

$$46) a_n = \frac{\sin^2 n}{2^n}$$

$$0 \leq \frac{\sin^2 n}{2^n} \leq \frac{1}{2^n}$$

10.1

1

$$\lim_{n \rightarrow \infty} \frac{\sin^2 n}{2^n} = 0 \quad \text{conv}$$

$$\lim_{n \rightarrow \infty} \left(\frac{1}{2}\right)^n = 0$$

$$59) a_n = \frac{\ln n}{n^{\frac{1}{n}}}$$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n^{\frac{1}{n}}} = \frac{\infty}{1} = \infty \quad \text{div}$$

Haykram Stekadete

$$68) a_n = \ln \left(1 + \frac{1}{n}\right)^n$$

$$\ln \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right) = \ln e = 1 \quad \text{conv}$$

$$84) a_n = \sqrt[n]{n^2 + n}$$

$$\lim_{n \rightarrow \infty} e^{\frac{\ln(n^2 + n)}{n}} = e^{\lim_{n \rightarrow \infty} \frac{2n+1}{n^2+n}} = e^0 = 1 \quad \text{conv}$$

$$6) a_n = \frac{2^n - 1}{2^n}$$

$$\cancel{a_n = \left(\frac{1}{2}\right)^n}$$

$$a_1 = \frac{2-1}{2} = \frac{1}{2}$$

$$a_2 = \frac{3}{4}$$

$$a_4 = \frac{15}{16}$$

$$a_3 = \frac{7}{8}$$

$$a_1 = -2, \quad a_{n+1} = \frac{n a_n}{(n+1)}$$

2

10)

$$a_2 = \frac{-2}{2} = -1$$

$$a_6 = \frac{-2}{6} = -\frac{1}{3}$$

$$a_{10} = \frac{-2}{10} = -\frac{1}{5}$$

$$a_3 = \frac{-2}{3}$$

$$a_7 = \frac{-2}{7}$$

$$a_4 = \frac{-2}{4} = -\frac{1}{2}$$

$$a_8 = \frac{-2}{8} = -\frac{1}{4}$$

$$a_5 = \frac{-2}{5}$$

$$a_9 = \frac{-2}{9}$$

22) 2, 6, 10, 14, ...

$$a_n = 4n - 2, \quad n = 1, 2, \dots$$

26) 0, 1, 1, 2, 2, 3, 3, 4, ...

~~$a_n = \lfloor \frac{n}{2} \rfloor$~~ $a_n = \lfloor \frac{n}{2} \rfloor \quad n = 1, 2, 3, \dots$

31) $a_n = \frac{1 - 5n^4}{n^4 + 8n^3}$

$$\lim_{n \rightarrow \infty} \frac{1 - 5n^4}{n^4 + 8n^3} = \frac{5}{1} = 5 \quad \text{conv}$$

35) $a_n = 1 + (-1)^n$

$$\lim_{n \rightarrow \infty} 1 + (-1)^n \rightarrow \text{does not exist}$$

div

$$a_1 = 0$$

$$a_2 = 1 + 1 = 2$$

$$a_3 = 1 - 1 = 0$$

$$a_4 = 1 + 1 = 2$$

$$a_5 = 1 - 1 = 0$$

$$36) a_n = (-1)^n \left(1 - \frac{1}{n}\right)$$

$$a_1 = 0$$

$$a_2 = \frac{1}{2}$$

$$a_3 = -\frac{2}{3}$$

$$a_4 = \frac{3}{4}$$

$$\lim_{n \rightarrow \infty} (-1)^n \left(1 - \frac{1}{n}\right)$$

does not exist

div

$$41) a_n = \sqrt{\frac{2n}{n+1}}$$

$$= \sqrt{\lim_{n \rightarrow \infty} \frac{2n}{n+1}} = \sqrt{2} \quad \text{conv}$$

$$44) a_n = n\pi \cos(n\pi)$$

$$a_n = n\pi (-1)^n$$

$$\lim_{n \rightarrow \infty} n\pi (-1)^n \quad \text{does not exist}$$

div

$$48) a_n = \frac{3^n}{n^3}$$

$$\lim_{n \rightarrow \infty} \frac{3^n \ln 3}{3n^2} = \lim_{n \rightarrow \infty} \frac{3^n (\ln 3)^2}{6n} = \lim_{n \rightarrow \infty} \frac{3^n (\ln 3)^3}{6} = \infty \quad \text{div}$$

$$50) a_n = \frac{\ln n}{\ln 2n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{2n}{2} = 1 \quad \text{conv}$$

$$a_n = \left(1 - \frac{1}{n}\right)^n$$

$$57] \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = e^{-1} = \frac{1}{e}$$

$$58] a_n = (n+4)^{\frac{1}{(n+4)}}$$

$$\lim_{x \rightarrow \infty} x^{\frac{1}{x}} = 1 = \lim_{n \rightarrow \infty} (n+4)^{\frac{1}{(n+4)}} = 1 \text{ conv}$$

$$60] a_n = \ln n - \ln(n+1)$$

$$\lim_{n \rightarrow \infty} \ln\left(\frac{n}{n+1}\right) = \ln\left(\lim_{n \rightarrow \infty} \frac{n}{n+1}\right) = \ln 1 = 0 \text{ conv}$$

$$63] a_n = \frac{n!}{n^n}$$

$$\frac{n!}{n^n} \leq \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\therefore \lim_{n \rightarrow \infty} \frac{n!}{n^n} = 0 \text{ conv}$$

$$70] a_n = \left(\frac{n}{n+1}\right)^n$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1}\right)^n$$

$$\lim_{u \rightarrow \infty} \left(1 - \frac{1}{u}\right)^{u-1}$$

$$\lim_{u \rightarrow \infty} \left(1 - \frac{1}{u}\right)^u \left(1 - \frac{1}{u}\right)^{-1} = e^{-1} = \frac{1}{e}$$

$$u = n+1$$

or

$$e^{\lim_{n \rightarrow \infty} \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}}}$$

$$e^{\lim_{n \rightarrow \infty} \frac{\frac{1}{(n+1)^2}}{\frac{1}{n} - \frac{1}{n+1}}}$$

$$e^{\lim_{n \rightarrow \infty} \frac{n}{-n-1}} = e^{-1} = \frac{1}{e} \text{ conv}$$

$$72] a_n = \left(1 - \frac{1}{n^2}\right)^n$$

$$u = n^2$$

$$\lim_{u \rightarrow \infty} \left(1 - \frac{1}{u}\right)^{u^{\frac{1}{2}}}$$

$$\lim_{u \rightarrow \infty} \left(1 - \frac{1}{u}\right)^u$$

$$e^{\lim_{n \rightarrow \infty} \frac{\ln\left(1 - \frac{1}{n^2}\right)}{\frac{1}{n}}} = e^{\lim_{n \rightarrow \infty} \frac{\frac{2}{n^3} \cdot \frac{n^2}{n^2-1} \cdot \frac{n^2}{-1}}{\frac{1}{n}}} = e^{\lim_{n \rightarrow \infty} \frac{2n}{-n^2+1}} = e^0 = 1 \text{ conv}$$

$$76] a_n = \sinh(\ln n)$$

$$a_n = \frac{e^{\ln n} - e^{-\ln n}}{2}$$

$$a_n = \frac{n - \frac{1}{n}}{2}$$

$$\lim_{n \rightarrow \infty} a_n = \infty \text{ div}$$

$$82] a_n = \frac{1}{\sqrt{n}} \tanh^{-1} n$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} 0 \cdot \frac{\pi}{2} = 0 \text{ conv}$$

$$86] a_n = \frac{(\ln n)^5}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{5}{n} \cdot \frac{2\sqrt{n}}{1}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{10}{\sqrt{n}} = 0 \text{ conv}$$

92] $a_1 = -1$, $a_{n+1} = \frac{a_n + 6}{a_n + 2}$

6

$a_2 = 5$

$a_3 = \frac{11}{7}$

$a_4 =$

$\lim_{n \rightarrow \infty} a_n = L \Rightarrow \text{conv}$

$\lim_{n \rightarrow \infty} \frac{a_n + 6}{a_n + 2} = L$

$\lim_{n \rightarrow \infty} \frac{L + 6}{L + 2} = L$

$L(L + 2) = L + 6$

$L^2 + 2L - L - 6 = 0$

$L^2 + L - 6 = 0$

$(L - 2)(L + 3) = 0$

$L = 2$

or $L = -3$

$a_n > 0 \therefore L = 2$

111] $a_n = \frac{3n+1}{n+1}$

$a_1 = \frac{4}{2} = 2$

$a_2 = \frac{7}{3}$

$a_3 = \frac{10}{4} = \frac{5}{2}$

$a_4 = \frac{13}{5}$

$M = 3, 4, 5$

$m = 1, 0, -1$

non decreasing

bounded from above