السير ، والتقدم والسبق إلى الله سبحانه انما هو بالهمم وصدق الرغبة والعزيمة ، فيتقدم صاحب الهمة ، مع سكونه ، صاحب العمل الكثير بمراحل)[1]

pole- zero placement method

geometric Interpretation pole/zero location :-

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{r=0}^{N} a_r z^{-r}},$$

$H(z)$ in pole/zero form $H(z) = \frac{b_0}{a_0} \frac{\prod_{k=0}^{M}(1-c_k z^{-1})}{\prod_{k=0}^{N}(1-d_k z^{-1})}$

stable system means that the ROC of $H(z)$ includes $z=1$.

The frequency Response

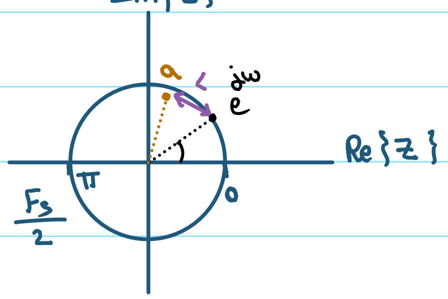
$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{b_0}{a_0} \frac{\prod_{k=0}^{M}(1-c_k e^{-j\omega})}{\prod_{k=0}^{N}(1-d_k e^{-j\omega})}$$

The magnitude Response

$$|H(e^{j\omega})| = \frac{|b_0|}{|a_0|} \left|e^{-j\omega(M-N)}\right|$$

$$= \left|\frac{b_0}{a_0}\right| \frac{\prod_{k=0}^{M}(e^{j\omega}-c_k)}{\prod_{k=0}^{N}(e^{j\omega}-d_k)}$$

Note the the magnitude $|H(e^{j\omega})|$ depends on term $|e^{j\omega}-a|$



* $e^{j\omega}-a$ is a vector Connecting $e^{j\omega}$ to a.
* $|L| = |e^{j\omega}-a|$ is the length of the vector L.

$$|H(e^{j\omega})| = \left|\frac{b_0}{a_0}\right| \frac{\prod_{k=0}^{M}(e^{j\omega}-c_k)}{\prod_{k=0}^{N}(e^{j\omega}-d_k)} = \left|\frac{b_0}{a_0}\right| \frac{\prod^{M} \text{"Distance from } e^{j\omega} \text{ to zeros"}}{\prod^{N} \text{"Distance from } e^{j\omega} \text{ to poles"}}$$

* when $e^{j\omega}$ close to a zero, $|H(e^{j\omega})|$ is "small".

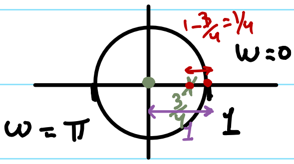
* when $e^{j\omega}$ close to a pole, $|H(e^{j\omega})|$ is "large"

Example 8: let $H(z) = \dfrac{1}{1-\frac{3}{4}z^{-1}}$, sketch $|H(e^{j\omega})|$, Evaluate $|H(e^{j\omega})|$ at $\omega = 0, \pi/4, \pi$.

$$H(z) = \frac{z}{z-\frac{3}{4}}$$, Zeros $= 0$

Poles $= \frac{3}{4}$

[1] at $\omega = 0$



* magnitude from $e^{j\omega}$ to the pole

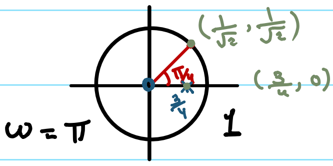
* the distance between 1 and $\frac{3}{4}$ is $1/4$

$$1-\frac{3}{4} = 1/4$$

* distance from $\omega=0$ to zero $= 1$

$$|H(e^{j\omega})| = \frac{1}{1/4} = 4$$

[2] at $\omega = \pi/4$

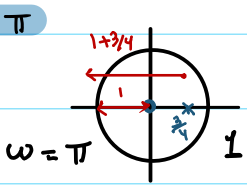


$r = \sqrt{(\frac{3}{4}-\frac{1}{\sqrt{2}})^2+(\frac{1}{\sqrt{2}})^2}$

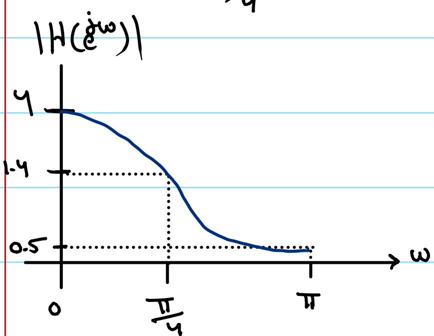
≈ 0.708

$$|H(e^{j\omega})| = \frac{\text{Distance from } e^{j\omega} \text{ to zero}}{\text{Distance from } e^{j\omega} \text{ to poles}} = \frac{1}{\approx 1/\sqrt{2}} = \sqrt{2} = 1.4$$

[3] at $\omega = \pi$



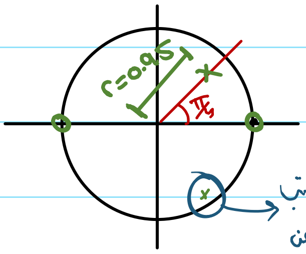
$$|H(e^{j\omega})| = \frac{1}{7/4} = 4/7 = 0.57$$



Example: $H(z) = \dfrac{1 - z^{-2}}{(1 - 0.95 e^{j\pi/4} z^{-1})(1 - 0.95 e^{-j\pi/4} z^{-1})}$, Find $|H(e^{jw})|$ at $w = 0$, $w = \pi/4$, $w = \pi$

Zeros $\rightarrow$ $z = \mp 1$

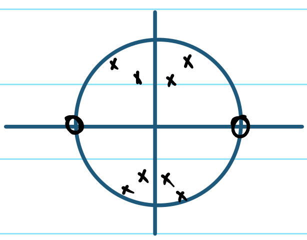
Poles $\rightarrow$ $z = 0.95 e^{j\pi/4}$, $z = 0.95 e^{-j\pi/4}$

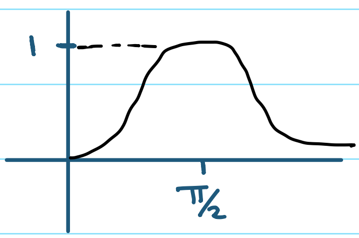


هون ما بنكتب *
لإنه عبارة عن
انعكاس للي فوقه

1. $w = 0$, Magnitude $= 0$
2. $w = \pi$, Magnitude $= 0$
3. $w = \pi/4$, Mag. $= \dfrac{\quad}{0.05}$

Example: Infer filter charectaristics from pole/zero.





filters design by poles zero placement

- When a Zero is placed at a given point on the Z-plane, the frequency Response will be Zero at the Corresponding point.

- A pole in the other hand produces a peak at the Corresponding frequency point.

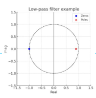
* Zeros ← مكان الفلتر "يخنق" أو يقلل الإشارة في هذا التردد تحديدًا

* poles ← مكان الفلتر "يضخم" أو يقوّي الإشارة في ذلك التردد.

* Unite Circle is most important

* الأقطاب القريبة جدًا من الدائرة الوحدية ← تنتج قمم كبيرة في استجابة التردد.

* الأصفار القريبة أو الموجودة على الدائرة الوحدية تنتج انخفاضات أو قيعان في استجابة التردد.



- الهدف من اختيار أماكن الأقطاب والأصفار ← نقدر نصمم فلتر
- كل فلتر بيختلف عن التاني بوجود أماكن الأصفار والأقطاب

A Narrowband Bandpass Filter

"Resonator".

- In general :- allows frequencies with in a Certain Range (band) to pass and Blocks frequencies outside this Range.
- A Narrow band Pass filter :- works the Same way, but the allowed Range is very narrow.

in other words, instead of allowing a wide Range of frequencies, it selects a small slice of frequencies and blocks all the Rest.

Features:

- passes only the Frequencies Very close to a Central frequency f_0
- Blocks frequencies far from f_0, whether they are higher or lower.
- Has high Selectivity due to the narrow band width.

On the Graph:

If you plot the Frequency Response :-

- you will see a peak at f_0 (the Center frequency).
- the width at half of the peak (Band width) is very Small.
- The shape looks like a thin mountain or spike in the middle of the axis.

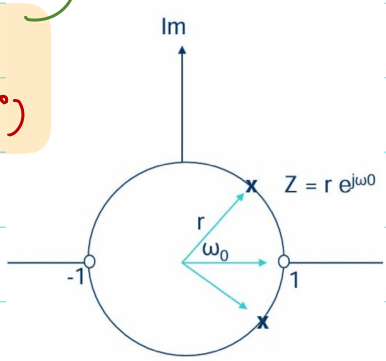
In pole-zero placement

To design a Narrow BPF :-

1. place poles close to the unite Circle at the desired frequency f_0, so they boost only the Region around it.
2. place Zeros on or near the unit Circle at other locations to Cancel unwanted frequencies.
3. the closer poles are to the unit Circle, the narrower and taller the peak becomes Resulting in a narrower band width filter. غير معطى في الامتحان

لازم تكوني حافظتيه

$$H(z) = \frac{K(z-1)(z+1)}{(z-re^{j\omega_0})(z-re^{-j\omega_0})} = \frac{K(z^2-1)}{(z-re^{j\omega_0})(z-re^{-j\omega_0})}$$



$$r = 1 - \frac{\Delta F}{F_s}\pi \quad , \quad 0.9 < r < 1$$

→ يتحكم في عرض ال Band width

كلما زاد r اقتربنا من فلتر ضيق النطاق

معطيات في الامتحان

"مش حفظ"!

$$K = \frac{(1-r)\sqrt{1-2r\cos 2\omega_0 + r^2}}{2|\sin\omega_0|}$$

K: is the gain factor.

$$\omega_0 = \frac{2\pi f_0}{F_s}$$

Examples:- Design a Second_order band pass filter using the pole_zero placement method and satisfying the following specifications :-

Sampling Rate = 8000 Hz

3_dB band width = 200 Hz

Passband Center frequency = 1000 Hz

Zero gain at Zero and 4000 Hz. $\longrightarrow$

* معلومة مش مهمة: لإننا
أصلاً عارفين إنه راح
يكون في zeros عند $0, \pi$

Ans :-

$F_s = 8000 \quad , \quad \Delta f = 200 \quad , \quad f_0 = 1000$

$$H(z) = \frac{K(z^2-1)}{(z - re^{j\omega})(z - re^{-j\omega})}$$

$$\omega = \frac{2\pi f_0}{F_s} = \frac{\cancel{2}\pi(1\cancel{000})}{4\,\cancel{8}\cancel{000}} = \pi/4$$

$$r = 1 - \frac{\Delta F}{F_s}\pi = 1 - \frac{200}{8000}\pi = 0.9214$$

$$K = \frac{(1-r)\sqrt{1-2r\cos 2\omega_0 + r^2}}{2\sin\omega_0} = \frac{(1-0.9214)\sqrt{1-(2)(0.9214)\cos\pi/2 + (0.9214)^2}}{2\sin(\pi/4)}$$

$$K = \frac{(0.0786)(1.359)}{0.3535} = 0.0755$$

A narrowband Band Stop filter

→ Also, Called Notch filter (Band Reject).

Definition: -

- Notch filter :- is a type of narrowband band Stop filter.
- it Removes or greatly attenuates one specific frequency (or avery narrow Range around it) while allowing all other frequencies to pass almost unaffected.

Frequency Response :

- the frequency Response is flat (high gain) for most frequencies.
- At the notch frequency f_0. there is a sharp dip to very low gain (ideally Zero).
- This "V-shaped" dip is very narrow for a nallowband notch filter.

Design in Digital Filters:

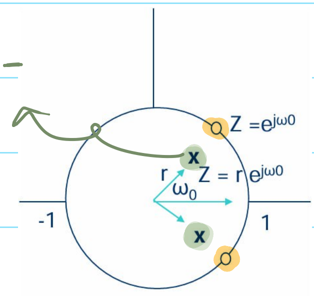
1. place Zeros on the unit Circle at the angle Corresponding f_0.

$$\omega_0 = \frac{2\pi f_0}{f_s}$$

2. Place poles just inside the unit Circle at the same angle
   - the Zeros Completly Remove f_0.
   - The poles Control the bandWidth of the notch
     - poles closer to the unit Circle → narrower notch.
     - poles further away → wider notch.

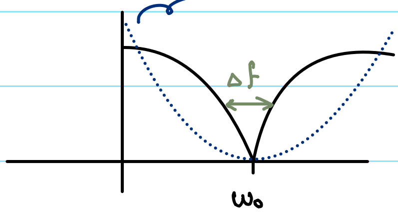
- ليش بنحط poles قريب من الـ Zero's ؟

لأنه احنا بدنا الفلتر ماياخذ f_0 فقط فعشان نقلل تأثير الـ Zeros على frequencies القريبة من الـ Zero's ..



$$H(z) = \frac{K(z-e^{j\omega})(z-e^{-j\omega})}{(z-re^{j\omega})(z-re^{-j\omega})}$$

without poles !!



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$$r = 1 - \frac{\Delta F}{F_s}\pi \quad , \quad 0.9 < r < 1$$

↳ يتحكم في عرض الـBand width

كلما زاد r اقتربنا من فلتر ضيق النطاق

معطيات في الامتحان

« مش حفظ »

$$K = \frac{|1 - 2r\cos w_0 + r^2|}{2|1 - \cos w_0|}$$

K: is the gain factor.

$$w_0 = \frac{2\pi f_0}{F_s}$$

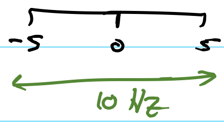
Example: ① obtain the Pole-Zero placement method,

② the transfer function of a sample digital Notch filter

Notch frequency :- 50 Hz.

3dB width of the Notch $= \mp 5$ Hz $\equiv$ 10 Hz

Sampling frequency :- 500 Hz

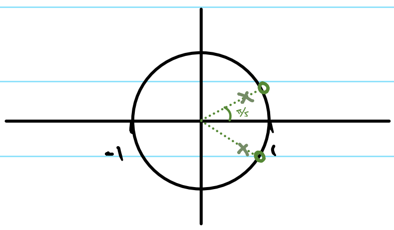


① $$H(z) = \frac{K(z - e^{jw})(z - e^{-jw})}{(z - re^{jw})(z - re^{-jw})}$$, $w_0 = \frac{2\pi(50)}{500} = \pi/5$

* $$r = 1 - \frac{\Delta f}{f_s}\pi = 1 - \frac{10}{500}\pi = 0.937$$

* $$K = \frac{1 - 2r\cos w_0 + r^2}{2|1 - \cos w_0|} = \frac{1 - 2(0.937)\cos(\pi/5) + (0.937)^2}{2\ |1 - \cos \pi/5|} = \frac{4.08\times10^{-3}}{1.2\times10^{-4}} = 34$$

* $$H(z) = \frac{(34)(z - e^{j\pi/5})(z - e^{-j\pi/5})}{(z - 0.37e^{j\pi/5})(z - 0.37e^{-j\pi/5})}$$



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Example:- Design a second-order Notch Filter using pole-zero placement method :-

$F_s = 8000\ Hz$

$3dB\_BW = 100\ Hz \rightarrow \Delta F = 100\ Hz$

Stop Band Center freq. $F_0 = 1500\ Hz$.

$$\omega = \frac{2\pi f_0}{F_s} = \frac{2\pi(1500)}{8000} = 1.178$$

$$H(z) = \frac{K(z - e^{j1.178})(z - e^{-j1.178})}{(z - re^{j1.178})(z - re^{-j1.178})}$$

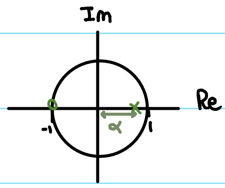
$$r = 1 - \frac{100}{8000}\pi = 0.9607$$

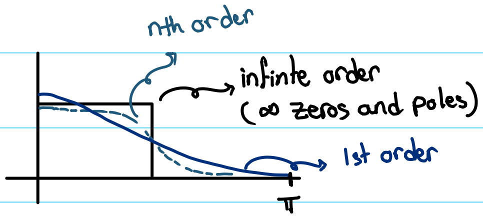
$$K = \frac{1 - 2(0.9607)\cos(1.178) + (0.9607)^2}{2\,|1 - \cos(1.178)|} = -806.78$$

من بكى بصدقه لموت الشهداء
ثم لم يتحرك بقوة
وصدق وعزيمة لإصلاح لنفسه
وإصلاح بلاده
← فحزنه كاذب. كاذب، كاذب

first order low pass filter

- A wide Band filter.
- pass frequency Components from 0 to F_c (cut off frequency).
- place a zero on the unit Circle at $Z = -1$.
- place a pole on the Real axis and inside the unit Circle





$$H(z)_{LPF} = \frac{K(Z+1)}{(Z-\alpha)}$$

$$K = \frac{1-a}{2}$$

$$\alpha = \begin{cases} 1-2\pi\left(\frac{F_c}{F_s}\right), & F_c < \left(\frac{F_s}{4}\right) \\ \pi-1-2\pi\left(\frac{F_c}{F_s}\right), & F_c > \frac{F_s}{4} \end{cases}$$

Example :- Design a first order low pass filter using the pole_zero placement method and satisfying the following specifications :- Sampling Rate = 8000 Hz
3_dB Cutoff frequency = 100 Hz.
Zero gain at 4000 Hz.

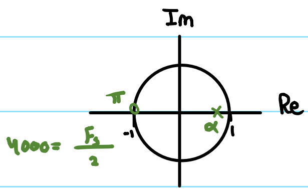
$f_c = 100$, $f_s = 8000$ Hz

$$H(z) = \frac{K(Z+1)}{(Z-\alpha)} = \frac{0.039(Z+1)}{(Z-0.921)}$$

$$\frac{F_s}{4} = \frac{8000}{4} = 2000 \Rightarrow \quad 100 < 2000 \rightarrow 1$$

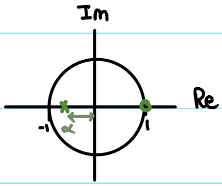
$$\alpha = 1 - 2\pi\left[\frac{100}{8000}\right] = 0.921$$

$$K = \frac{1-0.921}{2} = 0.039$$



First Order High pass filter

- A wide Band filter.
- Suppress frequency Components from 0 to F_c the Cut off frequency.
- place a Zero on the unit Circle at $Z = 1$
- place a pole on the Real axis and inside the unit circle.



$$\underset{LPF}{H(Z)} = \frac{K(Z-1)}{Z+\alpha}$$

$$K = \frac{1+a}{2}$$

$$\alpha = \begin{cases} 1 - 2\pi\left(\frac{F_c}{F_s}\right) & , F_c < \left(\frac{F_s}{4}\right) \\ \pi - 1 - 2\pi\left(\frac{F_c}{F_s}\right) & , F_c > \frac{F_s}{4} \end{cases}$$

Example :- Design a first_order highpass filter using the pole zero placement method and satisfying the following specifications :-

Sampling Rate = 8000 Hz

3-dB cut off frequency = 3800 Hz

Zero gain at Zero Hz.

$F_s = 8000$ Hz

$f_c = 3800$ Hz

3800 > 2000

$$\alpha = \pi - 1 - 2\pi\left(\frac{3800}{8000}\right)$$

$$= -0.842$$

$$K = \frac{1-0.842}{2} = 0.078$$

$$H(Z) = \frac{0.078(Z-1)}{Z+0.842}$$

ربنا تقبل منا إنك
أنت السميع العليم ..