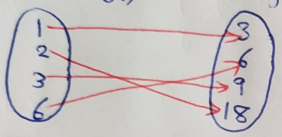
1.2 Functions

A function is a relation between two sets (domain الجال and range المدى) such that for each element in the domain there corresponds only one element of the range.

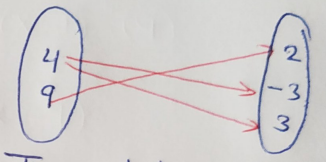
Example :

1) Domain(x) Range(y)



This is a function

2) Domain(x) Range(y)



This relation does not expresses y as a function of x.

A function may be defined by a set of ordered pairs (ازواج مرتبة), a table, a graph or an equation.

Example:

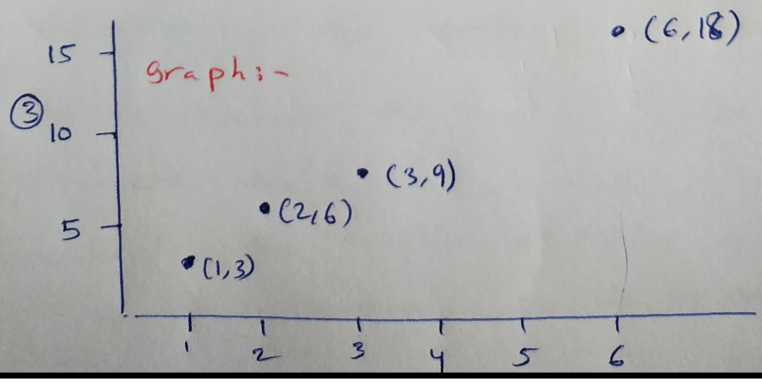
① table :-

Domain: Independent variable (x)	1	2	3	6
Range: dependent variable (y)	3	6	9	18

② ordered pairs:-

$\{(1,3), (2,6), (3,9), (6,18)\}$

③ graph:-

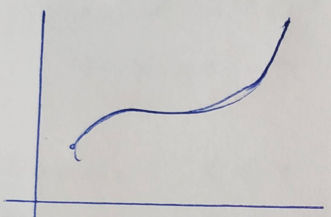


④ Equation : $y = 3x$, $x = 1, 2, 3, 6$

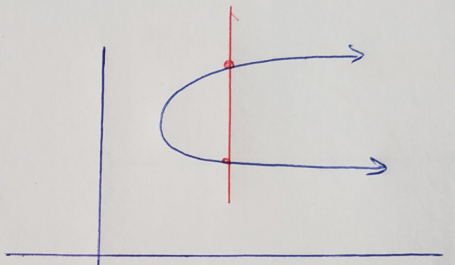
Vertical- line Test :- اختبار الخط العمودي

If no vertical line exists that intersects the graph at more than one point, then the graph is that of a function.

Example :



اي خط عمودي رح يقطع المنحنى بنقطة واحدة فقط اذن هو اقتران



ليس اقتران لانه يوجد خط عمودي قطع الاقتران في اكثر من نقطة

Letters other than f may also be used to denote functions For example $y = g(x)$ or $y = h(x)$.

If the domain is not specified then it is the set of all real numbers ($\mathbb{R}$).

The range is the set of all outputs obtained from evaluating all the domain.

Example : Consider the function $y = 3x$, $x = 1, 2, 3, 6$

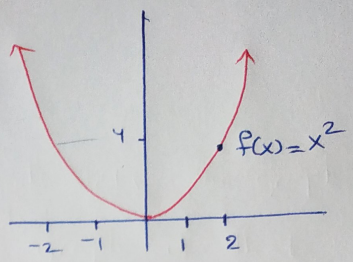
1) evaluate the function at $x = 2$, $y = 3(2) = 6$
2) evaluate the function at $x = 6$, $y = 3(6) = 18$
3) 9 is not in the domain , so we can not evaluate for 9

Example: Find the domain and the range of $f(x)=x^2$

Domain : $\mathbb{R}$ or $(-\infty,\infty)$

Range : $[0,\infty)$ or $y\geq 0$

Find the range for $x=2$: $f(2)=4$



Example: Find the domain of $f(x)=\sqrt{4-x}$

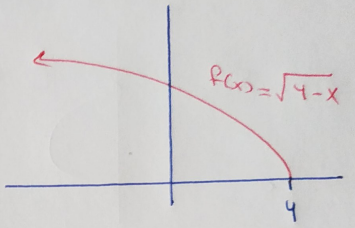
لازم ماتحت الجذر موجب

$4-x\geq 0 \rightarrow 4\geq x$

$+x \quad +x$

Domain : $x\leq 4$ or $(-\infty,4]$

Range : $y\geq 0$ or $[0,\infty)$



Example: Find the domain of $f(x)=1+\frac{1}{x-2}$

Domain : $(-\infty,\infty)/\{2\}$ ممنوع المقام صفر

ممنوع نعوض فيه 2

Example: State the domain and find $f(10), f(-1)$

$$f(x)=\frac{\sqrt{x-1}}{x-2}$$

Domain : ① ماتحت الجذر موجب $x-1\geq 0 \rightarrow x\geq 1$

② ممنوع المقام صفر $x-2=0 \rightarrow x=2$ ×

} Domain : $[1,\infty)/\{2\}$

$$f(10)=\frac{\sqrt{10-1}}{10-2}=\frac{\sqrt{9}}{8}=\frac{3}{8}$$

$$f(-1)=\frac{\sqrt{-1-1}}{-1-2}=\frac{\sqrt{-2}}{-3}$$ not Real.

Operations for functions:

① $(f+g)(x) = f(x) + g(x)$

② $(f-g)(x) = f(x) - g(x)$

③ $(f \cdot g)(x) = f(x) \cdot g(x)$

④ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, if $g(x) \neq 0$

Example: If $f(x) = 3x + 3$, $g(x) = 8 - 2x$.

Find ① $(f+g)(2) = f(2) + g(2)$

$= 3(2) + 3 + 8 - 2(2)$

$= 6 + 3 + 8 - 4 = 9 + 4 = 13$

② $(f-g)(1) = f(1) - g(1)$

$= 3(1) + 3 - (8 - 2(1))$

$= 3 + 3 - (6) = 6 - 6 = 0$

③ $(f \cdot g)(0) = f(0) \cdot g(0) = (3(0)+3) \cdot (8 - 2(0))$

$= (0+3)(8-0) = 3(8) = 24$

④ $\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{3(-1)+3}{8-2(-1)}$

$= \frac{-3+3}{8+2} = \frac{0}{10} = 0$

⑤ $\left(\frac{g}{f}\right)(-1) = \frac{g(-1)}{f(-1)}$

$= \frac{8-2(-1)}{3(-1)+3} = \frac{8+2}{-3+3} = \frac{10}{0}$ undefined.

* Composite Functions: الاقترانات المركبة

- f composed of g written as $(f \circ g)(x) = f(g(x))$
- g composed of f written as $(g \circ f)(x) = g(f(x))$

Example: If $f(x) = 3x + 3$, $g(x) = 8 - 2x$. Find:

1) $(f \circ g)(x) = f(g(x)) = f(8 - 2x)$

$= 3(8 - 2x) + 3$

$= 24 - 6x + 3 = 27 - 6x$

2) $(g \circ f)(x) = g(f(x)) = g(3x + 3)$

$= 8 - 2(3x + 3)$

$= 8 - 6x - 6 = 2 - 6x$

3) $(f \circ g)(2) = 27 - 6(2) = 27 - 12 = 15$

4) $(g \circ f)(0) = 2 - 6(0) = 2 - 0 = 2$

or $(g \circ f)(0) = g(f(0)) = g(3(0) + 3) = g(0 + 3)$

$= g(3) = 8 - 2(3) = 8 - 6 = 2$

5) $(f \circ f)(1) = f(f(1)) = f(3(1) + 3) = f(6)$

$= 3(6) + 3 = 18 + 3 = 21$