8.3 Determining the sample size

How to choose a sample size $\underline{n}$ to provide a desired margin of error?

- Recall that the margin of Error $= Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$E = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \iff {}^{*} n = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2 ,$$

n is rounded up because n will be the min sample size satisfies the E.

- $Z_{\alpha/2}$ is known as far as we choose the confidence level.
 " If we choose 95% confidence level, then $Z_{\alpha/2} = Z_{0.025} = 1.96$ "

- If σ is known, then we use * directely.
- If σ is unknown, then we use * also by estimating the planing value for σ as follows:

1) Use s as the planing value for σ, where $s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$, when the data are available in sample.

2) Select a sample and estimate s as the planing value for σ, when there is no sample.

3) Use judgment or "best guess" for σ. For example, if we know the largest value and the smallest value in the population, then the planning value for $\sigma = \frac{\text{Range}}{4} = \frac{\text{Max Value} - \text{Min Value}}{4}$.

Example (Q 23 page 312) How large a sample should be selected to provide 95% confidence level interval with margin of error of 10?

[1] Assume that the population standard deviation is 40?

$Z_{\alpha/2} = Z_{0.025} = 1.96$ and $\sigma = 40$ and $E = 10$

$$n = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\frac{1.96\,(40)}{10}\right)^2 = 61.46 \approx 62$$

[2] Assume that the range of data is estimated to be 124?

The planning value of $\sigma = \frac{\text{Range}}{4} = \frac{124}{4} = 31$

$$n = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\frac{1.96 \times 31}{10}\right)^2 = 36.9 \approx 37$$
