

8.7

improper integrals

types of improper integrals

type I

- one or both of integration limits are infinite

→ If $f(x)$ is continuous on $[a, \infty)$

$$\int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

→ If $f(x)$ is continuous on $(-\infty, a]$

$$\int_{-\infty}^a f(x) dx = \lim_{b \rightarrow -\infty} \int_b^a f(x) dx$$

→ If $f(x)$ is continuous on $(-\infty, \infty)$ Then

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^c f(x) dx + \int_c^{\infty} f(x) dx$$

when $-\infty < c < \infty$

$$\text{Then} = \lim_{a \rightarrow -\infty} \int_a^c f(x) dx + \lim_{b \rightarrow \infty} \int_c^b f(x) dx$$

• If The lim = finite Then The integral **converges**

• If The lim = ∞ or DNE Then The integral **diverges**

type II

• Discontinuity in the function

① → If $f(x)$ is cont on $[a, b)$ and discont at $x=b$

Then:

$$\int_a^b f(x) dx = \lim_{c \rightarrow b^-} \int_a^c f(x) dx$$

② → If $f(x)$ is cont on $(a, b]$ and discont at $x=a$

Then:

$$\int_a^b f(x) dx = \lim_{c \rightarrow a^+} \int_c^b f(x) dx$$

→ If $f(x)$ is cont on $[a, c) \cup (c, b]$ & discont at $x=c$

Then:-

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

① ②

$$= \lim_{d \rightarrow c^-} \int_a^d f(x) dx + \lim_{e \rightarrow c^+} \int_e^b f(x) dx$$

Alia Fatima

How To know if a function diverges or converges:-

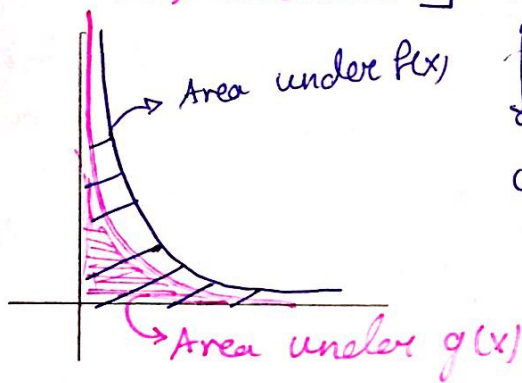
- 1- • P-integrals (The rules work only when $\int_a^\infty f(x) dx$)

$\int_1^\infty \frac{1}{x^p} dx$ is a P-integral where P can be $\pm 1, \pm 2, \pm 3, \dots$

→ Cases:-
→ 1 $\Rightarrow P=1$ integral diverges
2 $\Rightarrow P>1$ integral converges to $\frac{1}{P-1}$
3 $\Rightarrow P<1$ integral diverges

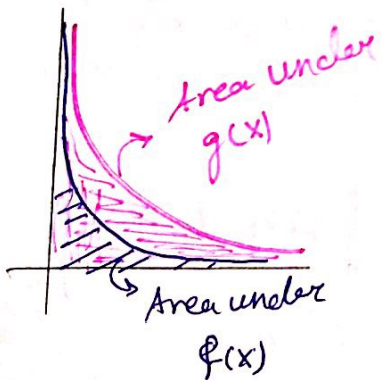
2- • Direct Comparison Test

→ Cases:- I- If $f(x), g(x)$ is ≥ 0 on $[a, \infty)$ &



$\int_a^\infty f(x) dx$ converges & $g(x) \leq f(x)$
where $a \leq x < \infty$ Then $\int_a^\infty g(x) dx$
also converges

II- If $f(x), g(x)$ is ≥ 0 on $[a, \infty)$ &
 $\int_a^\infty f(x) dx$ diverges and $g(x) \geq f(x)$
where $a \leq x < \infty$ Then $\int_a^\infty g(x) dx$
also diverges



3- • Limit Comparison Test

Cases:-

1 $\rightarrow$ If $f(x), g(x) \geq 0$ and $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = L$ $0 < L < \infty$
Then $\int_a^\infty f(x) dx$ and $\int_a^\infty g(x) dx$ both converge
or diverge

Haa Etaiwi

2-. If $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \infty$ This means

→ $f(x) \gg g(x)$ and if we knew That $\int_a^\infty g(x) dx$ diverges

Then $\int_a^\infty f(x) dx$ diverges

→ If we knew That $\int_a^\infty f(x) dx$ converges Then $\int_a^\infty g(x) dx$ converges

3-. If $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$ Then

→ If $\int_a^\infty g(x) dx$ conv Then $\int_a^\infty f(x) dx$ converges

→ If $\int_a^\infty f(x) dx$ diverges Then $\int_a^\infty g(x) dx$ diverges

Note : Comparison Tests can be used in integrals
lik $\int_a^b$

Alaa Elaini