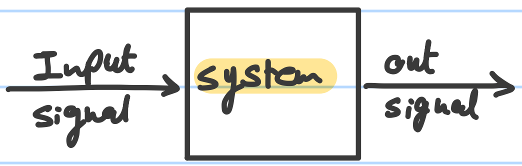
chapter

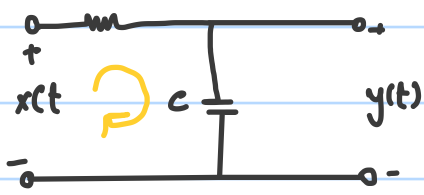
2

Chapter 2: Systems



→ Discrete signal ⟹ Discrete system.

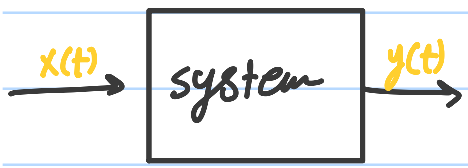
→ cont. signal ⟹ cont. system.



$$RC\frac{\partial y(t)}{\partial t} + y(t) = x(t)$$

Properties of the system:-

① Fixed (time invariant) and (time variant)



$$\frac{\partial y(t)}{\partial t} + t\,y(t) = x(t)$$

→ To check if the system is fixed or not

- delay of time → على اي (t) في العلاقة

$$\frac{\partial y_1(t-t_0)}{\partial t} + (t-t_0)\,y_1(t-t_0) = x_1(t-t_0) \rightarrow Eq_1$$

- delay function of time → فقط داخل x او y

$$\frac{\partial y_2(t-t_0)}{\partial t} + t\,y_2(t-t_0) = x_2(t-t_0) \rightarrow Eq_2$$

"بعدها بعمل مقارنة"

$Eq_1 \neq Eq_2 \Rightarrow$ not fixed (time variant)

Ex. $y(t) = x(t)u(t)$

① delay of time:

$y(t-t_0) = x((t-t_0)^2)\,u(t-t_0) \rightarrow Eq_1$

② delay fun. of time

$y(t-t_0) = x(t^2-t_0)\,u(t) \rightarrow Eq_2$

$Eq_1 \neq Eq_2 \Rightarrow$ not fixd (time variant)

$y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$

① delay of time:

$y(t-t_0) = \int_{-\infty}^{2(t-t_0)} x(\tau)\,d\tau \rightarrow Eq_1$

② delay fun. of time

$y(t-t_0) = \int_{-\infty}^{2t-t_0} x(\tau)\,d\tau \rightarrow Eq_2$

$Eq_1 \neq Eq_2 \Rightarrow$ not fixd (time variant)

Ex. $y(t) = x(t^2)$

① delay of time:

$y(t-t_0) = x((t-t_0)^2) \rightarrow Eq_1$

② delay fun. of time

$y(t-t_0) = x(t^2-t_0) \rightarrow Eq_2$

$Eq_1 \neq Eq_2 \Rightarrow$ not fixd (time variant)

$y(t) = \cos(3t)\, x(t)$

① delay of time:

$y(t-t_0) = \cos(3(t-t_0))\, x(t-t_0) \rightarrow Eq_1$

② delay fun. of time

$y(t-t_0) = \cos(3t)\, x(t-t_0) \rightarrow Eq_2$

② Causal and non causal → بناء على تنبؤ

↳ النتيجة في ال output تعتمد على نتيجة سابقة

$y(t) = x(t^2)$

when $t < 1$ ⟹ $y(0.1) = x(0.1^2)$ ⟹ causal

$y(0.1)$ → present, $x(0.1^2)$ → past

when $t > 1$ ⟹ $y(2) = x(4)$ ⟹ non causal

$y(2)$ → present, $x(4)$ → future

⟶ non causal

$y(t) = x(t + t_0)$

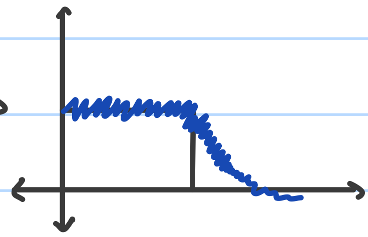
$y(t)$ → present, $x(t + t_0)$ → fut. ⟹ non causal

$y(t = x(t - t_0)$

$y(t$ → present, $x(t - t_0)$ → past ⟹ causal

$y(t) = \frac{x(t) + x(t - t_0) + x(t + t_0)}{3}$ ⟹ non causal ⟹

$y(t)$ → present, $x(t)$ → present, $x(t - t_0)$ → past, $x(t + t_0)$ → fut.



③ Dinamic (memory) and instan. (memory less) → القيم داخل الاقواس جميعها متساوية

اذا كان النظام inst ⇐ فهو causal

$y(t) = k\,x(t)$

$y(t)$ → present, $x(t)$ → present

$y(t) = x(t)x(t)$ ⟹ instan., memory les > causal

$x(t)$ → present, $x(t)$ → present

$y(t) = x(t+2) + x(t-2)$ ⟹ non causal * if all of them $= t+2$ ⟹ instant

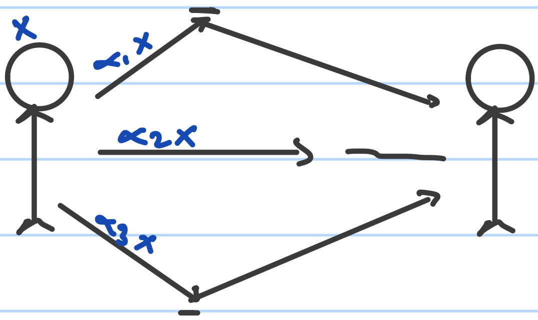
$y(t)$ → present, $x(t+2)$ → fut., $x(t-2)$ → past

$y(t) = x(t^2)$

$y(0) = x(0)$

$y(2) = x(4)$ => Dinamic

④ Linearity:



$$\frac{\alpha_4 x}{\alpha_1 x + \alpha_2 x + \alpha_3 x}$$

In System

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

To check the linearity

① scaling.

$$\alpha_1 \frac{dy(t)}{dt} + \alpha_1 y(t) = \alpha_1 x(t)$$

$$\alpha_2 \frac{dy(t)}{dt} + \alpha_2 y(t) = \alpha_2 x(t)$$

② $\pm$

$$\alpha_1 \frac{dy(t)}{dt} + \alpha_2 \frac{dy(t)}{dt} + \alpha_1 y(t) + \alpha_2 y(t) = \alpha_1 x(t) + \alpha_2 x(t)$$ => Eq1

③

$$\alpha_3 y(t) = \alpha_1 y(t) + \alpha_2 y(t)$$

$$\alpha_3 x(t) = \alpha_1 x(t) + \alpha_2 x(t)$$

$\alpha_3 \frac{dy(t)}{dt} + \alpha_3 y(t) = \alpha_3 x(t)$ => Eq_2

$$\underbrace{\alpha_1 \frac{dy(t)}{dt} + \alpha_2 \frac{dy(t)}{dt}}_{\alpha_3 \frac{dy(t)}{dt}} + \underbrace{\alpha_1 y(t) + \alpha_2 y(t)}_{\alpha_3 y_3(t)} = \underbrace{\alpha_1 x(t) + \alpha_2 x(t)}_{\alpha_3 x_3(t)}$$ => Eq_1

$Eq_1 = Eq_2$ => linear

Ex.

$y(t) = x^2(t)$

$\alpha_1 y(t) = \alpha_1 x^2(t)$

$\alpha_2 y(t) = \alpha_2 x^2(t)$

+ => $\alpha_1 y(t) + \alpha_2 y(t) = \alpha_1 x_1^2(t) + \alpha_2 x^2(t)$ => Eq_1

Now. let $\alpha_3 y(t) = \alpha_1 y(t) + \alpha_2 y(t)$

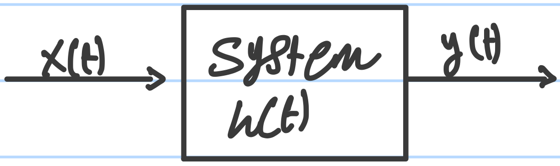
$\alpha_3 x(t) = \alpha_1 x(t) + \alpha_2 x(t)$

$\alpha_3 y(t) = \alpha_3 x^2(t)$

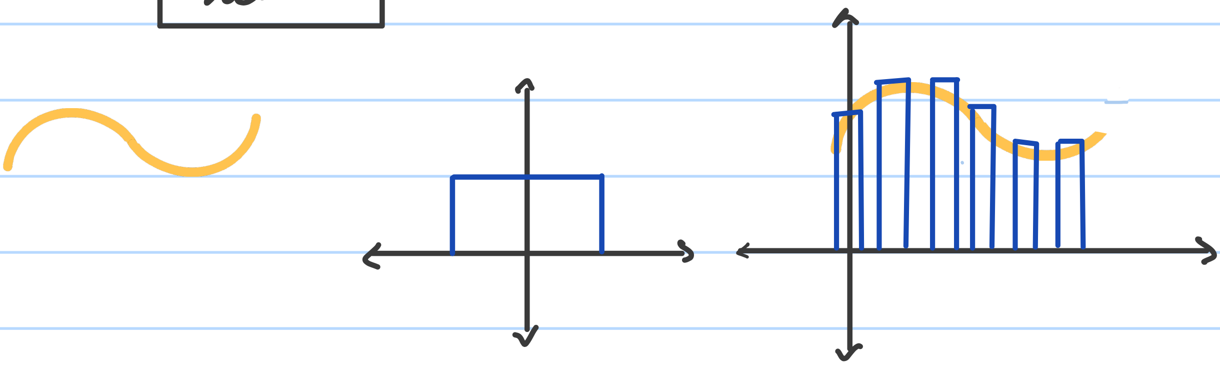
$\alpha_3 y(t) = \alpha_3 (\alpha_1 x(t) + \alpha_2 x(t))^2$ → Eq_2

$Eq_1 \neq Eq_2$ => non.linear

LTI: linear time invariant system



= Response of the system



$$y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

$$= x(t) * h(t)$$

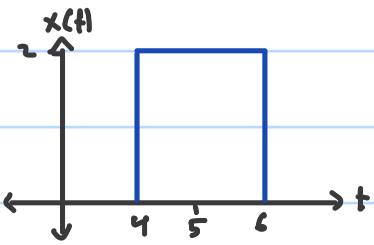
Ex. For LTI system if $x(t) = 2\Pi\left(\frac{t-5}{2}\right)$ and $h(t) = \Pi\left(\frac{t-2}{4}\right)$

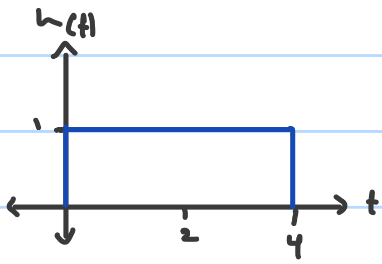
Find $y(t)$

Ans.

[1] For LTI system $y(t) = x(t) * h(t)$

[2]

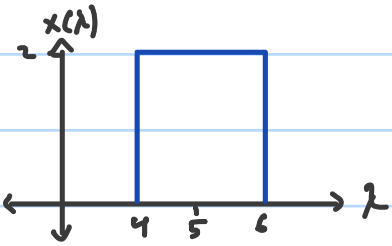


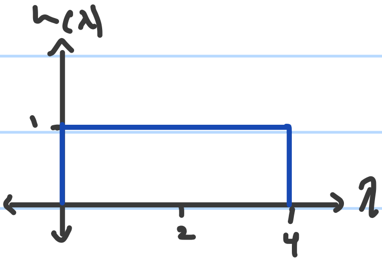


[3] $x(t) \Rightarrow [4, 6]$ $h(t) \Rightarrow [0, 4]$ $\Rightarrow [4+0, 4+4, 6+0, 6+4] = [4, 8, 6, 10]$

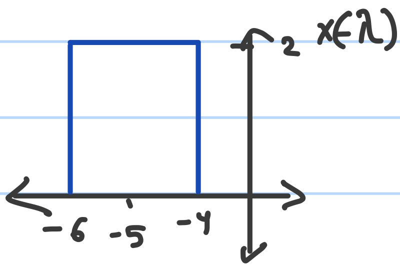
$\Rightarrow [4, 6, 8, 10]$

$\boxed{4}$

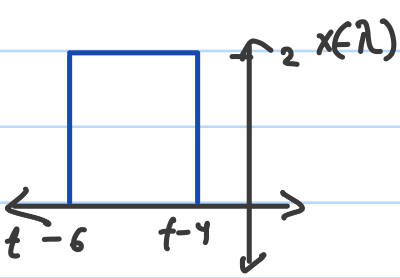




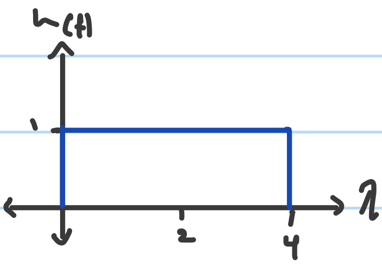
$\boxed{5}$ $y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$



→ بنقلب الاقتران حول محور الصادات

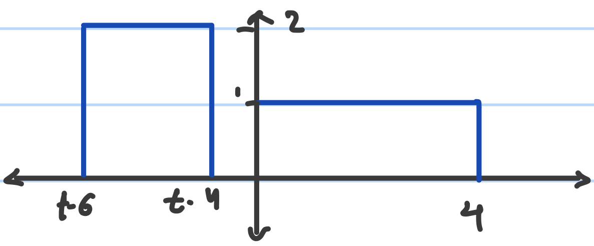


→ + t



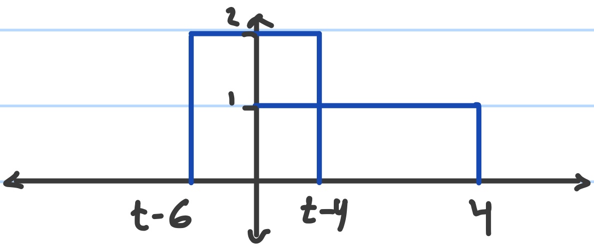
for $t<4$:

$y(t) = 0$



for $4 \le t < 6$

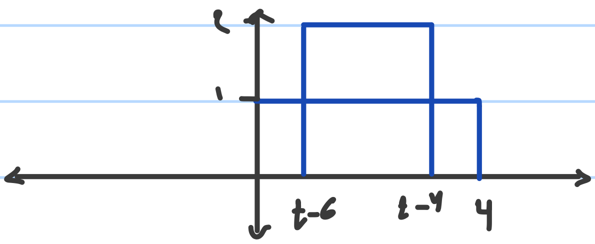
$\int_{0}^{t-4} (2)(1)\, d\lambda$



$2(t-4)$

for $6 \le t \le 8$

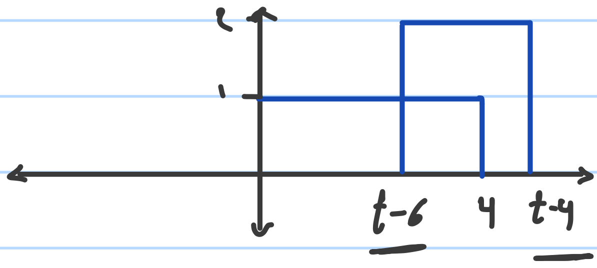
$$y(t) = \int_{t-6}^{t-4} (2)(1)\, d\lambda$$



$2\left[(t-4)-(t-6)\right]$

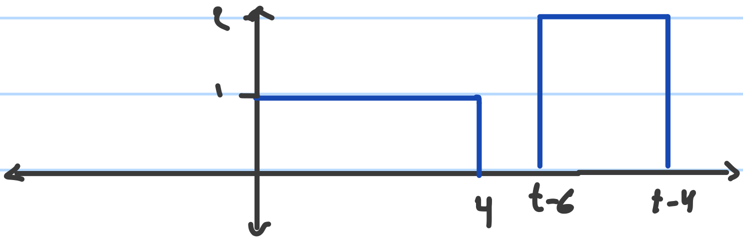
for $8 \le t < 10$

$$y(t) = \int_{t-6}^{4} (2)(1)\, d\lambda$$



$2\left[4-(t-6)\right]$

for $t > 10$

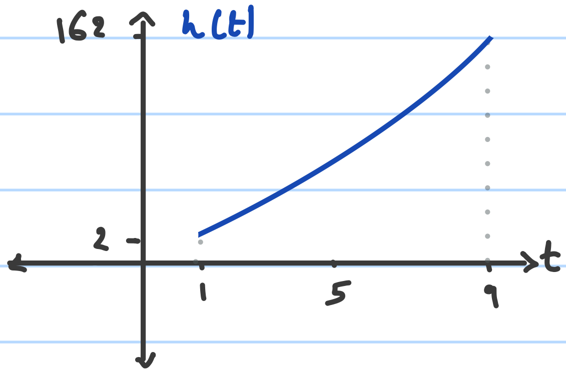


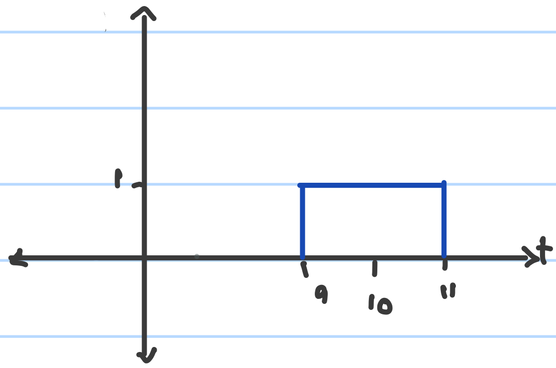
$y(t) = 0$

Ex. Consider LTI system with response $h(t) = 2t^2\,\Pi\left(\frac{t-5}{8}\right)$ and input $x(t) = \Pi\left(\frac{t-10}{2}\right)$

Ans.

For LTI system $\Rightarrow y(t) = x(t) * h(t)$

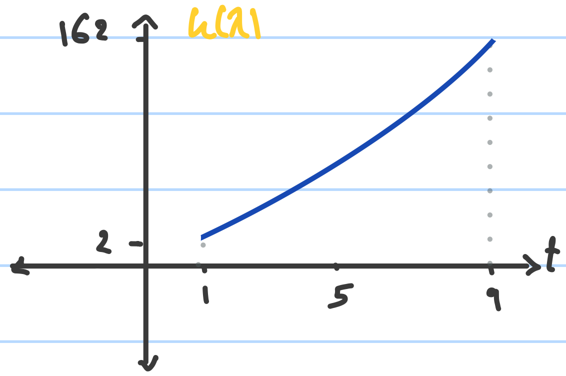


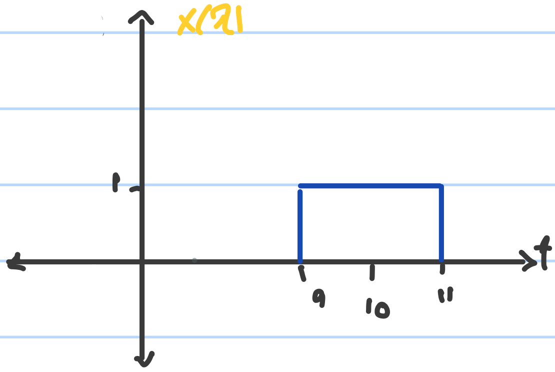


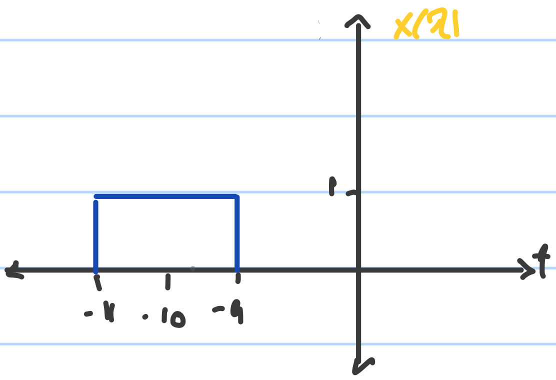
$h(t)$ [1,9] $x(t)$ [9,11]

$\Rightarrow$ [10,12,18,20]

$\Rightarrow y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\,h(\lambda)\,d\lambda$

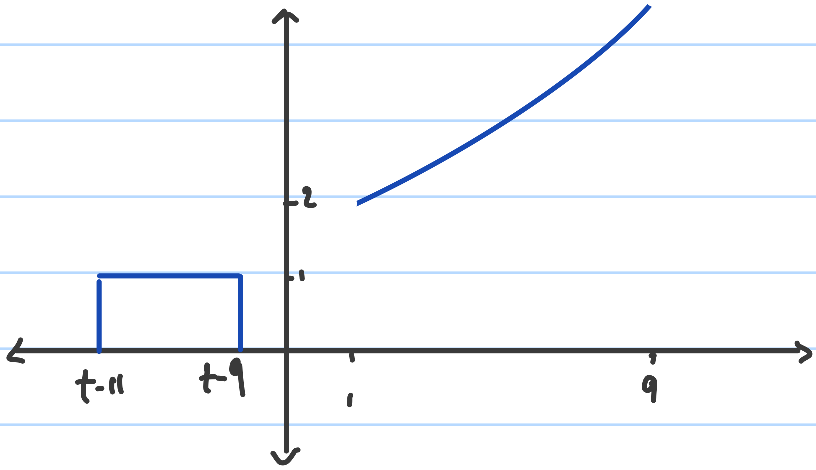






for $t < 10$

$y(t) = 0$

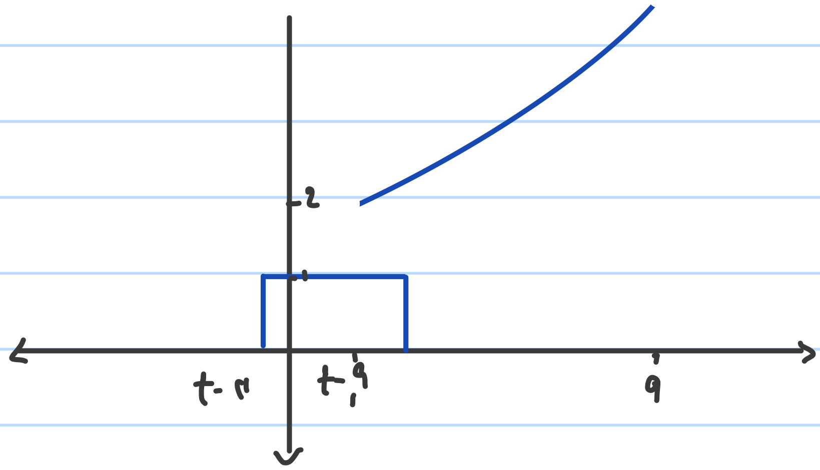


for $10 < t < 12$

$$y(t) = \int_{1}^{t-9} (1)(2\lambda^2)\, d\lambda$$

$$= \frac{2\lambda^3}{3} \Big|_{1}^{t-9}$$

$$= \frac{2}{3}\left[(t-9)^3 - (1)^3\right]$$

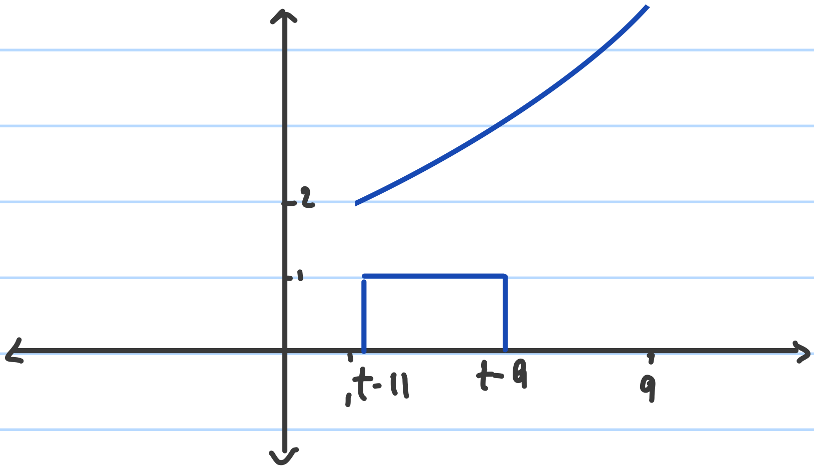


for $12 < t < 18$

$$y(t) = \int_{t-11}^{t-9} 2\lambda^2\, d\lambda$$

$$= \frac{2\lambda^3}{3} \Big|_{t-11}^{t-9}$$

$$= \frac{2}{3}\left[(t-9)^3 - (t-11)^3\right]$$

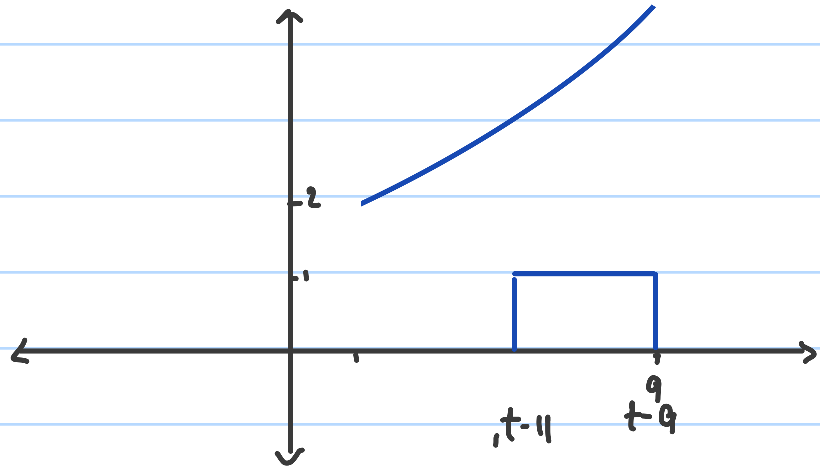


for $16 < t < 20$

$$y(t) = \int_{t-11}^{9} 2\lambda^2 \, d\lambda$$

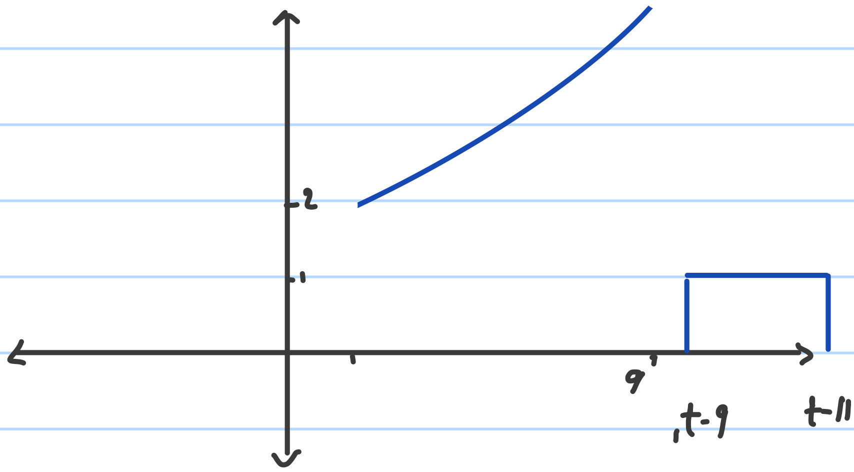
$$= \frac{2}{3}\lambda^3 \Big|_{t-11}^{9}$$

$$= \frac{2}{3}\left[81 - (t-11)^3\right]$$



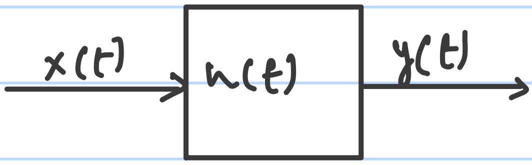
for $t > 20$

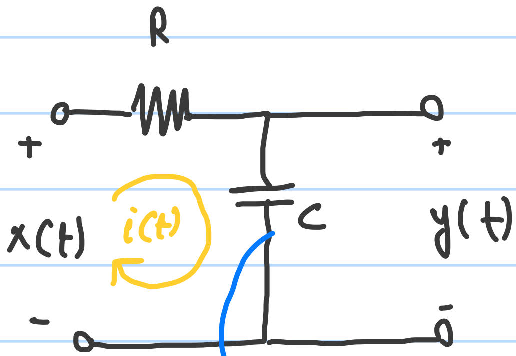
$$y(t) = 0$$



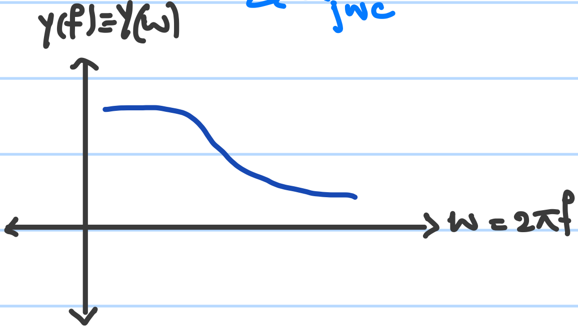
$$y(t) = \begin{cases} 0 & , t > 20 , t < 10 \\ \frac{2}{3}\left[(t-9)^3 - 1\right] & , 10 \leq t < 12 \\ \frac{2}{3}\left[(t-9)^3 - (t-11)^3\right] & , 12 \leq t < 18 \\ \frac{2}{3}\left[81 - (t-11)^3\right] & , 18 \leq t < 20 \end{cases}$$

Impulse response of LTI:





$Z_c = \frac{1}{jwC}$



ODE between (x(t), y(t)):

$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

$$-x(t) + R\,i(t) + y(t) = 0$$

$$i(t) = i_c(t) = C\frac{dV_c}{dt}$$

$$= C\frac{dy(t)}{dt}$$

$$-x(t) + RC\frac{dy(t)}{dt} + y(t) = 0$$

$$\Rightarrow RC\frac{dy(t)}{dt} + y(t) = x(t)$$

* احسن خيار ال capaci لأنه

ال Z = $\frac{1}{jwC}$ بالتالي

عند ال w تساوي صفر

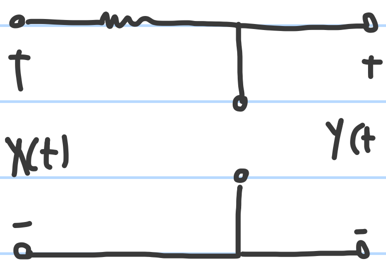
⇐ ∞ معناها O.C

في هاي الحالة y(t) = x(t)

عند w تساوي ∞

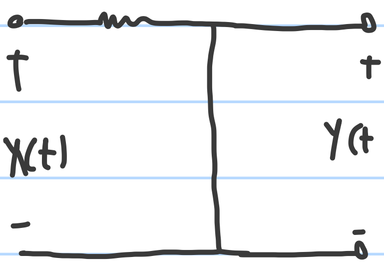
⇐ 0 معناها S.C

$\boxed{1}$ $w = 0$



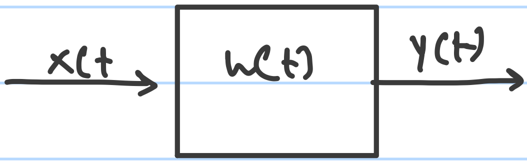
$\Rightarrow y(t) = x(t)$

$\boxed{2}$ $w \to \infty$



$\Rightarrow y(t) = 0$

For LTI system   Impulse fun. ⇐ input بنعتبر حجة ال



$y(t) = x(t) * h(t)$

$*$ IF $x(t) = \delta(t)$

$y(t) = \delta(t)\, h(t)$

$$= \int_{-\infty}^{\infty} \delta(t-\lambda)\, h(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} \delta(\lambda - t)\, h(\lambda)\, d\lambda \qquad \rightarrow \delta(-(\lambda - t)) \quad \text{even fun}$$

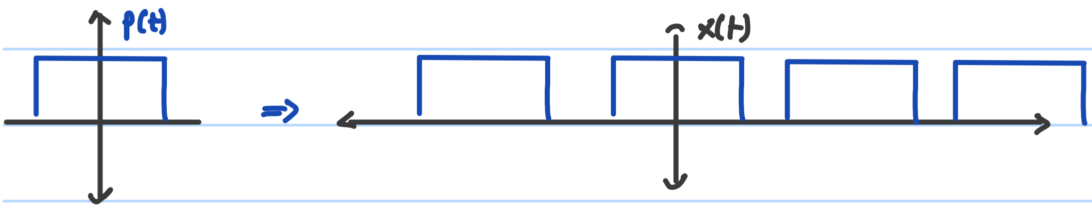
$= h(t)$

In general "Remember"

① $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$ "sampling theorem"

② $\int_{-t_1}^{t_2} x(t)\,\delta(t-t_0) = \begin{cases} x(t_0), & t_1 < t_0 < t_2 \\ 0, & o.w \end{cases}$ "Sifting theorem"

③ $x(t) * \delta(t-t_0) = x(t-t_0)$ "convolution theorem"



$x(t) = p(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$

when $n=0$

$x(t) = p(t) * \delta(t) = p(t)$

when $n=1$

$x(t) = p(t) * \delta(t - T_0) = p(t - T_0)$

Ex Evaluate $x(t)\,\delta(-2t+3)$, $x(t)=3t+1$

$x(t)\,\delta(\cancel{-}2(t-\frac{3}{2}))$ even fun.

$x(t)\cdot\frac{1}{2}\,\delta(t-\frac{3}{2})$

$= \frac{1}{2}\cdot x(\frac{3}{2})\cdot\delta(t-\frac{3}{2}) \quad = \frac{1}{2}\left[3\cdot\frac{3}{2}+1\right]\delta(t-\frac{3}{2})$

$= \frac{11}{4}\,\delta(t-\frac{3}{2})$

Ex. $x(t) * \delta(-2t+3)$ $\quad x(t)=3t+1$

$x(t)\,\delta(\cancel{-}2(t-\frac{3}{2}))$ even fun.

$x(t)\cdot\frac{1}{2}\,\delta(t-\frac{3}{2})$

$= \frac{1}{2}x(t-\frac{3}{2}) \quad = \boxed{\frac{1}{2}\left[3(t-\frac{3}{2})+1\right]}$

In our example $RC\frac{dy}{dt}+y(t)=x(t)$

1. For impulse response $x(t)=\delta(t)$, $y(t)=h(t)$

For LTI $y(t)=x(t)*h(t)$

$= \delta(t)*h(t) \Rightarrow t_0=0$

$= h(t)$

* نعتبر في حالة Impulse

2. $RC\frac{dh(t)}{dt}+h(t)=\delta(t)$

3. let $\lambda^n=\frac{d^n}{dt}$

$RC\,\lambda+1=0 \Rightarrow \lambda=\frac{-1}{RC}$

$h(t) = A e^{\lambda t} u(t)$

$= A e^{-t/RC} u(t) \Rightarrow$ let $h(t) = g(t)\, u(t)$

4 $\dfrac{d\,h(t)}{dt} = g(t)\,\delta(t) + \dot{g}(t)\,u(t)$

$\Rightarrow RC\left[g(t)\,\delta(t) + \dot{g}(t)\,u(t)\right] + g(t)\,u(t) = \delta(t)$

$= RC\,g(t)\,\delta(t) + RC\,\dot{g}(t)\,u(t) + g(t)\,u(t) = \delta(t)$

$= RC\,g(0)\,\delta(t) + RC\,\dot{g}(t)\,u(t) + g(t)\,u(t) = \delta(t)$

5 $RC\,g(0) = 1$

$RC \cdot A = 1$

$A = \dfrac{1}{RC}$

$y(t) = \dfrac{1}{RC} e^{-t/RC} u(t)$

Ex. Determine the response of the following LTI

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 18\,\dot{x}(t-2) \quad , \text{ where } x(t) = \delta(t)$$

For LTI system and impulse response

$x(t) = \delta(t) \quad , y(t) = h(t)$

$$\frac{d^2h(t)}{dt^2} + 6\frac{dh(t)}{dt} + 8h(t) = 18\,\dot{\delta}(t-2)$$

$$\frac{d^2h_1(t)}{dt^2} + 6\frac{dh_1(t)}{dt} + 8h_1(t) = 18\,\dot{\delta}(t) \rightarrow$$

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$$h(t) = h_1(t-2)$$

let $\lambda = \frac{d}{dt}$

$$\lambda^2 + 6\lambda + 8 = 0$$

$$(\lambda+4)(\lambda+2) \Rightarrow \lambda = -4 \ , \lambda = -2$$

$$h_1(t) = \left[Ae^{\lambda_1 t} + Be^{\lambda_2 t}\right]u(t)$$

$$h_1(t) = \left[Ae^{-4t} + Be^{-2t}\right]u(t) = g(t)\,u(t) \ ; \ g(t) = Ae^{-4t} + Be^{-2t}$$

$$\frac{dh_1(t)}{dt} = g(t)\delta(t) + \dot{g}(t)u(t)$$

$$\frac{d^2h_1(t)}{dt} = g(t)\dot{\delta}(t) + \dot{g}(t)\delta(t) + \dot{g}(t)\delta(t) + \ddot{g}(t)u(t)$$

$$\underbrace{g(t)\dot{\delta}(t) + \dot{g}(t)\delta(t) + \dot{g}(t)\delta(t) + \ddot{g}(t)u(t)}_{\frac{d^2h_1(t)}{dt}} + \underbrace{6\left[g(t)\delta(t) + \dot{g}(t)u(t)\right]}_{6\frac{dh_1(t)}{dt}} + \underbrace{8\left[g(t)u(t)\right]}_{8h_1(t)} = 18\,\dot{\delta}(t)$$

$$g(0)\dot{\delta}(t)+\dot{g}(0)\delta(t)+\dot{g}(0)\delta(t)+\ddot{g}(t)u(t)+6[g(0)\delta(t)+\dot{g}(t)u(t)]+8[g(t)u(t)]=18\dot{\delta}(t)$$

$g(0)=18$

$A+B=18 \rightarrow$ Eq 1

$2g(0)+6g(0)=0$

$=2[-4A-2B]+6[A+B]=0$

$=-2A+2B=0 \rightarrow$ Eq 2

$A=9,\ B=9$

$$h_1(t)=[9e^{-4t}+9e^{-2t}]u(t)$$

$* \ h(t)=h_1(t-2)$

$$\boxed{h(t)=[9e^{-4(t-2)}+9e^{-2(t-2)}]u(t-2)}$$

Super position Integral in terms of step response

$x(t) = u(t)$

Remember:-

- impulse response $\Rightarrow$ $x(t) = \delta(t)$
  $y(t) = h(t)$

- step response $\Rightarrow$ $x(t) = u(t)$
  $y_s(t) = ??$

- ramp response $\Rightarrow$ $y_R(t) = ??$

$$* \; y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} u(t-\lambda)\, h(\lambda)\, d\lambda$$

by using Integral by parts

$u = x(t-\lambda) \qquad dv = h(\lambda)\, d\lambda$

$du = -x(t-\lambda) \qquad v = \int h(\lambda)\, d\lambda = a(\lambda)$

$$= \cancel{-x(t-\lambda) \cdot a(t)}^{\,0} \Big|_{-\infty}^{\infty} + \underbrace{\int_{-\infty}^{\infty} \dot{x}(t-\lambda)\, a(\lambda)\, d\lambda} = a(t)$$

$$\int_{-\infty}^{\infty} \dot{x}(t-\lambda)\, a(\lambda)\, d\lambda = \int_{-\infty}^{\infty} \delta(t-\lambda)\, a(\lambda) \Rightarrow a(t) = \int_{-\infty} h(\lambda)\, d\lambda$$

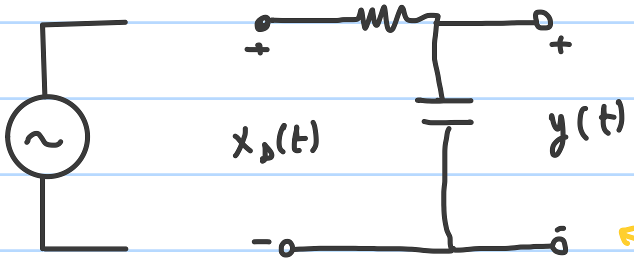
Note:-

- impulse response $\Rightarrow$ $x(t) = \delta(t)$
  $y(t) = h(t)$

- step response $\Rightarrow$ $x(t) = u(t)$
  $y_s(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$

- ramp response $\Rightarrow$ $x(t) = r(t)$
  $y_R(t) = \int_{-\infty}^{t} y_s(t)\, d\lambda$

Ex. Find the response of the RC circuit shown below of triangle signal.



$x_\Delta(t) = r(t) - 2u(t+1) + 3\delta(t-2)$

$\leftarrow$ LTI

$$RC\frac{dy}{dt} + y(t) = x_\Delta(t)$$

$$RC\frac{dy}{dt} + y(t) = r(t) - 2u(t+1) + 3\delta(t-2)$$

$$y(t) = y_R(t) - 2y_s(t+1) + 3h(t-2)$$

For impulse $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$, $y_s(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$

$$y_s(t) = \int_{-\infty}^{\infty} h(\lambda)\, d\lambda = \int_{-\infty}^{t} \frac{1}{RC} e^{-\lambda/RC}\, d\lambda = \frac{1}{RC} \cdot \left[-RC\, e^{-\lambda/RC} \Big|_{0}^{t} \right]$$

$$= \left[1 - e^{-t/RC} \right] u(t)$$

$$y_R(t) = \int_{-\infty}^{t} y_s(t)\, dt = \int_{0}^{t} \left[1 - e^{-\lambda/RC} \right] d\lambda$$

$$= \left[t - RC\left[1 - e^{-t/RC} \right] \right] u(t)$$

$$= \underbrace{t\,u(t)}_{\downarrow} - \left[RC\left[1 - \exp\left(\frac{-t}{RC} \right) \right] \right] u(t)$$

$$= r(t) - RC\left[1 - \exp\left(\frac{-t}{RC} \right) \right] u(t)$$

$$y(t) = y_R(t) - 2y_s(t+1) + 3h(t-2)$$

Frequency response function for LTI:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$x(t) = e^{j\omega t}$ LTI ; $x(t) = \cos(\omega t)$

$$= \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda = \int_{-\infty}^{\infty} e^{j\omega(t-\lambda)}\, h(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} e^{j\omega t} \cdot e^{-j\omega\lambda}\, h(\lambda)\, d\lambda = e^{j\omega t} \underbrace{\int_{-\infty}^{\infty} e^{-j\omega\lambda}\, h(\lambda)\, d\lambda}_{H(\omega)} \rightarrow \frac{1}{RC} e^{-\lambda/RC} u(\lambda)$$

$$H(\omega) = \int_{-\infty}^{\infty} h(\lambda)\, e^{-j\omega t}\, d\lambda$$

For RC circuit $\Rightarrow$ impulse response

$$H(\omega) = \int_{-\infty}^{\infty} \frac{1}{RC} e^{-\lambda/RC} \cdot e^{j\omega\lambda}\, u(\lambda)\, d\lambda$$

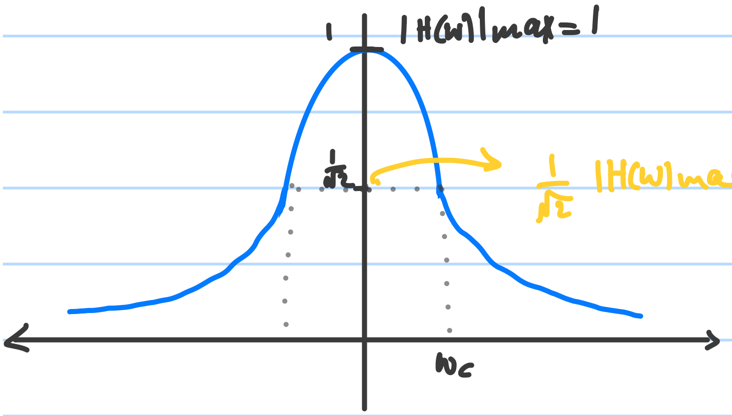
$$= \int_{0}^{\infty} \frac{1}{RC} e^{-(j\omega + \frac{1}{RC})\lambda}\, d\lambda = \frac{1}{RC} \cdot \frac{1}{j\omega + \frac{1}{RC}} e^{-(j\omega + \frac{1}{RC})\lambda} \Big|_{0}^{\infty}$$

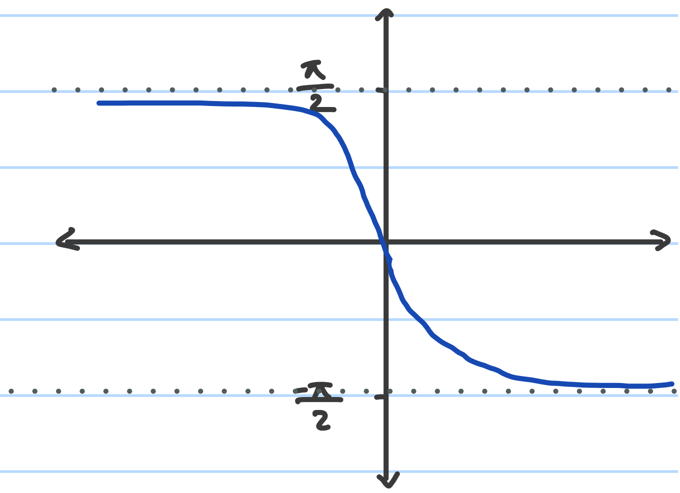
$$= \frac{1}{RC} \cdot \frac{RC}{1 + j\omega RC} = \boxed{\frac{1}{1 + j\omega RC}}$$

$$H(\omega) = \frac{1\angle 0}{\sqrt{1^2+(\omega RC)^2}\ \angle \tan^{-1}\left(\frac{\omega RC}{1}\right)}$$

$$= |H(\omega)| \angle \theta_{H(\omega)}$$

where $|H(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$, $\theta_{H(\omega)} = -\tan(\omega RC)$



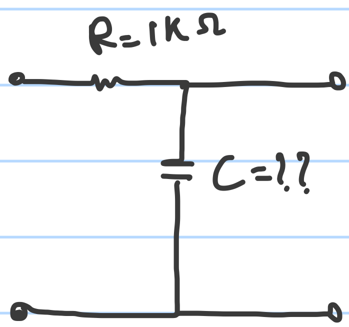


$$\frac{1}{\sqrt{2}} |H(\omega)|_{max} = \frac{1}{\sqrt{1+(\omega_{3dB} RC)^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(\omega_{3dB} RC)^2}} \Rightarrow 2 = 1 + (\omega_{3dB} RC)^2$$

$$\Rightarrow \omega_{3dB} RC = 1$$

$$\omega_{3dB} = \frac{1}{RC}$$



$$2\pi f_{3dB} = \frac{1}{RC} \qquad f_{3dB} = \frac{1}{2\pi RC}$$

$\Rightarrow C = (\ )F$

Stability

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

LTI

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda \right|$$

$$\leq \int_{-\infty}^{\infty} |x(t-\lambda)| \cdot |h(\lambda)|\, d\lambda$$

In general

$$|x(t-\lambda)| = M < \infty$$ "Bounded input"

↳ const.

$$|y(t)| = M \int_{-\infty}^{\infty} h(\lambda)\, d\lambda$$

To obtain bound input bounded output we have to check:

$$\int_{-\infty}^{\infty} |h(\lambda)|\, d\lambda < \infty$$

Ex. check the stability of the following system which has impulse response :-

$$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

$$\int_{-\infty}^{\infty} h(\lambda)\, d(\lambda) = \int_{0}^{\infty} \frac{1}{RC} e^{-\frac{\lambda}{RC}} d\lambda$$

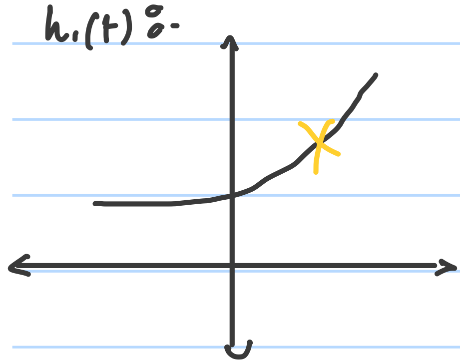
$$= e^{-\lambda/RC} \Big|_{0}^{\infty} = 1$$

$1 < \infty$ BIBO

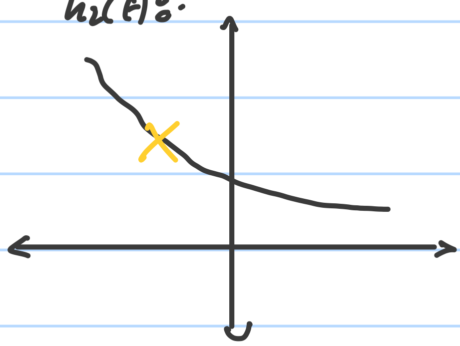
Ex. check the stability of the following system

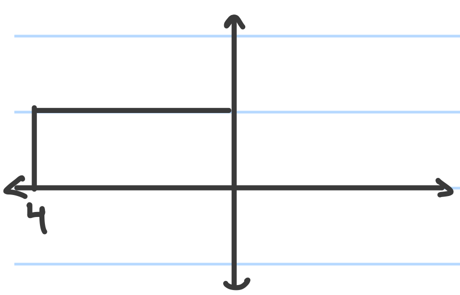
$$h(t) = \underbrace{e^{t}\, \Pi\left(\frac{t+2}{4}\right)}_{h_1(t)} + \underbrace{e^{-t}\, \Pi\left(\frac{t-2}{4}\right)}_{h_2(t)}$$

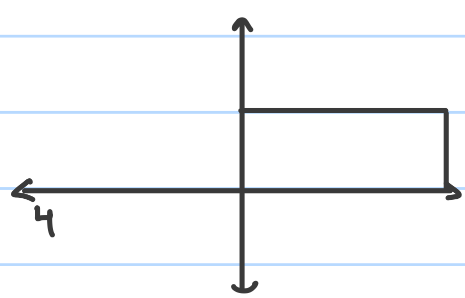
$h_1(t)$:-



$h_2(t)$:-



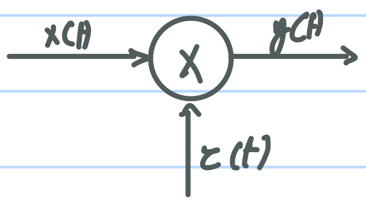


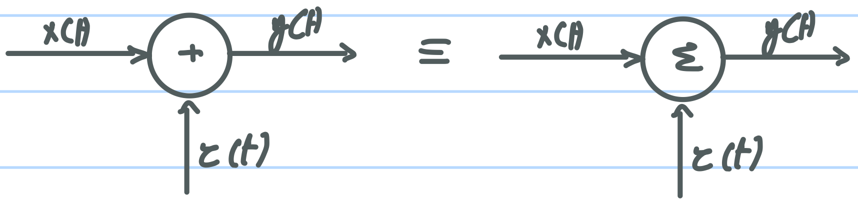


$$y(t) = h_1(t) + h_2(t)$$

$$y(t) = \int_{-4}^{0} e^{\lambda} d\lambda + \int_{0}^{4} e^{-\lambda} d\lambda$$

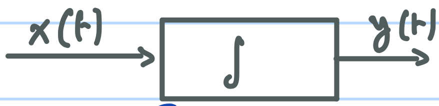
Simulink model :-





$y(t) = x(t) \cdot z(t)$

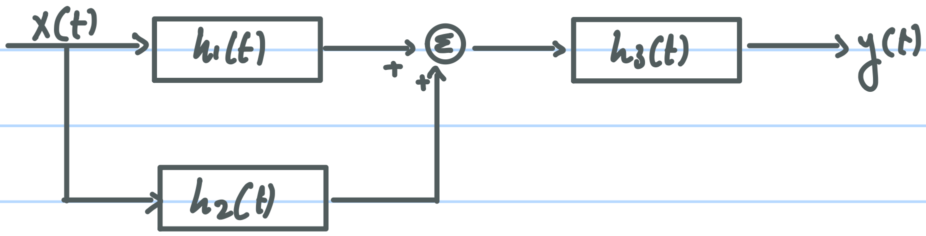
$y(t) = x(t) + z(t)$



$y(t) = \int_{\square}^{\square} x(\lambda)\, d\lambda$

$y(t) = K\, x(t)$

Ex. For LTI system :-



Where $x(t) = u(t)$, $h_1(t) = \delta(t-1)$, $h_2(t) = \delta(t-2)$, $h_3(t) = \delta(t-4)$.

Evaluate $y(t)$. Evaluate the total response, $h(t)$

$$
\begin{aligned}
y(t) &= [x(t) * h_1(t) + x(t) * h_2(t)] * h_3(t) \\
&= [u(t) * \delta(t-1) + u(t) * \delta(t-2)] * \delta(t-4) \\
&= [u(t-1) + u(t-2)] * \delta(t-4) \\
&= u(t-5) + u(t-6)
\end{aligned}
$$

$$
\begin{aligned}
h(t) &= [h_1(t) + h_2(t)] * h_3(t) \\
&= [\delta(t-1) + \delta(t-2)] * \delta(t-4) \\
&= \delta(t-5) + \delta(t-6)
\end{aligned}
$$

Plot the Simulink model for the following differential equation

$$6\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = 3\frac{dx(t)}{dt} + 5x(t)$$

$$6\frac{d^2y(t)}{dt} + 3\frac{dy(t)}{dt} + y(t) = 3\frac{dx(t)}{dt} + 5x(t)$$

① /6 معامل اعلى مشتقة في ال out يساوي 1 →

$$\frac{d^2y(t)}{dt^2} + \frac{1}{2}\frac{dy(t)}{dt} + \frac{1}{6}y(t) = \frac{1}{2}\frac{dx(t)}{dt} + \frac{5}{6}x(t)$$

② اي مشتقة بطرف والباقي بالطرف الثاني

$$\frac{d^2y(t)}{dt^2} + \frac{1}{2}\frac{dy(t)}{dt} - \frac{1}{2}\frac{dx(t)}{dt} = \underbrace{\frac{5}{6}x(t) - \frac{1}{6}y(t)}_{q_0}$$

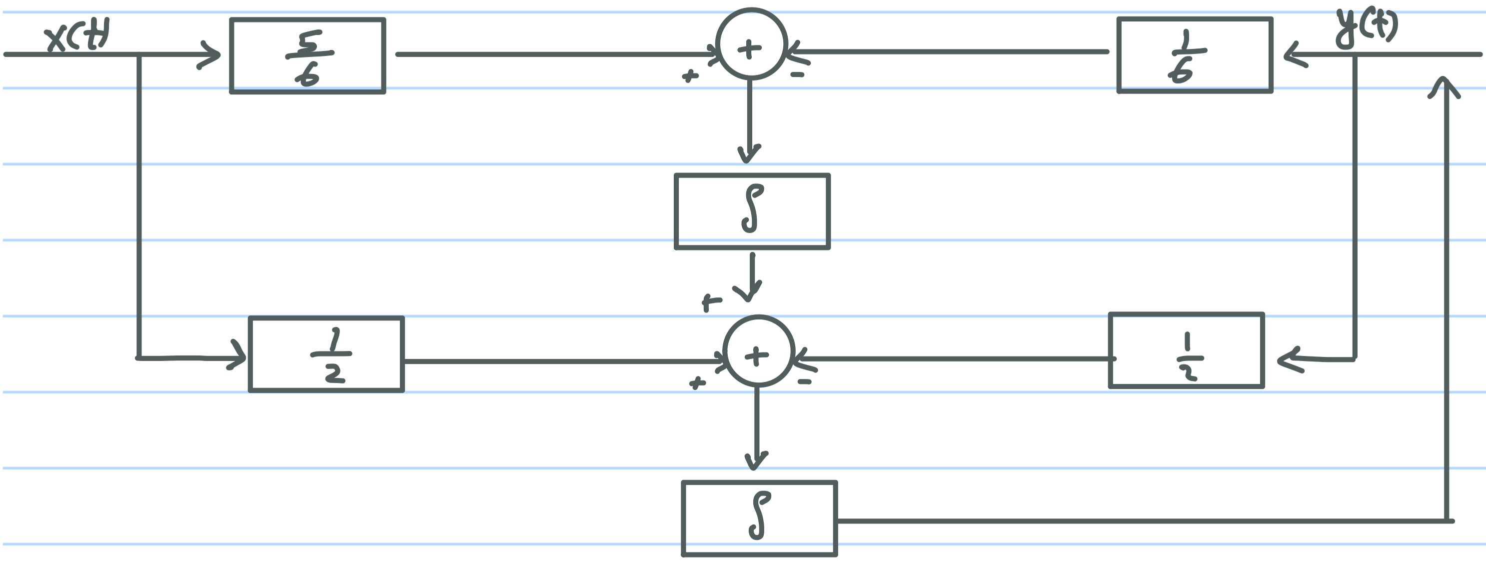
③ تكامل

$$\frac{dy(t)}{dt} + \frac{1}{2}y(t) - \frac{1}{2}x(t) = \int q_0$$

④ بكرر الخطوات ②,③ لحد ما اطلع ال y(t)

$$\frac{dy(t)}{dt} = \underbrace{\int q_0 + \frac{1}{2}x(t) - \frac{1}{2}y(t)}_{q_1}$$

$$y(t) = \int q_1$$



Ex. $2\frac{d^4y(t)}{dt^4} + 3\frac{d^2y(t)}{dt^2} + 6y(t) = \frac{d^2x(t)}{dt} + 3x(t)$

$2\frac{d^4y(t)}{dt^4} + 3\frac{d^2y(t)}{dt^2} + 6y(t) = \frac{d^2x(t)}{dt} + 3x(t)$

$\frac{d^4y(t)}{dt^4} + \frac{3}{2}\frac{d^2y(t)}{dt^2} + 3y(t) = \frac{1}{2}\frac{d^2x(t)}{dt} + \frac{3}{2}x(t)$

$\frac{d^4y(t)}{dt^4} + \frac{3}{2}\frac{d^2y(t)}{dt^2} - \frac{1}{2}\frac{d^2x(t)}{dt} = \underbrace{\frac{3x(t)}{2} - 3y(t)}_{q_0}$

$\frac{d^3y(t)}{dt^3} + \frac{3}{2}\frac{d\,y(t)}{dt} - \frac{1}{2}\frac{d\,x(t)}{dt} = \underbrace{\int q_0}_{q_1}$

$\frac{d^2y(t)}{dt^2} + \frac{3}{2}y(t) - \frac{1}{2}x(t) = \int q_1 \Rightarrow \frac{d^2y(t)}{dt^2} = \underbrace{\int q_1 - \frac{3}{2}y(t) + \frac{1}{2}x(t)}_{q_2}$

$\frac{dy(t)}{dt} = \underbrace{\int q_2}_{q_3}$

$y(t) = \int q_3$