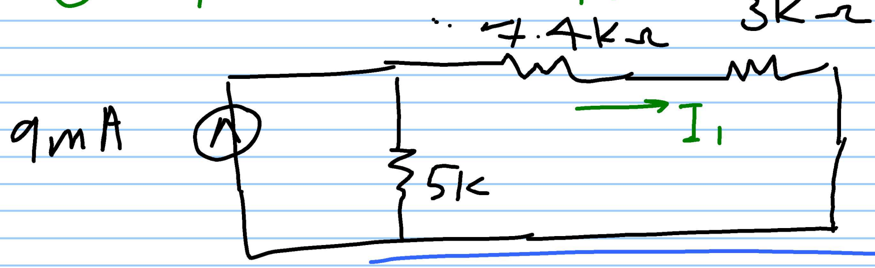


Figure15 "a"

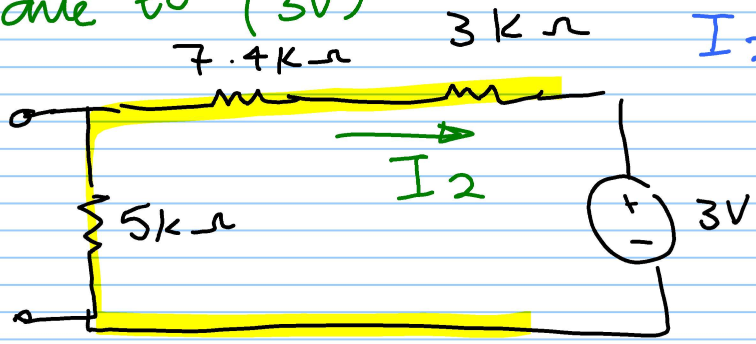
① I_1 due to (9mA)



$$I_1 = \frac{5k}{5k+7.4k+3k} \times 9m$$

$$= 2.922 \text{ mA}$$

② I_2 due to (3V)



$$I_2 = \frac{-3}{15.4k}$$

$$= -0.194 \text{ mA}$$

$\therefore\ I = I_1 + I_2$

$= 2.992 - 0.194$

$= 2.728 \text{ mA}$

Use superposition to solve for ix

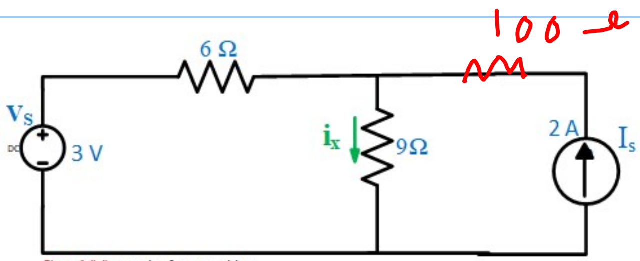
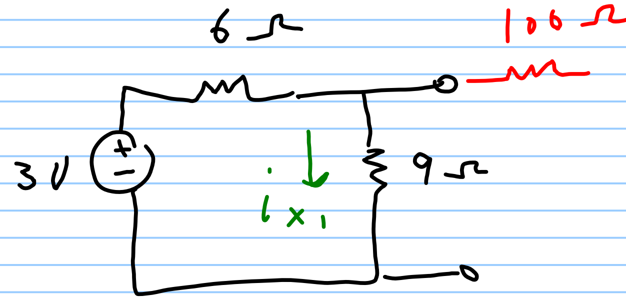
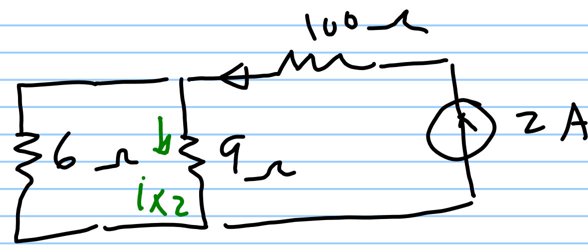


Figure1:"a":example of superposition



① i_{x_1} due to (3 V)

$$i_{x_1} = \frac{3}{9+6} = 0.2\,A$$

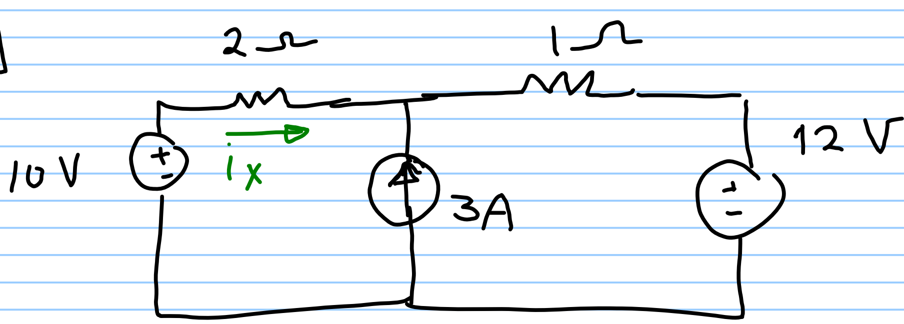


② i_{x_2} due to 2A

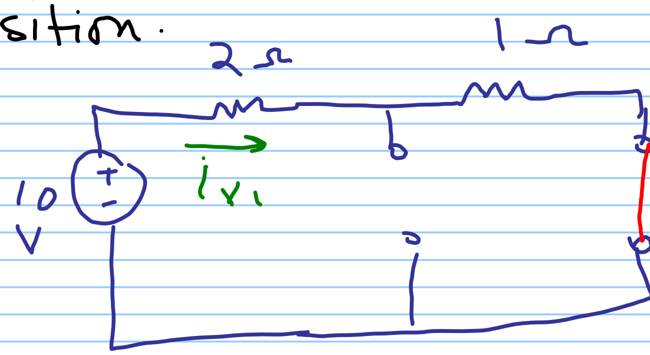
$$i_{x_2} = \frac{6}{6+9}\,2A = 0.8\,A$$

$$\text{so } i_x = i_{x_1} + i_{x_2} = 0.2 + 0.8 = \underline{1\,A}$$

EX



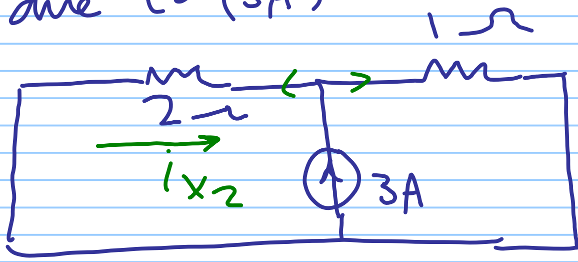
find Ix using superposition.



① i_{x_1} due to (10 V)

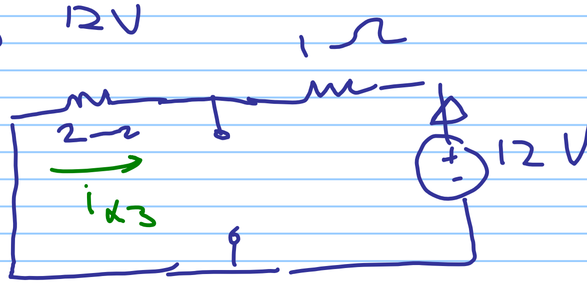
$$i_{x_1} = \frac{10}{3}\,A$$

② i_{x_2} due to (3A)



$$i_{x_2} = \frac{1}{3} 3 = -1A$$

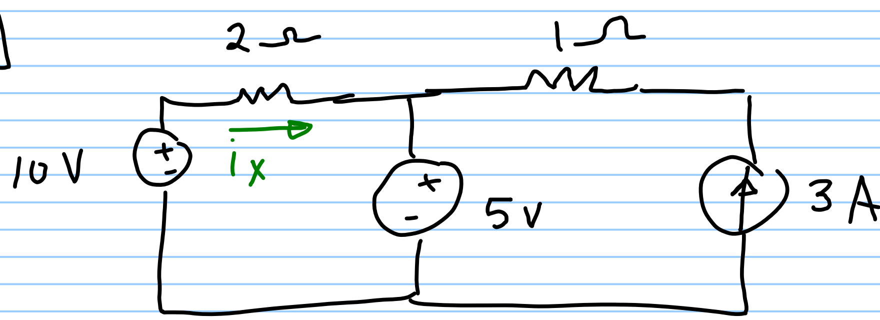
③ i_{x_3} due to 12V



$$i_{x_3} = -\frac{12}{3}$$
$$= -4A$$

$$\therefore I_x = i_{x_1} + i_{x_2} + i_{x_3}$$
$$= 3.33 - 1 - 4$$
$$= -1.666 \text{ A}$$

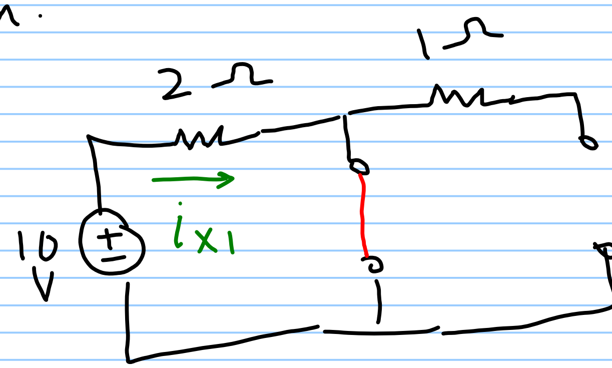
EX1



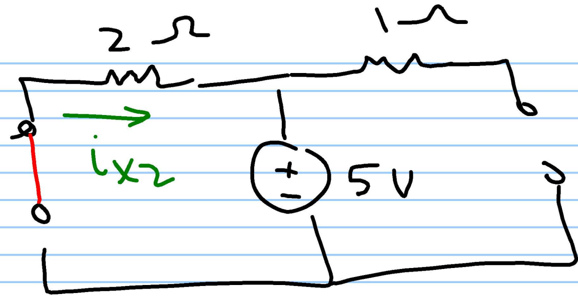
find Ix using superposition.

① i_{x_1} due to (10V)

$$i_{x_1} = \frac{10}{2} = 5A$$

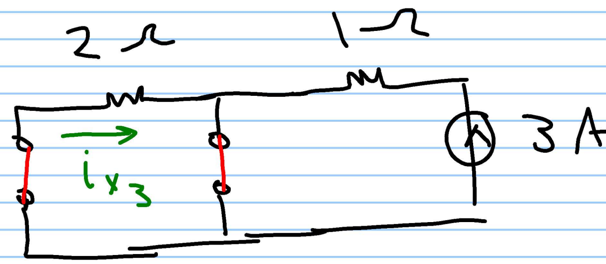


② i_{x_2} due to (5 V)



$$i_{x_2} = -\frac{5}{2} = -2.5A$$

③ i_{x_3} due to (3A)



$$i_{x_3} = \text{Zero}$$

or $i_x = i_{x_1} + i_{x_2} + i_{x_3}$

$= 5 - 2.5 + 0$

$= 2.5A$

Find *Ix* using superposition

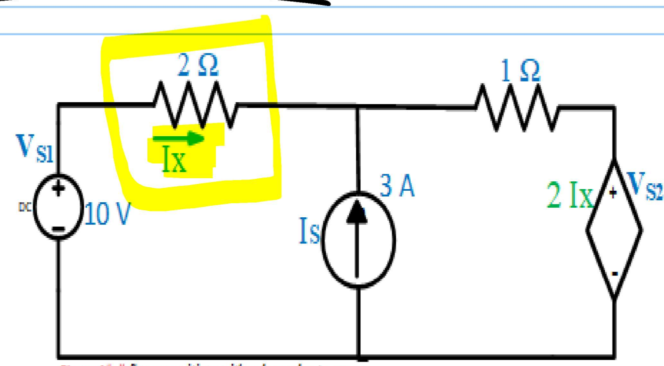


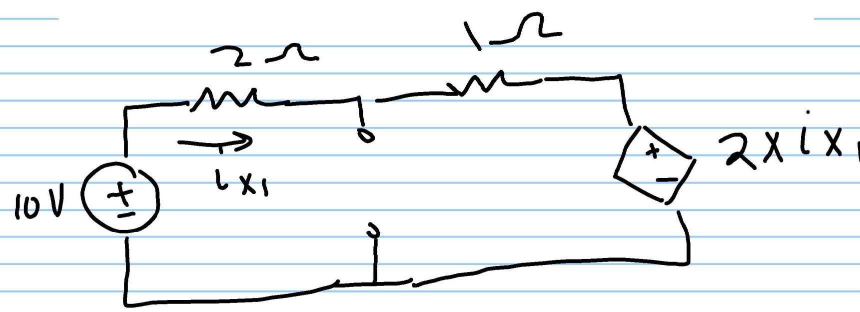
Figure1"a":Superposition with a dependent source

① i_{x_1} due to 10V

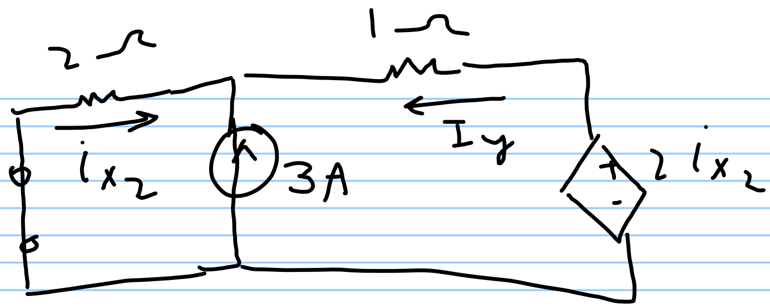
KVL

$$-10 + 3i_{x_1} + 2i_{x_1} = 0$$

$$i_{x_1} = 2A$$



(2) i_{x_2} due to (3A)



KVL

$$2I_{x_2} - I_y + 2i_{x_2} = 0$$

$$4i_{x_2} - I_y = 0$$

KCL $\quad I_{x_2} + I_y = -3$

$$5I_{x_2} = -3$$

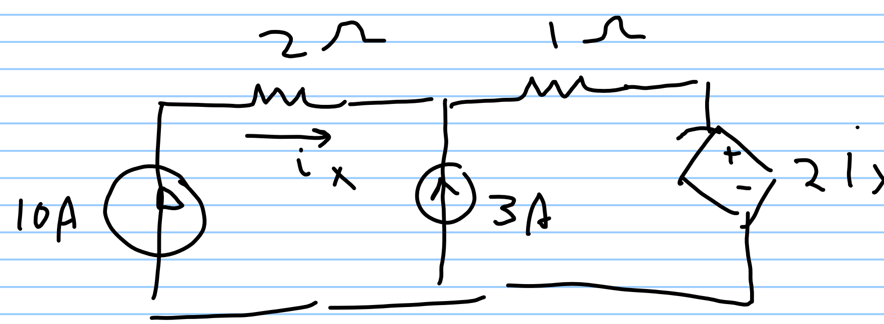
$$i_{x_2} = -3/5 = -0.6A$$

$$\therefore I_x = i_{x_1} + i_{x_2} = 2 - 0.6$$

$$= 1.4A$$

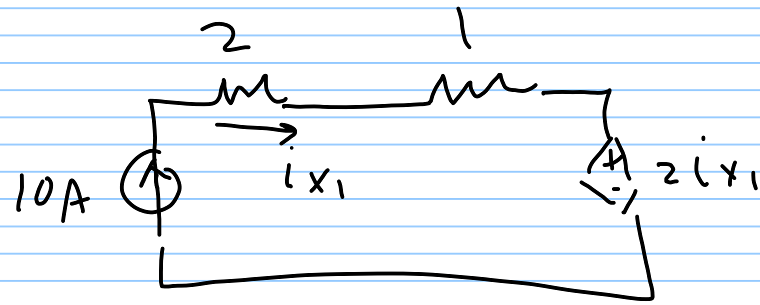
Ex) find i_x

$$\boxed{i_x = 10A}$$



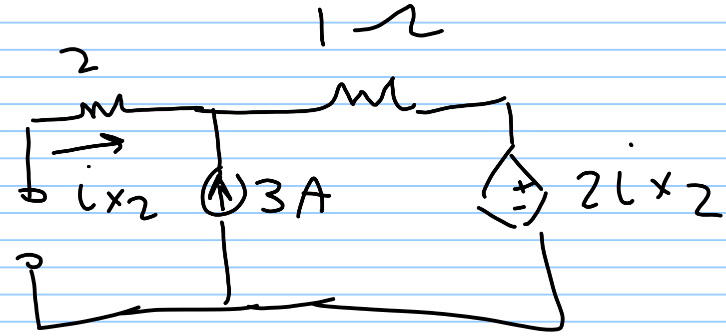
(1) i_{x_1} due to (10A)

$$i_{x_1} = 10A$$



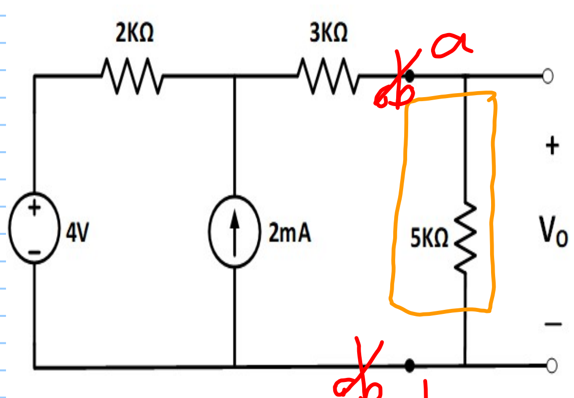
② i_{x2} due to (3A)

$i_{x2} = 0$

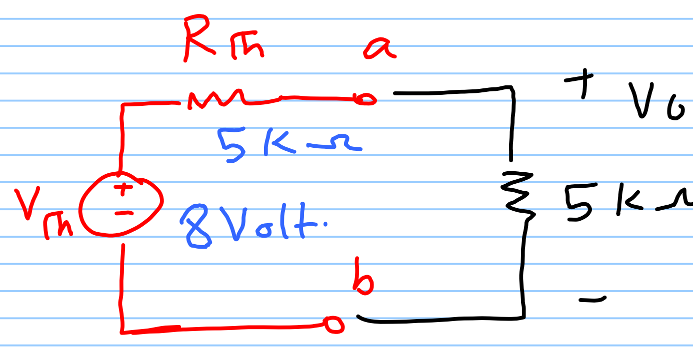


Thevenin & Norton eq. circuits

Example: Find Vo using thevenin's equivalent circuit

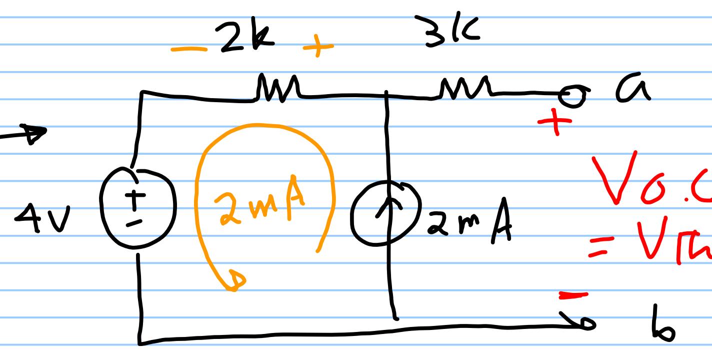


V_{Th}, R_{Th}



→ to find V_{Th}

$V_{Th} = V_{O.C.}$



$\therefore V_{Th} = (2K)(2m) + 4$

$= 8$ Volt.

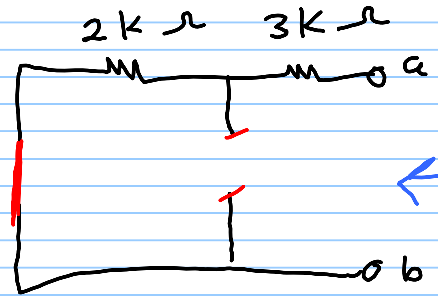
How to find R_{Th}.

(No dependent sources) — Best method

kill all independent sources → set to zero

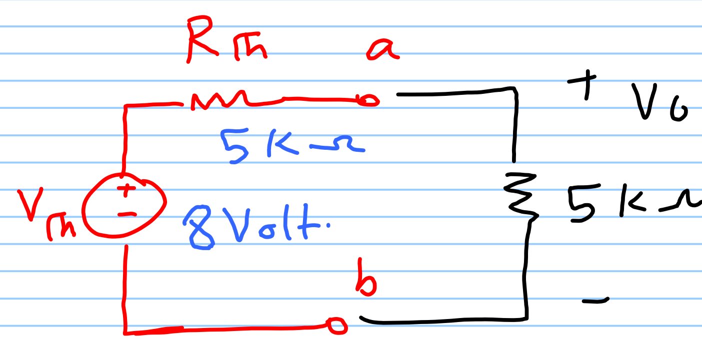
all indep. V sources 0 V → S.C. (short circuit)

all indep. I sources 0 A → O.C. (open circuit)



$$R_{Th} = 3K + 2K$$
$$= 5\ \Omega$$

أسهل طريقة



$$\therefore V_o = \frac{5}{5+5} * 8$$
$$= 4V$$

to find R_{Th} using $V_{o.c.}$ & $I_{s.c.}$ — dep. sources ✓, indep. sources ✓

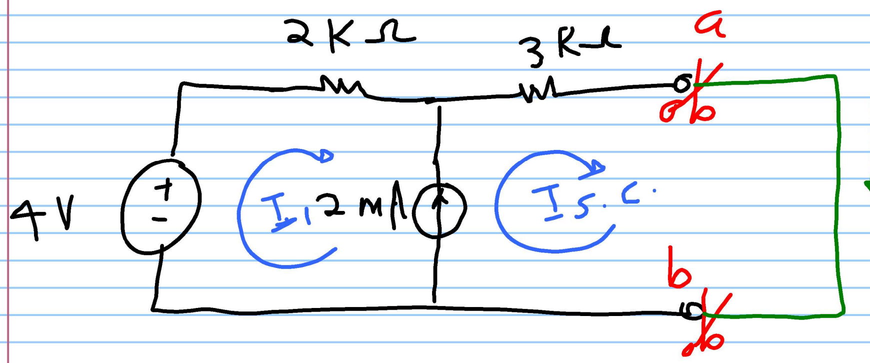
(Note: you can use this method when there is NO dep. sources, But → time consuming!!)

method ②

$V_{o.c.} = 8V$ (the same as before)

$$R_{Th} = \frac{V_{o.c.}}{I_{s.c.}}$$

→ find $I_{s.c.}$



$2k \times (I_{s.c.} - I_1 = 2mA)$ — ①

$-4 + 2k I_1 + 3k I_{s.c.} = 0$

$+$

$(3k I_{s.c.} + 2k I_1 = 4)$ — ②

$5k I_{s.c} = 8$

$$\boxed{I_{s.c.} = \frac{8}{5}\ mA}$$

$= I_N.$

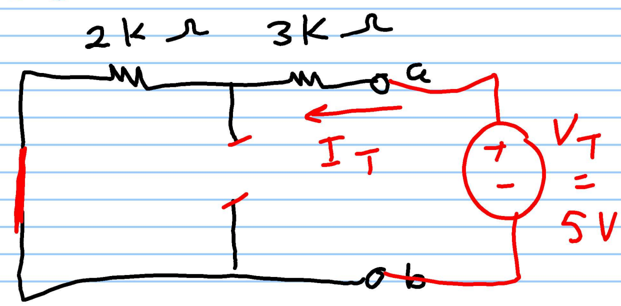
$\therefore R_{Th} = \frac{V_{o.c.}}{I_{s.c.}} = \frac{8}{\frac{8}{5} m}$

$= 5 k\Omega$

method ③ R_{Th} using test sources

dep & indep sources ✓

$R_{Th} = \frac{V_T}{I_T}\Big|_{\text{all indep. sources are set to zero}}$



let $V_T = 5V$

then $I_T = \frac{V_T}{R_{eq}} = \frac{5}{5K\Omega} = 1mA$

$\therefore R_{Th} = \frac{5}{1m} = 5k\Omega$ ✓

* methods to Find R_{Th} :-

indep. method 1 | if there is no dep. sources (best method) (easy one)

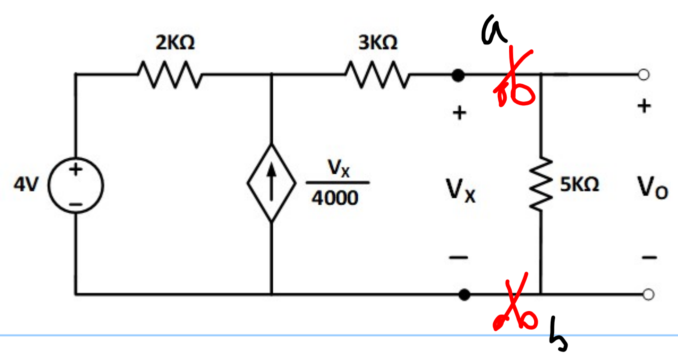
$\rightarrow$ kill all indep. sources, then find R_{Th} (series/parallel combination)

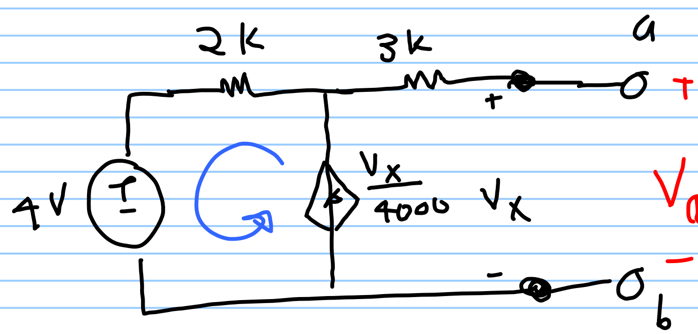
dep. indep. method 2 | $R_{Th} = \frac{V_{o.c.}}{I_{s.c.}}$ General method

can be applied if the circuit has dep. source (also it can be applied if the circuit doesn't have dep. sources)

dep. indep. method 3 | $R_{Th} = \frac{V_T}{I_T}\Big|_{\text{all indep. Sources}=0}$ (dep. sources are left intact.)

Find V_o using Thevenin's theorem





$V_{Th} = V_{o.c.}$

for this circuit $V_{Th} = V_{o.c.} = V_x$!

$$V_{o.c} = \left(\frac{V_x}{4000}\right)2k + 4V = \left(\frac{V_{o.c}}{4000}\right)2k + 4$$

$$V_{o.c.} = \frac{1}{2}V_{o.c} + 4$$

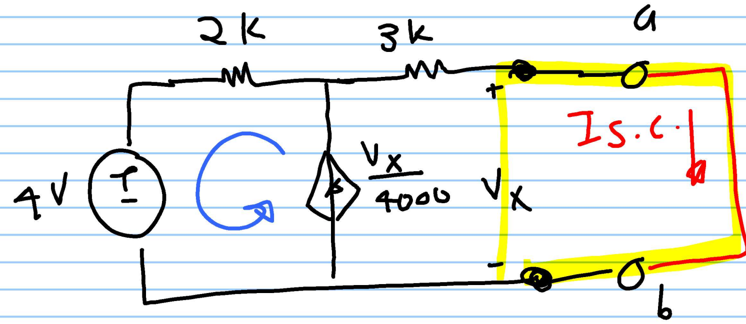
$\boxed{V_{o.c.} = 8 \text{ Volt.}}$

to find R_{Th} – method 1 X

method 2 or method 3 ✓ (dep. source)

method ② $V_{o.c.} = 8V$.

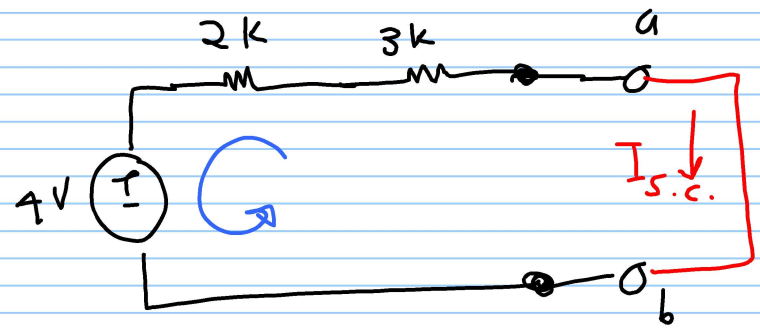
find I_{sc}.



$V_x = $ Zero !!

$\frac{V_x}{4000} = 0\ A\ (o.c.)$

$\therefore I_{s.c.} = \frac{4}{5k} = 0.8\ mA$



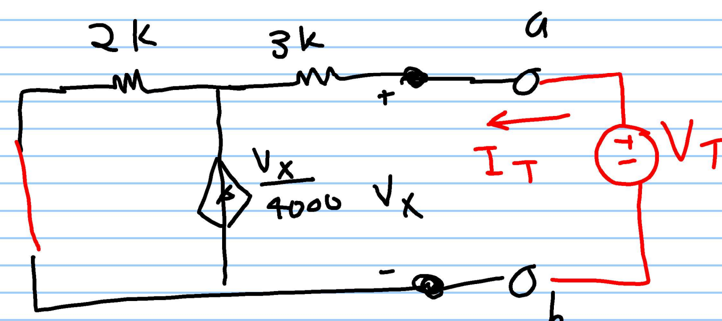
$\therefore R_{Th} = \frac{V_{o.c.}}{I_{s.c.}} = \frac{8}{0.8m}$

$= 10\ k\Omega$ ✓

method ③ $R_{Th} = \frac{V_T}{I_T}$

all indep. sources killed

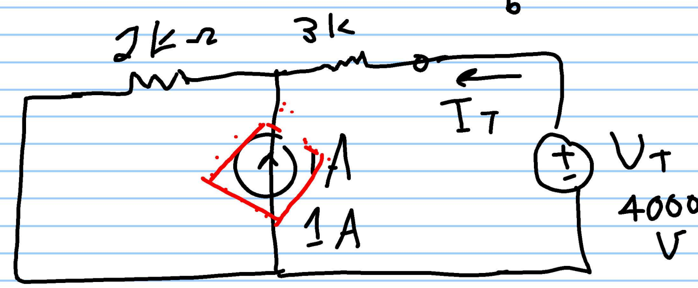
let $V_T = 4000\ V$



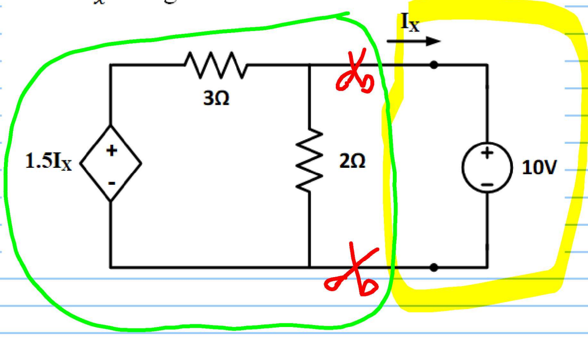
$\therefore I_T = \frac{4000 - 2000}{5000}$

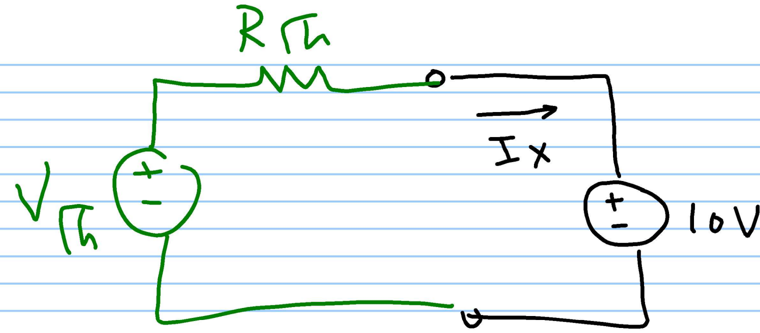
$= 0.4\ A$

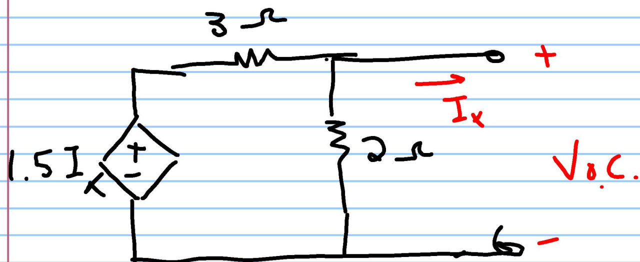
$\therefore R = \frac{4000}{0.4} = 10\ k\Omega$



Find I_x using Thevenin's theorem







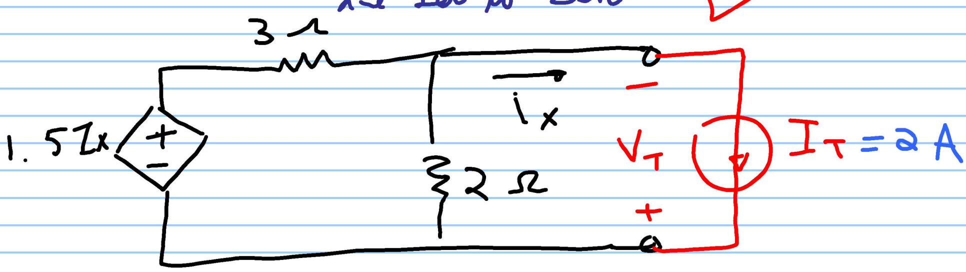
Since there is only dep. sources

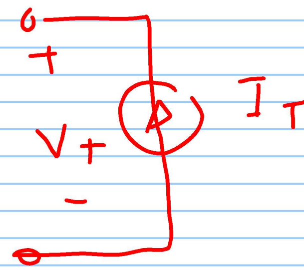
$$\boxed{\therefore V_{o.c.} = V_{Th} = \text{zero}}$$

method ② ✗

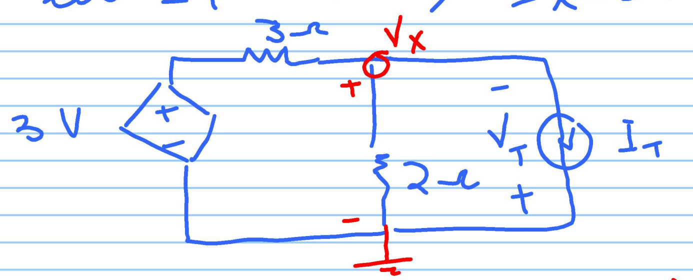
✓ to find R_{Th}

$$R_{Th} = \frac{V_T}{I_T}\Big|_{\text{all indep. sources are set to zero}}$$





let $I_T = 2A$, $I_x = I_T$



Nodal

$$\frac{V_x - 3}{3} + \frac{V_x}{2} + 2 = 0$$

$$V_x\left(\frac{1}{3} + \frac{1}{2}\right) = -2 + 1$$

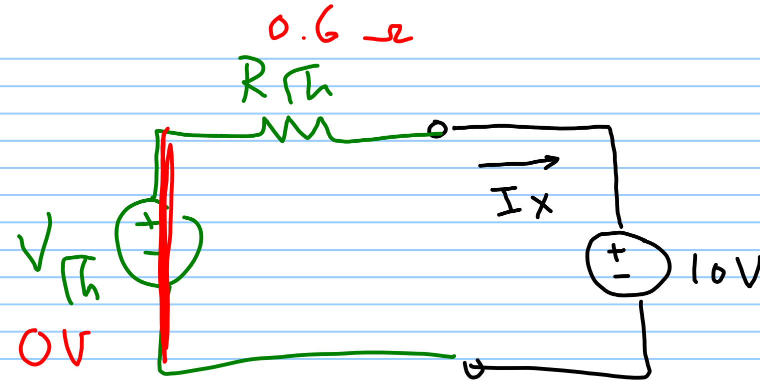
$$V_x = -\frac{6}{5}\ V$$

$$\therefore V_T = -V_X = \frac{6}{5}\ V$$

$$\therefore R_{Th} = \frac{V_T}{I_T}\Big| = \frac{6/5}{2} = 0.6\ \Omega$$

$\therefore I_x = \frac{-10}{0.6}$

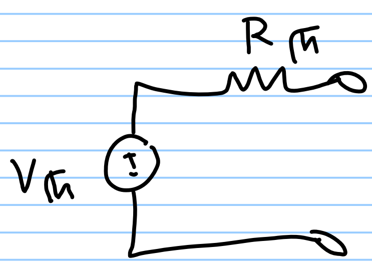
$= -16.67\ A$

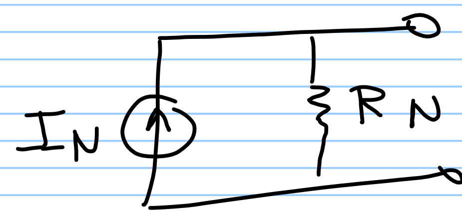


$$\boxed{R_{Th} = R_N}$$

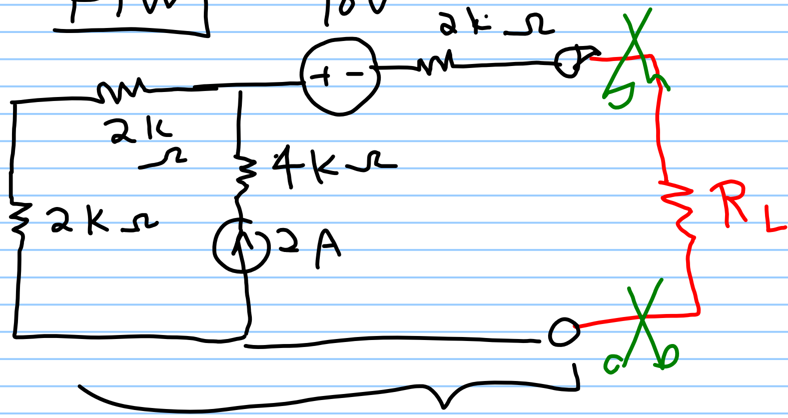
$V_{Th} \rightarrow V_{o.c.}$

$I_N \rightarrow I_{s.c.}$



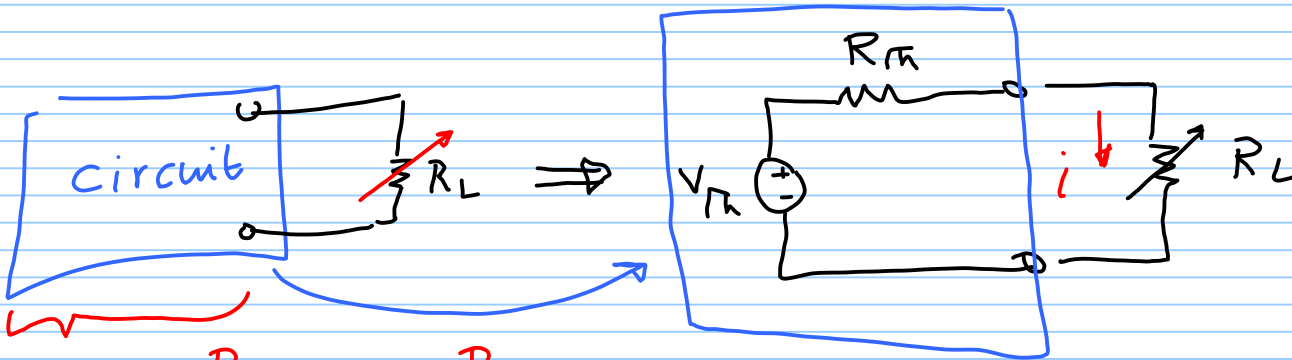


HW



Find the thevenin eq. circuit.

4.12 Maximum Power Transfer



max. Power → R_L

$$P = i^2 R_L = \left(\frac{V_{Th}}{R_L + R_{Th}} \right)^2 R_L$$

P is max when

$$\frac{dP}{dR_L} = \text{Zero}$$

$$P = \frac{V_{Th}^2 \, R_L}{(R_L + R_{Th})^2}$$

$$\frac{dP}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th} + R_L)^2 - R_L \times 2 \times (R_L + R_{Th})}{(R_{Th} + R_L)^4} \right] = 0$$

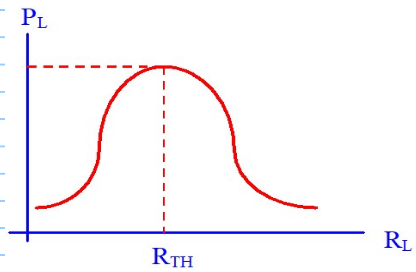
$$(R_{Th} + R_L)^2 = 2 R_L (R_L + R_{Th})$$

$$R_{Th}^2 + R_L^2 + \cancel{2 R_L R_{Th}} = 2 R_L^2 + \cancel{2 R_{Th}} R_{Th}$$

$$\boxed{R_L = R_{Th}}$$

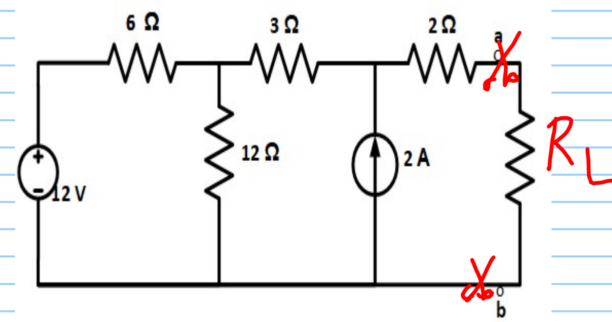
→ The max. power transfer occurs when the load resistance $\boxed{R_L = R_{Th}}$ or $P_{max} = \left(\frac{V_{Th}}{2 R_L} \right)^2 R_L$

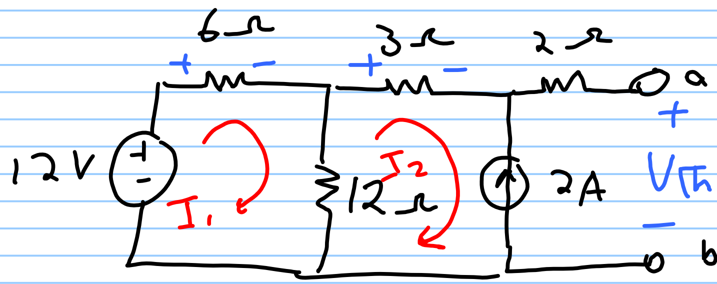
$$\boxed{P_{max} = \frac{V_{Th}^2}{4R_L}}$$



Find the value of R_L for maximum power transfer in the circuit shown .

Find the maximum power .





$I_2 = -2A$

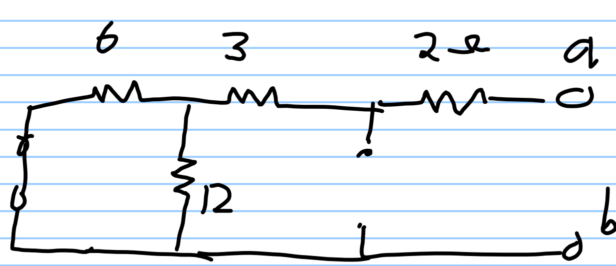
$$-12 + 6I_1 + 12(I_1 - I_2) = 0$$

$$-12 + 6I_1 + 12I_1 + 12 \times 2 = 0$$

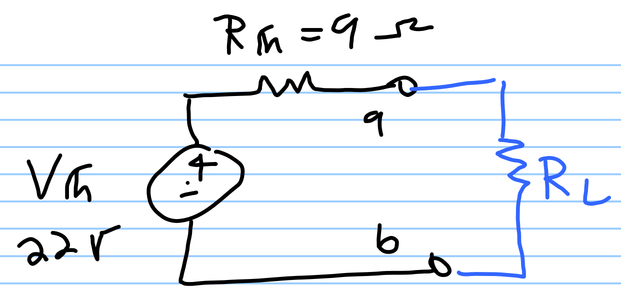
$$I_1 = -2/3 \text{ A}$$

$$V_{Th} = -3 \times I_2 - 6 \times I_1 + 12 = \underline{\underline{22 \text{ V}}}$$

$\rightarrow R_{Th}$



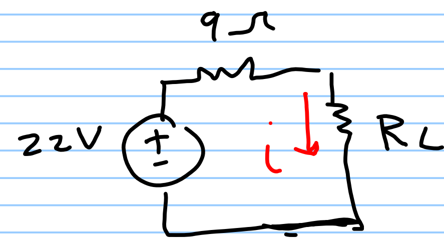
$$R_{Th} = (6 // 12) + 3 + 2 = 9\,\Omega$$



∴ for max. power transfer

$$R_L = R_{th} = 9\,\Omega$$

$$\therefore P_{max} = \frac{V_{th}^2}{4R_{th}} = 13.44\text{ W}$$

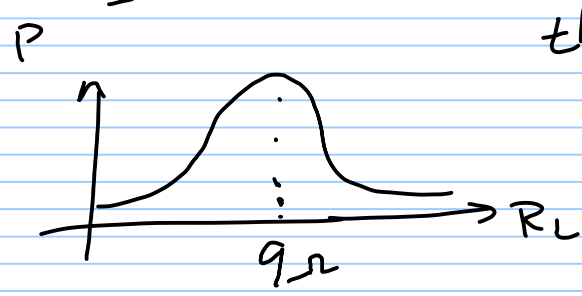


if $R_L = 6\,\Omega$

$$P_{R_L} = i^2 R_L$$

$$= \left(\frac{22}{9+6}\right)^2 \times 6$$

$$= 12.9\text{ W}$$



if $R_L = 13\,\Omega$

then $P_{R_L} = \left(\frac{22}{9+13}\right)^2 \times 13$

$$= 13\text{ W}$$