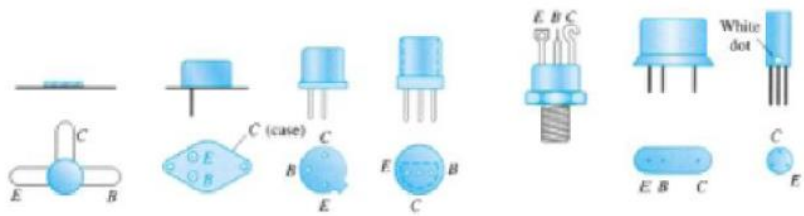

ENEE2360 Analog Electronics

T8: BJT AC Models and Analysis

Instructor : Nasser Ismail

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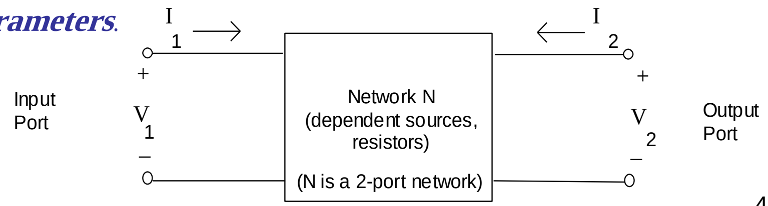
Small Signal ac Equivalent Circuit

- In order to simplify the analysis, we replace the Transistor by an equivalent circuit (model)
- An AC model represents the AC characteristics of the transistor.
- A model uses circuit elements that approximate the behavior of the transistor.
- There are two models commonly used in small signal AC analysis of a transistor:
  - **r_e model**
  - **Hybrid equivalent model**

ENEE236

Two-port networks

- Suppose that a network N has two ports as shown below. How could it be represented or modeled?
- A common way to represent such a network is to use one of 6 possible ***two-port networks***.
- These networks are circuits that are based on one of 6 possible sets of ***two-port equations***. These equations are simply different combinations of two equations that relate the variables V_1 , V_2 , I_1 , and I_2 to one another. The coefficients in these equations are referred to as ***two-port parameters***.



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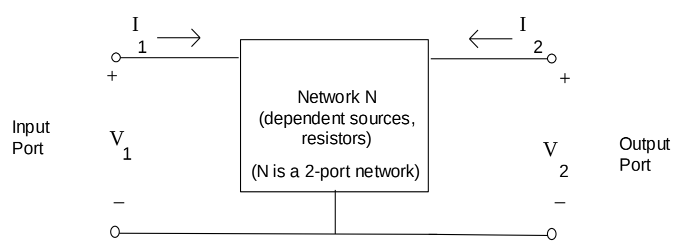
ENEE234 – Circuit Analysis

Note that I_1 , I_2 , V_1 , and V_2 are labeled as shown by convention. Often there is a common negative terminal between the input and the output so the figure above could be redrawn as:



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ENEE236

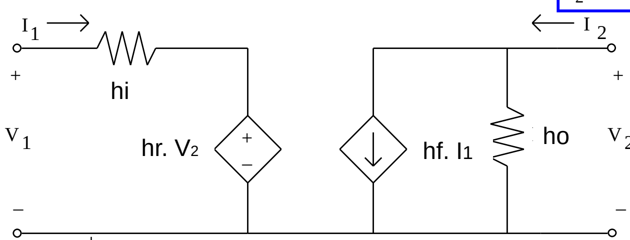
Development of the h-parameter model of BJT:

For A BJT the equivalent h parameter model can be described by the following equations:

h - parameter equations :

$$V_1 = h_i \cdot I_1 + h_r \cdot V_2$$

$$I_2 = h_f \cdot I_1 + h_o \cdot V_2$$



$$h_i = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_r = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_f = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_o = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

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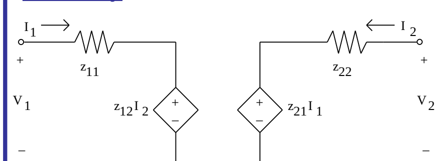
Summary:

Note: This page is for information only

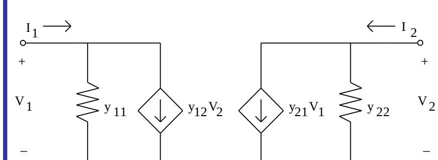


z-parameter model

z - parameter equations :

$$V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

$$V_2 = z_{21} \cdot I_1 + z_{22} \cdot I_2$$

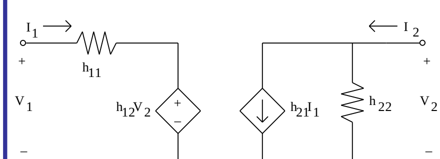


y-parameter model

y - parameter equations :

$$I_1 = y_{11} \cdot V_1 + y_{12} \cdot V_2$$

$$I_2 = y_{21} \cdot V_1 + y_{22} \cdot V_2$$



h-parameter model

h - parameter equations :

$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2$$

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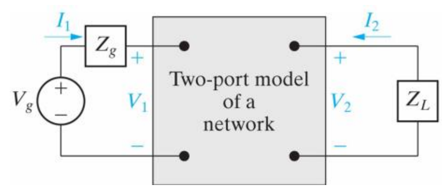
BJT Configurations

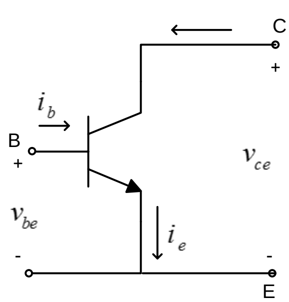
- Common Emitter
- Common Base
- Common Collector



Terminated Two port network
Includes source and load

Common Emitter Configuration

(inverting configuration, provides voltage and current gain)

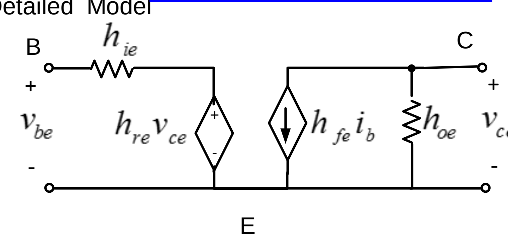


h - parameter equations:

$$V_{be} = h_{ie} \cdot I_b + h_{re} \cdot V_{ce}$$

$$I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_{ce}$$

Detailed Model



Typical Data sheet parameter values

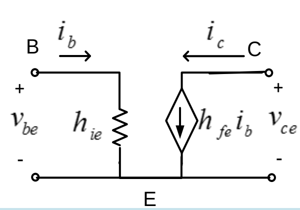
$h_{ie} \approx 1600\,\Omega$

$h_{re} \approx 0.0002$

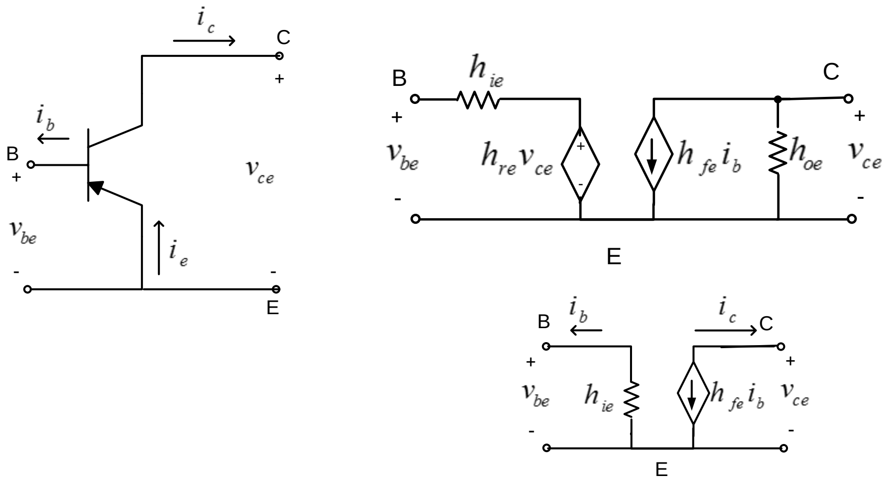
$h_{fe} \approx 80$

$h_{oe} \approx 20.10^{-6}$ Siemens

Simplified Model



Common Emitter and Common Collector Configuration



Value of hie

Base Emitter is a pn junction similar to a diode
hie is the dynamic resistance of the pn junction

In a diode:

$$r_d = \frac{V_T}{I_{DQ}} \Rightarrow$$

$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{V_T}{\frac{I_{CQ}}{h_{fe}}} = \frac{h_{fe} V_T}{I_{CQ}}$$

I_{BQ} dc value of base current
I_{CQ} dc value of collector current

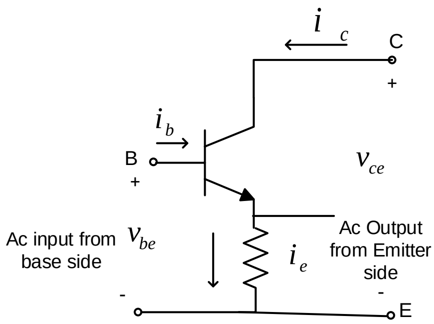
$$h_{fe} = \beta$$

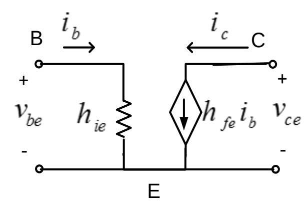
$$V_T = 25.69 \text{ mV} \ @ \ 25\ ^oC$$

Common Collector

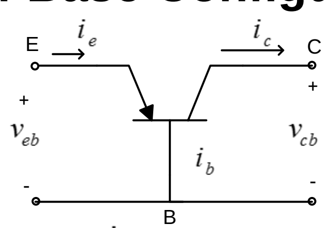
(provides current gain and no voltage gain)

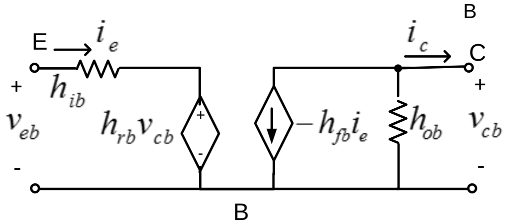
Same Model of Common Emitter will be used due to the similarities between them and for simplicity





Common-Base Configuration





h - parameter equations:

$$V_{eb} = h_{ib} \cdot I_e + h_{rb} \cdot V_{cb}$$

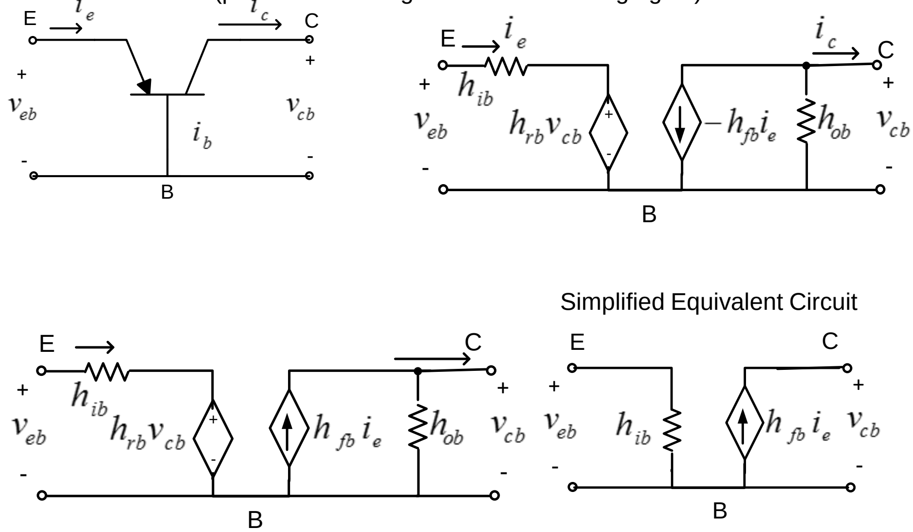
$$I_c = h_{fb} \cdot I_e + h_{ob} \cdot V_{cb}$$

$$h_{ib} = \frac{V_{EB}}{I_E}\bigg|_{V_{CB}=0} \qquad h_{rb} = \frac{V_{EB}}{V_{CB}}\bigg|_{I_E=0}$$

$$h_{fb} = \alpha = \frac{I_C}{I_E}\bigg|_{V_{CB}=0} \qquad h_{ob} = \frac{I_C}{V_{CB}}\bigg|_{I_E=0}$$

Common-Base Configuration

(provides current gain and some voltage gain)



Common-Base Configuration

$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha$$

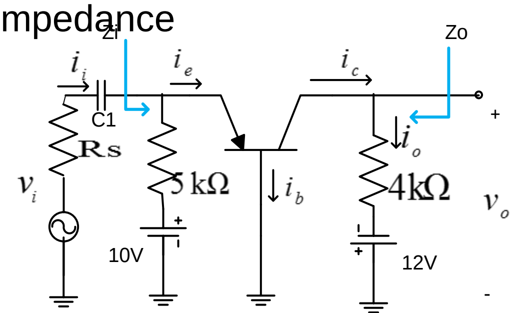
$$V_T = 25.69 \text{ mV} \ @ \ 25\ ^{o}C$$

$$h_{ie} > h_{ib}$$

BJT Amplifier Analysis

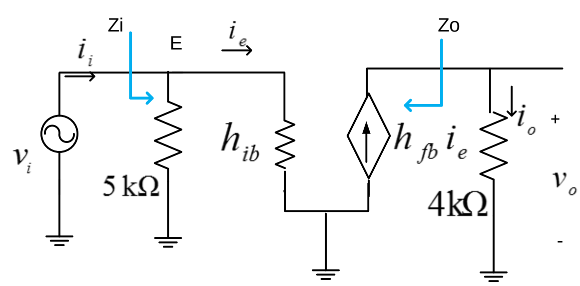
When Analyzing Amplifier Circuits, we usually want to find some or all of the following quantities **with and without Rs**:

1) Av=Vo/Vi, small signal voltage gain
2) Ai=io/ii, small signal current gain
3) Zi Input Impedance
4) Zo Output Impedance



BJT Amplifier Analysis

Solution: (with Rs=0)
We draw the ac small signal equivalent circuit
Capacitors ==> replaced by short circuit
DC sources are killed ,



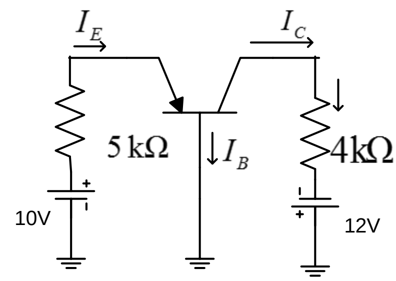
$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha \cong 1$$

I_{EQ} must be calculated from DC analysis

DC Analysis

DC Equivalent Circuit:
-Cap ==> open
-Kill ac sources ==>



$$10 = 5\,k\Omega\,.I_{EQ} + V_{EB}$$

$$I_{EQ} = \frac{10 - 0.7}{5\,k\Omega} = 1.86\,mA$$

$$h_{ib} = \frac{V_T}{I_{EQ}} = \frac{25.69\,mV}{1.86\,mA} = 13.98\,\Omega$$

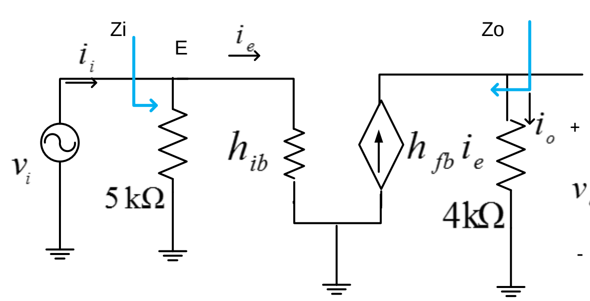
Ac ss equivalent circuit

1) $A_V = \frac{v_o}{v_i}$

$v_o = i_o.\,4\,k\Omega$

$i_o = h_{fb}.i_e$

$i_e = \frac{v_i}{h_{ib}}$



$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_o}.\frac{i_o}{i_e}.\frac{i_e}{v_i}$$

$$\Rightarrow \quad A_V = (4\,k\Omega).(h_{fb}).\left(\frac{1}{h_{ib}}\right)$$

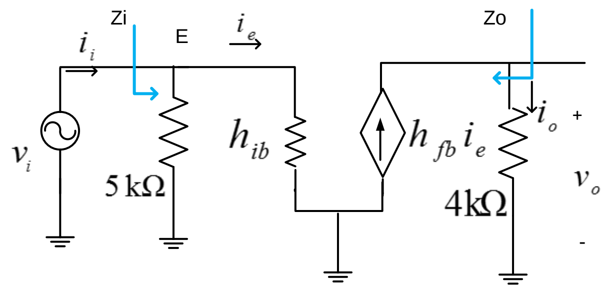
$$= (4\,k\Omega).(1).\left(\frac{1}{13.98}\right) = 286 > 1$$

Current Gain Ai

2) $\mathrm{A_i} = \dfrac{i_o}{i_i}$

$i_o = h_{fb}.i_e$

$i_e = i_i \dfrac{5\,\mathrm{k}\Omega}{5\,\mathrm{k}\Omega + h_{ib}}$

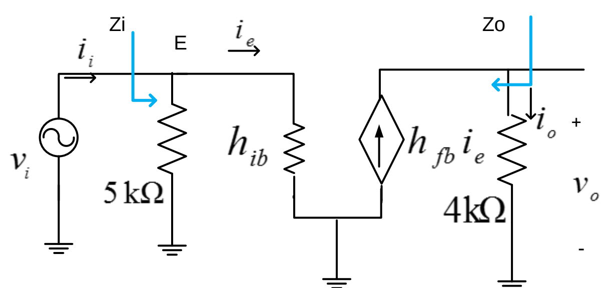


$$\Rightarrow \mathrm{A_i} = \frac{i_o}{i_i} = \frac{i_o}{i_e}.\frac{i_e}{i_i}$$

$$\Rightarrow \mathrm{A_i} = (h_{fb})\left(\frac{5\,\mathrm{k}\Omega}{5\,\mathrm{k}\Omega + h_{ib}}\right)$$

$$= (1)\left(\frac{5\,\mathrm{k}\Omega}{5\,\mathrm{k}\Omega + 13.98}\right) < 1$$

Zi & Zo



3) Input Impedance

$$Z_\mathrm{i} = (h_{ib} // 5\,\mathrm{k}\Omega) = \left(\frac{h_{ib}.5\,\mathrm{k}\Omega}{5\,\mathrm{k}\Omega + h_{ib}}\right)$$

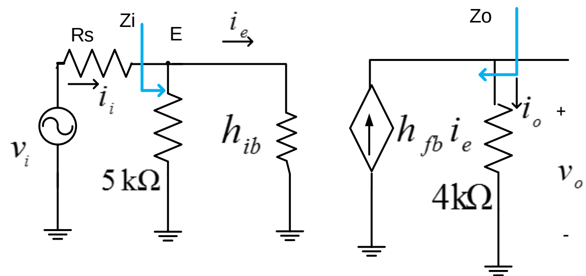
4) Output Impedance

$$Z_o\big|_{\text{all independant sources killed (i.e. Vi=0 or short)}} = 4\,\mathrm{k}\Omega$$

With Presence of Rs

with R_S

$$i_i = \frac{v_i}{Z_i + R_s}$$



For Rs = 50 Ω

$A_V = 62.5$

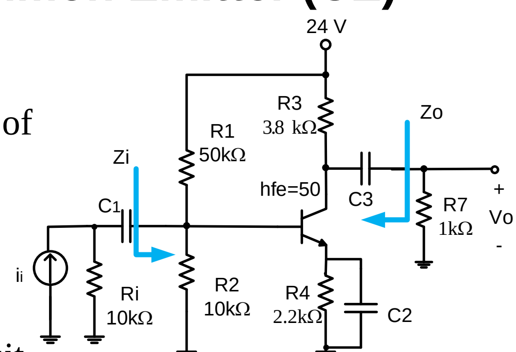
For Rs = 10 kΩ

$A_V = 0.4$

Example: Common Emitter (CE)

1) From DC Analysis, we find Q - point and value of

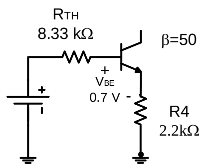
$$h_{ie} = \frac{V_T}{I_{BQ}}$$



Thevenin's equivalent circuit as seen from the base

$$V_{TH} = \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 50\ \text{k}\Omega} . 24\ \text{V} = 4\ \text{V}$$

$$R_{TH} = 10\ \text{k}\Omega\Omega // 5\text{k}\Omega = 8.33\ \text{k}\Omega$$

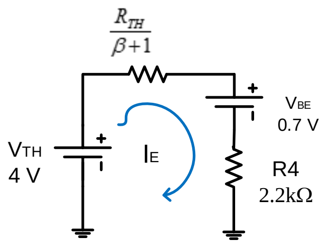


$$4 = 8.33\,\text{k}\Omega \,.\, I_B + V_{BE} + 2.2\,\text{k}\Omega \,.\, I_E$$

But, $I_E = (1+\beta) I_B$

$$\text{Solve for } I_E = \frac{4 - 0.7}{\frac{8.33\,\text{k}\Omega}{(1+50)} + 2.2\,\text{k}\Omega} = 1.4\,\text{mA}$$

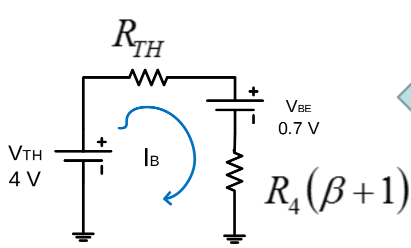
$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{25.69\,\text{mV}}{\frac{1.4\,\text{mA}}{51}} = 928\,\Omega$$



Here we have base reflected to emitter

$$I_B \Rightarrow I_E = (\beta+1) I_B$$

$$R_B \Rightarrow \frac{R_B}{\beta+1}$$

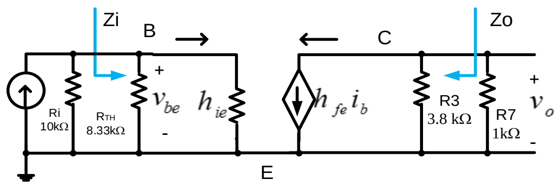


Here we have emitter reflected to base

$$I_E \Rightarrow I_B = \frac{I_E}{(\beta+1)}$$

$$R_E \Rightarrow R_E(\beta+1)$$

AC small signal Equivalent Circuit



1) $A_V = \dfrac{v_o}{v_i}$

$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_b} \cdot \frac{i_b}{v_i}$$

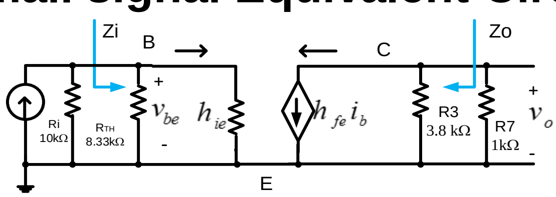
$v_o = -h_{fe} i_b . (R_3 // R_7)$

$i_b = \dfrac{v_i}{h_{ie}}$

$$= -h_{fe} . (R_3 // R_7) . \left(\frac{1}{h_{ie}} \right)$$

$$= -50 . (3.8\,k\Omega // 1\,k\Omega) . \left(\frac{1}{928\,\Omega} \right) = -42.7$$

AC small signal Equivalent Circuit



2) $Z_I = R_{TH} // h_{ie}$

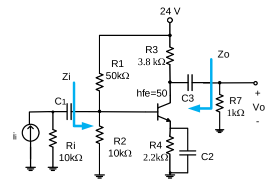
$= 8.33\,k\Omega // 928\,\Omega$

only elements to the right of arrow are considered
according to the given direction of the arrow

3) $Z_o \big|_{\text{all independant sources killed (i.e. Vi=0 or short)}} = 3.8\,k\Omega$

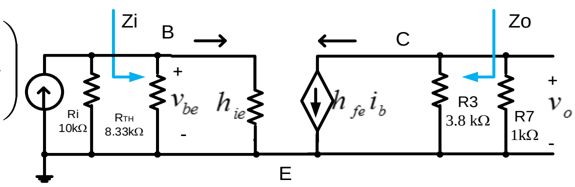
here $h_{fe} . i_b = 0$ since $i_b = 0$ (vi = 0 - killed)

AC small signal Equivalent Circuit



4) $A_i = \dfrac{i_o}{i_i}$

$$i_b = (i_i)\left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}}\right)$$

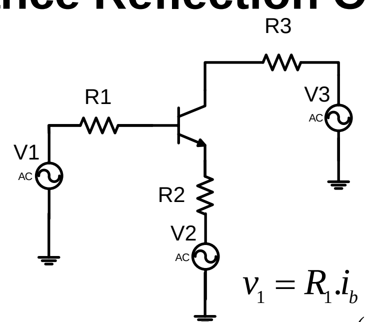


$$i_o = -h_{fe} i_b \left(\frac{R_3}{R_3 + R_7}\right)$$

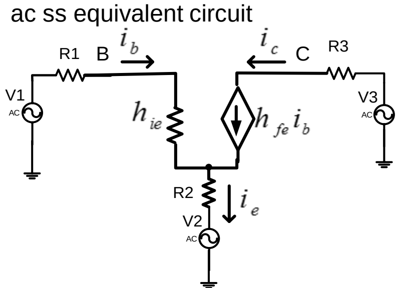
$$A_i = \frac{i_o}{i_i} = \frac{i_o}{i_b} \cdot \frac{i_b}{i_i} = -h_{fe}\left(\frac{R_3}{R_3 + R_7}\right) \cdot \left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}}\right) = -33$$

Impedance Reflection Concept

v1,v2,v3 are all ac sources



ac ss equivalent circuit



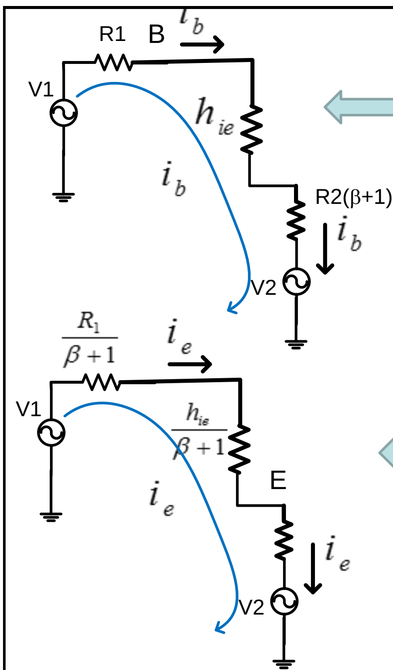
$$v_1 = R_1 . i_b + h_{ie} . i_b + R_2 . i_e + v_2$$

but $i_e = (\beta + 1) i_b$

$$v_1 = R_1 . i_b + h_{ie} . i_b + R_2 . (\beta + 1) i_b + v_2$$

$$i_b = \frac{v_1 - v_2}{R_1 + h_{ie} + R_2 . (\beta + 1)} \Leftarrow \text{base loop}$$

equivalent circuit equation



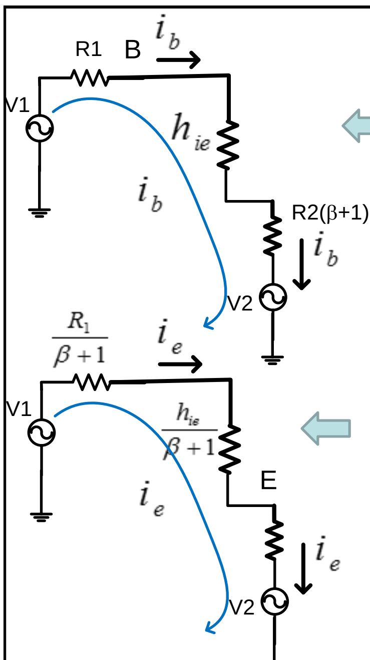
$$i_b = \frac{v_1 - v_2}{R_1 + h_{ie} + R_2.(\beta+1)} \Leftarrow \text{base loop}$$

equivalent circuit equation

Instead of i_b use i_e

$$i_e = \frac{v_1 - v_2}{\frac{R_1}{(\beta+1)} + \frac{h_{ie}}{(\beta+1)} + R_2} \Leftarrow \text{Emitter loop}$$

equivalent circuit equation



base equivalent circuit

(reflection from emitter to base)

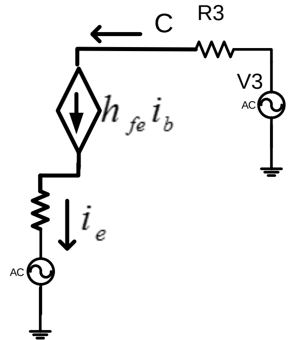
Here we must change i_e to i_b which requires division by $(h_{fe}+1)$, but voltage must remain the same and thus the resistance must be multiplied by the same factor $(h_{fe}+1)$

Emitter equivalent circuit

(reflection from base to emitter)

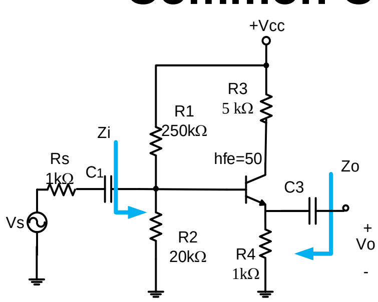
Here we must change i_b to i_e which requires multiplication by $(h_{fe}+1)$, but voltage must remain the same and thus the resistance must be divided by the same factor $(h_{fe}+1)$

Collector Equivalent Circuit



Note: there is no reflection from emitter to collector or vise versa since the ie and ic are almost the same

Common Collector Amplifier



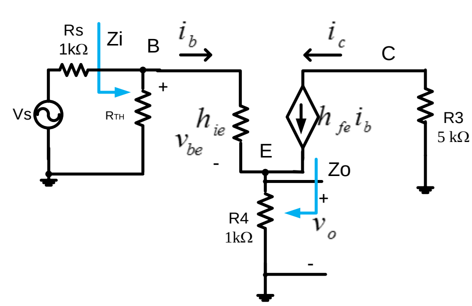
Given

$$h_{ie} = 1\text{k}\Omega$$

$$h_{fe} = \beta = 50$$

Find Av, Ai, Zi, Zo

AC small signal Equivalent Circuit

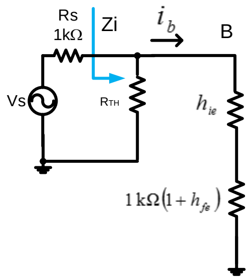


$$1)\ A_V = \frac{v_o}{v_s}$$

$$v_o = 1\text{k}\Omega \,.\, i_e$$

$$i_e = i_b(h_{fe} + 1)$$

i_b can be found from base equivalent circuit



$$R_{TH} = 20\,k\Omega // 250\,k\Omega$$

$$i_b = i_i \frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1\,k\Omega(h_{fe}+1))}$$

$$i_i = \frac{V_S}{R_S + (R_{TH} // (h_{ie} + 1\,k\Omega(h_{fe}+1)))}$$

$$\therefore A_V = \frac{v_o}{v_S} = \frac{v_o}{i_e} \cdot \frac{i_e}{i_b} \cdot \frac{i_b}{i_i} \cdot \frac{i_i}{v_S}$$

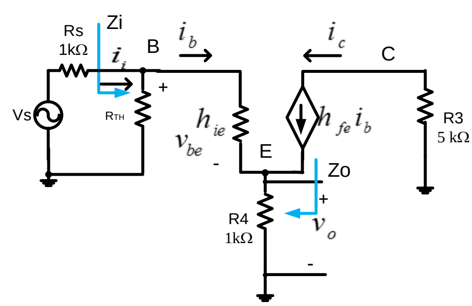
$$= (1k\Omega).(h_{fe}+1)\left(\frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1\,k\Omega(h_{fe}+1))}\right)\left(\frac{1}{R_S + (R_{TH} // (h_{ie} + 1\,k\Omega(h_{fe}+1)))}\right)$$

$$= 0.915 < 1$$

$$2) A_i = \frac{i_o}{i_i}$$

$$i_o = \frac{v_o}{1\,k\Omega}$$

$$i_o = i_e = i_b(h_{fe}+1)$$

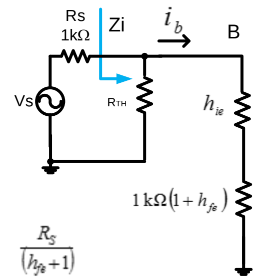


$$i_b = i_i \frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1\,k\Omega(h_{fe}+1))}$$

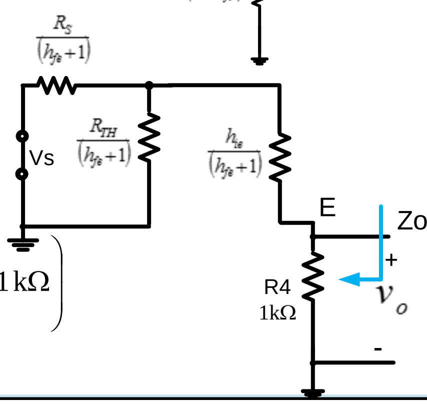
$$A_i = \frac{i_o}{i_i} = \frac{i_o}{i_e} \cdot \frac{i_e}{i_b} \cdot \frac{i_b}{i_i}$$

$$= 1(h_{fe}+1)\left(\frac{R_{TH}}{R_{TH} + [hie + 1k(hfe+1)]}\right) = 13.39 > 1$$

$$3)\ Z_I = \left(R_{TH} // \left(h_{ie} + 1\,\text{k}\Omega(h_{fe}+1)\right)\right)$$
$$= 13.66\,\text{k}\Omega \quad (\text{high})$$



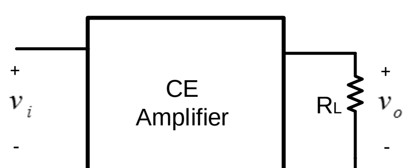
Emitter Equivalent Circuit
$\& V_S = 0$



$$Z_o\big|_{Vs=0} = \left(\frac{(R_S // R_{TH}) + h_{ie}}{(h_{fe}+1)} // 1\,\text{k}\Omega\right)$$
$$= \left(\left(\left(\frac{R_S}{(h_{fe}+1)} // \frac{R_{TH}}{(h_{fe}+1)}\right) + \frac{h_{ie}}{(h_{fe}+1)}\right) // 1\,\text{k}\Omega\right)$$
$$= 36.8\,\Omega \quad (\text{low})$$

CC Amplifier as a Buffer

- The value of load resistor RL affects the voltage gain Av,
- This effect is called loading effect and can be substantial



$h_{ie} = 1000\,\Omega$
$h_{fe} = 140$

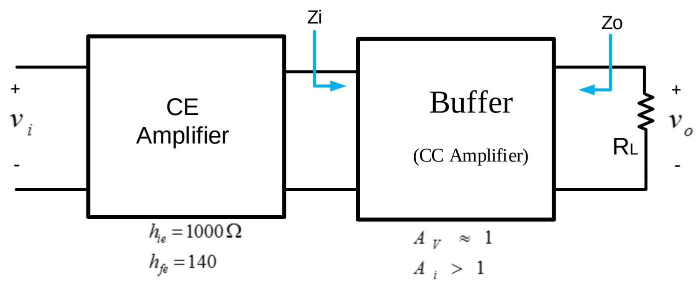
- A buffer (interface) can be used between the amplifier and the load to reduce this loading effect and keep the high gain
- CC Amplifier is also known as Emitter Follower

CC Amplifier as a Buffer

- The buffer must have the following characteristic:

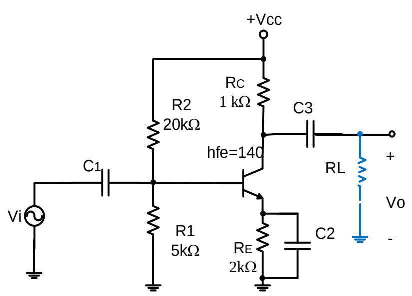
$$A_V \approx 1$$
$$A_I > 1$$
$$Z_I >> \text{high}$$
$$Z_o << \text{low}$$

- The above characteristic are present in the CC amplifier the load to reduce this loading effect and keep the high gain



Example

- First we consider effect of load (RL) on amplifier voltage gain
- Then we use a buffer and see its effect on reducing effect of RL



1) with $R_L = \infty$

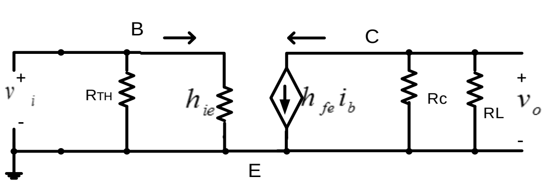
$$v_o = -h_{fe} i_b .(R_C)$$

$$i_b = \frac{v_i}{h_{ie}}$$

$$A_V = \frac{v_o}{v_i} = \left(-h_{fe} R_C\right).\frac{1}{h_{ie}} = -140$$

2) with $R_L = 50\,\Omega$

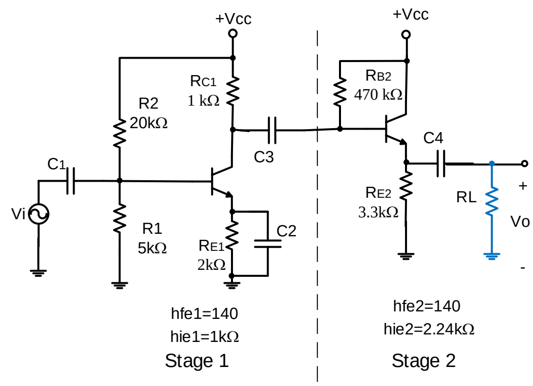
$$A_V = \frac{v_o}{v_i} = \left(-h_{fe} R_C // R_L\right).\frac{1}{h_{ie}} = -6.87$$



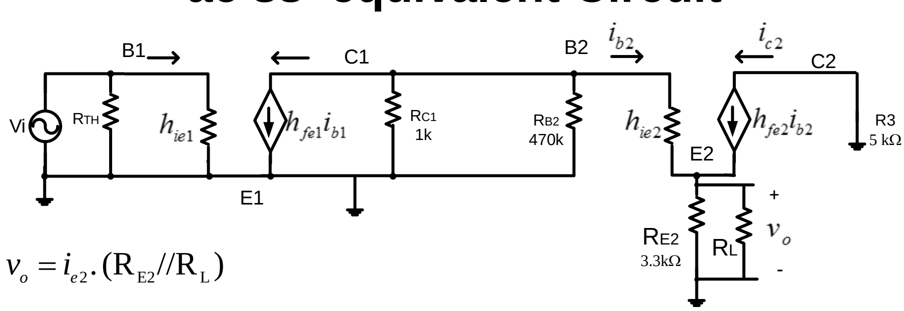
Av have been reduced from -140 to -6.87

Amplifier + Buffer + Load

Now let us look at the new circuit with the buffer



ac ss equivalent Circuit



$$v_o = i_{e2}.(R_{E2}//R_L)$$

$$i_{e2} = i_{b2}(1 + h_{fe2})$$

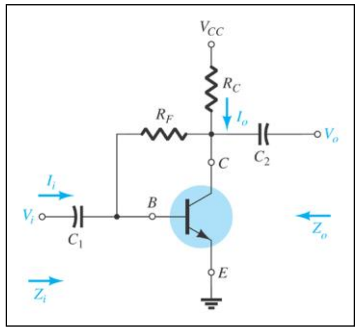
$$i_{b2} = -h_{fe1}.i_{b1}\frac{(R_{C1}//R_{B2})}{((R_{C1}//R_{B2}) + (h_{ie2} + (R_{E2}//R_L)(1 + h_{fe2})))}$$

$$i_{b1} = \frac{V_i}{h_{ie1}}$$

$$\Rightarrow Av = \frac{v_o}{v_i} = \frac{v_o}{i_{e2}}.\frac{i_{e2}}{i_{b2}}.\frac{i_{b2}}{i_{b1}}.\frac{i_{b1}}{v_i} = -95.6$$

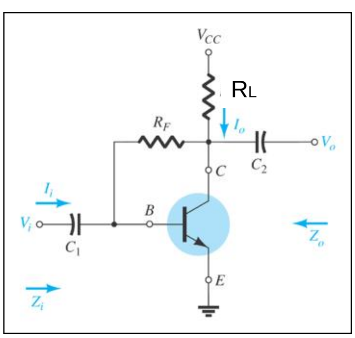
This is much better than the case without buffer

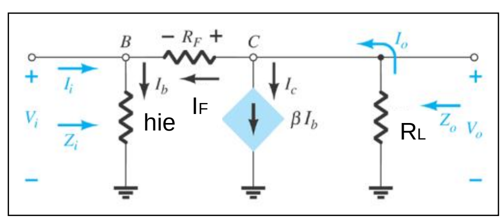
Base To Collector Feedback



Exercise : Find A_V, Zi and Zo

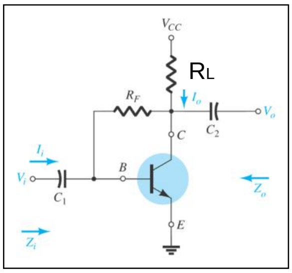
Base To Collector Feedback

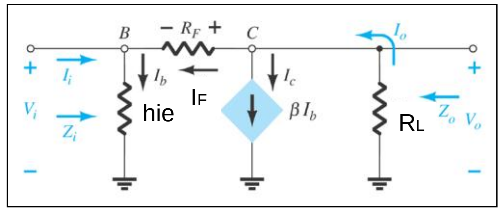




Exercise : Find A_V, Zi and Zo

Base To Collector Feedback





$$v_0 = -i_o.R_L$$

$$i_o = h_{fe}.i_b + i_F$$

$$i_F = \frac{v_o - v_i}{R_F}$$

$$i_b = \frac{v_i}{h_{ie}}$$

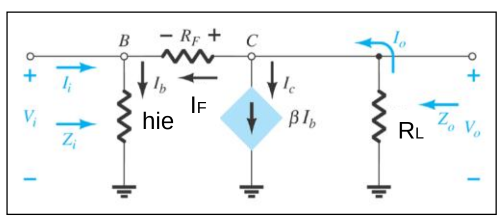
$$v_0 = -\left(h_{fe}.\frac{v_i}{h_{ie}} + \frac{v_o - v_i}{R_F}\right).R_L$$

$$v_0 = -R_L h_{fe}.\frac{v_i}{h_{ie}} - \frac{v_o R_L}{R_F} + \frac{v_i R_L}{R_F}$$

$$v_0\left(1 + \frac{R_L}{R_F}\right) = v_i\left(\frac{R_L}{R_F} - R_L.\frac{h_{fe}}{h_{ie}}\right)$$

$$Av = \frac{\left(\frac{R_L}{R_F} - R_L.\frac{h_{fe}}{h_{ie}}\right)}{\left(1 + \frac{R_L}{R_F}\right)}$$

$$Z_0\Big|_{v_i=0} = R_F // R_L$$



$$Z_i = \frac{v_i}{i_i}$$

$$i_i = i_b - i_F = \left(\frac{v_i}{h_{ie}} - \frac{v_o - v_i}{R_F}\right)$$

$$Z_i = \frac{v_i}{i_i} = \frac{v_i}{\left(\frac{v_i}{h_{ie}} - \frac{v_o - v_i}{R_F}\right)}$$

$$= \frac{v_i}{\left(\frac{R_F v_i - h_{ie}(v_o - v_i)}{R_F h_{ie}}\right)}$$

$$= \frac{v_i R_F h_{ie}}{(R_F v_i - h_{ie}(v_o - v_i))}$$

$$= \frac{v_i R_F h_{ie}}{((R_F + h_{ie})v_i - h_{ie} v_o)}$$

$$= \frac{R_F h_{ie}}{\left((R_F + h_{ie}) - h_{ie}\frac{v_o}{v_i}\right)}$$

$$= \frac{R_F h_{ie}}{((R_F + h_{ie}) - h_{ie} A_v)}$$

The common emitter amplifier design:

Design a common emitter amplifier using a transistor having

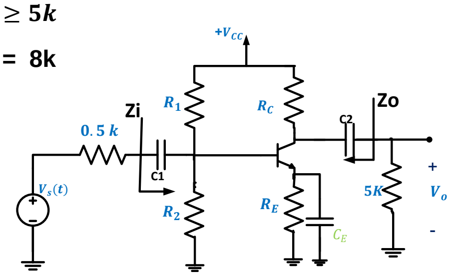
$\beta(min) = 480$, $\beta(max) = 1500$

To provide a voltage gain $\left|\frac{V_o}{V_S}\right| \geq 200$**, between a small signal voltage source having a resistance 500Ω and load** $R_L = 5k$

Its specified that $Z_{in} \geq 5k$

Its specified that Z_o **= 8k**

Solution :



Solution:

Ac small signal equivalent circuit:

$$V_o = -(R_C \backslash\backslash R_L) h_{fe} i_b$$

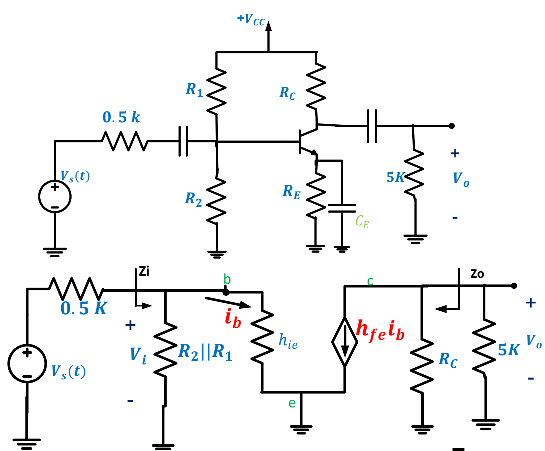
$$i_b = \frac{V_i}{h_{ie}}$$

$$V_i = \frac{Z_i}{Z_i + R_s} V_s$$

❖ $|A_v| = \frac{h_{fe}}{h_{ie}} \frac{Z_i}{Z_i + R_s} (R_C \backslash\backslash R_L)$

$$1 > \frac{Z_i}{Z_i + R_s} > 0.9$$

❖ $|A_v| = \frac{h_{fe}}{h_{ie}} (0.9) (R_C \backslash\backslash 5k)$



Zo = R_C

Z_i = R_{th} || hie = 5k

Let $g_m = \frac{h_{fe}}{h_{ie}} = \frac{ICQ}{VT} = \frac{1}{25.69m} = 38.92\ I_{CQ}$

$$h_{ie} = \frac{\beta V_T}{I_C} = \frac{h_{fe} V_T}{I_C}$$

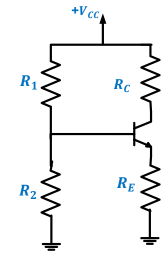
❖ $|A_v| = (g_m)(0.9)(R_C \backslash\backslash 5k) \geq 200$

Its specified that $Z_o = 8\ k$

$\therefore\ R_C = 8k,$ then $g_m \geq \frac{200}{0.9(3k ll 8k)} = 72.2$

Let $g_m = 77.86$, then $I_{CQ} = 2mA$

Since $V_{RC} = 16V$; let $V_{CC} = 30V$



Let $V_{RE} = \frac{V_{CC}}{5} = 6volt$

$R_E = \frac{V_{RE}}{I_E} = 3k\Omega$

Z_i = R_{th} || hie = 5k

$h_{ie} = \frac{\beta V_T}{I_C}$ = 6.165K $\therefore R_{th}$ = 26.45k

From : $I_E = \frac{V_{th} - V_{BE}}{\frac{R_{th}}{\beta+1} + R_E}$

$V_{th} = 6.81\ volt$

$$V_{th} = \frac{R_2}{R_1 + R_2} V_{CC}$$

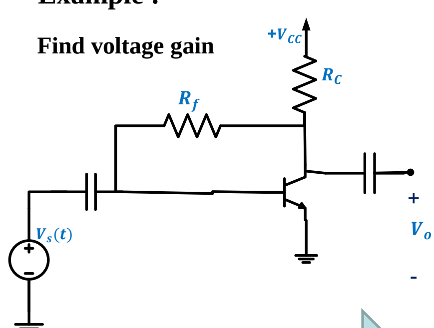
$$R_{th} = \frac{R_1 R_2}{R_1 + R_2}$$

➢ $R_1 = 34.22k$

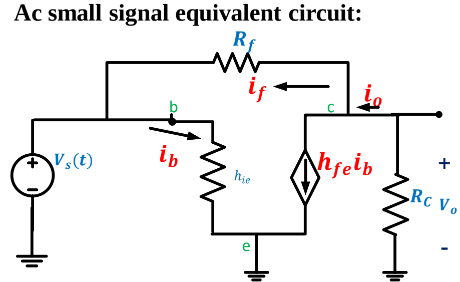
➢ $R_2 = 116.5.6k$

Example :

Find voltage gain



Ac small signal equivalent circuit:



$V_o = -R_C i_o$

$i_o = h_{fe} i_b + i_f$

$i_f = \frac{V_o - V_s}{R_f}$

$$i_b = \frac{V_s}{h_{ie}}$$

$$A_v = -\frac{\frac{R_C}{R_E} - R_C \frac{h_{fe}}{h_{ie}}}{1 + \frac{R_C}{R_E}}$$