Thermal Fluid Engineering
ENMC4411
Chapter 9 Convection heat transfer

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Outline

- Convection principles
- External flow convection
- Internal flow convection
- Free convection
- Heat exchangers

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- External flow convection
- Internal flow convection
- Free convection
- Heat exchangers

Concept of convection

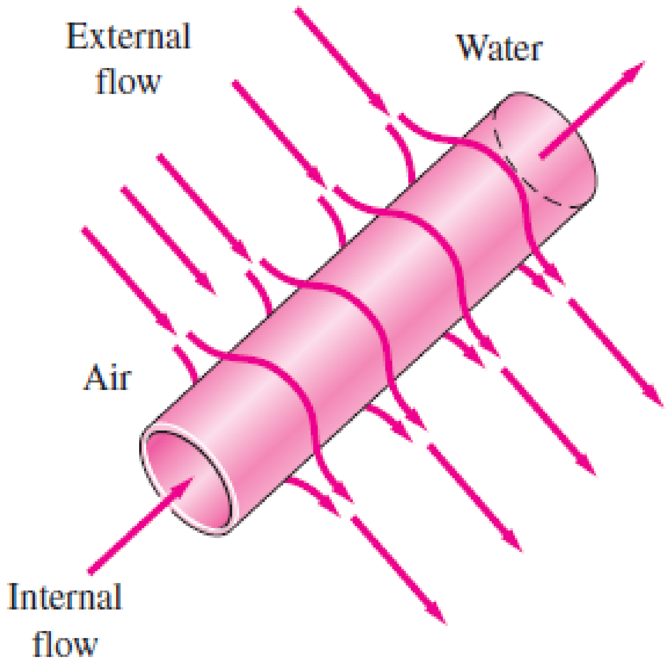
- Convection is heat transfer between a surface and an adjacent fluid (stationary or moving)when they are at different temperatures.
- The convection coefficient depends on fluid properties, surface geometry and flow conditions.

$$q_{conv} = hA\Delta T$$

- The dominant contribution is due to the bulk (or gross) motion of fluid particles.
- What is the physical mechanisms underlying convection?
- As convection heat transfer implies fluid flow and heat transfer problems, solution of such cases require identifying relevant dominant dimensionless parameters based on grouping and dimensional analysis techniques.

External and internal flows

- A fluid flow is classified as being internal and external, depending on whether the fluid is forced to flow in a confined channel or over a surface.
- The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is external flow.
- The flow in a pipe or duct is internal flow if the fluid is completely bounded by solid surfaces. Water flow in a pipe, for example, is internal flow, and air flow over an exposed pipe during a windy day is external flow.

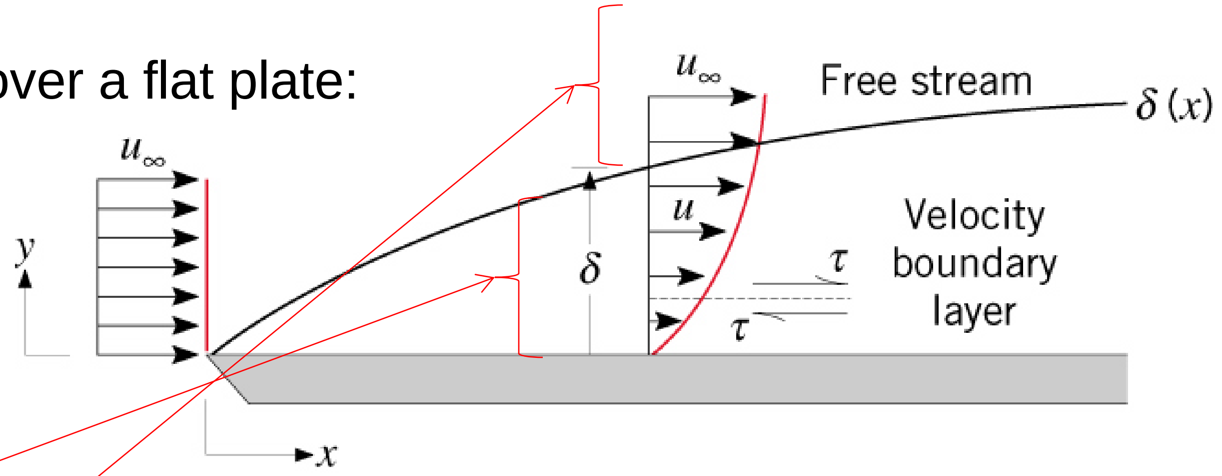


Boundary layer

- Heat transfer is determined by boundary layer that develops on the surface.
- Boundary layer results from velocity distribution as affected by the solid surface.
- Velocity changes from zero at surface to free stream velocity away from surface and this region is known as hydraulic boundary layer.
- The region extending from surface to "δ"{ where velocity almost u_∞} is known as the boundary layer, actually δ is taken when [δ=y] U=.099U_∞.
- In the B.L a velocity profile exists u= f(y) .

Boundary layer

Consider flow of a fluid over a flat plate:



The flow is characterized by two regions:

- A **thin fluid layer** (boundary layer) in which velocity gradients (***du***) and shear stresses (τ_s) are large. Its thickness δ is defined as the value of ***y*** for which ***u = 0.99***
- An outer region in which velocity gradients and shear stresses are negligible

For Newtonian fluids: $\tau_S = \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0}$ and $C_f = \dfrac{\tau_S}{\rho u_\infty^2 / 2}$ where C_f is the local friction coefficient

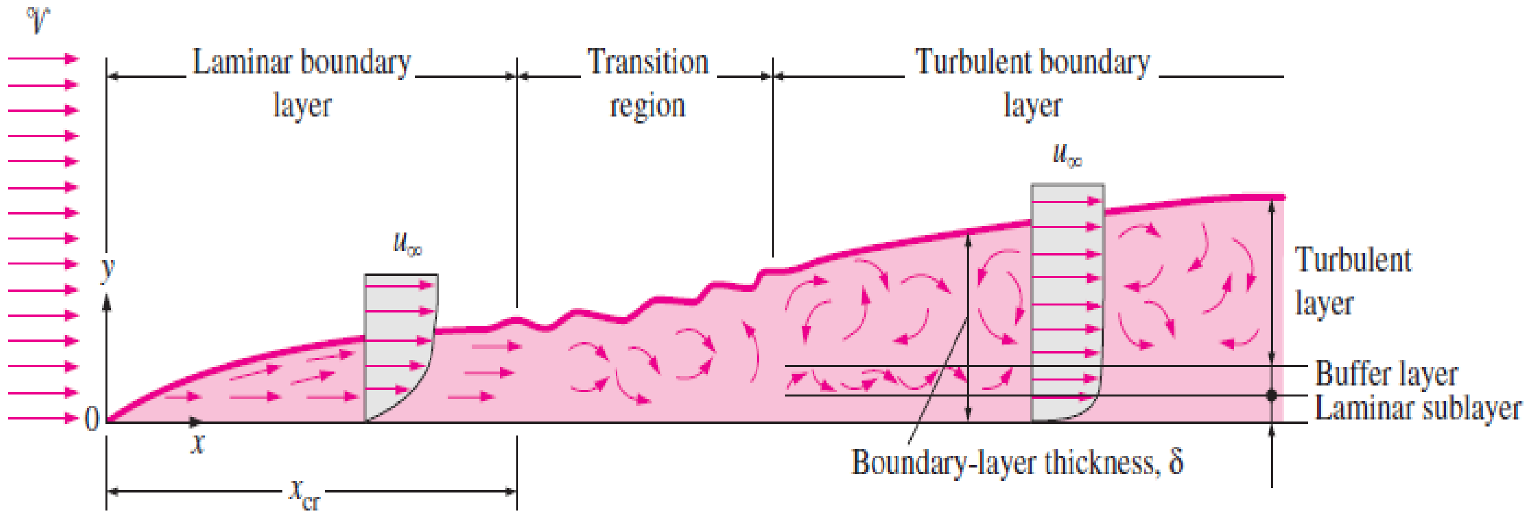
Laminar & turbulent flow

- The highly ordered fluid motion characterized by smooth streamlines is called laminar. The flow of high-viscosity fluids such as oils at low velocities is typically laminar.
- The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called turbulent. The flow of low-viscosity fluids such as air at high velocities is typically turbulent.
- The flow regime greatly influences the heat transfer rates.
- In laminar flow: fluid motion is highly ordered and stream lines are well defined along which particles move.
- While in turbulent flow: fluid motion is highly irregular and is characterized by velocity fluctuation. These fluctuations enhance the transfer of momentum, energy and species, thus it increases surface friction and convection heat transfer.
- Fluid mixing due to fluctuation increase B.L thickness (velocity thermal B.L) and profile is flatter than is laminar flow.

Reynolds number

- Transition begins at some xc ,which is determined from Reynolds number

$$\mathrm{Re}_x = \rho \frac{U_\infty x}{\mu} \qquad \mathrm{Re}_{x,c} = \rho \frac{U_\infty X_c}{\mu} = 5\times 10^5$$

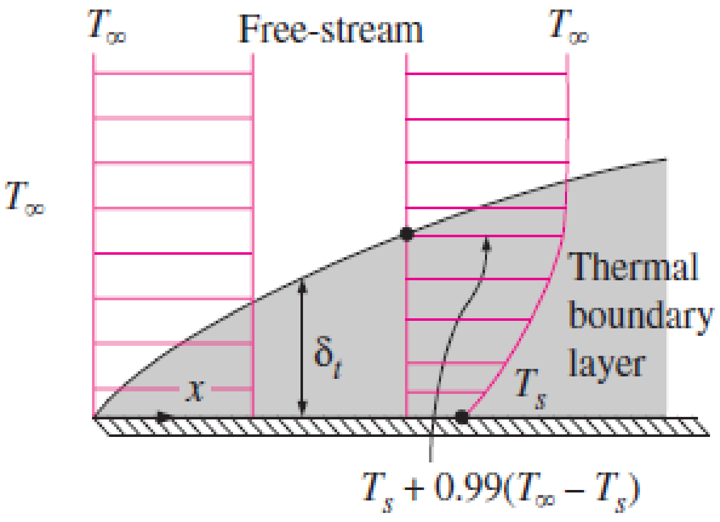


Thermal B.L

Thermal B.L similar to velocity B.L develops in regions adjacent to surfaces. Velocity B.L develops because of velocity difference (u=o------- u=u∞), similarity thermal B.L T=T_S at y = 0 and T_∞ at for free steam .

- Thermal B.L is the region where temperature gradient prevails.
- Thickness of thermal B.L δ_t is defined as "y" where

$$\left(T_S - T\right)/\left(T_S - T_\infty\right) = 0.99$$



Local and average convection coefficients

- Local heat flux at surface and in the boundary layer is given by Fourier law of conduction to the fluid. At surface particles (u=0) are still (no motion) and conduction prevails;

$$q_s'' = h_x(T_S - T_\infty) = -k_f \left.\frac{\partial T}{\partial y}\right|_{Y=O}$$

$$h_x = \frac{-kf \left.\frac{\partial T}{\partial y}\right|_{y=o}}{(T_S - T_\infty)}$$

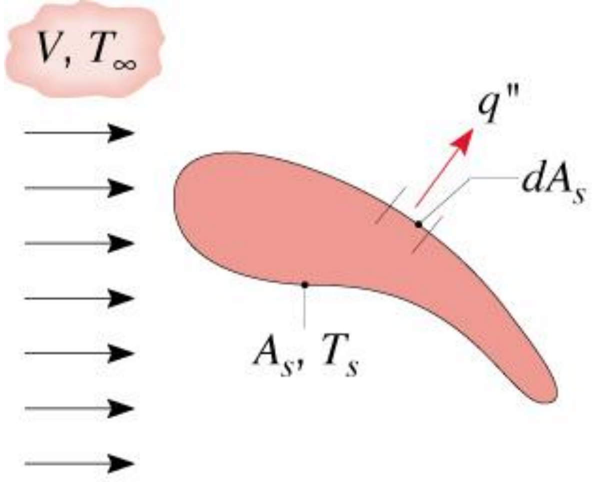
Total heat transfer q over the surface

$$q = \int_{As} q'' dA_\infty = \int_{As} h(T_s - T_\infty) dA_S = (T_S - T_\infty)\int_{As} h dA_s$$

Local and average convection coefficients

Recall Newton's law of cooling for heat transfer between a surface of arbitrary shape, area $\boldsymbol{A_s}$ and temperature $\boldsymbol{T_s}$ and a fluid:

$$q'' = h(T_S - T_\infty)$$



- Generally flow conditions will vary along the surface, so ***q″*** is a local heat flux and ***h*** a local convection coefficient.
- The total heat transfer rate is

$$q = \int_{A_S} q''\, dA_S = (T_S - T_\infty)\int_{A_S} h\, dA_S = \bar{h} A_S (T_S - T_\infty)$$

where $\boxed{\bar{h} = \frac{1}{A_S}\int_{A_S} h\, \mathrm{dA_S}}$ => *average heat transfer coefficient*

For special case of that plate with length L $\bar{h} = \frac{1}{A_S}\int_{As} h_x dA_S = \frac{1}{L}\int_0^l h_x dx$

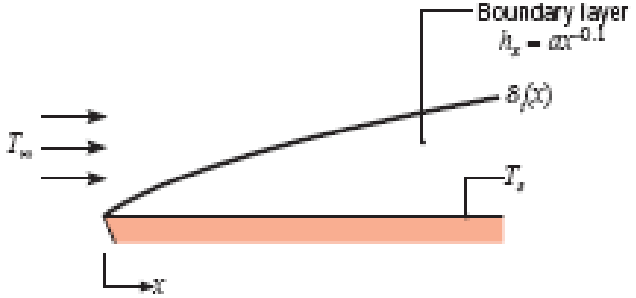
Example 17.1 Average coefficient

Experimental results for the local heat transfer coefficient $\boldsymbol{h_x}$ for flow over a flat plate with an extremely rough surface were found to fit the relation;

$$h_x(x) = ax^{-0.1}$$

where ***a*** is a coefficient ***(W/m^1.9^.K)*** and ***x*** *(in **m**)* is the distance from the leading edge of the plate.

1. Develop an expression for the ratio of the average heat transfer coefficient ***x*** *for a* plate of length ***x*** to the local heat transfer coefficient $\boldsymbol{h_x}$ at ***x****?*
2. Plot the variation of $\boldsymbol{h_x}$ and ***x*** as a function of ***x****?*

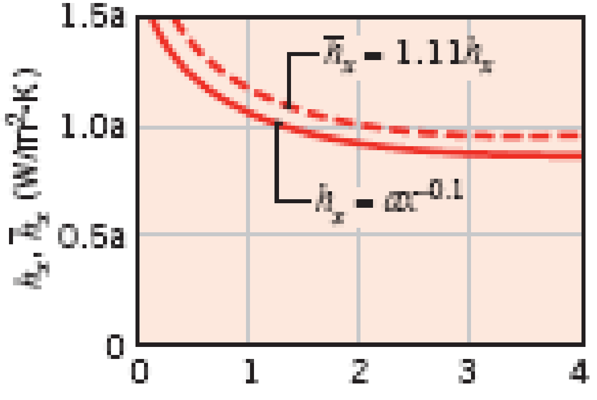


The average value of the convection heat transfer coefficient over the region from ***0*** to ***x*** is:

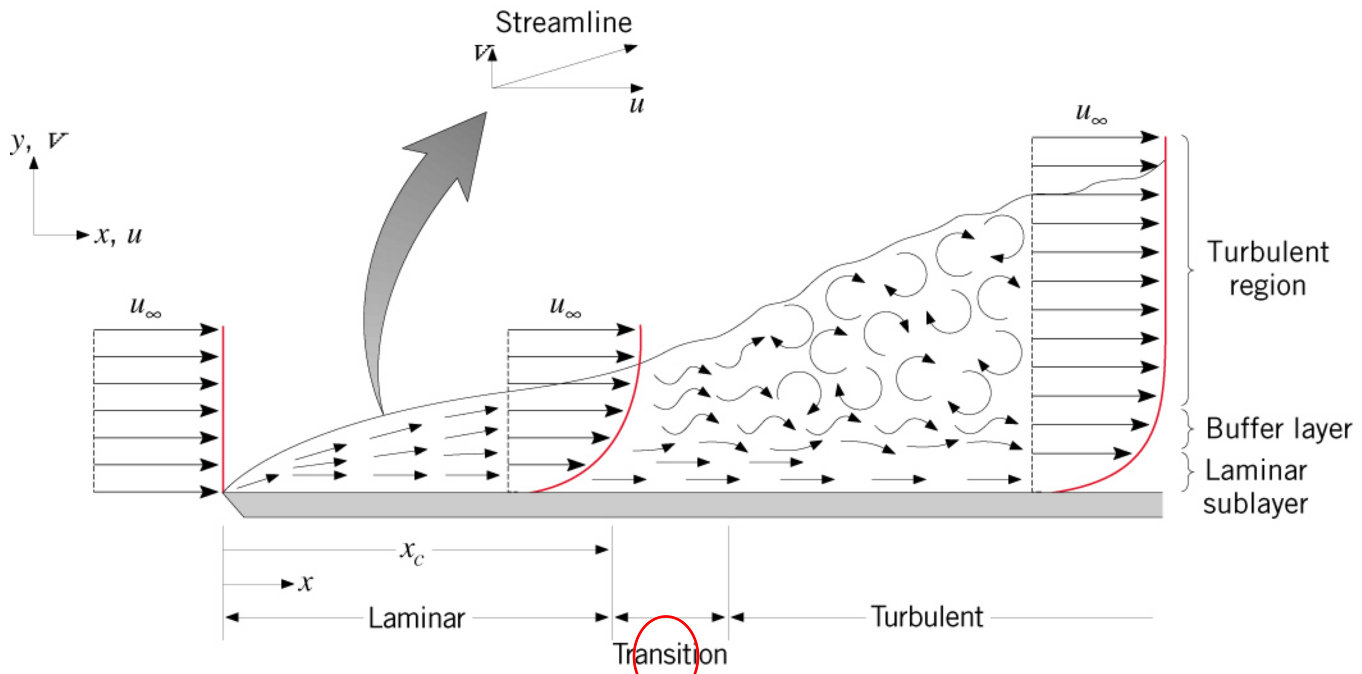
$$\bar{h} = \frac{1}{L}\int_0^L h\,\mathrm{dx}$$

$$\bar{h} = \frac{1}{L}\int_0^L h\,\mathrm{dx} \qquad \bar{h}_x = \bar{h}_x(x) = \frac{1}{x}\int_0^x h_x(x)dx$$

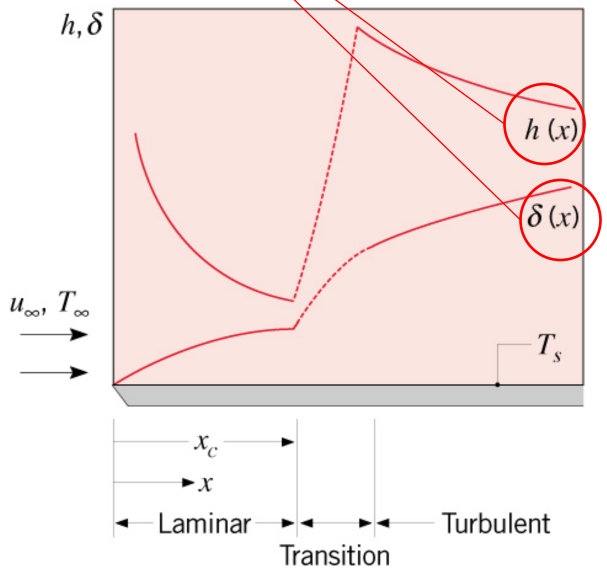
$$\bar{h}_x = \bar{h}_x(x) = \frac{1}{x}\int_0^x h_x(x)dx = \frac{a}{x}\int_0^x x^{-0.1}dx = \frac{a}{x}\left(\frac{x^{+0.9}}{0.9}\right) = 1.11ax^{-0.1}$$



Laminar & turbulent , h



$$Re_x = \frac{\rho u_\infty x}{\mu}$$



Transition criterion at ***Re**_{critical}*:

$$Re_{x,c} = \frac{\rho u_\infty x_c}{\mu} = 5\times10^5$$

Dimensionless numbers

- Nusselt number represents the dimensionless temperature gradient at the surface, and is defined as,

$$Nu_x = \frac{hX}{k_f}$$

  k_f Conductivity of fluid

  h: convection heat transfer

- Average Nusselt number

$$\bar{Nu} = \frac{\bar{h}l}{k_f}$$

Nu$_L$= hL/k_f

- Note Nu$_L$ is same as the average number
- Prandtl number Pr = momentum diffusivity/ Thermal diffusivity
  - Pr = υ/α $\alpha = \rho\, c/k$

Important dimensionless groups

Table 17.1 Important Dimensionless Groups in Convection Heat Transfer

Group[a]	Definition[a]		Interpretation/Application
Nusselt number, Nu_L	$\frac{hL}{k}$	(17.9)	Dimensionless temperature gradient at the surface. Measure of the convection heat transfer coefficient.
Reynolds number, Re_L	$\frac{VL}{\nu}$	(17.12)	Ratio of the inertia and viscous forces. Characterizes forced convection flows.
Prandtl number, Pr	$\frac{c_p \mu}{k} = \frac{\nu}{\alpha}$	(17.13)	Ratio of the momentum and thermal diffusivities. Property of the fluid.
Grashof number, Gr_L	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	(17.16)	Ratio of buoyancy to viscous forces. Characterizes free convection flows.
Rayleigh number, Ra_L	$\frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$	(17.19)	Product of Grashof and Prandtl numbers, Gr · Pr. Characterizes free convection flows.

[a]The subscript L represents the characteristic length on the surface of interest.

Correlation Selection Rules

The selection and application of convection correlations *for any flow situation* are facilitated by following a few simple *rules:*

- *Identify the flow surface geometry*. Does the problem involve flow over a flat plate, a cylinder, or a sphere? Or flow through a tube of circular or non-circular cross-sectional area?
- *Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature*. For moderate boundary layer temperature differences, the ***film temperature,*** *Tf*, defined as the average of the surface and free stream temperatures

$$T_f = \frac{T_s + T_\infty}{2}$$

Correlation Selection Rules

- *Calculate the Reynolds number*. Using the appropriate characteristic length, calculate the Reynolds number to determine the boundary layer flow conditions. If the geometry is the flat plate in parallel flow, determine whether the flow is laminar, turbulent, or mixed.
- *Decide whether a local or surface average coefficient is required*. The local coefficient is used to determine the heat flux at a point on the surface; the average coefficient is used to determine the heat transfer rate for the entire surface.
- *Select the appropriate correlation*.

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Empirical correlations

- General external flow forced convection correlation is given as

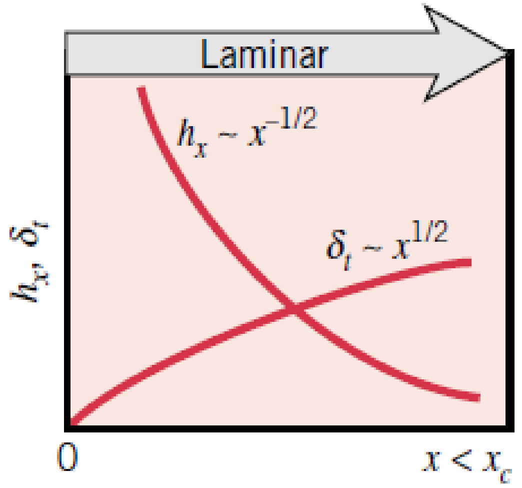
$$Nu_L = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

- Where constants C, m , and n experimentally determined constants and depend on surface geometry and flow conditions.
- Fluid properties are assumed constant and evaluated at average film temperature.

$$T_f = \frac{T_s + T_\infty}{2}$$

- Laminar flow over flat plate, local; $Nu_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{\frac{1}{2}}.\mathrm{Pr}^{\frac{1}{3}}$
- And average coefficient;

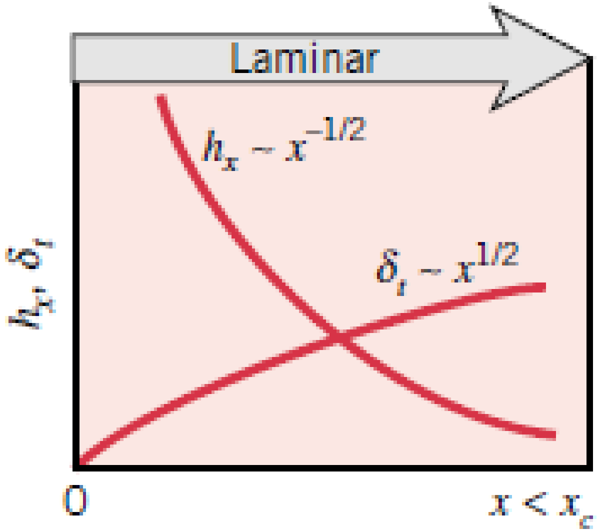
$$\overline{Nu_x} = \frac{\bar{h}_x x}{k} = 0.664\,\mathrm{Re}_x^{\frac{1}{2}}.\mathrm{Pr}^{\frac{1}{3}}$$



Example convection – Laminar Flow

The expression for the *average convection coefficient for any surface shorter than* $\mathbf{x_c}$ *is given by the integration below;*

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x\,dx = 0.332\left(\frac{k}{x}\right)\mathrm{Pr}^{1/3}\left(\frac{u_\infty}{\nu}\right)^{1/2}\int_0^x \frac{dx}{x^{1/2}}$$



since the definite integral has the value $2x^{1/2}$, then we my use the expression;

$$\bar{h}_x = 2h_x.$$

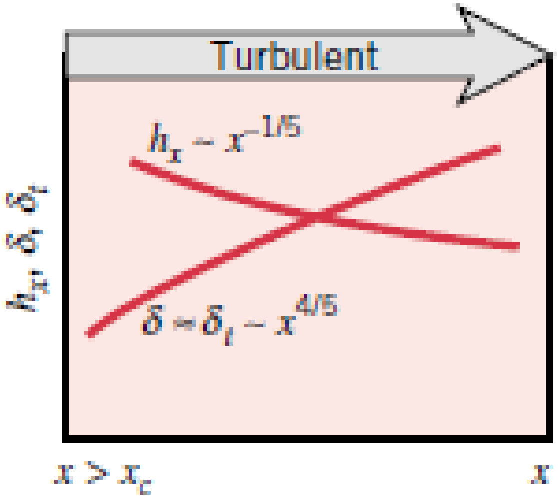
And average Nusselt is the written as;

$$\overline{\mathrm{Nu}}_x = \frac{\bar{h}_x x}{k} = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \qquad [0.6 \le \mathrm{Pr} \le 50]$$

Forced Convection – Turbulent Flow

Local Nusselt is given by;

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \qquad \begin{bmatrix} \mathrm{Re}_x \le 10^8 \\ 0.6 < \mathrm{Pr} < 60 \end{bmatrix}$$



For a fully turbulent from the leading edge over the entire plate;

$$\overline{\mathrm{Nu}}_L = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3} \qquad \begin{bmatrix} \mathrm{Re}_{x,c} = 0 \\ 0.6 \le \mathrm{Pr} \le 50 \end{bmatrix}$$

Forced Convection – Mixed BL Flow

When transition of flow type occurs sufficiently upstream of the trailing edge, ***(x_c/L)=0.95***, the average coefficient will be influenced by laminar and turbulent boundary layers or Mixed Boundary Layer, then average convection heat transfer coefficient over the plate will be the sum of the laminar and the turbulent, i.e.;

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{\text{lam}}\,dx + \int_{x_c}^{L} h_{\text{turb}}\,dx\right) \qquad \bar{h}_L = \left(\frac{k}{L}\right)\left[0.332\left(\frac{u_\infty}{\nu}\right)^{1/2}\int_0^{x_c}\frac{dx}{x^{1/2}} + 0.0296\left(\frac{u_\infty}{\nu}\right)^{4/5}\int_{x_c}^{L}\frac{dx}{x^{1/5}}\right]\text{Pr}^{1/3}$$

And Nusslet is given by, $\overline{\text{Nu}}_L = [0.664\,\text{Re}_{x,c}^{1/2} + 0.037(\text{Re}_L^{4/5} - \text{Re}_{x,c}^{4/5})]\text{Pr}^{1/3}$

$$\overline{\text{Nu}}_L = (0.037\,\text{Re}_L^{4/5} - A)\text{Pr}^{1/3} \qquad A = 0.037\,\text{Re}_{x,c}^{4/5} - 0.664\,\text{Re}_{x,c}^{1/2}$$

$$\overline{\text{Nu}}_L = (0.037\,\text{Re}_L^{4/5} - 871)\text{Pr}^{1/3} \qquad \begin{bmatrix} 0.6 < \text{Pr} < 60 \\ 5\times 10^5 < \text{Re}_L \le 10^8 \\ \text{Re}_{x,c} = 5\times 10^5 \end{bmatrix}$$

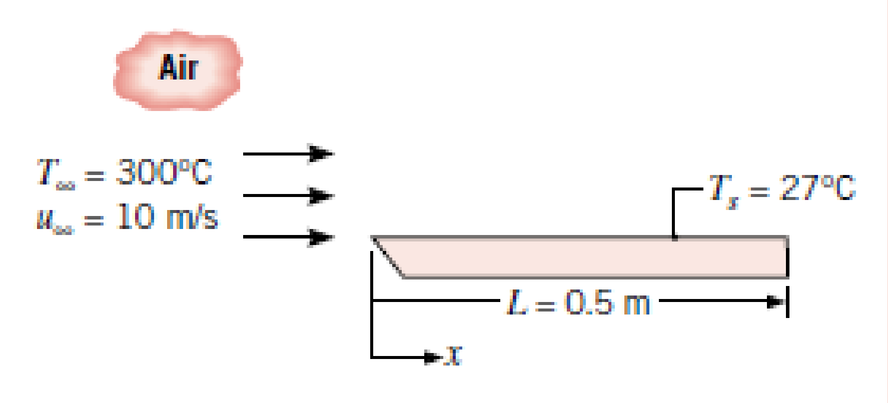
If Re>> Rex,c then above may be reasonably approximated by

$$\overline{\text{Nu}}_L = 0.037\,\text{Re}_L^{4/5}\,\text{Pr}^{1/3} \qquad \begin{bmatrix} \text{Re}_{x,c} = 0 \\ 0.6 \le \text{Pr} \le 50 \end{bmatrix}$$

Example 17.2 Laminar Flow over a Flat Plate

Air at atmospheric pressure and a temperature of 300ºC flows steadily with a velocity of 10 m/s over a flat plate of length 0.5 m. Estimate the cooling rate per unit width of the plate needed to maintain a surface temperature of 27ºC.

Properties: Table HT-3, air (T_f = 437 K, p = 1 atm): $\nu = 30.84 \times 10^{-6}$ m²/s, $k = 36.4 \times 10^{-3}$ W/m · K, Pr = 0.687.



Analysis: For a plate of unit width, it follows from Newton's law of cooling that the rate of convection heat transfer *to* the plate is

$$q' = \bar{h}L(T_\infty - T_s)$$

To select the appropriate convection correlation for estimating $\bar{h}$, the Reynolds number must be determined to characterize the flow

$$\mathrm{Re}_L = \frac{u_\infty L}{\nu} = \frac{10 \text{ m/s} \times 0.5 \text{ m}}{30.84 \times 10^{-6} \text{ m}^2/\text{s}} = 1.62 \times 10^5$$

Since $\mathrm{Re}_L < \mathrm{Re}_{x,c} = 5 \times 10^5$, the flow is laminar over the entire plate, and the appropriate correlation is given by Eq. 17.26 (see also Table 17.3, page 423)

$$\overline{\mathrm{Nu}}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} = 0.664(1.62 \times 10^5)^{1/2}(0.687)^{1/3} = 236$$

The average convection coefficient is then

$$\bar{h} = \frac{\overline{\mathrm{Nu}}_L\, k}{L} = \frac{236 \times 0.0364 \text{ W/m} \cdot \text{K}}{0.5 \text{ m}} = 17.2 \text{ W/m}^2 \cdot \text{K}$$

and the required cooling rate per unit width of plate is

$$q' = 17.2 \text{ W/m}^2 \cdot \text{K} \times 0.5 \text{ m}(300 - 27)^\circ\text{C} = 2348 \text{ W/m} \quad \triangleleft$$

3. If upstream turbulence is promoted by a fan or grill, or a trip wire were placed at the leading edge, a *turbulent boundary condition could exist over the entire plate.* For such a condition, Eq. 17.32 is the appropriate correlation to estimate the convection coefficient

$$\overline{\mathrm{Nu}}_L = 0.037\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3} = 0.037(1.62 \times 10^5)^{4/5}(0.687)^{1/3} = 480$$

$$\bar{h}_L = 480(36.4 \times 10^{-3} \text{ W/m} \cdot \text{K})/0.5 \text{ m} = 35.0 \text{ W/m}^2 \cdot \text{K}$$

The cooling rate per unit plate width is

$$q' = 35 \text{ W/m}^2 \cdot \text{K} \times 0.5 \text{ m}(300 - 27)^\circ\text{C} = 4778 \text{ W/m}$$

Forced Convection – Cylinder in Cross-Flow

Reynolds number based upon the cylinder diameter as the characteristic length;

$$Re_D \equiv \frac{\rho u_\infty D}{\mu} = \frac{u_\infty D}{\nu}$$

The Hilpert correlation;

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\,Re_D^m\,Pr^{1/3} \qquad [Pr \geq 0.7]$$

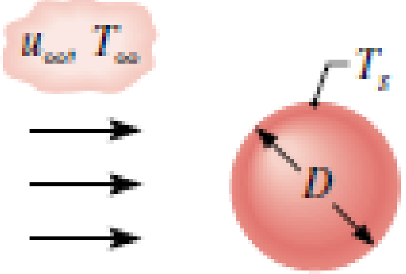
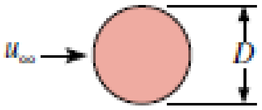
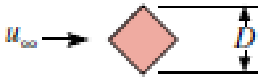
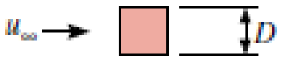
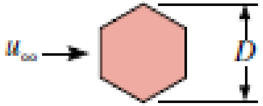
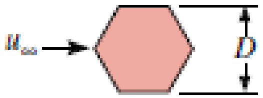
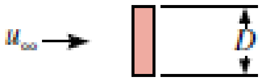


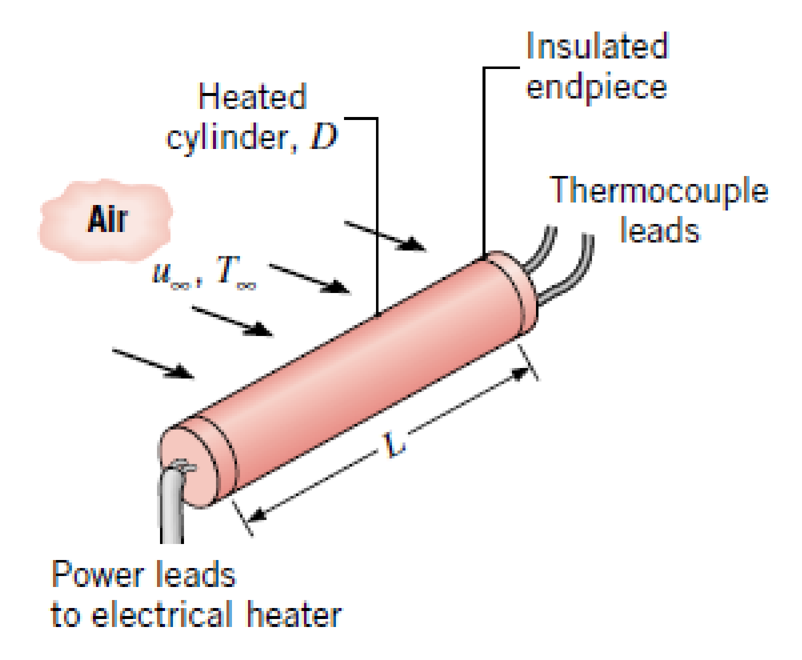
Table 17.2 Constants for the Hilpert Correlation, Eq. 17.34, for Circular ($Pr \geq 0.7$) and Noncircular (Gases only) Cylinders in Cross Flow

Geometry	Re_D	C	m
Circular 	0.4–4	0.989	0.330
	4–40	0.911	0.385
	40–4000	0.683	0.466
	4000–40,000	0.193	0.618
	40,000–400,000	0.027	0.805

Geometry	Re_D	C	m
Square 	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon 	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
	5×10^3–10^5	0.153	0.638
Vertical plate 	4×10^3–1.5×10^4	0.228	0.731

Example 17.4 Cylindrical Test Section: Measurement of the Convection Coefficient

Experiments have been conducted to measure the convection coefficient on a polished metallic cylinder 12.7 mm in diameter and 94 mm long (Fig. E17.4*a*). The cylinder is heated internally by an electrical resistance heater and is subjected to a cross flow of air in a low-speed wind tunnel. Under a specific set of operating conditions for which the free stream air velocity and temperature were maintained at $u_\infty = 10$ m/s and 26.2°C, respectively, the heater power dissipation was measured to be $P_e =$ 46 W, while the average cylinder surface temperature was determined to be $T_s = 128.4$°C. It is estimated that 15% of the power dissipation is lost by conduction through the endpieces.



(a) Determine the convection heat transfer coefficient from the experimental observations.

(b) Compare the experimental result with the convection coefficient computed from an appropriate correlation.

Figure E17.4a

Solution

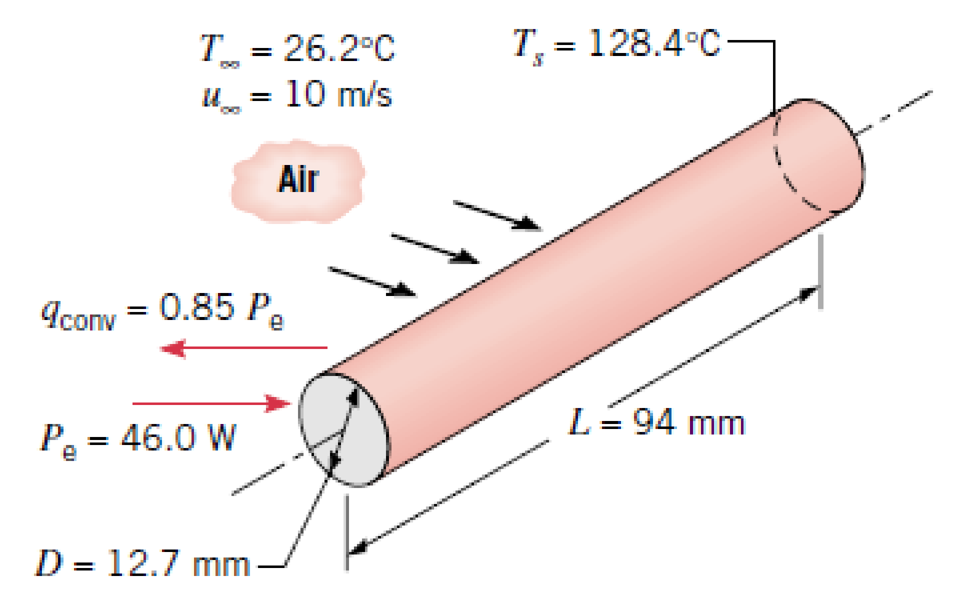
Known: Operating conditions for a heated cylinder.

Find:

(a) Convection coefficient associated with the operating conditions.

(b) Convection coefficient from an appropriate correlation.

Schematic and Given Data:



Assumptions:

1. Steady-state conditions.
2. Uniform cylinder surface temperature.
3. Negligible radiation exchange with surroundings.

Figure E17.4b

Properties: Table HT-3, air ($T_f \approx 350$ K): $\nu = 20.92 \times 10^{-6}$ m^2/s, $k = 30 \times 10^{-3}$ W/m · K, Pr = 0.700.

Analysis:

(a) The convection heat transfer coefficient may be determined from the *experimental observations* by using Newton's law of cooling. That is

$$\bar{h} = \frac{q_{\text{conv}}}{A(T_s - T_\infty)}$$

Since 15% of the electrical power is transfered by conduction from the test section, it follows that $q_{\text{conv}} = 0.85P_e$, and with $A = \pi DL$

$$\bar{h} = \frac{0.85 \times 46 \text{ W}}{\pi \times 0.0127 \text{ m} \times 0.094 \text{ m } (128.4 - 26.2)°\text{C}} = 102 \text{ W/m}^2 \cdot \text{K} \quad \triangleleft$$

(b) Using the *Churchill-Bernstein correlation*, Eq. 17.35

$$\overline{\text{Nu}}_D = 0.3 + \frac{0.62\,\text{Re}_D^{1/2}\,\text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

With all properties evaluated at T_f, Pr = 0.70 and

$$\text{Re}_D = \frac{u_\infty D}{\nu} = \frac{10 \text{ m/s} \times 0.0127 \text{ m}}{20.92 \times 10^{-6} \text{ m}^2\text{/s}} = 6071$$

Note that $Re_D \, Pr = 6071 \times 0.700 = 4250 > 0.2$, so that the correlation is within the recommended range. Hence, the Nusselt number and the convection coefficient are

$$\overline{Nu}_D = 0.3 + \frac{0.62(6071)^{1/2}(0.70)^{1/3}}{[1 + (0.4/0.70)^{2/3}]^{1/4}}\left[1 + \left(\frac{6071}{282{,}000}\right)^{5/8}\right]^{4/5} = 40.6$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 40.6 \frac{0.30 \text{ W/m} \cdot \text{K}}{0.0127 \text{ m}} = 96 \text{ W/m}^2 \cdot \text{K} \quad \triangleleft$$

Comments:

1. The Hilpert correlation, Eq. 17.34, is also appropriate for estimating the convection coefficient

$$\overline{Nu}_D = C \, Re_D^m \, Pr^{1/3}$$

With all properties evaluated at the film temperature, $Re_D = 6071$ and $Pr = 0.70$. Hence, from Table 17.2, find for the given Reynolds number that $C = 0.193$ and $m = 0.618$. The Nusselt number and the convection coefficient are then

$$\overline{Nu}_D = 0.193(6071)^{0.618}(0.700)^{0.333} = 37.3$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 37.3 \frac{0.030 \text{ W/m} \cdot \text{K}}{0.0127 \text{ m}} = 88 \text{ W/m}^2 \cdot \text{K}$$

Forced Convection – The Sphere

For the flow around a sphere, the ***Whitaker correlation*** is recommended and has the form;

$$\overline{Nu}_D = 2 + (0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3})Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4} \qquad \left[\begin{array}{l} 0.71 < Pr < 380 \\ 3.5 < Re_D < 7.6 \times 10^4 \end{array}\right]$$

Evaluated at the surface temperature

Example 17.5 p.421.

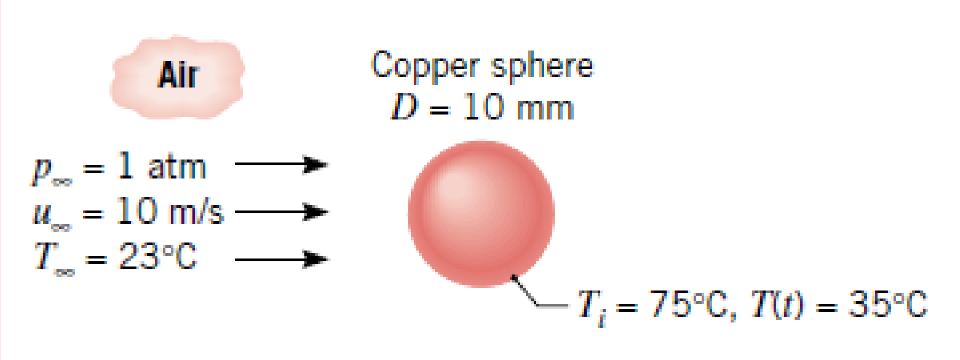
Example 17.5 Time to Cool a Sphere in an Air Stream

The decorative plastic film on a copper sphere of 10 mm in diameter is cured in an oven at 75°C. Upon removal from the oven, the sphere is subjected to an air stream at 1 atm and 23°C having a velocity of 10 m/s. Estimate how long it will take to cool the sphere to 35°C.

Solution

Known: Sphere cooling in an air stream.

Find: Time t required to cool from $T_i = 75°C$ to $T(t) = 35°C$.



Assumptions:
1. Negligible thermal resistance and capacitance for the plastic film.
2. Spatially isothermal sphere with $Bi \ll 1$.
3. Negligible radiation effects.

Figure E17.5

Properties: Table HT-1, copper ($\overline{T}_s = 328$ K): $\rho = 8933$ kg/m^3, $k = 399$ W/m · K, $c = 387$ J/kg · K. Table HT-3, air ($T_\infty = 296$ K): $\mu = 181.6 \times 10^{-7}$ N · s/m^2, $\nu = 15.36 \times 10^{-6}$ m^2/s, $k = 0.0258$ W/m · K, Pr = 0.709. Table HT-3, air ($T_s \approx 328$ K): $\mu = 197.8 \times 10^{-7}$ N · s/m^2.

Analysis: The time required to complete the cooling process may be obtained from results for a lumped capacitance (see Comment 1). In particular, from Eq. 16.84

$$t = \frac{\rho V c}{\bar{h} A_s} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

or, with $V = \pi D^3/6$ and $A_s = \pi D^2$

$$t = \frac{\rho c D}{6\bar{h}} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

To estimate the average convection coefficient, use the *Whitaker correlation*, Eq. 17.36

$$\overline{\text{Nu}}_D = 2 + (0.4\,\text{Re}_D^{1/2} + 0.06\,\text{Re}_D^{2/3})\text{Pr}^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

where the Reynolds number is

$$\text{Re}_D = \frac{u_\infty D}{\nu} = \frac{10\text{ m/s} \times 0.01\text{ m}}{15.36 \times 10^{-6}\text{ m}^2\text{/s}} = 6510$$

Hence the Nusselt number and the convection coefficient are

$$\overline{\text{Nu}}_D = 2 + [0.4(6510)^{1/2} + 0.06(6510)^{2/3}](0.709)^{0.4} \times \left(\frac{181.6 \times 10^{-7}\text{ N}\cdot\text{s/m}^2}{197.8 \times 10^{-7}\text{ N}\cdot\text{s/m}^2}\right)^{1/4} = 47.4$$

$$\bar{h} = \overline{\text{Nu}}_D \frac{k}{D} = 47.4\frac{0.0258\text{ W/m}\cdot\text{K}}{0.01\text{ m}} = 122\text{ W/m}^2\cdot\text{K}$$

The time required for cooling is then

$$t = \frac{8933\ \mathrm{kg/m^3} \times 387\ \mathrm{J/kg \cdot K} \times 0.01\ \mathrm{m}}{6 \times 122\ \mathrm{W/m^2 \cdot K}} \ln\left(\frac{75 - 23}{35 - 23}\right) = 69.2\ \mathrm{s} \quad \triangleleft$$

Comments:

1. The validity of the lumped capacitance method may be determined by calculating the Biot number. With Eqs. 16.89 and 16.90

$$Bi = \frac{\overline{h}L_c}{k_s} = \frac{\overline{h}(r_o/3)}{k_s} = \frac{122\ \mathrm{W/m^2 \cdot K} \times 0.005\ \mathrm{m/3}}{399\ \mathrm{W/m \cdot K}} = 5.1 \times 10^{-4}$$

and since $Bi < 0.1$, the criterion is satisfied.

2. Note that the thermophysical properties of copper and air corresponding to the average surface temperature were evaluated at $\overline{T}_s = (T_i + T(t))/2 = (75 + 35)°\mathrm{C}/2 = 328\ \mathrm{K}$.
3. Although their definitions are similar, the Nusselt number is defined in terms of the thermal conductivity of the fluid, whereas the Biot number is defined in terms of the thermal conductivity of the solid.

Summary

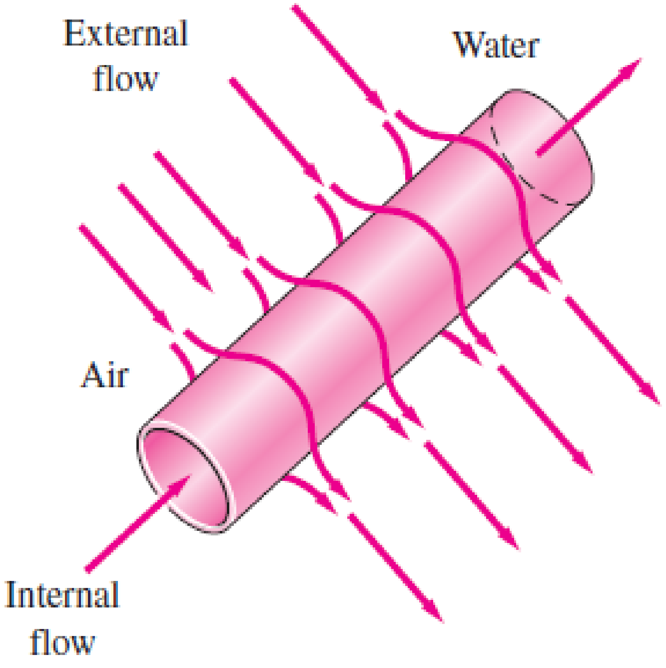
Table 17.3 Summary of Convection Heat Transfer Correlations for External Flow

Flow	Coefficient	Correlation[a]		Range of Applicability
Flat plate				
Laminar	—	$\delta = 5x\,Re_x^{-1/2}$	(17.21)	
	Local	$Nu_x = 0.332\,Re_x^{1/2}\,Pr^{1/3}$	(17.23)	$0.6 \le Pr \le 50$
	—	$\delta_t = \delta\,Pr^{-1/3}$	(17.24)	
	Average	$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3}$	(17.26)	$0.6 \le Pr \le 50$
Turbulent	Local	$\delta = 0.37x\,Re_x^{-1/5}$	(17.27)	$Re_x \le 10^8$
	Local	$Nu_x = 0.0296\,Re_x^{4/5}\,Pr^{1/3}$	(17.28)	$Re_x \le 10^8$, $0.6 \le Pr \le 60$
	Average	$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3}$	(17.32)	$Re_{x,c} = 0$, $0.6 \le Pr \le 60$
Mixed	Average	$\overline{Nu}_L = (0.037\,Re_L^{4/5} - 871)Pr^{1/3}$	(17.31)	$Re_{x,c} = 5 \times 10^5$, $10^5 \le Re_L \le 10^8$ $0.6 \le Pr \le 60$
Cylinders[b]	Average	$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3}$ (Table 7.2)	(17.34)	$Pr \ge 0.70$
	Average	$\overline{Nu}_D = 0.3 + \{0.62\,Re_D^{1/2}\,Pr^{1/3} \times [1 + (0.4/Pr)^{2/3}]^{-1/4}\} \times [1 + (Re_D/282{,}000)^{5/8}]^{4/5}$	(17.35)	$Re_D\,Pr > 0.2$
Sphere	Average	$\overline{Nu}_D = 2 + (0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3})Pr^{0.4}(\mu/\mu_s)^{1/4}$	(17.36)	$3.5 < Re_D < 7.6 \times 10^4$ $0.71 < Pr < 380$

[a]Thermophysical properties are evaluated at the film temperature, $T_f = (T_\infty + T_s)/2$, for all the correlations except Eq. 17.36. For that correlation, properties are evaluated at the free stream temperature T_∞ or at the surface temperature T_s if designated with the subscript s.
[b]For the cylinder with noncircular cross section, use Eq. 17.34 with the constants listed in Table 17.2.

Internal flow ;hydrodynamic entrance length

- For internal flow in a pipe or tube, the fluid is constrained by a surface, and hence eventually the boundary layer development will be constrained.
- When flow enters a tube, a hydrodynamic boundary layer forms in the entrance region, growing in thickness to eventually fill the tube. Beyond this location, referred to as the fully developed region, the velocity profile no longer changes in the flow direction.
- Due to viscous effects, the uniform velocity profile at the entrance will gradually change to a parabolic distribution as the boundary layer _ begins to fill the tube in the entrance region.
- Beyond the hydrodynamic entrance length, $x_{fd,h}$, the velocity profile no longer changes, and we speak of the flow as hydrodynamically fully developed.



Internal Flow; regions

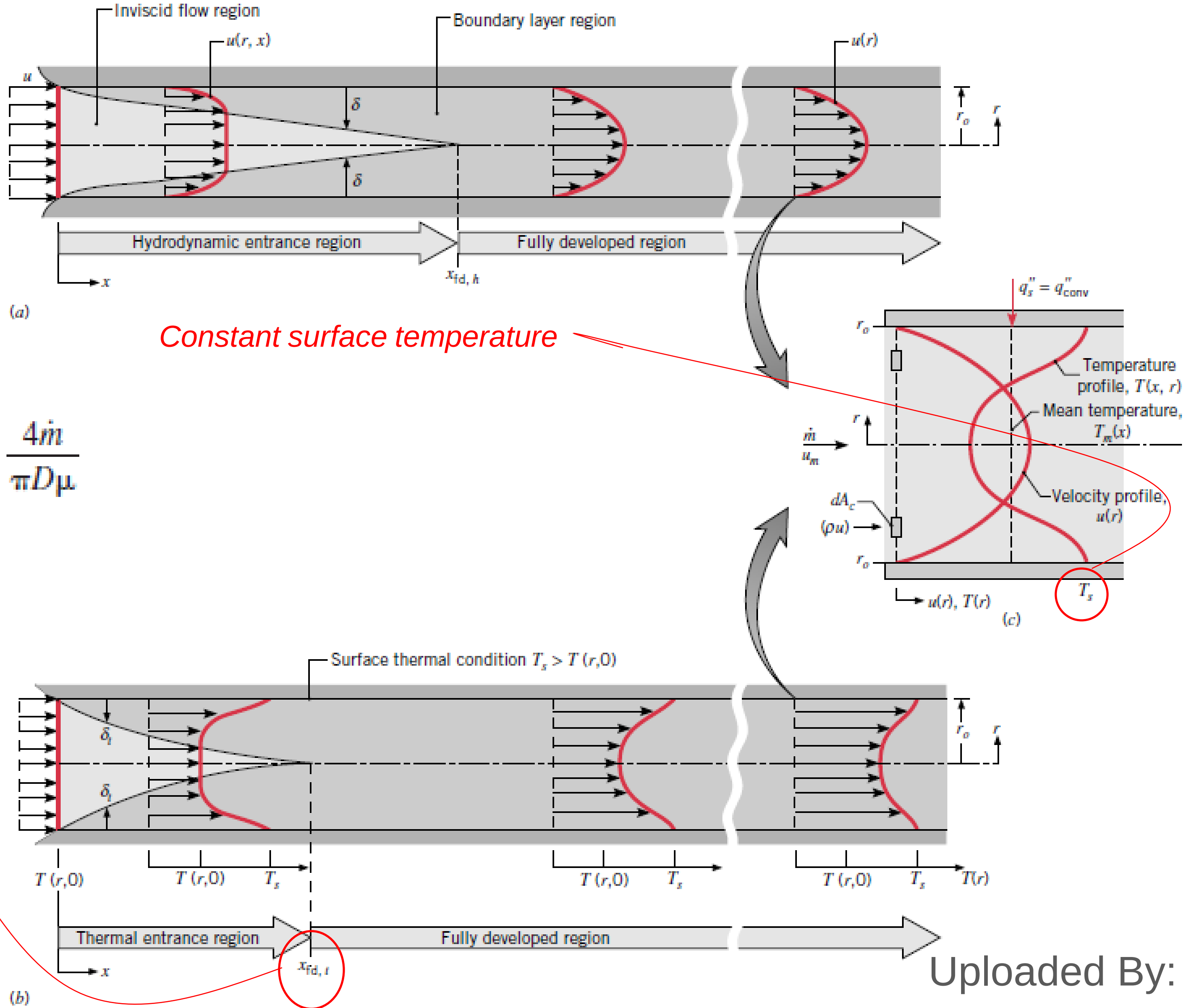
mean (average) velocity

$$Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} = \frac{4\dot{m}}{\pi D \mu}$$

$$Re_{D,c} \approx 2300$$

Thermal Entrance Length

Constant surface temperature



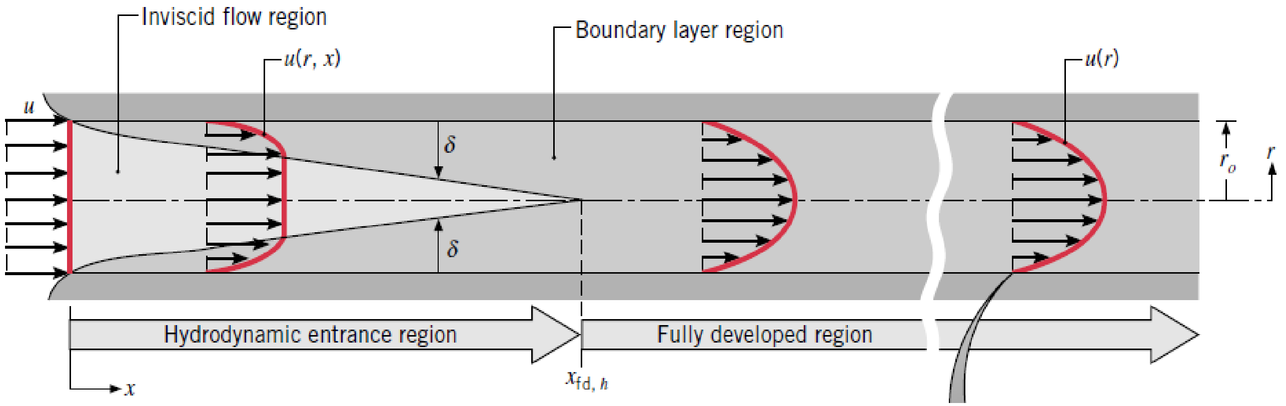
hydrodynamic entry length

- For *laminar flow* ($Re_D \leq 2300$), the *hydrodynamic entry length* has the form,

$$\left(\frac{x_{fd,h}}{D}\right)_{lam} \leq 0.05\, Re_D$$

- While for *turbulent flow,* the entry length is approximately independent of Reynolds number and that, as a first approximation entrance length is

$$(x/D) \leq 10$$



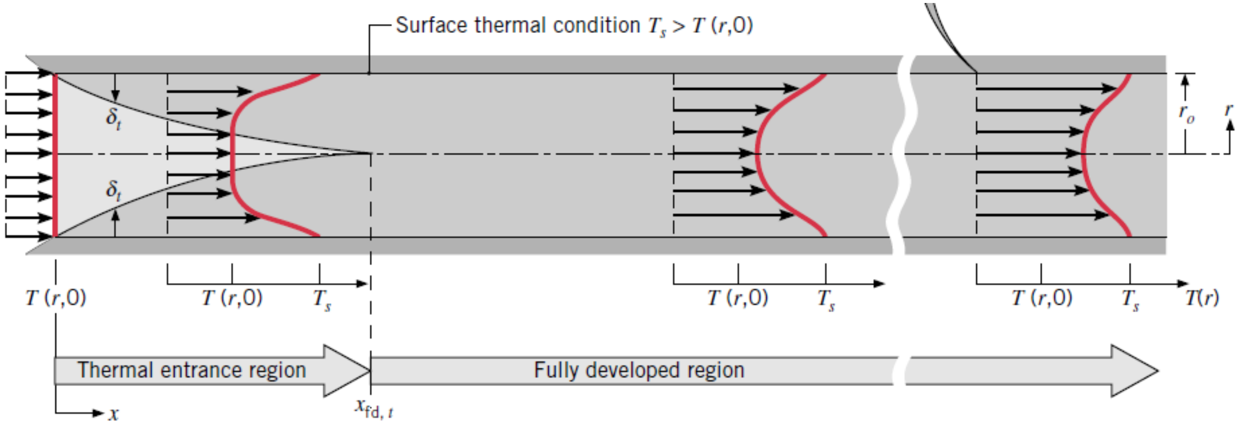
Internal Flow – Laminar and Turbulent Flows' Parameters

The thermal entry length for <u>Laminar Flow</u> may be expressed as;

$$\left(\frac{x_{fd,t}}{D}\right)_{\text{lam}} \approx 0.05\, Re_D\, Pr \qquad [Re_D < 2300]$$

And for <u>Turbulent Flow</u> as;

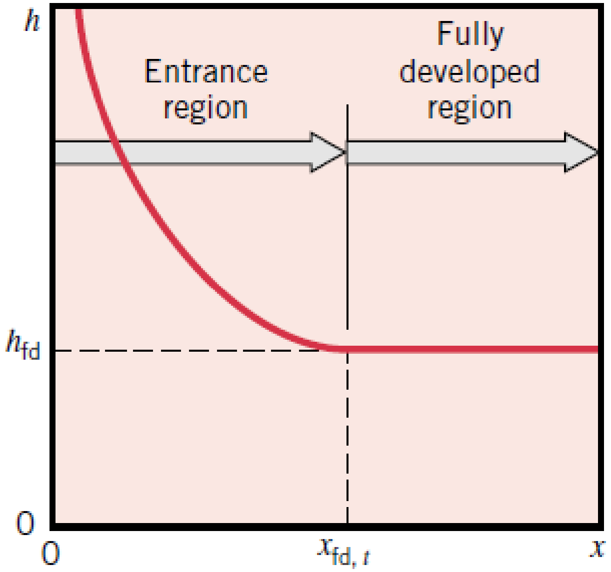
$$\left(\frac{x_{fd,t}}{D}\right)_{\text{turb}} = 10 \qquad [Re_D \geq 10{,}000]$$



Local convection coefficient

- Hence, in the thermally fully developed flow of a fluid with constant properties, the local convection coefficient is a constant, independent of x, while in the entrance region where h varies with x.

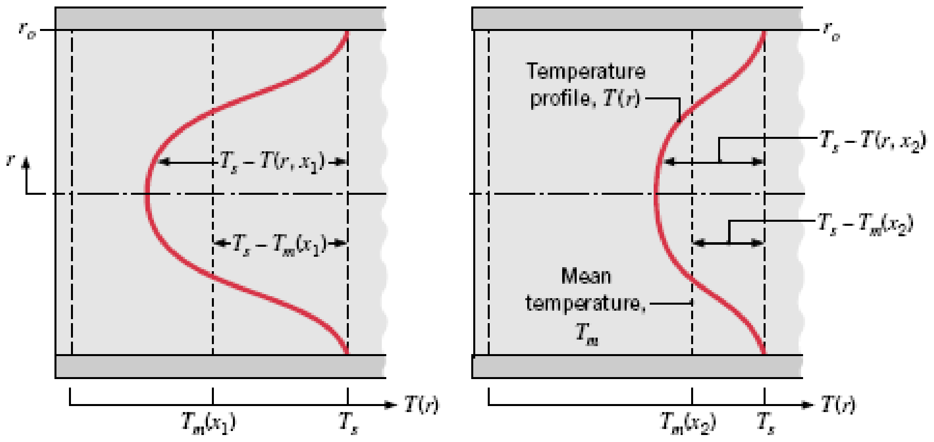
The mean temperature is the fluid reference temperature used for determining the convection heat rate with Newton’s law of cooling and the overall energy balance



Mean Temperature

- **The Mean Temperature** in the flow direction **x** depends on radius, **r**. It is referred to as the average or bulk temperature,
- For incompressible flow, with constant specific heat $\mathbf{c_p}$, the mean temperature is found from

$$T_m = \frac{\int_{A_c} uTdA_c}{u_m A_c}$$



(a)

For a circular tube it becomes;

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} uTrdr$$

Hence, the convection heat transfer is calculated from Newton's cooling law;

$$q_s'' = q_{conv}'' = h(T_s - T_m)$$

Mean temperature a long the pipe

- Assuming that fluid kinetic and potential energy changes are negligible, there is no shaft work, and regarding Cp as constant, the energy rate balance, reduces to give;

$$q_{conv} = \dot{m}c_p(T_{m,o} - T_{m,i})$$

- Tm denotes the mean fluid temperature and the subscripts i and o denote inlet and outlet conditions, respectively.

Differential control volume

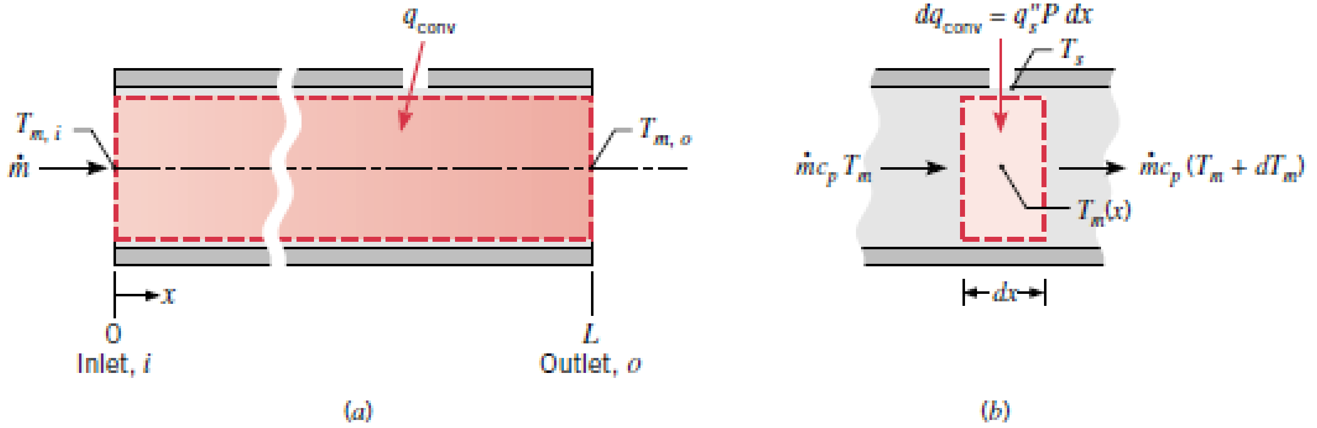
- apply the same analysis to a differential control volume within the tube as shown

$$dq_{\text{conv}} = \dot{m}c_p dT_m$$

$$dq_{\text{conv}} = q_s'' P dx$$

$$q_s'' P dx = \dot{m}c_p dT_m$$

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m}c_p} \qquad \left[\text{surface heat flux, } q_s''\right]$$



Constant surface heat flux

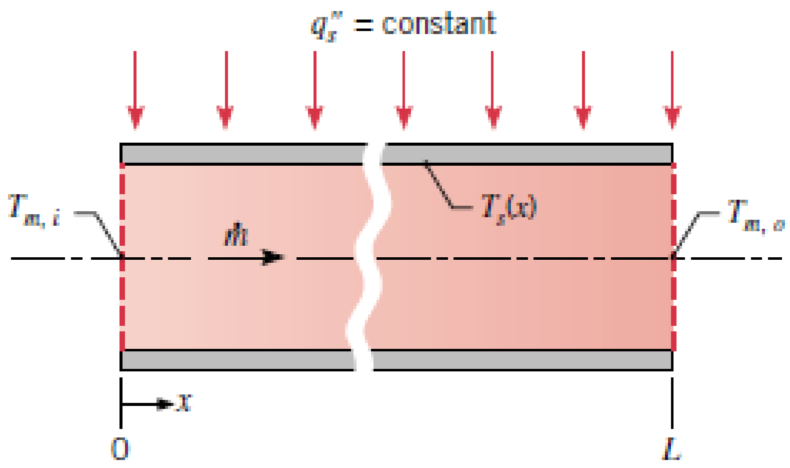
$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} \qquad [\text{surface heat flux, } q_s'']$$

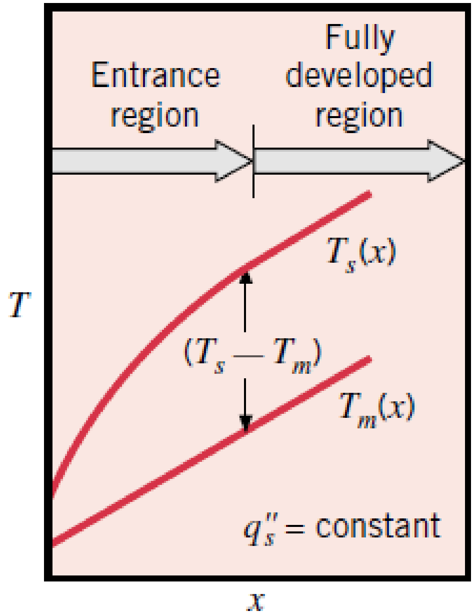
$$q_{\text{conv}} = q_s''(P \cdot L)$$

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} = \text{constant}$$

Integrating from x = 0 to some axial position x, we obtain the mean temperature distribution, Tm(x)

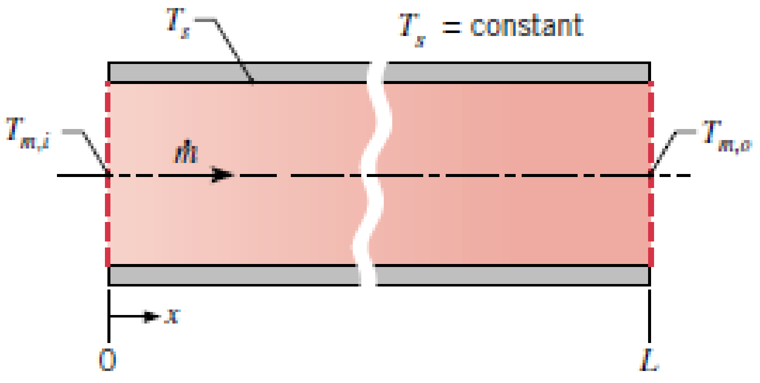
$$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}c_p}x \qquad [q_s'' = \text{constant}]$$





Constant Surface Temperature, Ts

$$\frac{dT_m}{dx} = \frac{P}{\dot{m}c_p} h(T_s - T_m) \qquad \text{[surface temperature, } T_s\text{]}$$



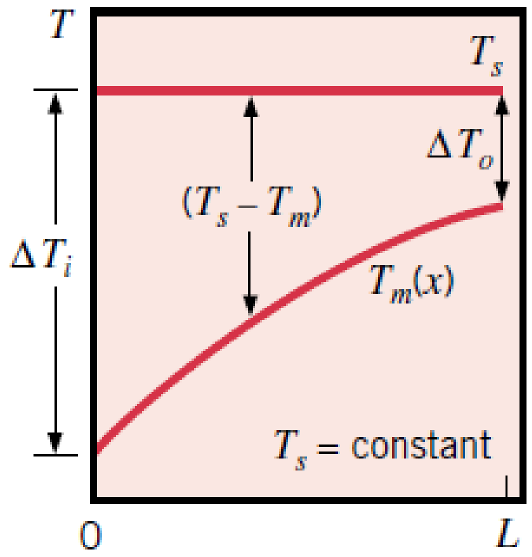
- Defining ΔT as (Ts - Tm), above may be expressed as

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\, \Delta T$$

$$\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m}c_p} \int_0^L h\, dx$$

$$\ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\left(\frac{1}{L}\int_0^L h\, dx\right) \qquad \ln\frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\bar{h}_L \qquad [T_s = \text{constant}]$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right) \qquad [T_s = \text{constant}]$$



Mean log temperature

$$q_{conv} = \dot{m}c_p(T_{m,o} - T_{m,i})$$

$$q_{conv} = \dot{m}c_p[(T_s - T_{m,i}) - (T_s - T_{m,o})] = \dot{m}c_p(\Delta T_i - \Delta T_o)$$

By substituting; $\dot{m}c_p$ from equation;

$$\ln \frac{\Delta T_o}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\bar{h}_L \qquad [T_s = \text{constant}]$$

$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right) \qquad [T_s = \text{constant}]$$

$$q_{conv} = \bar{h}A_s\Delta T_{lm} \qquad [T_s = \text{constant}]$$

A_s is the tube surface area (As =P . L)

$$\Delta T_{lm} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o/\Delta T_i)}$$

Is the log mean temperature difference (LMTD)

Example 17.7 Thermal Condition: Constant Surface Temperature, Ts

Example 17.6 Thermal Condition: Constant Surface Heat Flux q_s

A system for heating water from an inlet temperature of $T_{m,i} = 20°C$ to an outlet temperature of $T_{m,o} = 60°C$ involves passing the water through a tube having inner and outer diameters of 20 and 40 mm. The outer surface of the tube is well insulated, and electrical power dissipation within the wall provides for a uniform volumetric generation rate of $\dot{q} = 10^6$ W/m^3.

(a) For a water mass flow rate of $\dot{m} = 0.1$ kg/s, how long must the tube be to achieve the desired outlet temperature?

(b) Do fully developed hydrodynamic and thermal conditions exist in the flow?

(c) If the inner surface temperature of the tube is $T_s = 70°C$ at the outlet ($x = L$), what is the local convection heat transfer coefficient at the outlet?

Solution

Known: Internal flow through thick-walled tube having uniform volumetric energy generation.

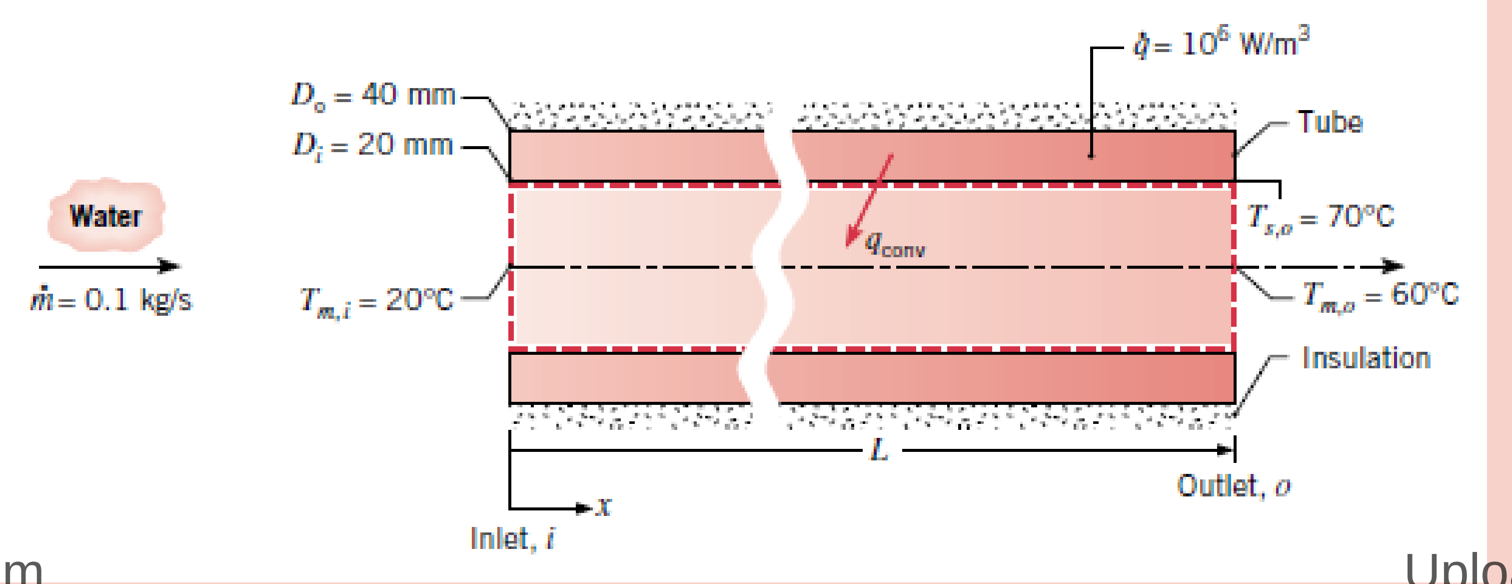
Find:

(a) Length of tube needed to achieve the desired outlet temperature.

(b) Whether fully developed hydrodynamic and thermal conditions exist.

(c) Local convection coefficient at the outlet.

Schematic and Given Data:



Assumptions:

1. Steady-state conditions.
2. Uniform heat flux.
3. Negligible potential energy and kinetic energy effects. No shaft work.
4. Constant properties.
5. Adiabatic outer tube surface.

Properties: Table HT-5, water ($\overline{T}_m = (T_{m,i} + T_{m,o})/2 = 313$ K): $c_p = 4.179$ kJ/kg · K, $\mu = 5.56 \times 10^{-4}$ N · s/m^2.

Analysis:

(a) Since the outer surface of the tube is adiabatic, the rate at which energy is generated within the tube wall must equal the rate at which it is convected to the water ($\dot{E}_g = q_{\text{conv}}$)

$$\dot{q}\frac{\pi}{4}(D_o^2 - D_i^2)L = q_{\text{conv}}$$

From the overall tube energy balance, Eq. 17.48, it follows that

$$\dot{q}\frac{\pi}{4}(D_o^2 - D_i^2)L = \dot{m}c_p(T_{m,o} - T_{m,i})$$

Solving for L and substituting numerical values with c_p evaluated at $\overline{T}_m = (T_{m,i} + T_{m,o})/2$, the required tube length is

$$L = \frac{4\dot{m}c_p}{\pi(D_o^2 - D_i^2)\dot{q}}(T_{m,o} - T_{m,i}) = \frac{4 \times 0.1 \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K}}{\pi(0.04^2 - 0.02^2)\text{ m}^2 \times 10^6 \text{ W/m}^3}(60 - 20)^\circ\text{C} = 17.7 \text{ m} \quad \triangleleft$$

(b) To determine whether fully developed conditions exist, calculate the Reynolds number to characterize the flow. From Eq. 17.37

$$\text{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.1 \text{ kg/s}}{\pi(0.020 \text{ m})(6.57 \times 10^{-4} \text{N} \cdot \text{s/m}^2)} = 9696$$

Since Re_D is nearly 10,000, the flow is fully turbulent. The hydrodynamic and thermal entry length is given as $x_{fd}/D = 10$ so that $x_{fd} = 10D = 10 \times 0.020\text{ m} = 0.2\text{ m}$. We conclude that, to a good approximation, fully developed conditions exist over the entire tube since $L \gg x_{fd}$ (17.7 m vs. 0.2 m).

(c) From Newton's law of cooling, Eq. 17.45, the local convection coefficient at the tube exit is

$$h_o = \frac{q_s''}{T_{s,o} - T_{m,o}}$$

Assuming that uniform generation in the wall provides a constant surface heat flux, with

$$q_s'' = \frac{q_{\text{conv}}}{\pi D_i L} = \dot{q}\,\frac{D_o^2 - D_i^2}{4D_i} = 10^6\ \text{W/m}^3\,\frac{(0.04^2 - 0.02^2)\,\text{m}^2}{4 \times 0.02\ \text{m}} = 1.5 \times 10^4\ \text{W/m}^2$$

it follows that the local coefficient at the outlet is

$$h_o = \frac{1.5 \times 10^4\ \text{W/m}^2}{(70 - 60)^\circ\text{C}} = 1500\ \text{W/m}^2 \cdot \text{K} \quad \triangleleft$$

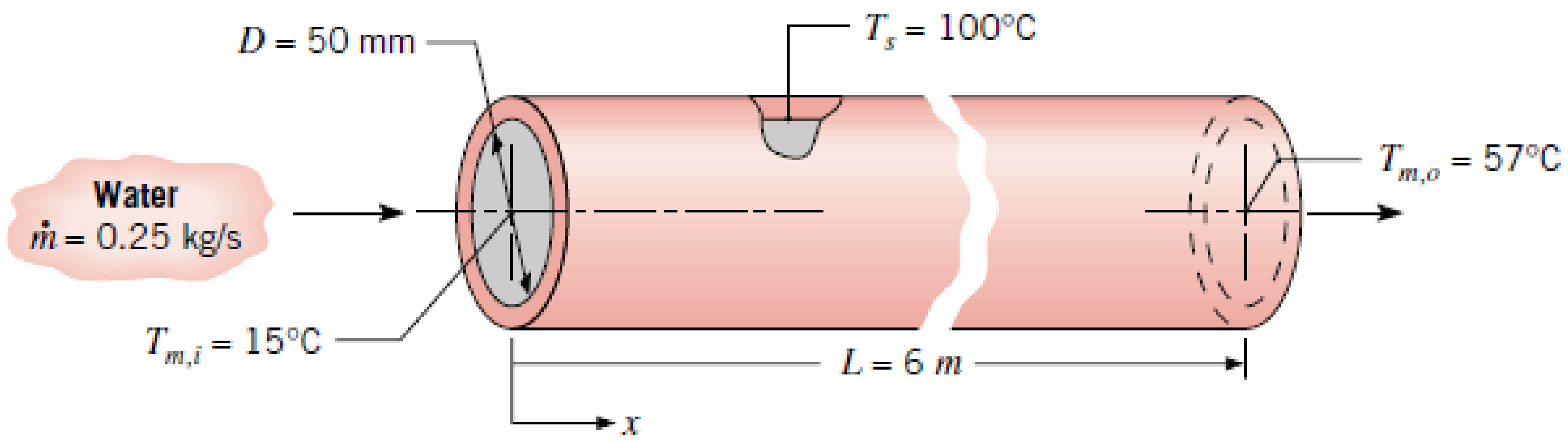
1. Since conditions are *fully developed over the entire tube,* the local convection coefficient and the temperature difference $(T_s - T_m)$ are independent of x for this *constant heat flux condition.* Hence, $h = 1500\ \text{W/m}^2 \cdot \text{K}$ and $(T_s - T_m) = 10^\circ\text{C}$ over the entire tube. The inner surface temperature at the tube inlet is then $T_{s,i} = 30^\circ\text{C}$. The fluid and tube surface temperature distributions are shown in Fig. E17.6*b*.
2. For the constant surface heat flux condition, the *exact shape* of the temperature profile in the *fully developed region* does not change in the flow direction ($x_2 > x_1$) as illustrated in Fig. E17.6*c*. Compare this behavior to that for *constant surface temperature condition,* Fig. 17.12*a*, where it is the *relative* shape that remains unchanged in the fully developed region.

Example 17.7 Constant Surface Temperature, *Ts*

Steam condensing on the outer surface of a thin-walled circular tube of 50-mm diameter and 6-m length maintains a uniform surface temperature of 100C. Water flows through the tube at a rate of and its inlet and outlet temperatures are *Tm,i* =15C and *Tm,o*= 57C. What is the average convection coefficient associated with the water flow?

Assumptions:

1. Negligible outer surface convection resistance and tube wall conduction resistance; hence, tube inner surface is at *Ts* 100C.
2. Negligible kinetic and potential energy effects. No shaft work.
3. Constant properties



Properties: Table HT-5, water ($\overline{T}_m = (T_{m,i} + T_{m,o})/2 = 36°C = 309$ K): $c_p = 4178$ J/kg · K.

Analysis: Combining the overall tube energy balance, Eq. 17.48, with the convection rate equation, Eq. 17.57, the average convection coefficient is given by

$$\bar{h} = \frac{\dot{m}c_p}{\pi DL}\frac{(T_{m,o} - T_{m,i})}{\Delta T_{\mathrm{lm}}}$$

From Eq. 17.58, the log mean temperature difference is

$$\Delta T_{\mathrm{lm}} = \frac{(T_s - T_{m,o}) - (T_s - T_{m,i})}{\ln[(T_s - T_{m,o})/(T_s - T_{m,i})]} = \frac{(100 - 57) - (100 - 15)}{\ln[(100 - 57)/(100 - 15)]} = 61.6°\mathrm{C}$$

Hence, the average convection coefficient is

$$\bar{h} = \frac{0.25\ \mathrm{kg/s} \times 4178\ \mathrm{J/kg \cdot K}}{\pi \times 0.05\ \mathrm{m} \times 6\ \mathrm{m}}\frac{(57 - 15)°\mathrm{C}}{61.6°\mathrm{C}} = 756\ \mathrm{W/m^2 \cdot K} \quad \triangleleft$$

Comments: Note that the properties for use in the energy balance and convection correlations are evaluated at the *average* mean temperature defined as $\bar{T}_m = (T_{m,i} + T_{m,o})/2$.

Internal Flow – Convection correlation for Tubes with Fully Developed flow

Laminar Flow

For flow in a circular tube characterized by uniform surface heat flux and laminar, fully developed conditions, the Nusselt number is a constant, independent of $\boldsymbol{Re_D}$, $\boldsymbol{Pr}$, and axial location.

For Constant Heat Flux;

$$\mathrm{Nu}_D = \frac{hD}{k} = 4.36 \qquad [q_s'' = \text{constant}]$$

For Constant Surface Temperature

$$\mathrm{Nu}_D = \frac{hD}{k} = 3.66 \qquad [T_s = \text{constant}]$$

Example 17.9 Laminar Flow Application: Solar Collector

One concept used for solar energy collection involves placing a tube at the focal point of a parabolic reflector (concentrator) and passing a fluid through the tube.

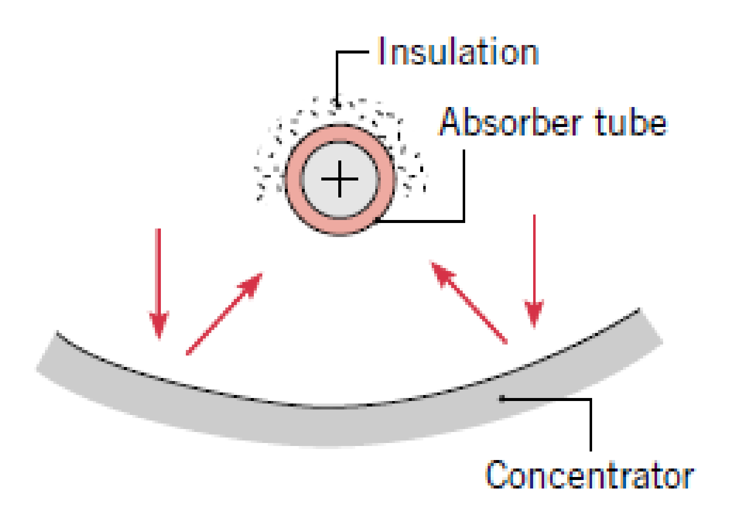


Figure E17.9a

The net effect of this arrangement *can be approximated* as one of creating a condition of uniform heating at the surface of the tube. That is, the resulting heat flux to the fluid q_s'' can be assumed to be a constant along the circumference and axis of the tube. Consider operation with a tube of diameter $D = 60$ mm on a sunny day for which $q_s'' = 2000\ \text{W/m}^2$.

(a) If pressurized water enters the tube at $\dot{m} = 0.01$ kg/s and $T_{m,i} = 20°\text{C}$, what tube length L is required to obtain an exit temperature of 80°C?

(b) What is the surface temperature at the outlet of the tube, where fully developed conditions can be assumed to exist?

Solution

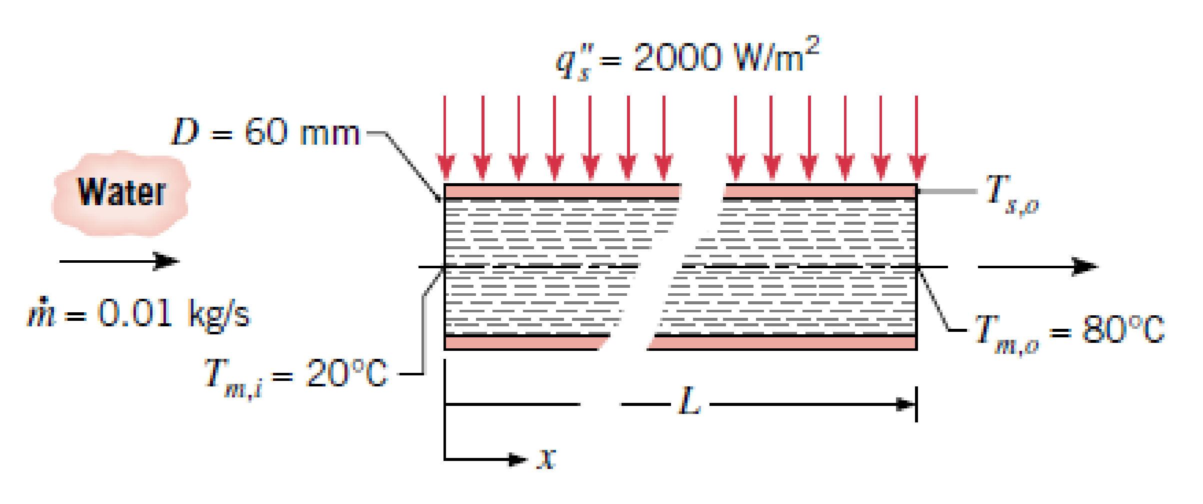
Known: Internal flow with uniform surface heat flux.

Find:

(a) Length of tube L to achieve required heating.

(b) Surface temperature $T_s(L)$ at the outlet section, $x = L$.

Schematic and Given Data:



Assumptions:

1. Steady-state conditions.
2. Incompressible flow with constant properties.
3. Negligible kinetic and potential energy effects. No shaft work.
4. Constant properties.
5. Fully developed conditions at tube outlet.

Figure E17.9b

Properties: Table HT-5, water ($\overline{T}_m = (T_{m,i} + T_{m,o})/2 = 323$ K): $c_p = 4181$ J/kg · K. Table HT-5, water ($T_{m,o} = 353$ K): $k = 0.670$ W/m · K, $\mu = 352 \times 10^{-6}$ N · s/m^2, Pr = 2.2.

Analysis:

(a) For constant surface heat flux, Eq. 17.53 can be used with the overall tube energy balance, Eq. 17.48, to obtain

$$A_s = \pi DL = \frac{\dot{m}c_p(T_{m,o} - T_{m,i})}{q_s''} \qquad \text{or} \qquad L = \frac{\dot{m}c_p}{\pi D q_s''}(T_{m,o} - T_{m,i})$$

Substituting numerical values, the required tube length is

$$L = \frac{0.01\ \text{kg/s} \times 4181\ \text{J/kg}\cdot\text{K}}{\pi \times 0.060\ \text{m} \times 2000\ \text{W/m}^2}(80 - 20)^\circ\text{C} = 6.65\ \text{m} \quad \triangleleft$$

(b) The surface temperature at the outlet can be obtained from Newton's law of cooling, Eq. 17.45, where

$$T_{s,o} = \frac{q_s''}{h} + T_{m,o}$$

To find the local convection coefficient at the tube outlet, the nature of the flow condition must first be established. From Equation 17.37

$$\mathrm{Re}_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.01\ \mathrm{kg/s}}{\pi \times 0.060\ \mathrm{m}(352 \times 10^{-6}\ \mathrm{N \cdot s/m^2})} = 603$$

Hence the flow is laminar. With the assumption of fully developed conditions, the appropriate heat transfer correlation is Eq. 17.61

$$\mathrm{Nu}_D = \frac{hD}{k} = 4.36$$

and the local coefficient is

$$h = 4.36\frac{k}{D} = 4.36\frac{0.670\ \mathrm{W/m \cdot K}}{0.06\ \mathrm{m}} = 48.7\ \mathrm{W/m^2 \cdot K}$$

The surface temperature at the tube outlet is then

$$T_{s,o} = \frac{2000\ \mathrm{W/m^2}}{48.7\ \mathrm{W/m^2 \cdot K}} + 80°\mathrm{C} = 121°\mathrm{C} \quad \triangleleft$$

Comments: For this laminar flow condition, from Eq. 17.41, we find the thermal entry length, $(x_{fd}/D) = 0.05\,\mathrm{Re}_D\,\mathrm{Pr} = 66.3$, while $L/D = 110$. Hence the assumption of fully developed conditions is reasonable. Because the water is pressurized, we assume that local boiling does not occur even though $T_{s,o} > 100°\mathrm{C}$.

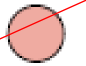
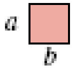
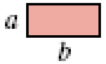
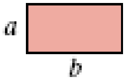
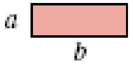
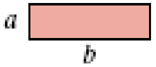
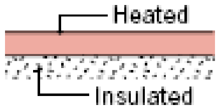
$$\mathrm{Nu}_D = \frac{hD}{k} = 4.36 \qquad [q''_s = \text{constant}]$$

Internal Flow – Convection correlation for Tubes with Fully Developed flow

$$D_h \equiv \frac{4A_c}{P}$$

hydraulic diameter

Table 17.4 Nusselt Numbers for Fully Developed Laminar Flow in Noncircular Tubes for Constant T_s and q_s'' Surface Thermal Conditions[a]

		$\mathrm{Nu}_D \equiv \frac{hD_h}{k}$	
Cross Section	$\frac{b}{a}$	Constant q_s''	Constant T_s
	—	4.36	3.66
	1.0	3.61	2.98
	1.43	3.73	3.08
	2.0	4.12	3.39
	3.0	4.79	3.96
	4.0	5.33	4.44
	8.0	6.49	5.60
	∞	8.23	7.54
	∞	5.39	4.86
	—	3.11	2.47

[a]The characteristic length is the hydraulic diameter, D_h, Eq. 17.63.

Internal Flow – Convection correlation for Tubes with Fully Developed flow

Turbulent Flow

the local Nusselt number for fully developed (hydrodynamically and thermally) turbulent flow in a smooth circular tube is the *Dittus-Boelter correlation* of the form

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^n \qquad \begin{bmatrix} 0.6 \le Pr \le 160 \\ Re_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10 \end{bmatrix}$$

where ***n=0.4*** for heating ***(Ts >Tm)*** and ***n=0.3*** for cooling ***(Ts <Tm)***

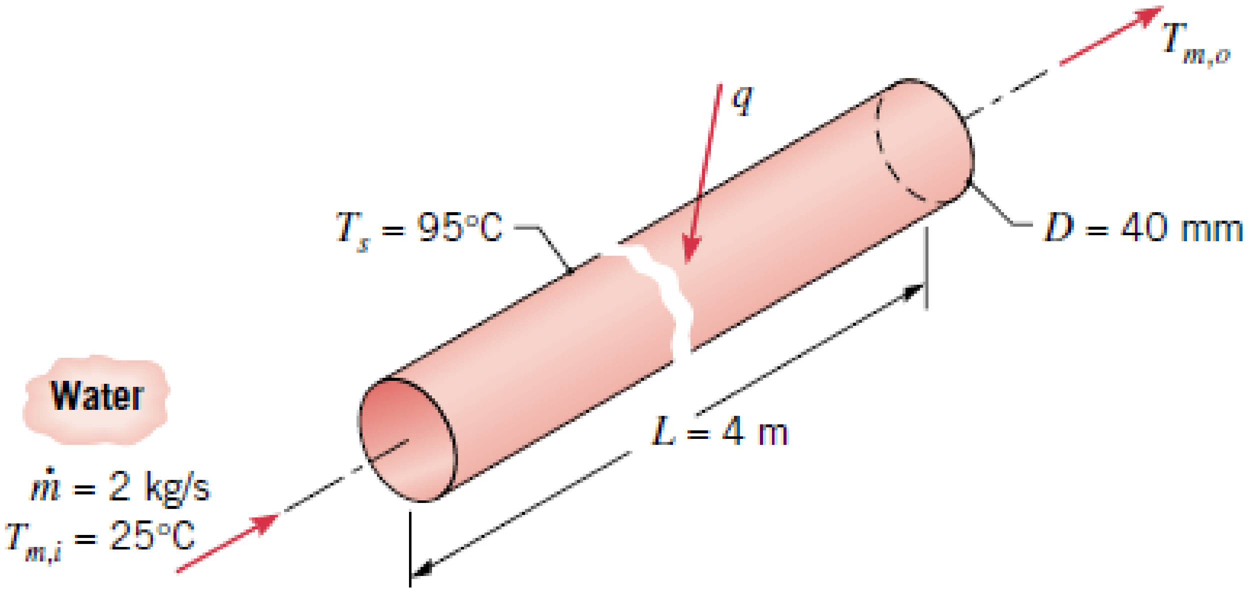
For flows characterized by large property variations, the *Sieder-Tate correlation* is recommended;

$$Nu_D = 0.027\, Re_D^{4/5}\, Pr^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad \begin{bmatrix} 0.7 \le Pr \le 16{,}700 \\ Re_D \gtrsim 10{,}000 \\ \dfrac{L}{D} \gtrsim 10 \end{bmatrix}$$

Example 17.10 Turbulent Flow Application: Hot Water Supply

Water flows steadily at 2 kg/s through a 40-mm-diameter tube that is 4 m long. The water enters at 25C, and the tube temperature is maintained at 95C by steam condensing on the exterior surface. Determine the outlet temperature of the water and the rate of heat transfer to the water.

Assumptions:
1. Steady-state conditions.
2. Negligible kinetic and potential energy effects. No shaft work.
3. Constant properties.
4. Fully developed flow conditions since *LD* 100



Properties: Table HT-5, water (assume $T_{m,o} = 50°C$; hence $\overline{T}_m = (T_{m,o} + T_{m,i})/2 = 37.5°C \approx 310K$): $c_p = 4178$ J/kg · K, $\mu = 695 \times 10^{-6}$ N · s/m^2, $k = 0.628$ W/m · K, Pr = 4.62. Table HT-5, water ($T_s = 95°C = 368$ K): $\mu_s = 296 \times 10^{-6}$ N · s/m^2.

Analysis: Since the tube surface temperature is constant, the water outlet temperature $T_{m,o}$ can be calculated from the energy rate expression of Eq. 17.55b

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\overline{h}\right) \tag{1}$$

Knowing $T_{m,o}$, the heat rate to the water follows from the overall energy balance, Eq. 17.48

$$q = \dot{m}c_p(T_{m,o} - T_{m,i}) \tag{2}$$

To select an appropriate correlation for estimating the average convection coefficient $\overline{h}$, calculate the Reynolds number, Eq. 17.37, to characterize the flow

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 2\ \text{kg/s}}{\pi(0.040\ \text{m})695 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2} = 9.16 \times 10^4$$

Hence the flow is turbulent, and with the assumption of fully developed conditions, we select the *Dittus-Boelter correlation*, Eq. 17.64, with $n = 0.4$ since $T_s > T_m$

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = 0.023\, Re_D^{4/5}\, Pr^{0.4} = 0.023\,(9.16 \times 10^4)^{4/5}\,(4.62)^{0.4} = 396$$

$$\overline{h} = \frac{\overline{Nu}_D k}{D} = \frac{396 \times 0.628\ \text{W/m} \cdot \text{K}}{0.040\ \text{m}} = 6210\ \text{W/m}^2 \cdot \text{K}$$

Using the energy rate expression, Eq. (1) with $P = \pi D$, find $T_{m,o}$

$$\frac{95°\text{C} - T_{m,o}}{95°\text{C} - 25°\text{C}} = \exp\left(\frac{-\pi(0.040\ \text{m})4\ \text{m}}{2\ \text{kg/s} \times 4178\ \text{J/kg} \cdot \text{K}} 6210\ \text{W/m}^2 \cdot \text{K}\right)$$

$$T_{m,o} = 46.8°\text{C} \quad \triangleleft$$

From the overall energy balance, Eq. (2), the heat rate *to* the water is

$$q = 2\ \text{kg/s} \times 4176\ \text{J/kg} \cdot \text{K}\,(46.8 - 25)°\text{C} = 182\ \text{kW} \quad \triangleleft$$

Comments:

1. Since the flow is turbulent and $L/D = 100$, the assumption of fully developed conditions is justified according to Eq. 17.42.
2. In using the energy relations for the entire tube, properties are evaluated at $\overline{T}_m$. Not knowing $T_{m,o}$ at the outset, we *guessed* $T_{m,o} = 50°\text{C}$ and used $\overline{T}_m = 310$ K. This was a good guess since the analysis shows $\overline{T}_m = (T_{m,i} + T_{m,o})/2 = (25 + 46.8)°\text{C}/2 = 309$ K. For such a situation, recognize that you may have to iterate your analysis until the *guessed* and *calculated* temperatures are in satisfactory agreement.
3. The *Sieder-Tate* correlation, Eq. 17.65, would also be appropriate for this situation. Substituting numerical values, find

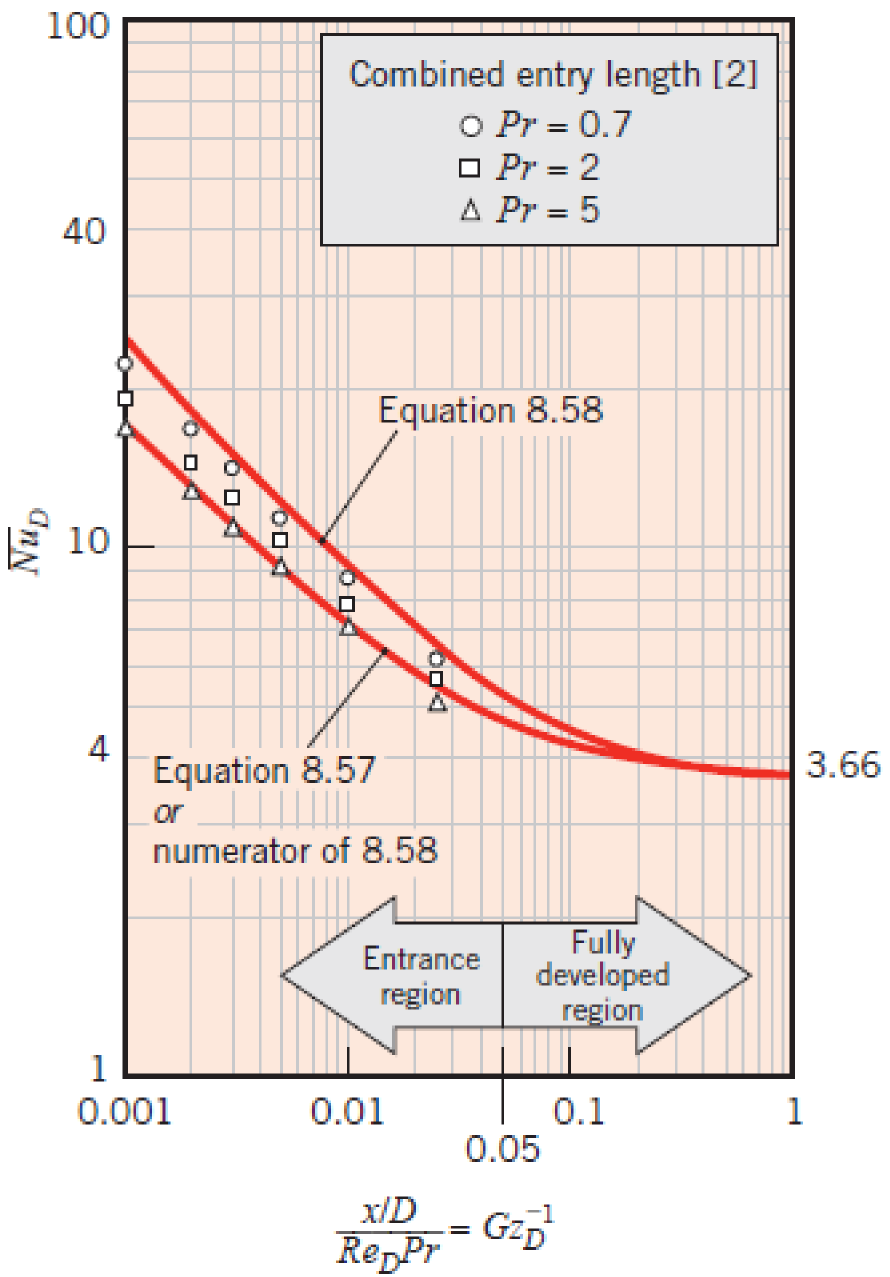
$$\overline{\text{Nu}}_D = 0.027\,\text{Re}_D^{4/5}\,\text{Pr}^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} = 0.027(9.16 \times 10^4)^{4/5}\,4.62^{1/3}\left(\frac{695 \times 10^{-6}}{695 \times 10^{-6}}\right)^{0.14} = 523$$

$$\overline{h} = \overline{\text{Nu}}_D \frac{k}{D} = 523\,\frac{0.628\ \text{W/m}\cdot\text{K}}{0.040\ \text{m}} = 8214\ \text{W/m}\cdot\text{K}$$

Using Eqs. (1) and (2), find $T_{m,o} = 50.3°\text{C}$ and $q = 212$ kW. The results of the two correlations differ by approximately 15%, which is within the uncertainty normally associated with such correlations. Note that all properties are evaluated at $\overline{T}_m$, except for μ_s, which is evaluated at T_s.

Entry region

- Entry length solutions for laminar flow in a circular tube with constant surface temperature



Average Nusselt numbers

Internal Flow – Convection correlation for Tubes with Fully Developed Flow

Table 17.5 Summary of Forced Convection Heat Transfer Correlations for Internal Flow in Smooth Circular Tubes[c]

Flow/Surface Thermal Conditions	Correlation[a,b]		Restrictions on Applicability
Laminar, fully developed, $(x_{fd}/D) > 0.05\,Re_D Pr$			
Constant q''_s	$Nu_D = 4.36$	(17.61)	$Pr \geq 0.6$, $Re_D \leq 2300$
Constant T_s	$Nu_D = 3.66$	(17.62)	$Pr \geq 0.6$, $Re_D \leq 2300$
Turbulent, fully developed, $(x_{fd}/D) > 10$			
Constant q''_s or T_s (*Dittus-Boelter*)	$Nu_D = 0.023\,Re_D^{4/5}\,Pr^n$	(17.64)	$0.6 \leq Pr \leq 160$, $Re_D \geq 10{,}000$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
Constant q''_s or T_s (*Sieder-Tate*)	$Nu_D = 0.027\,Re_D^{4/5}\,Pr^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(17.65)	$0.7 \leq Pr \leq 16{,}700$, $Re_D \geq 10{,}000$

[a]Thermophysical properties in Eqs. 17.61, 17.62, and 17.64 are based upon the mean temperature, T_m. If the correlations are used to estimate the *average* Nusselt number over the entire tube length, the properties should be based upon the average of the mean temperatures, $\bar{T}_m = (T_{m,i} + T_{m,o})/2$.

[b]Thermophysical properties in Eq. 17.65 should be evaluated at T_m or $\bar{T}_m$, except for μ_s, which is evaluated at the tube wall temperature T_s or $\bar{T}_s$.

[c]For tubes of *noncircular cross section*, use the hydraulic diameter, D_h, Eq. 17.63, as the characteristic length for the Reynolds and Nusselt numbers. Results for fully developed *laminar* flow are provided in Table 17.4. For *turbulent* flow, Eq. 17.64 may be used as a first approximation.

Outline

- Convection principles
- External flow convection
- Internal flow convection
- Free convection
- Heat exchangers

Fee Convection

- When there is no forced motion, heat transfer occurs because of convection currents that are induced by buoyancy forces, which arise from density differences caused by temperature variations in the fluid. Heat transfer by this means is referred to as free (or natural) convection.

- Since free convection flow velocities are generally much smaller than those associated with forced convection, the corresponding heat transfer rates are also smaller.

- In many thermal systems, free convection may provide the largest resistance to heat transfer and therefore plays an important role in the design or performance of the system.

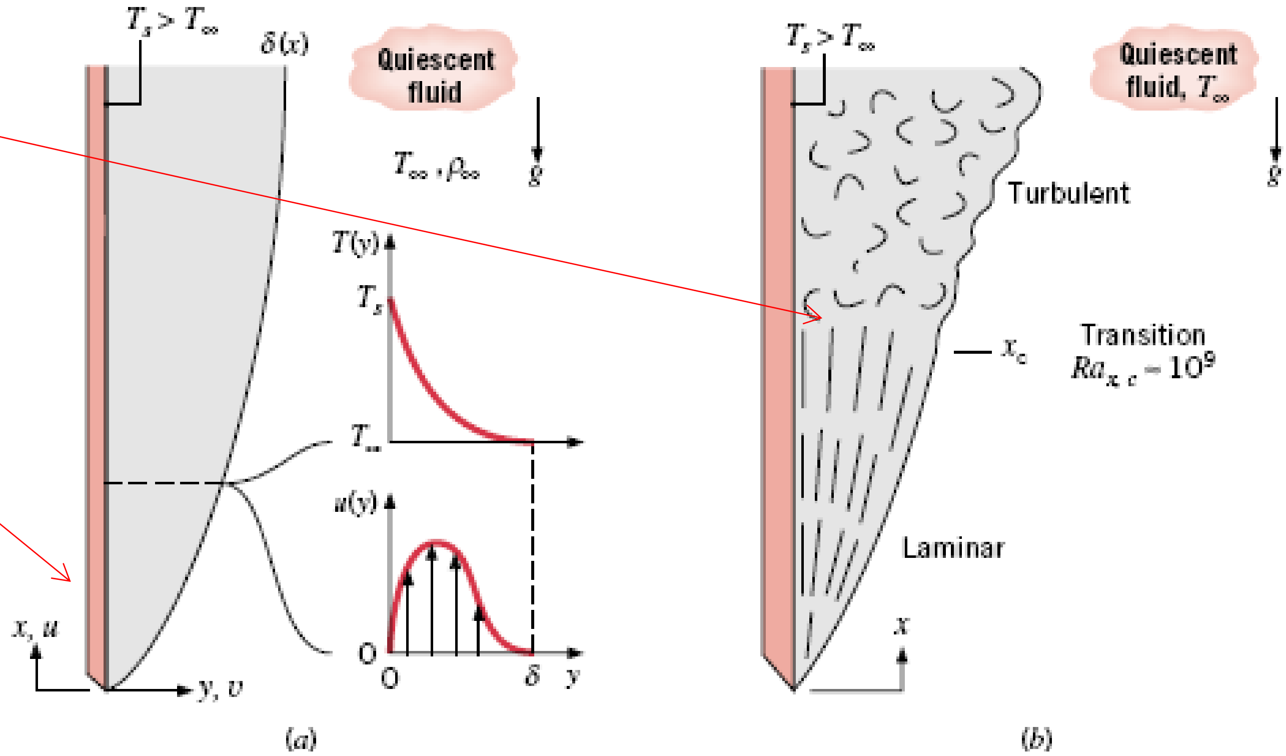
- Free convection is often the preferred mode of convection heat transfer, especially in electronic systems, for reasons of space limitations, maintenance-free operation, and reduced operating costs.

- Free convection strongly influences heat transfer from pipes, transmission lines, transformers, baseboard heaters, as well as appliances such as your stereo, television and laptop computer.

Free Convection

Buoyancy forces is due to the combined effect of fluid density gradient $\frac{\partial \rho}{\partial T} < 0$ and body force (gravitational) main sources of density gradient is temperature gradient

Boundary layer development on a heated vertical plate.
(a) Velocity and temperature profiles in the boundary layer at the location ***x***.
(b) Boundary layer transitional flow conditions.



Velocity distribution :u(y=0)=0 at surface then at returns to zero at edge of B.L u(y=δ)=0.

Grashof number

Grashof number Gr: $$Gr = \frac{buoyancy}{viscousforces}$$

$$F_B = \frac{g(\rho_\infty - \rho)}{\rho} = \frac{g}{\rho}\rho\beta(T - T_\infty) = g\beta(T - T_\infty) \qquad Gr_L = \frac{g\beta(T_s - T_\infty)}{\upsilon^2}L^3$$

Volume expansion coefficient β

$$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p = \frac{1}{\rho}\frac{p}{RT^2} = \frac{1}{\rho}\left(\frac{P}{RT}\right)\frac{1}{T} = \frac{1}{T}$$

"Gr_L" corresponds to "Re" in forced flow.

Laminar & turbulent flow

- Transition from laminar to turbulent flow depends on the relative magnitude of buoyancy forces & viscous forces define Rayleigh number Ra as GrPr

$$Ra_L = Gr\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\upsilon}$$

$Ra_{critical}=10^9$

Free Convection correlations

For free convection flows, we expect that the convection coefficient can be functionally expressed by equations of the form;

$$\overline{Nu}_L = f(Gr_L, Pr)$$

The most common empirical correlations suitable for engineering calculations have the form;

$$\overline{Nu}_L = \frac{\bar{h}L}{\boxed{k}} = C\,Ra_L^n$$

At $\boldsymbol{T_f}$

$$Ra_L = Gr_L\,Pr = \frac{g\beta(T_s - T_\infty)L^3}{\boxed{\nu\alpha}}$$

Vertical plate & cylinders isothermal surface

$$\overline{Nu}_L = C Ra_L^n$$

Constants C ,n as follows

i) $10^4 < Ra < 10^9$ laminar C=0.59, n=1/4

ii) $10^9 \le Ra \le 10^{13}$ turbulent . C=0.1, n=1/3.

- Churchill-Chu correlations 17.74 p.440 for entire range of Ra_L:

$$\overline{Nu}_L = \left[0.825 + \frac{0.387 Ra_L^{1/16}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{8/27}} \right]^2$$

Vertical cylinder same as vertical plates when $\dfrac{D}{L} \ge \dfrac{35}{Gr_L^{1/4}}$

$$Nu = \frac{hL}{k} \qquad Ra = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\upsilon}$$

L: height of cylinder

Example 17.11 Vertical Plate: Glass-Door Firescreen

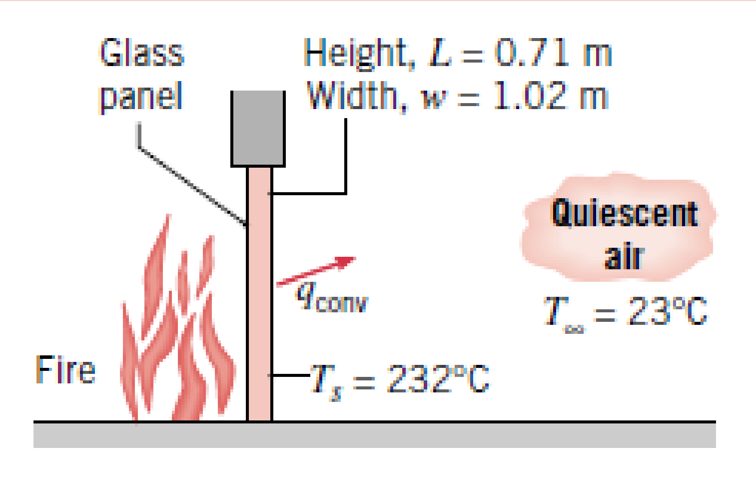
A glass-door firescreen, used to reduce loss of room air through a chimney, has a height of 0.71 m and a width of 1.02 m and reaches a temperature of 232°C. If the room temperature is 23°C, estimate the convection heat rate from the fireplace to the room.

Solution

Known: Glass screen situated in fireplace opening.

Find: Heat transfer by free convection between firescreen and room air.

Schematic and Given Data:



Assumptions:

1. Firescreen is at a uniform temperature T_s.
2. Room air is quiescent.
3. Costant properties.

Figure E17.11

Properties: Table HT-3, air ($T_f = (T_s + T_\infty)/2 = 400$ K): $k = 33.8 \times 10^{-3}$ W/m · K, $\nu = 26.4 \times 10^{-6}$ m^2/s, $\alpha = 38.3 \times 10^{-6}$ m^2/s, Pr = 0.690, $\beta = (1/T_f) = 0.0025$ K^{-1}.

Analysis: The rate of heat transfer by free convection from the firescreen to the room is given by Newton's law of cooling

$$q = \bar{h} A_s (T_s - T_\infty)$$

where $\bar{h}$ may be obtained from knowledge of the Rayleigh number. Using Eq. 17.71

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu}$$

$$\mathrm{Ra}_L = \frac{9.8\ \mathrm{m/s^2}\,(1/400\ \mathrm{K})(232 - 23)^\circ\mathrm{C} \times (0.71\ \mathrm{m})^3}{(38.3 \times 10^{-6}\ \mathrm{m^2/s})(26.4 \times 10^{-6}\ \mathrm{m^2/s})} = 1.813 \times 10^9$$

and from Eq. 17.66 it follows that transition to turbulence occurs on the panel. Using the *Churchill-Chu correlation*, Eq. 17.74, and substituting for the Rayleigh number, find

$$\overline{\mathrm{Nu}}_L = \left\{0.825 + \frac{0.387\,\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$\overline{\mathrm{Nu}}_L = \left\{0.825 + \frac{0.387(1.813 \times 10^9)^{1/6}}{[1 + (0.492/0.690)^{9/16}]^{8/27}}\right\}^2 = 147$$

Hence, the average convection coefficient is

$$\bar{h} = \frac{\overline{\mathrm{Nu}}_L k}{L} = \frac{147(33.8 \times 10^{-3}\ \mathrm{W/m \cdot K})}{0.71\ \mathrm{m}} = 7.0\ \mathrm{W/m^2 \cdot K}$$

and the heat transfer by free convection between the firescreen and room air is

$$q = 7.0\ \mathrm{W/m^2 \cdot K}(1.02 \times 0.71)\mathrm{m^2}\,(232 - 23)^\circ\mathrm{C} = 1060\ \mathrm{W} \quad \triangleleft$$

Comments:

1. If $\overline{h}$ were computed using the simpler correlation of Eq. 17.73, we would obtain $\overline{h} = 5.8\ \text{W/m}^2 \cdot \text{K}$, and the heat transfer prediction would be approximately 20% lower than the foregoing result. This difference is within the uncertainty normally associated with using such correlations.
2. Radiation heat transfer effects are often significant relative to free convection. Using the radiative exchange rate equation, Eq. 15.7, and assuming $\varepsilon = 1.0$ for the glass surface and $T_{sur} = 23°\text{C}$, the net rate of radiation heat transfer between the firescreen and the surroundings is

$$q_{rad} = \varepsilon A_s \sigma (T_s^4 - T_{sur}^4) = 1(1.02 \times 0.71)\text{m}^2\,(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)\,(505^4 - 296^4)\text{K}^4$$

$$q_{rad} = 2355\ \text{W}$$

The linearized radiation coefficient is given by Eq. 15.9

$$h_{rad} = \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2) = 1(5.67 \times 10^5\ \text{W/m}^2 \cdot \text{K}^4)(505 + 296)(505^2 + 296^2)\text{K}^3$$

$$h_{rad} = 15.6\ \text{W/m}^2 \cdot \text{K}$$

Note that the radiation coefficient (radiation heat rate) is more than twice the convection coefficient (convection heat rate) for this application.

Horizontal plates

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^{\ 3}}{\alpha\upsilon}$$

where L_c is characteristic length.

Characteristic length $L_c = A_s/p$= plate area/plate perimeter

Disk= $\dfrac{\pi D^2/4}{\pi D} = \dfrac{D}{4} = L_c$

Square: $L_c = \dfrac{L^2}{4L} = \dfrac{L}{4}$

Rectangle: $L_c = \dfrac{Lw}{2L + 2w}$ if w<<L $\Rightarrow L_c = \dfrac{Lw}{2L} = \dfrac{w}{2}$

Horizontal plates: four cases

Hot Surface Facing downward (Case A)
Cold Surface Facing Upward (Cases B)
Hot Surface Facing upward (Case c)
Cold Surface Facing Downward (Case D)

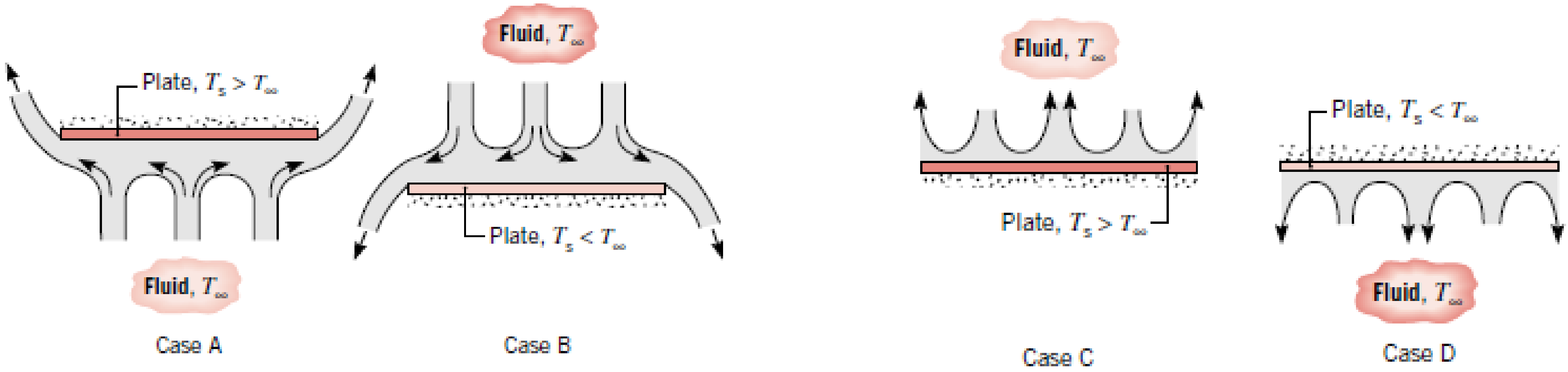


Figure 17.21 Free convection buoyancy-driven flows for hot ($T_s > T_\infty$) and cold ($T_s < T_\infty$) horizontal plates: Case A — hot surface facing downwards, Case B — cold surface facing upwards, Case C — hot surface facing upwards, and Case D — cold surface facing downwards.

Horizontal plates

Surface is heated (hot) when $T_s > T_\infty$, its surface is cooled (cold) when ($T_s < T_\infty$), its surface at temperature lower than ambient.

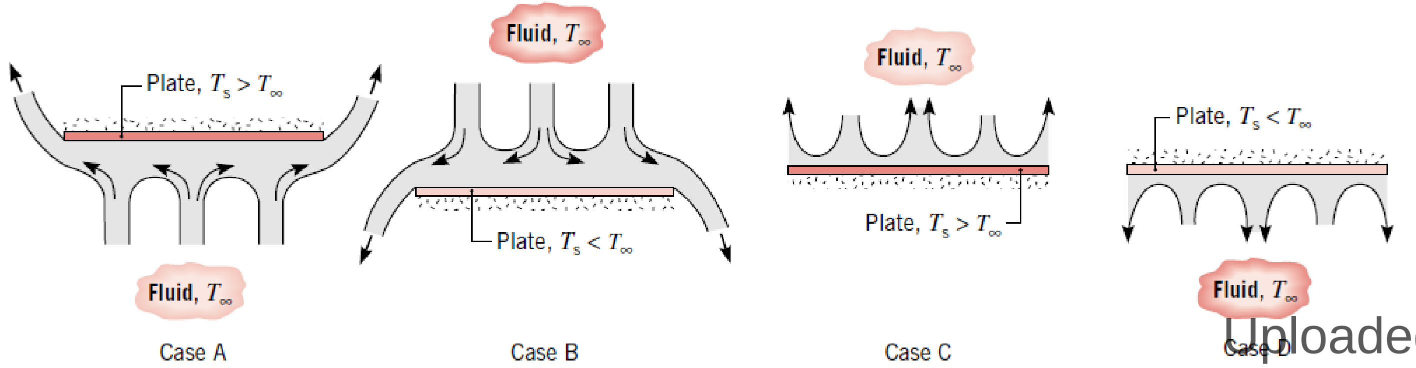
Hot surface facing downward (Case A) or cold surface facing upward (Case B)

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \qquad 10^5 < Ra_L < 10^{10}$$

Hot surface facing upward (Case C) or cold surface facing downward (Case D)

$$\overline{Nu}_L = 0.54 Ra_L^{1/4} \qquad 10^4 < Ra_L < 10^7$$

$$\overline{Nu}_L = 0.15 Ra_L^{1/3} \qquad 10^7 < Ra_L < 10^{11}$$



Example 17.12 Horizontal Plate: Cooling an Electronic Equipment Enclosure

An array of power-dissipating electrical components is mounted on the bottom side of a 1.2 m by 1.2 m horizontal aluminum alloy plate ($\varepsilon = 0.25$), while the top side is cooled by free convection with ambient, quiescent air at $T_\infty = 300$ K and by radiation exchange with the surroundings at $T_{sur} = 300$ K. The plate is sufficiently thick to ensure a nearly uniform upper surface temperature and is attached to a well-insulated enclosure.

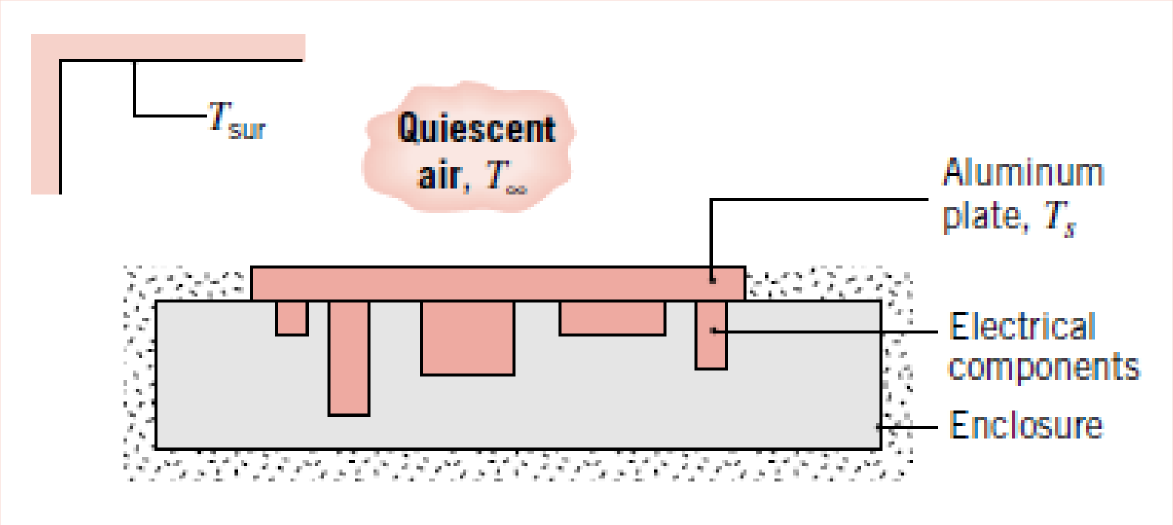
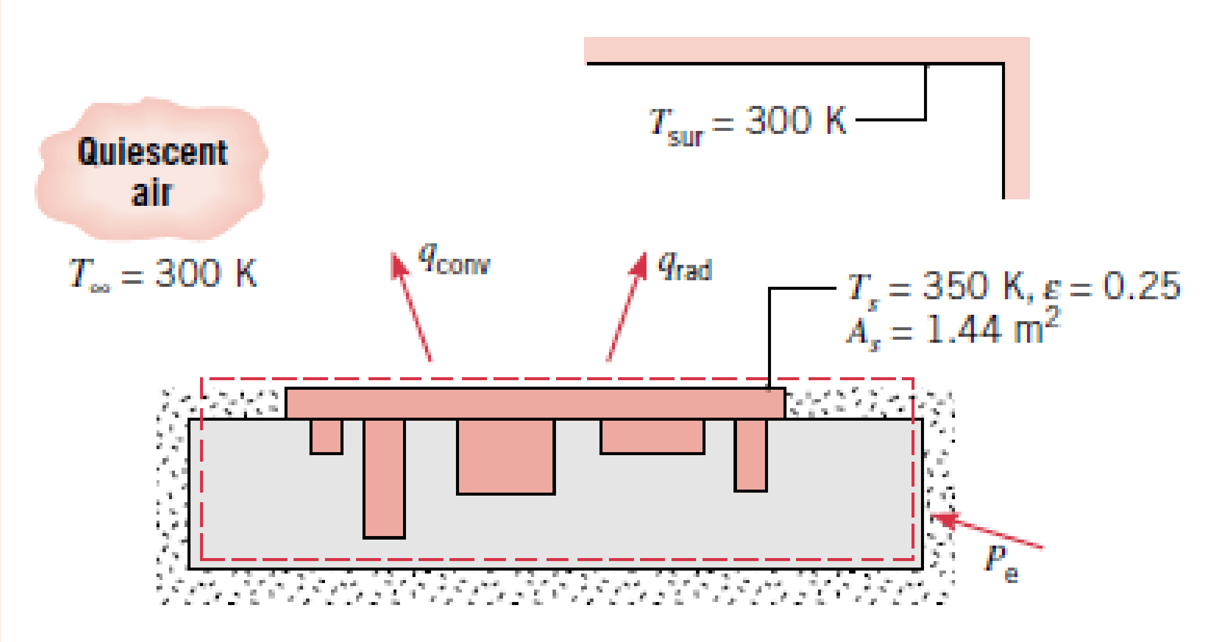


Figure E17.12a

If the temperature of the plate is not to exceed 57°C, what is the maximum allowable power dissipation in the electrical components?

Schematic and Given Data:



Assumptions:

1. Steady-state conditions.
2. Plate is isothermal.
3. Negligible heat transfer from other surfaces of the enclosure.
4. Radiation exchange is between a small, gray object (plate) and large isothermal surroundings.
5. Constant properties.

Figure E17.12b

Properties: Table HT-3, air ($T_f = 325$ K, 1 atm): $\nu = 18.4 \times 10^{-6}$ m^2/s, $k = 0.028$ W/m · K, $\alpha = 26.2 \times 10^{-6}$ m^2/s.

Analysis: From an overall energy balance on the enclosure and plate, the electrical power dissipation is the sum of the heat transfer rates by free convection and radiation exchange (Eq. 15.7)

$$P_e = q_{conv} + q_{rad}$$

$$P_e = \bar{h} A_s (T_s - T_\infty) + \varepsilon A_s \sigma (T_s^4 - T_{sur}^4)$$

For free convection from the horizontal plate, the characteristic length from Eq. 17.77 is

$$L = A_s/P = (1.2 \times 1.2\ \text{m}^2)/(4 \times 1.2\ \text{m}) = 0.3\ \text{m}$$

and from Eq. 17.71, the Rayleigh number with $\beta = 1/T_f$ (Eq. 17.69) is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{9.8\ \text{m/s}^2(325\ \text{K})^{-1}(50\ \text{K})(0.3\ \text{m})^3}{(18.4 \times 10^{-6}\ \text{m}^2/\text{s})(26.2 \times 10^{-6}\ \text{m}^2/\text{s})} = 8.44 \times 10^7$$

Using the correlation of Eq. 17.80 for a *hot surface facing upward* (Case C), find the average convection coefficient

$$\overline{\text{Nu}}_L = \frac{\bar{h}L}{k} = 0.15\,\text{Ra}_L^{1/3} = 0.15(8.44 \times 10^7)^{1/3} = 65.8$$

$$\bar{h}_L = 65.8\,\frac{0.028\ \text{W/m}\cdot\text{K}}{0.3\ \text{m}} = 6.2\ \text{W/m}^2\cdot\text{K}$$

The allowable electrical power is

$$P_e = [6.1\ \text{W/m}^2\cdot\text{K}(350 - 300)\text{K} + 0.25(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(350^4 - 300^4)\text{K}^4](1.44\ \text{m}^2)$$

$$P_e = 446\ \text{W} + 141\ \text{W} = 587\ \text{W}$$

Comments: Note that heat transfer by free convection and radiation exchange comprise 76% and 24%, respectively, of the total heat rate. It would be beneficial to apply a high emissivity coating to the plate as a means to enhance radiative heat transfer, and hence, the allowable electrical power.

Horizontal cylinders

$$\overline{Nu}_D == 0.850 Ra_D^{0.188}$$ $10^2 \le Ra \le 10^4$

$$\overline{Nu}_D == 0.480 Ra_D^{0.25}$$ $10^4 \le Ra \le 10^7$

$$\overline{Nu}_D == 0.125 Ra_D^{0.333}$$ $10^7 \le Ra \le 10^{12}$

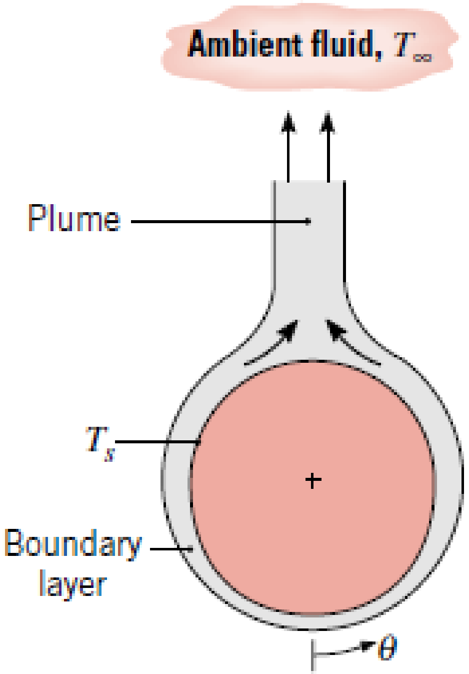
For the **Cylinder** shape, the *Churchill-Chu* correlation is recommended;

$$\overline{\mathrm{Nu}}_D = \left\{ 0.60 + \frac{0.387\,\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}} \right\}^2 \quad [\mathrm{Ra}_D \lesssim 10^{12}]$$

Free Convection –Sphere

For the **Sphere** shape, use;

$$\overline{Nu}_D = 2 + 0.43 Ra_D^{1/4}$$



$$\overline{Nu}_D = 2 + \frac{0.589\, Ra_D^{1/4}}{[1 + (0.469\, Pr)^{9/16}]^{4/9}} \qquad [Pr \geq 0.7,\ Ra_D \lesssim 10^{11}]$$

Example 17.13 Horizontal Cylinder: High Pressure Steam Line

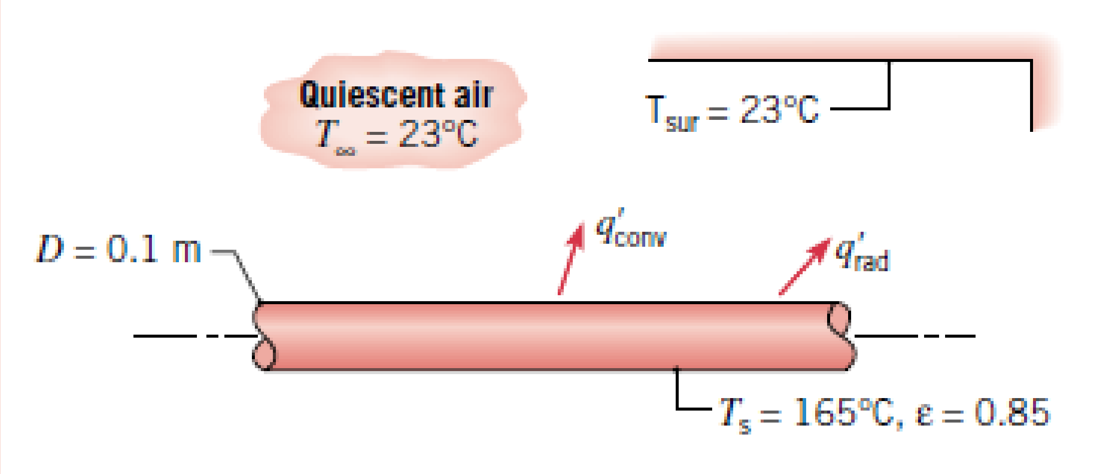
A horizontal, high-pressure steam pipe of 0.1-m outside diameter passes through a large room whose wall and air temperatures are 23°C. The pipe has an outside surface temperature of 165°C and an emissivity of $\varepsilon = 0.85$. Estimate the heat transfer from the pipe per unit length.

Solution

Known: Surface temperature of a horizontal steam pipe.

Find: Heat transfer from the pipe per unit length q' (W/m).

Schematic and Given Data:



Assumptions:

1. Pipe surface area is small compared to surroundings.
2. Room air is quiescent.
3. Radiation exchange is between a small, gray surface (pipe) and large isothermal surroundings.
4. Constant properties.

Figure E17.13

Properties: Table HT-3, air ($T_f = 367$ K): $k = 0.0313$ W/m · K, $\nu = 22.8 \times 10^{-6}$ m²/s, $\alpha = 32.8 \times 10^{-6}$ m²/s, Pr = 0.697, $\beta = 2.725 \times 10^{-3}$ K^{-1}.

Analysis: The total heat transfer per unit length of pipe due to convection and radiation exchange (Eq. 15.7) is

$$q' = q'_{\text{conv}} + q'_{\text{rad}} = \bar{h}\pi D(T_s - T_\infty) + \varepsilon\pi D\sigma(T_s^4 - T_{\text{sur}}^4)$$

The free convection coefficient may be estimated with the *Churchill-Chu correlation,* Eq. 17.84

$$\overline{\text{Nu}}_D = \left\{0.60 + \frac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

where the Rayleigh number from Eq. 17.71 is

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

$$\text{Ra}_D = \frac{9.8\ \text{m/s}^2(2.725 \times 10^{-3}\ \text{K}^{-1})(165 - 23)^\circ\text{C}\ (0.1\ \text{m})^3}{(22.8 \times 10^{-6}\ \text{m}^2/\text{s})(32.8 \times 10^{-6}\ \text{m}^2/\text{s})} = 5.073 \times 10^6$$

Substituting for the Rayleigh number into the correlation, find

$$\overline{\text{Nu}}_D = \left\{0.60 + \frac{0.387(5.073 \times 10^6)^{1/6}}{[1 + (0.559/0.697)^{9/16}]^{8/27}}\right\}^2 = 23.3$$

and the average convection coefficient for the cylinder is

$$\bar{h} = \frac{k}{D}\overline{\text{Nu}}_D = \frac{0.0313\ \text{W/m}\cdot\text{K}}{0.1\ \text{m}} \times 23.3 = 7.29\ \text{W/m}^2\cdot\text{K}$$

The total heat transfer rate from the pipe is

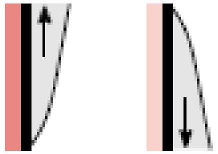
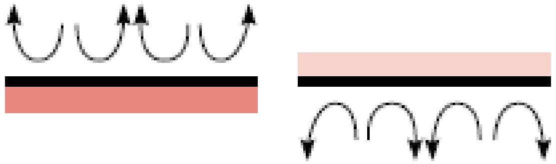
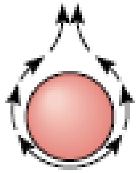
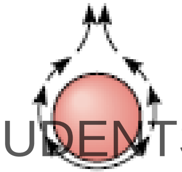
$$q' = 7.29\ \mathrm{W/m^2 \cdot K}\ (\pi \times 0.1\ \mathrm{m})(165 - 23)^\circ\mathrm{C}$$
$$+ 0.85\ (\pi \times 0.1\ \mathrm{m})(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(438^4 - 296^4)\ \mathrm{K^4}$$

$$q' = (325 + 441)\ \mathrm{W/m} = 766\ \mathrm{W/m} \quad \triangleleft$$

Comments: 1. Note that the heat transfer by free convection and radiation exchange comprise 42 and 58%, respectively, of the total heat rate. It would be beneficial to apply a low emissivity coating to the pipe as a means to reduce the radiation exchange, and hence the heat transfer from the pipe to the room.

2. Equation 17.82 could also be used to estimate the Nusselt number and the convection coefficient, with the result that $\overline{Nu}_D = 22.8$ and $\bar{h} = 7.14\ \mathrm{W/m^2 \cdot K}$. These results are about 2% lower than the foregoing ones. Generally we expect differences between correlation results of 10–15%, rather than the excellent agreement found here.

Table 17.6 Summary of Free Convection Correlations for Immersed Geometries

Geometry	Recommended Correlation		Restrictions
Vertical plates[a] 	$\overline{\mathrm{Nu}}_L = \left\{0.825 + \dfrac{0.387\,\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$	(17.74)	$\mathrm{Ra}_L \lesssim 10^{13}$
Horizontal plates[b] Case A or B: Hot surface down or cold surface up 	$\overline{\mathrm{Nu}}_L = 0.27\,\mathrm{Ra}_L^{1/4}$	(17.78)	$10^5 \lesssim \mathrm{Ra}_L \lesssim 10^{10}$
Case C or D: Hot surface up or cold surface down 	$\overline{\mathrm{Nu}}_L = 0.54\,\mathrm{Ra}_L^{1/4}$	(17.79)	$10^4 \lesssim \mathrm{Ra}_L \lesssim 10^7$
	$\overline{\mathrm{Nu}}_L = 0.15\,\mathrm{Ra}_L^{1/3}$	(17.80)	$10^7 \lesssim \mathrm{Ra}_L \lesssim 10^{11}$
Horizontal cylinder 	$\overline{\mathrm{Nu}}_D = \left\{0.60 + \dfrac{0.387\,\mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$	(17.84)	$\mathrm{Ra}_D \lesssim 10^{12}$
Sphere 	$\overline{\mathrm{Nu}}_D = 2 + \dfrac{0.589\,\mathrm{Ra}_D^{1/4}}{[1 + (0.469\,\mathrm{Pr})^{9/16}]^{4/9}}$	(17.85)	$\mathrm{Ra}_D \lesssim 10^{11}$ $\mathrm{Pr} \gtrsim 0.7$

The characteristic length is defined as $L = A_s/P$,

Outline

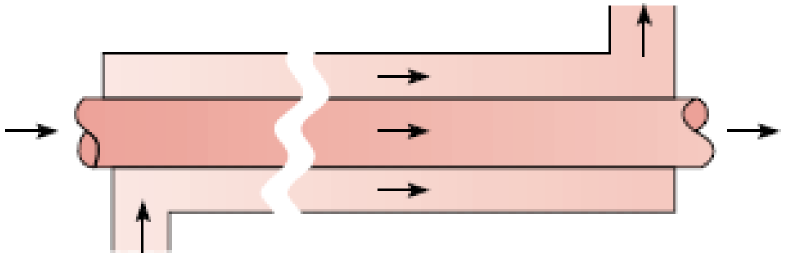
- Convection principles
- External flow convection
- Internal flow convection
- Free convection
- Heat exchangers

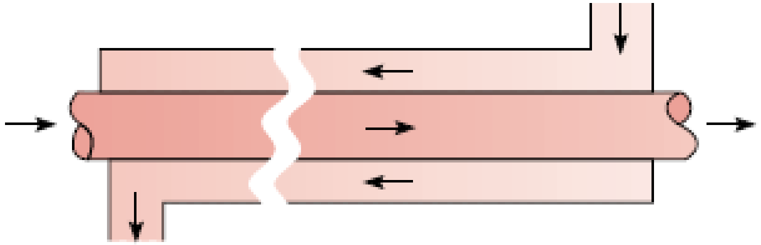
Introduction

- A heat exchanger is used to exchange heat between two fluids of different temperatures, which are separated by a solid wall.
- Heat exchangers encompass a wide range of flow configurations.
- Applications in heating and air conditioning, power production, waste heat recovery, chemical processing, food processing, sterilization in bio-processes.
- Heat exchangers are classified according to flow arrangement and type of construction.
- In this chapter we will learn how our previous knowledge can be applied to do heat exchanger calculations, discuss methodologies for design and introduce performance parameters.

Heat exchangers types

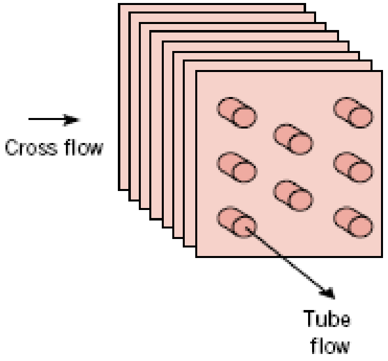
- Heat exchangers may be classified according to: Flow arrangements , and construction, industrial exchangers include;
- concentric tube heat exchanger (tube-in- tube Double pipe)
  - Parallel flow
  - Counter flows





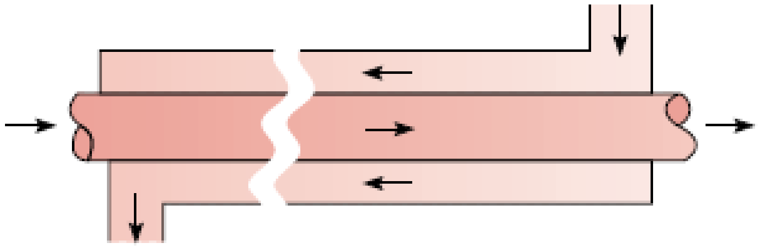
- *Cross flow heat exchangers*

  Fluids move perpendicular to each other.
  - Finned tube heat exchanger both fluids unmixed.
  - Unfinned tube heat exchanger with one fluid mixed (outside tubes unmixed

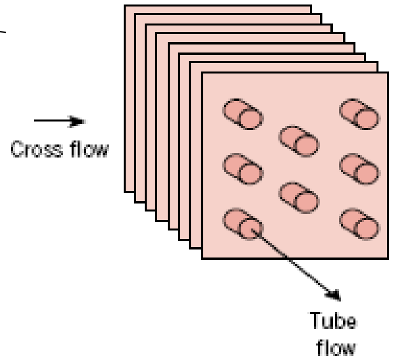


Heat Exchanger Types

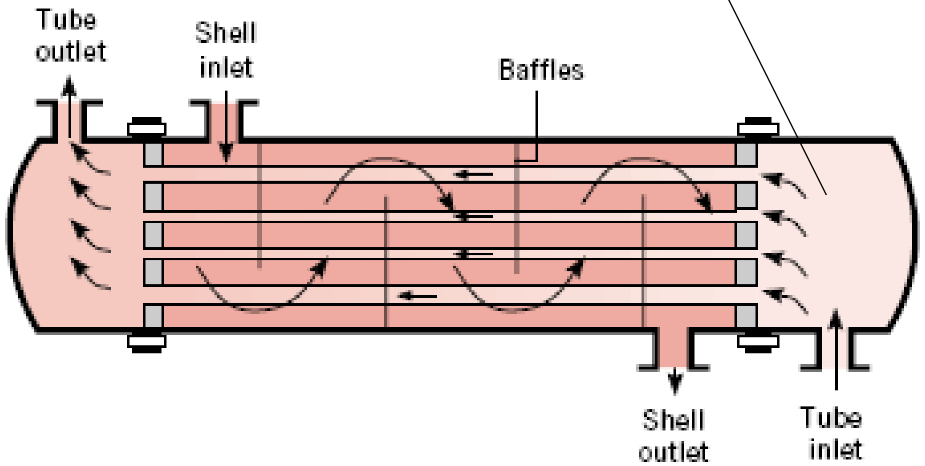
I. Concentric-tube (double-pipe) Heat Exchanger:

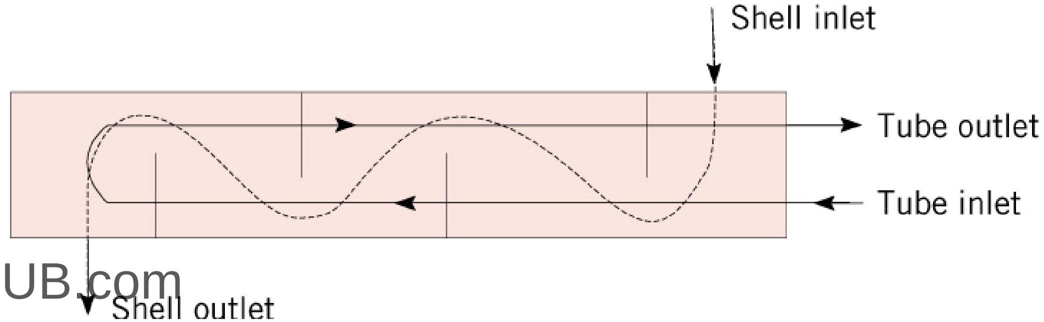


II. Cross-flow Heat Exchanger



III. Shell and Tube Heat Exchanger





Heat Exchanger Analysis

The Fluid Energy Balances: expressed in hot and cold fluids and given as;

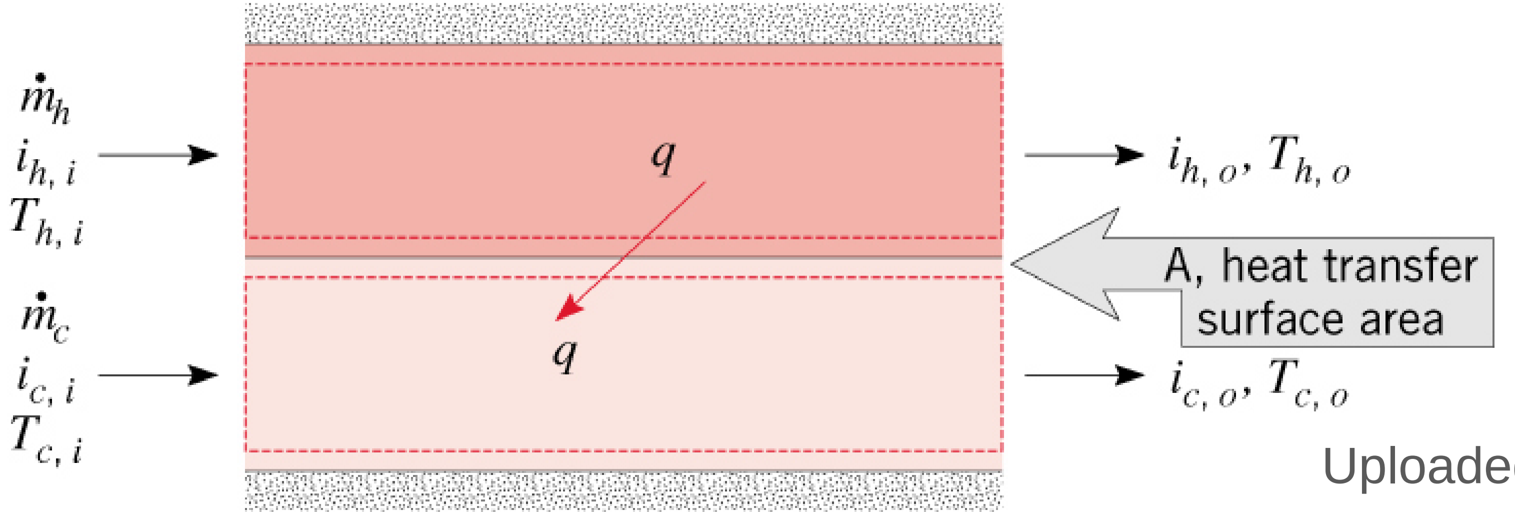
$$q = \dot{m}_h C_{p,h}(T_{h,i} - T_{h,o})$$

$$q = \dot{m}_c C_{p,c}(T_{c,i} - T_{c,o})$$

Or in terms of Capacity Rates $\boldsymbol{C_h}$ and $\boldsymbol{C_c}$ heat balance equation could be written as;

$$q = C_h(T_{h,i} - T_{h,o})$$

$$q = C_c(T_{c,i} - T_{c,o})$$



Heat Exchanger Analysis

The convection rate equation: expressed as;

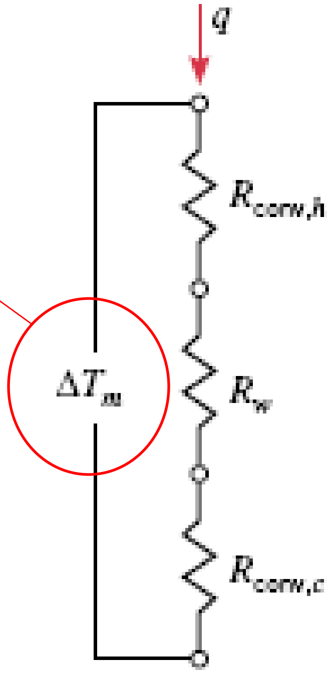
$$q = UA\Delta T_{lm}$$

And the **Overall Heat Coefficient** is;

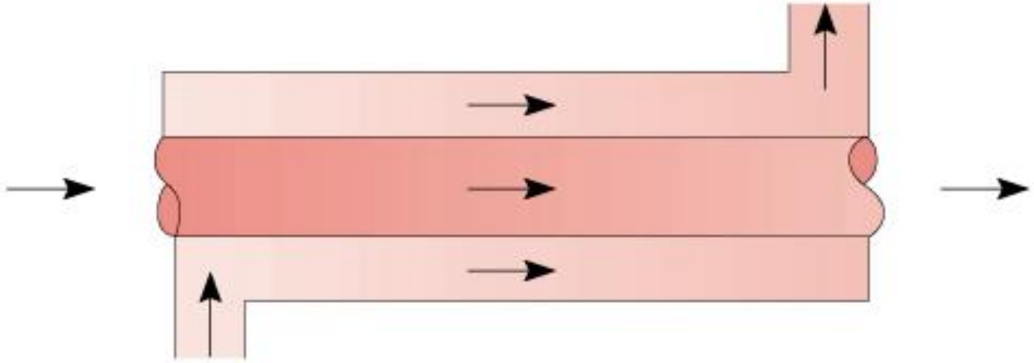
$$\frac{1}{UA} = R_{conv,h} + R_w + R_{conv,c}$$

$$\frac{1}{UA} = \left(\frac{1}{hA}\right)_h + R_w + \left(\frac{1}{hA}\right)_c$$

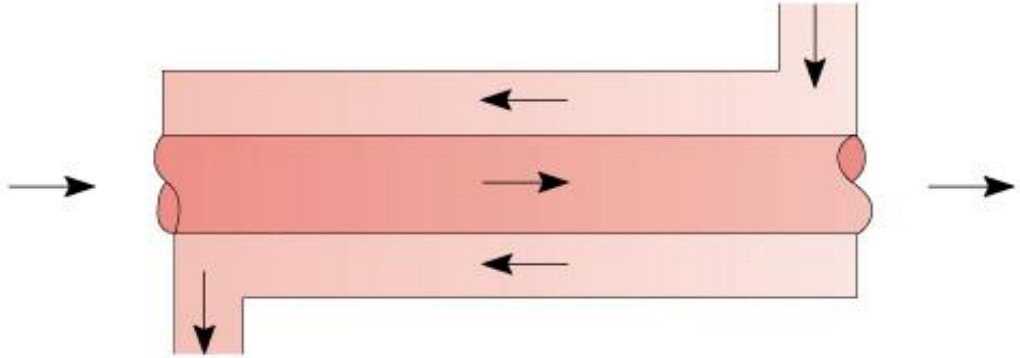
Mean Temperature Difference



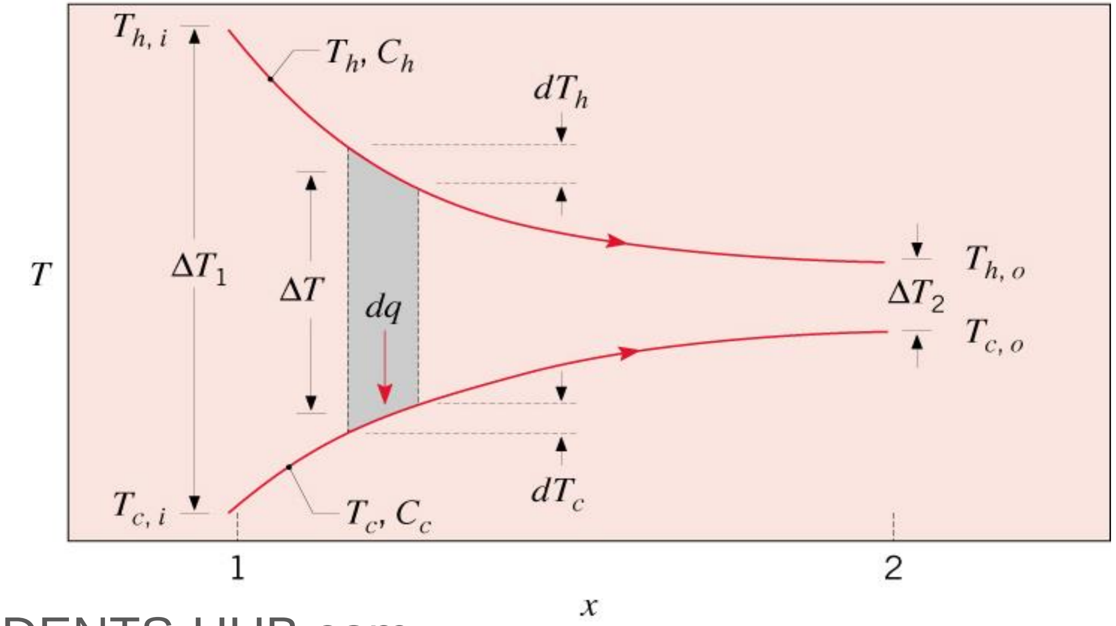
Concentric Tube Temperature Profile

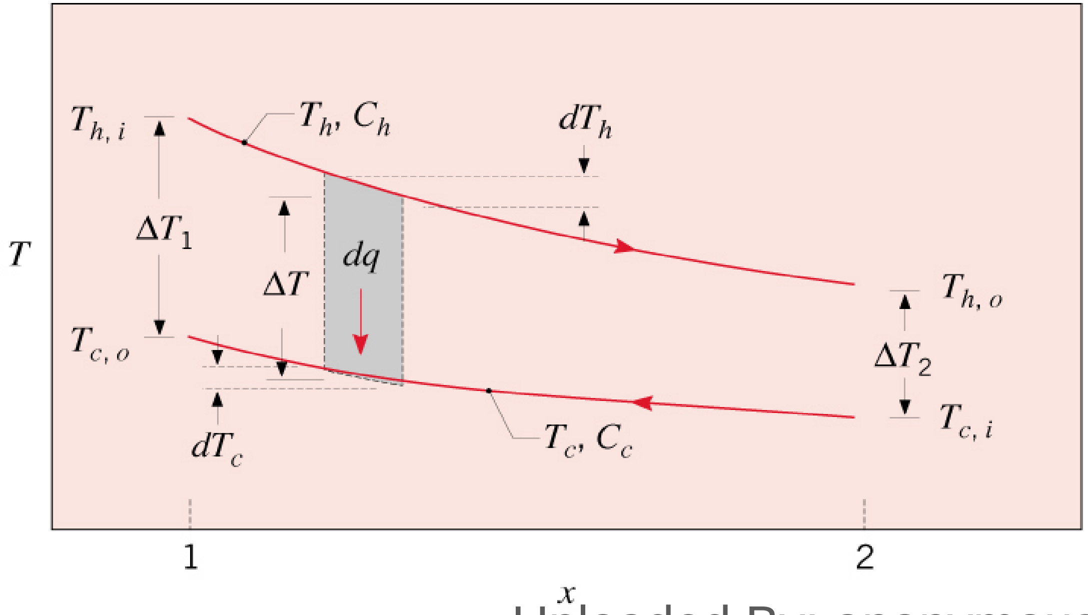


Parallel Flow



Counterflow





Heat Exchanger Analysis

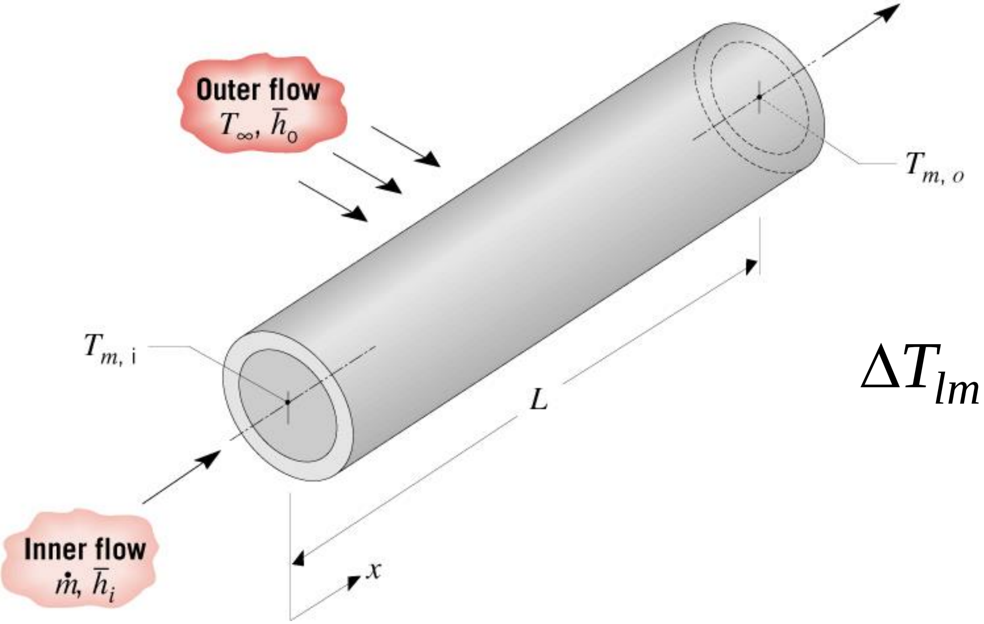
Recalling;

Expression for convection heat transfer for flow of a fluid inside a tube:

$$q_{conv} = \dot{m}c_p(T_{m,o} - T_{m,i})$$

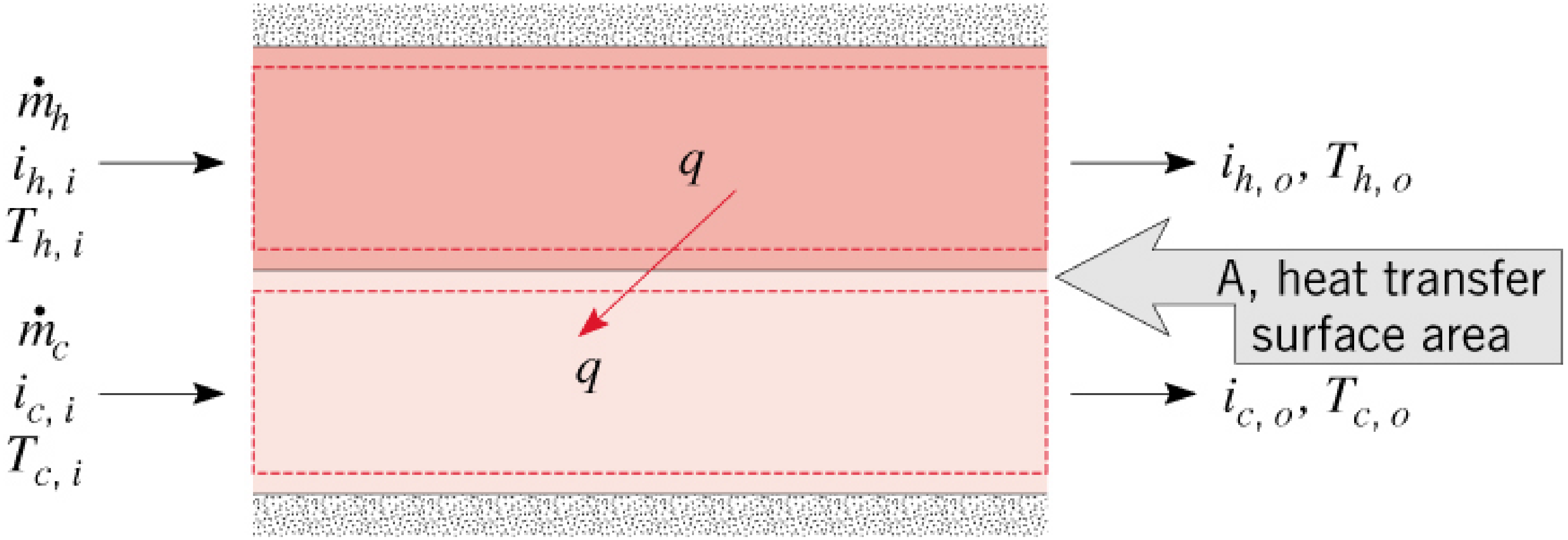
- For case 3 involving constant surrounding fluid temperature:

$$q = \overline{U}A_s\Delta T_{lm}$$



$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}$$

Heat Exchanger Analysis



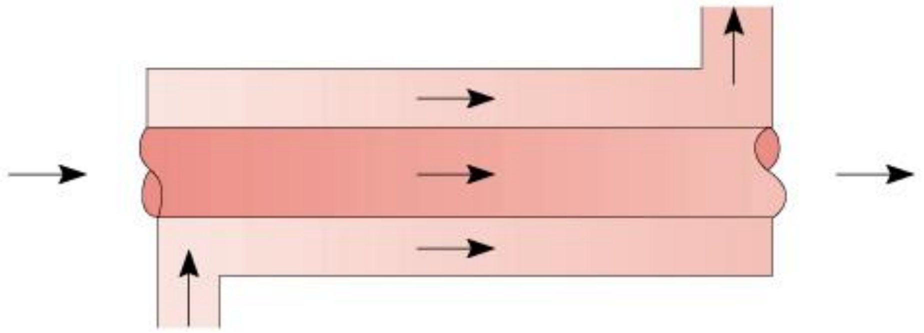
In a **two-fluid heat exchanger**, consider the hot and cold fluids separately:

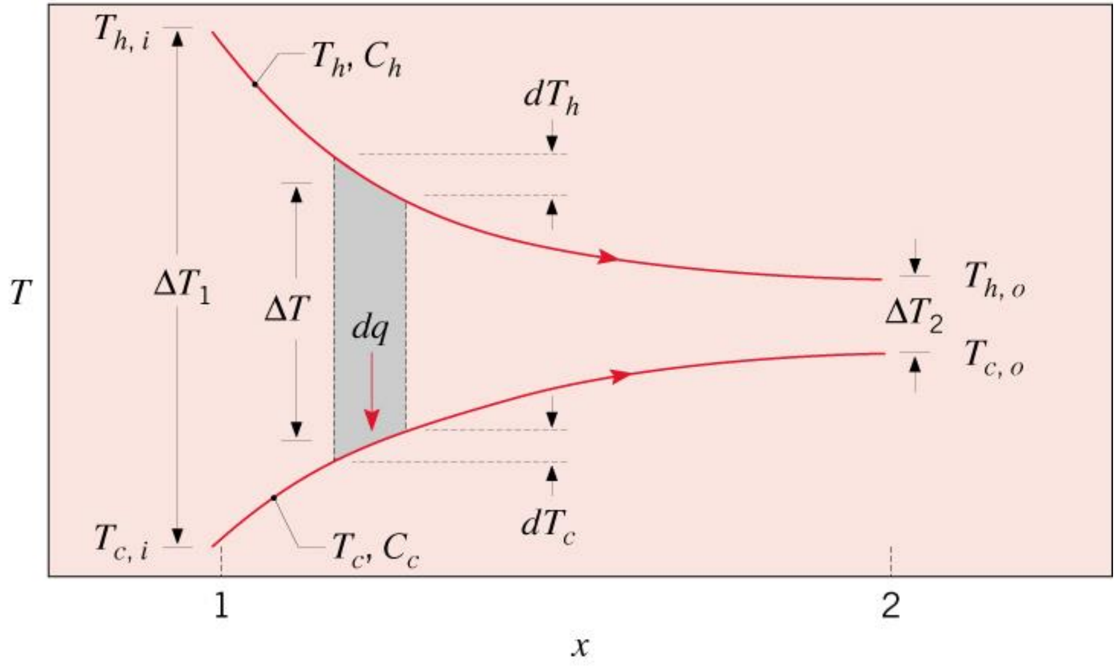
$$q_h = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o})$$
$$q_c = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i})$$

and

$$q = UA\,\Delta T_{lm}$$

ΔT_{lm}: Parallel-Flow Heat Exchanger





$$q = UA\,\Delta T_{lm}$$

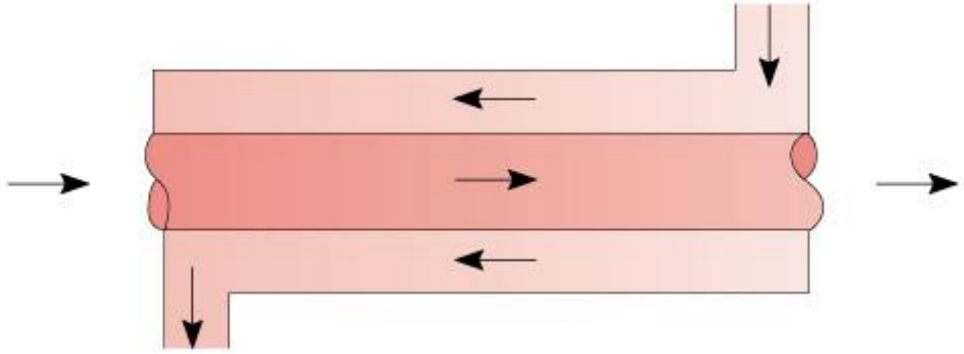
$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

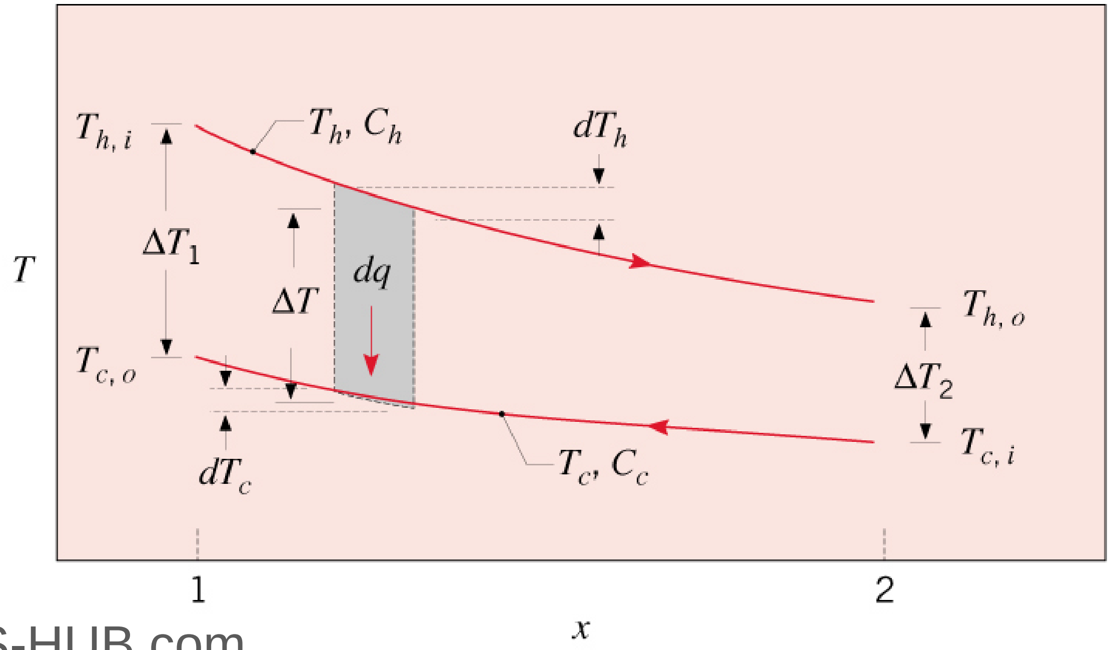
Where,

$$\Delta T_1 = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,o} - T_{c,o}$$

ΔT_{lm}: Counter-Flow Heat Exchanger





$$q = UA\,\Delta T_{lm}$$

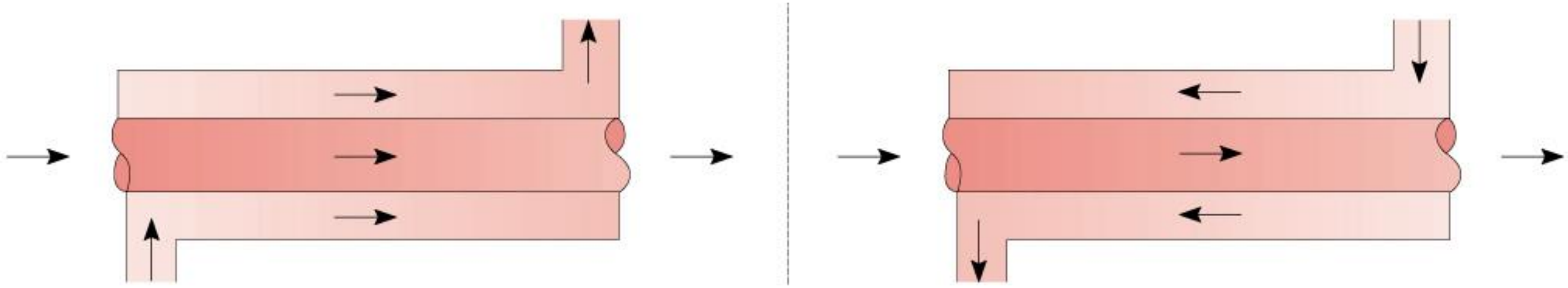
$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

where

$$\Delta T_1 = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,o} - T_{c,i}$$

Overall Heat Transfer Coefficient



For tubular heat exchangers we must take into account the conduction resistance in the wall and convection resistances of the fluids at the inner and outer tube surfaces.

$$\frac{1}{UA} = \frac{1}{h_i A_i} + R_{cond} + \frac{1}{h_o A_o}$$

$$R_{cond} = \frac{\ln(D_o / D_i)}{2\pi k L}$$

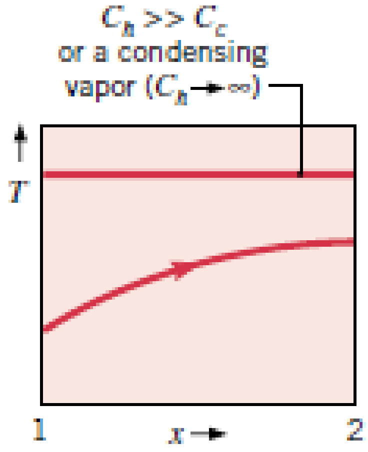
$$\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

where inner tube surface $A_i = \pi D_i L$

outer tube surface $A_o = \pi D_o L$

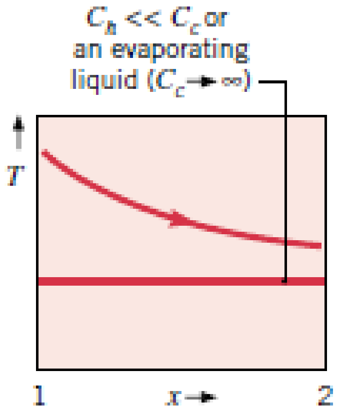
Special operation conditions

Ch » *Cc*. For this case, the hot fluid capacity rate *Ch* is much larger than the cold fluid capacity rate *Cc*. As shown in Fig. 17.28*a*, the hot fluid temperature remains approximately constant throughout the exchanger, while the temperature of the cold fluid increases. The same condition could be achieved if the hot fluid is a *condensing vapor*. Condensation occurs at a constant temperature, and for all practical purposes, *Ch*→ ∞.



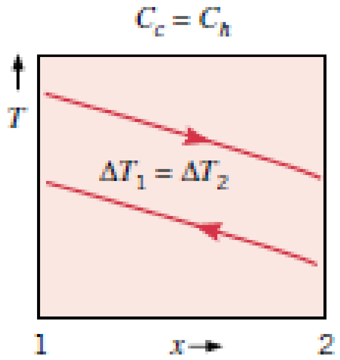
(*a*)

Ch <*Cc*. For this case, as shown in Fig. 17.28*b*, the cold fluid temperature remains
approximately constant throughout the exchanger, while the temperature of the hot fluid decreases. The same effect is achieved if the cold fluid experiences *evaporation* for which Note that *with evaporation and condensation, the fluid energy balances would be written in terms of the phase change enthalpies.*



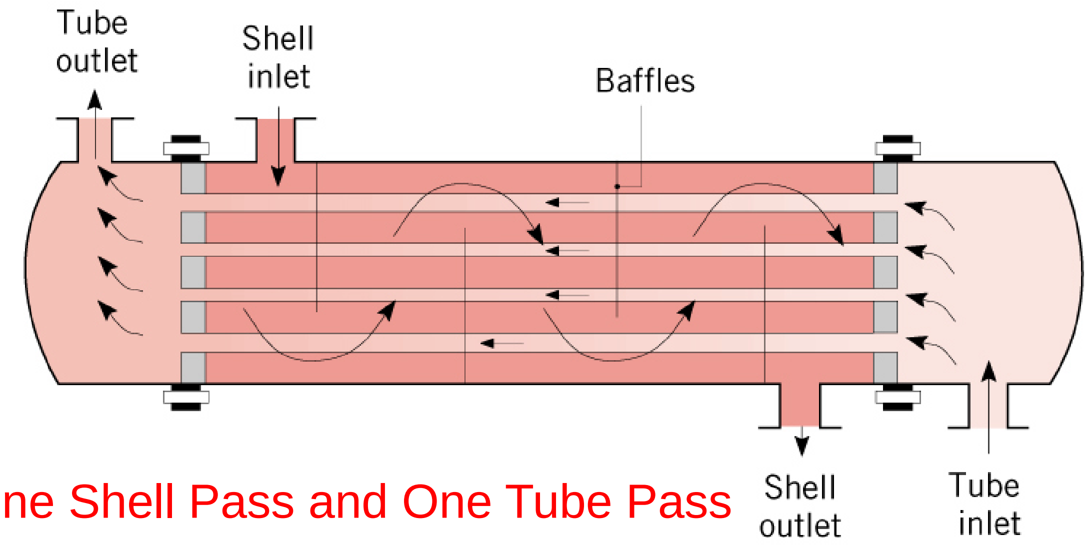
(*b*)

Ch = *Cc*. The third case, Fig. 17.28*c*, involves a *counterflow exchanger* for which the heat capacity rates are equal. The temperature difference *T* must be constant throughout the exchanger, in which case, $\Delta T1 = \Delta\ T2 = \Delta\ T\text{lm}$.



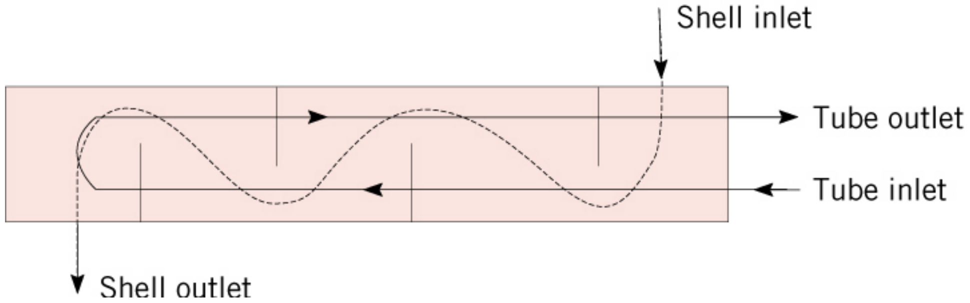
(*c*)

Shell and Tube Heat Exchanger

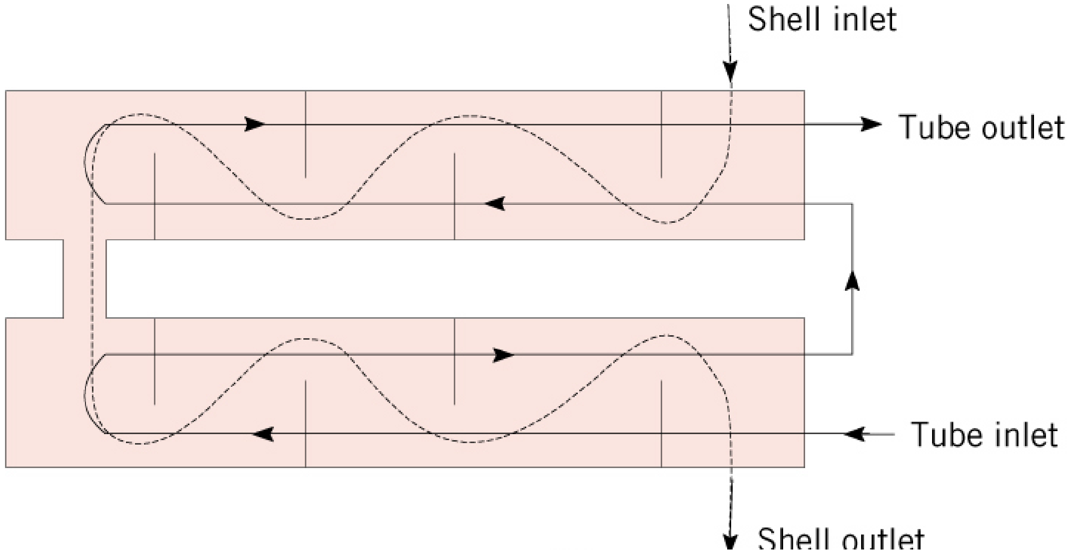


One Shell Pass and One Tube Pass

- Baffles are used to establish a cross-flow and to induce turbulent mixing of the shell-side fluid, both of which enhance convection.
- The number of tube and shell passes may be varied



One Shell Pass, Two Tube Passes



Two Shell Passes, Four Tube Passes

Shell and Tube Heat Exchanger Correction Factor (F)

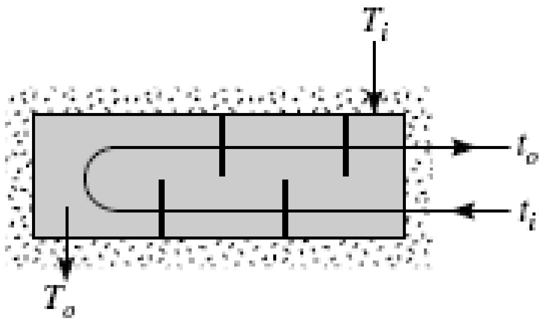
Use $$\Delta T_{lm} = F\Delta T_{lm}$$

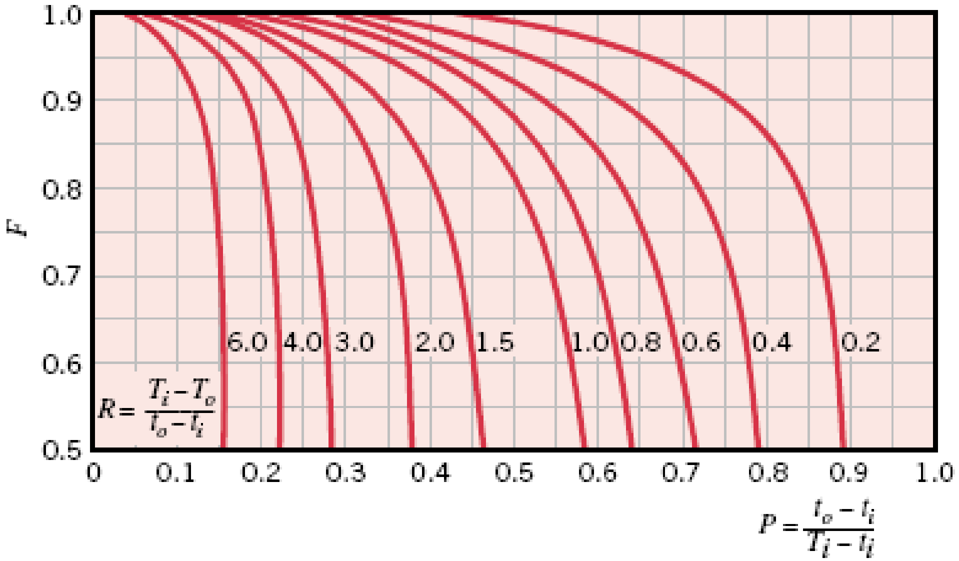
"F" is a correction factor from graphs *Fig 17.29, p.454* .

$$\Delta T_1 = T_{hi} - T_{c0}$$

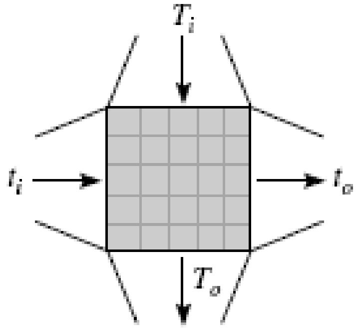
$$\Delta T_2 = T_{ho} - T_{ci}$$

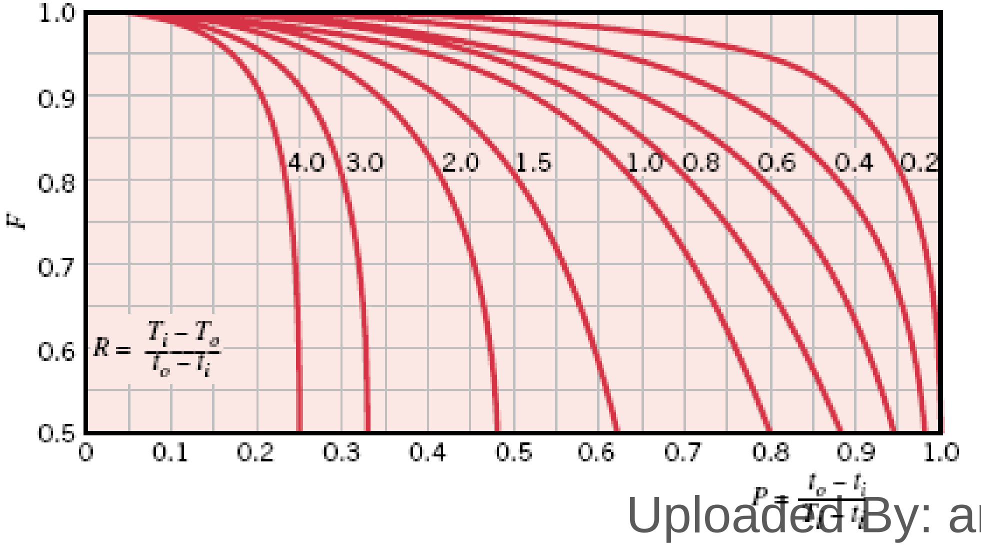
Shell-and-tube configuration with one shell and any multiple of two tube passes





cross-flow configuration





Example 17.14 Counterflow, Concentric Tube Heat Exchanger Analysis

A counterflow, concentric tube heat exchanger is used to cool the lubricating oil for a large industrial gas turbine engine. The flow rate of cooling water through the inner tube (D_i = 25 mm) is 0.2 kg/s. The flow rate of hot oil through the outer annulus (D_o = 45 mm) is 0.1 kg/s. The convection coefficient associated with the oil flow is h_o = 40 W/m$^2 \cdot$ K. The oil and water enter at temperatures of 100 and 30°C, respectively. What is the required tube length for an oil outlet temperature of 60°C?

Solution

Known: Fluid flow rates and inlet temperatures for a counterflow, concentric tube heat exchanger of prescribed inner and outer diameter.

Find: Tube length to achieve a desired hot fluid outlet temperature, $T_{h,o}$ = 60°C.

Schematic and Given Data:

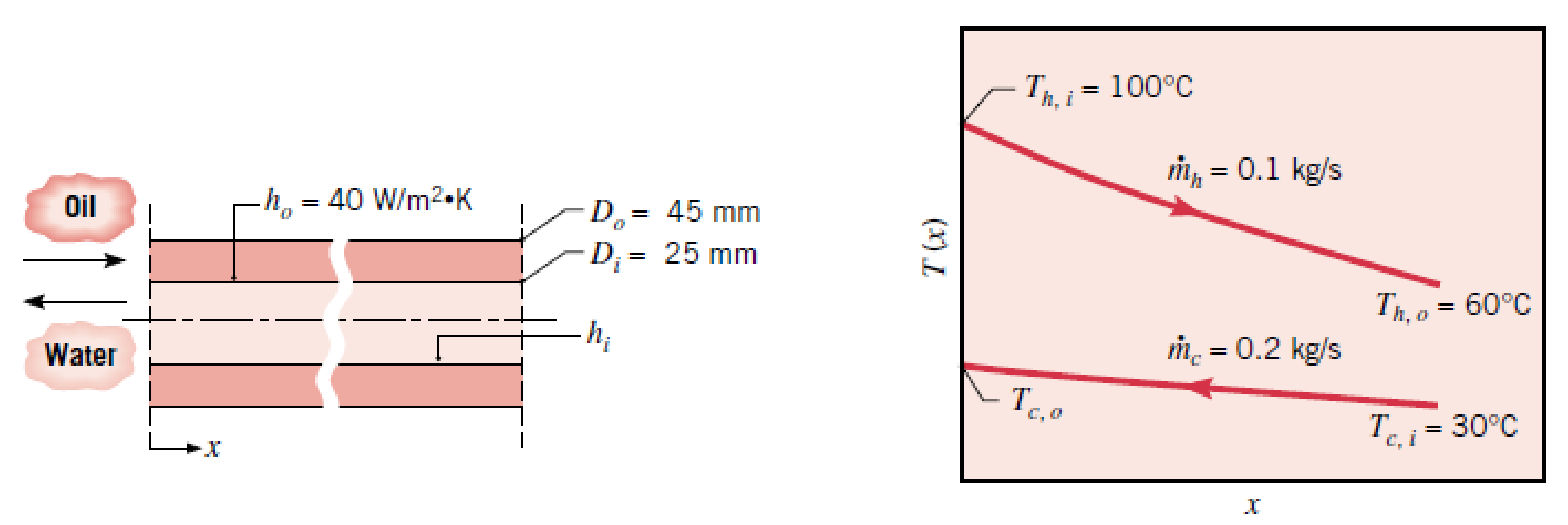


Figure E17.14

Assumptions:

1. Negligible heat loss to the surroundings.
2. Negligible kinetic and potential energy effects. No shaft work.
3. Constant properties.
4. Negligible tube wall thermal resistance and fouling factors.
5. Fully developed conditions for water flow.

Properties: Table HT-5, water (assume $\overline{T}_c = 35°C = 308$ K): $c_p = 4178$ J/kg · K, $\mu = 725 \times 10^{-6}$ N · s/m^2, $k = 0.625$ W/m · K, Pr = 4.85. Table HT-4, oil ($\overline{T}_h = 80°C = 353$ K): $c_p = 2131$ J/kg · K.

Analysis: The heat transfer rate can be obtained from the hot (oil) fluid energy balance, Eq. 17.86a

$$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = 0.1 \text{ kg/s} \times 2131 \text{ J/kg} \cdot \text{K} \, (100 - 60)°\text{C} = 8524 \text{ W}$$

Applying the cold fluid energy balance, Eq. 17.87a, the water outlet temperature is

$$T_{c,o} = \frac{q}{\dot{m}_c c_{p,c}} + T_{c,i} = \frac{8524 \text{ W}}{0.2 \text{ kg/s} \times 4178 \text{ J/kg} \cdot \text{K}} + 30°\text{C} = 40.2°\text{C}$$

Accordingly, the use of $\overline{T}_c = 35°C$, the average temperature of the cold fluid, to evaluate the water properties was a good choice. The required heat exchanger length may now be obtained from the convection rate equation, Eq. 17.89

$$q = UA \, \Delta T_{\text{lm}}$$

where $A = \pi D_i L$, and from Eqs. 17.96 and 17.98, the log mean temperature difference is

$$\Delta T_{\text{lm}} = \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\ln[(T_{h,i} - T_{c,o})/(T_{h,o} - T_{c,i})]} = \frac{59.8 - 30}{\ln(59.8/30)} = 43.2°\text{C}$$

From Eq. 17.90b, the overall heat transfer coefficient in terms of the water-side (i) and oil-side (o) convection coefficients is

$$U = \frac{1}{(1/h_i) + (1/h_o)}$$

To estimate h_i for the *water-side* (cold fluid), calculate the Reynolds number from Eq. 17.37 to characterize the flow and select a correlation

$$\text{Re}_D = \frac{4\dot{m}_c}{\pi D_i \mu} = \frac{4 \times 0.2\ \text{kg/s}}{\pi(0.025\ \text{m})(725 \times 10^6\ \text{N} \cdot \text{s/m}^2)} = 14{,}050$$

Accordingly, the flow is turbulent, and the convection coefficient may be estimated using the *Dittus-Boelter correlation*, Eq. 17.64, with $n = 0.4$ since $T_s > T_m$

$$\text{Nu}_D = 0.023\,\text{Re}_D^{4/5}\,\text{Pr}^{0.4} = 0.023(14{,}050)^{4/5}(4.85)^{0.4} = 90$$

$$h_i = \text{Nu}_D \frac{k}{D_i} = \frac{90 \times 0.625\ \text{W/m} \cdot \text{K}}{0.025\ \text{m}} = 2250\ \text{W/m}^2 \cdot \text{K}$$

Since the convection coefficient for the *oil-side* (hot fluid) is $h_o = 40\ \mathrm{W/m^2 \cdot K}$, the overall coefficient is then

$$U = \frac{1}{(1/2250\ \mathrm{W/m^2 \cdot K}) + (1/40\ \mathrm{W/m^2 \cdot K})} = 39.3\ \mathrm{W/m^2 \cdot K}$$

and from the convection rate equation it follows that the required length of the exchanger is

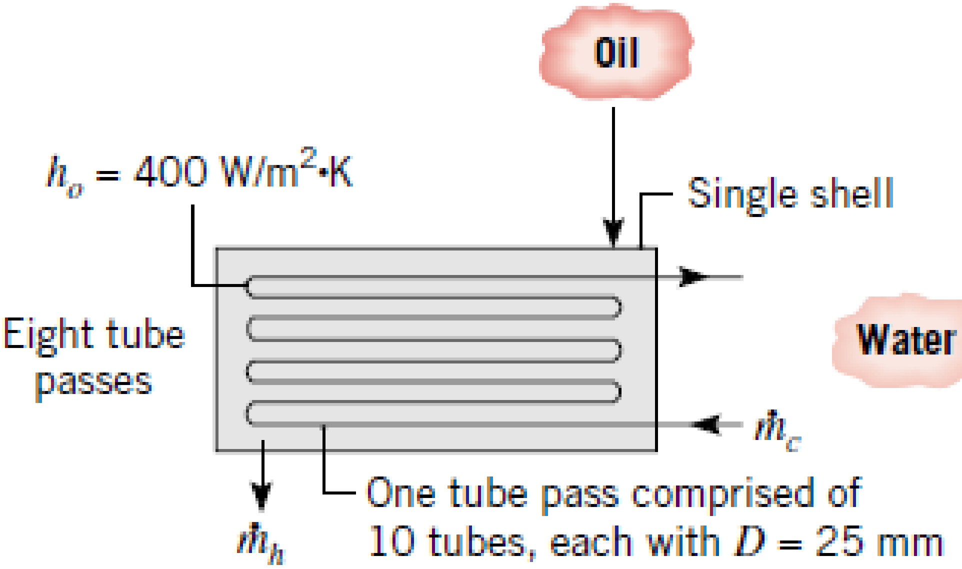
$$L = \frac{q}{U\pi D_i \Delta T_{\mathrm{lm}}} = \frac{8524\ \mathrm{W}}{39.3\ \mathrm{W/m^2 \cdot K}\ \pi(0.025\ \mathrm{m})(43.2^\circ\mathrm{C})} = 63.9\ \mathrm{m} \quad \triangleleft$$

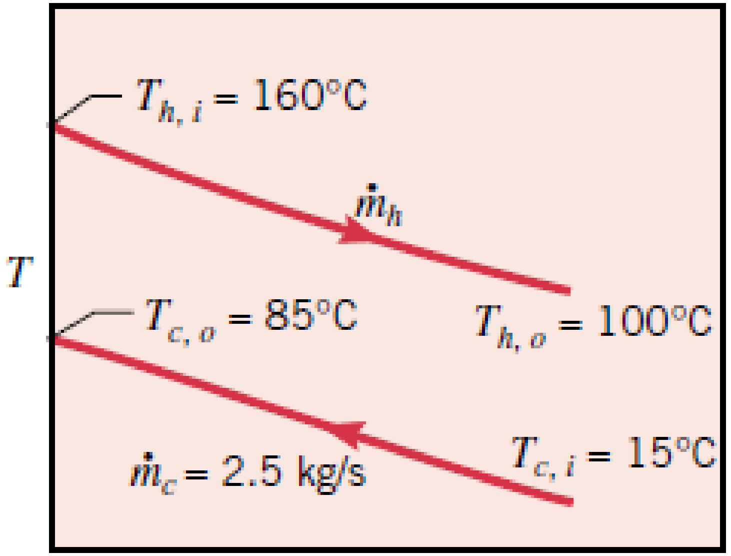
Comments: 1. The oil-side convection coefficient controls the rate of heat transfer between the two fluids, and the low value of h_o is responsible for the large value of L. In practice, multiple-pass construction would be required for a concentric tube exchanger with such a large tube length. Alternately, another exchanger type should be considered for this application.

2. Since the water flow is turbulent and $L/D = 2556$, the assumption of fully developed flow is justified according to Eq. 17.42.

Example 17.15 Shell-and-Tube Heat Exchanger Analysis

A shell-and-tube heat exchanger must be designed to heat 2.5 kg/s of water from 15 to 85°C. The heating is to be accomplished by passing hot engine oil, which is available at 160°C, through the shell side of the exchanger. The oil is known to provide an average convection coefficient of $h_o = 400\ \mathrm{W/m^2 \cdot K}$ on the outside of the tubes. Ten tubes pass the water through the shell. Each tube is thin walled, of diameter $D = 25$ mm, and makes eight passes through the shell. If the oil leaves the exchanger at 100°C, what is its flow rate? How long must each tube be to accomplish the desired heating?





Assumptions:
1. Negligible heat loss to the surroundings.
2. Negligible kinetic and potential energy effects. No shaft work.
3. Constant properties.
4. Negligible tube wall thermal resistance and fouling effects.
5. Fully developed water flow in tubes.

Properties: Table HT-4, unused engine oil ($\overline{T}_h = 130°C = 403$ K): $c_p = 2350$ J/kg · K. Table HT-5, water ($\overline{T}_c = 50°C = 323$ K): $c_p = 4181$ J/kg · K, $\mu = 548 \times 10^{-6}$ N · s/m^2, $k = 0.643$ W/m · K, Pr = 3.56.

Analysis:
(a) From an energy balance on the cold fluid (water), Eq. 17.87a, the heat transfer required of the exchanger is

$$q = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i}) = 2.5 \text{ kg/s} \times 4181 \text{ J/kg} \cdot \text{K} \, (85 - 15)°\text{C} = 7.317 \times 10^5 \text{ W}$$

Hence, from an energy balance on the hot fluid, Eq. 17.86a, the required oil flow rate is

$$\dot{m}_h = \frac{q}{c_{p,h}(T_{h,i} - T_{h,o})} = \frac{7.317 \times 10^5 \text{ W}}{2350 \text{ J/kg} \cdot \text{K} \times (160 - 100)°\text{C}} = 5.19 \text{ kg/s} \quad \triangleleft$$

(b) The required tube length can be obtained from the convection rate equation, Eq. 17.89, using the mean temperature difference from Eq. 17.99, where

$$q = UAF \, \Delta T_{\text{lm,CF}}$$

From Eq. 17.90b, the overall coefficient can be expressed in terms of the convection coefficients on the inside (water-side), h_i, and outside (oil-side), h_o, of the tube

$$U = \frac{1}{(1/h_i) + (1/h_o)}$$

where h_i may be obtained by first calculating Re_D. With $\dot{m}_1 \equiv \dot{m}_c/N = 0.25$ kg/s defined as the water flow rate per tube, Eq. 17.37 yields

$$\mathrm{Re}_D = \frac{4\dot{m}_1}{\pi D \mu} = \frac{4 \times 0.25 \text{ kg/s}}{\pi(0.025 \text{ m})548 \times 10^{-6} \text{ kg/s} \cdot \text{m}} = 23{,}234$$

Since $\mathrm{Re}_D > 2300$, the water flow is turbulent, and an appropriate correlation is Eq. 17.64 (*Dittus-Boelter*) with $n = 0.4$ since $T_s > T_m$

$$\mathrm{Nu}_D = 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^{0.4} = 0.023(23{,}234)^{4/5}(3.56)^{0.4} = 119$$

$$h_i = \frac{k}{D}\mathrm{Nu}_D = \frac{0.643 \text{ W/m} \cdot \text{K}}{0.025 \text{ m}}119 = 3061 \text{ W/m}^2 \cdot \text{K}$$

Hence, the overall coefficient is

$$U = \frac{1}{(1/400) + (1/3061)} = 354 \text{ W/m}^2 \cdot \text{K}$$

Associating T with the oil and t with the water, the correction factor F may be obtained from Fig. 17.29a, where

$$R = \frac{160 - 100}{85 - 15} = 0.86 \qquad P = \frac{85 - 15}{160 - 15} = 0.48$$

Hence, $F = 0.87$. From Eqs. 17.96 and 17.98, the log mean temperature difference for counterflow conditions is

$$\Delta T_{\text{lm,CF}} = \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\ln\left[(T_{h,i} - T_{c,o})/(T_{h,o} - T_{c,i})\right]} = \frac{75 - 85}{\ln(75/85)} = 79.9°\text{C}$$

Solving the convection rate equation for L, with $A = N\pi DL$, where $N = 10$ is the number of tubes, and substituting numerical values, find the required tube length

$$L = \frac{q}{UN(\pi D)F\,\Delta T_{\text{lm,CF}}} = \frac{7.317 \times 10^5\ \text{W}}{354\ \text{W/m}^2\cdot\text{K} \times 10(\pi 0.025\ \text{m}) \times 0.87(79.9°\text{C})}$$

$$L = 37.9\ \text{m} \quad \triangleleft$$

Comments:

1. With $(L/D) = 37.9\ \text{m}/0.025\ \text{m} = 1516$, the assumption of fully developed conditions throughout the tube for the water flow is justified.
2. With eight passes, the shell length is approximately $L/8 = 4.7$ m.

Shell and Tube Heat Exchanger Solution Methods

- The LMTD method may be applied to design problems for which the fluid flow rates and inlet temperatures, as well as a desired outlet temperature, are prescribed. For a specified HX type, the required size (surface area), as well as the other outlet temperature, are readily determined.

- If the LMTD method is used in performance calculations for which both outlet temperatures must be determined from knowledge of the inlet temperatures, the solution procedure is iterative.

- For both design and performance calculations, the effectiveness-NTU method may be used without iteration.

- End of heat exchanger
- End of convection heat transfer