Chapter (4) Applications of derivatives.

Q1. Find the intervals in which the following functions are increasing, decreasing, concave up and concave down. Then find the extreme values and inflection points and sketch their graphs.

a) $y = 1 - (x+1)^3$

$D: (-\infty, \infty)$

$y' = -3(x+1)^2$

$y' = -3(x+1)^2 = 0$

$(x+1)^2 = 0$

$x+1 = 0 \rightarrow x = -1$

y has a critical value at $x = -1$ (no extreme values)

sign of y' : $- - - - - - 0 - - - - - -$ (-1)

decreasing intervals : $(-\infty, \infty)$

$y'' = -3(2)(x+1) = -6(x+1)$

$y'' = 0$ if $-6(x+1) = 0 \rightarrow x = -1$

sign of y'' $+ + + 0 - - -$ (-1)

$\cup \quad \cap$

y concave up on $(-\infty, -1]$

y concave down on $[-1, \infty)$

at $x=-1$, $y = 1-(-1+1)^3 = 1-0=1$

so $(-1, 1)$ is an inflection point of y

y-intercept:

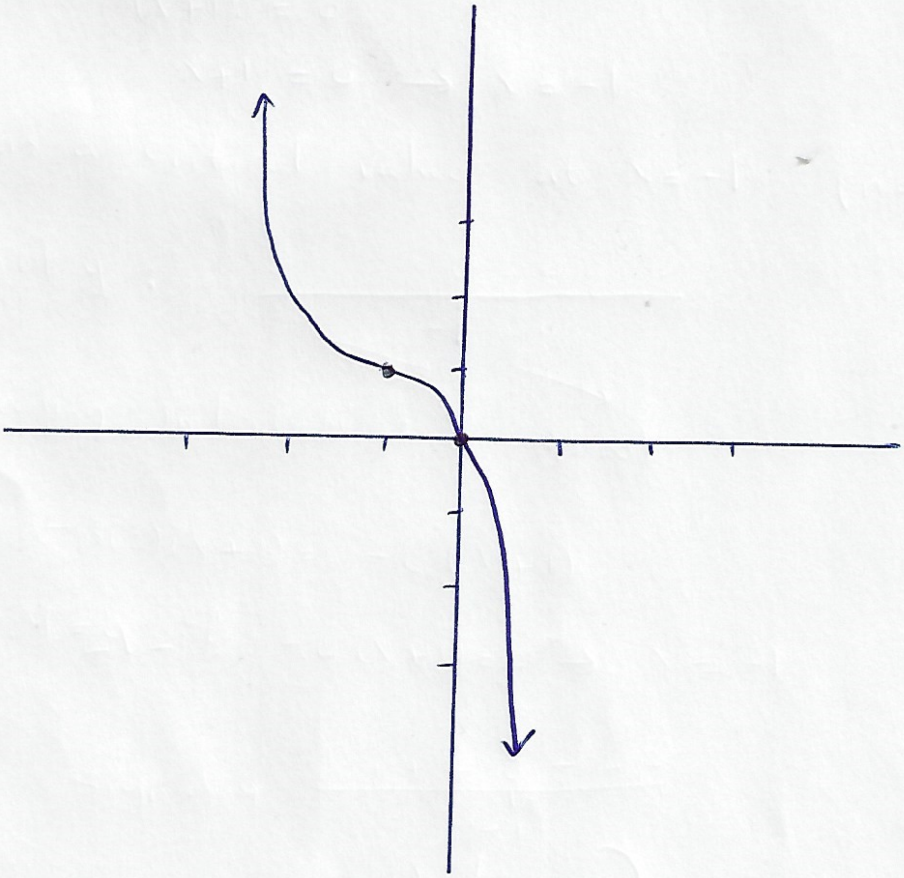
at $x=0$, $y = 1-(0+1)^3 = 1-1=0 \rightarrow (0,0)$

x-intercept:

at $y=0$, $0 = 1-(x+1)^3 \rightarrow 1 = (x+1)^3$

$1 = x+1$

$0 = x \rightarrow (0,0)$



b) $y = \dfrac{x^2+1}{x}$

- $D : (-\infty, \infty) \setminus \{0\}$

- Horizontal asymptotes:

  no horizonta asymptotes. (لا يوجد لأن درجة البسط أكبر من درجة المقام)

- Vertical asymptotes: (ممكن عند اصفار المقام)

$$\lim_{x \to 0^+} \frac{x^2+1}{x} = \frac{+}{0^+} = \infty$$

$$\lim_{x \to 0^-} \frac{x^2+1}{x} = \frac{+}{0^-} = -\infty$$

$x = 0$ is a vertical asymptote.

- Oblique asymptotes: (يوجد O.A لأن درجة البسط أكبر من درجة المقام)

$$\frac{x^2+1}{x} = \overset{\text{الناتج}}{x} + \frac{\overset{\text{الباقي}}{1}}{\underset{\text{المقسوم عليه}}{x}}$$

$$\begin{array}{r|l} & x \\ \hline x & \cancel{x^2} + 1 \\ & \mp \cancel{x^2} \\ \hline & 1 \end{array}$$

$y = x$ is an oblique asymptote.

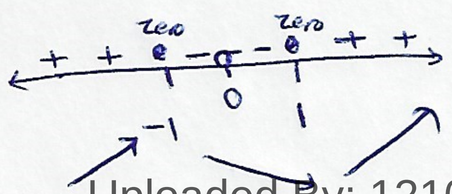
- $$y' = \frac{x \cdot (2x) - (x^2+1) \cdot 1}{(x)^2} = \frac{2x^2 - x^2 - 1}{x^2} = \frac{x^2-1}{x^2}$$

y' undefine at $x = 0$ (but $0 \notin D$)

$y' = 0$ if $\dfrac{x^2-1}{x^2} = 0 \to x^2 - 1 = 0 \to x^2 = 1 \to x = \pm 1$

so y has critical values at $x = -1, 1$

sign of y'



increasing intervals: $(-\infty, -1] \cup [1, \infty)$

decreasing intervals : $[-1, 0) \cup (0, 1]$

(Note that $0 \notin D$)

f has a Local max at $x=-1$, the local maximum value $= f(-1) = \frac{(-1)^2+1}{-1} = \frac{2}{-1} = -2$ $\quad (-1, -2)$

f has a Local minimum at $x=1$, the local minimum value $f(1) = \frac{(1)^2+1}{+1} = \frac{2}{+1} = +2$ $\quad (1, +2)$

$$- \; y'' = \frac{x^2 \cdot 2x - (x^2-1)\cdot 2x}{(x^2)^2} = \frac{\cancel{2x^3} - \cancel{2x^3} + 2x}{x^4} = \frac{2x}{x^4} = \frac{2}{x^3}$$

y'' undefined at $x=0$ (but $0 \notin D$)

$y''=0$ if $\frac{2}{x^3} = 0 \rightarrow 2 = 0$ impossible.

y has no inflection points.

sign of y'' $\quad$ $- - - - \underset{0}{|} + + +$

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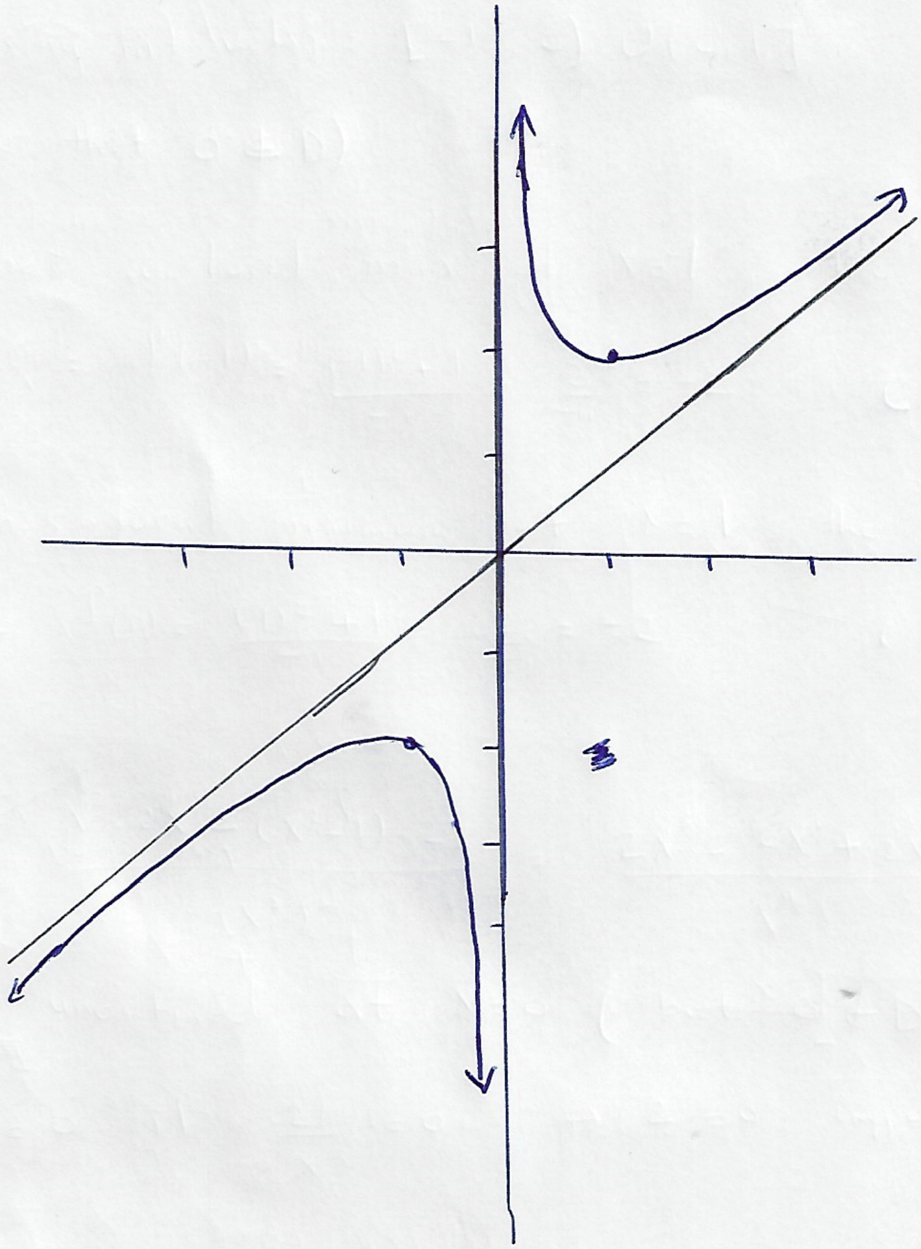
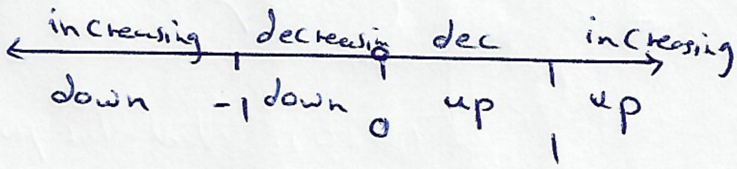
concave up intervals : $(0, \infty)$

concave down intervals : $(-\infty, 0)$





increasing
decreasing
dec
increasing
down
-1
down
0
up
1
up

c) $y = x^4 - 2x^2$

- $D: (-\infty, \infty)$

- $y' = 4x^3 - 2.2x = 4x^3 - 4x$

- critical values:

y' is always defined.

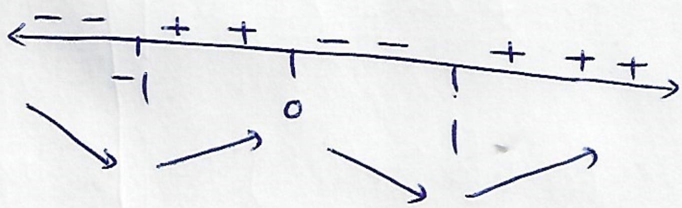
$y' = 0$ if $4x^3 - 4x = 0 \rightarrow 4x(x^2-1) = 0$

$\rightarrow 4x(x-1)(x+1) = 0$

$\rightarrow x = 0, 1, -1$

y has critical values at $x = 0, 1, -1$.

- sign of y'



- increasing intervals: $[-1, 0] \cup [1, \infty)$

- decreasing intervals: $(-\infty, -1] \cup [0, 1]$

- y has a local maximum value at $x = 0$

the local maximum value is $y(0) = (0)^4 - 2(0)^2 = 0$

$(0, 0)$

- y has local minimum values at $x = -1, 1$

the local minimum values are $\rightarrow f(-1) = (-1)^4 - 2(-1)^2 = -1$

$(-1, -1)$

$\rightarrow f(1) = (1)^4 - 2(1)^2 = -1$

$(1, -1)$

$- \; y'' = 4 \cdot 3x^2 - 4 = 12x^2 - 4$

$y'' = 0$ if $12x^2 - 4 = 0 \rightarrow 4(3x^2 - 1) = 0$

$\rightarrow 3x^2 - 1 = 0$

$x^2 = \frac{1}{3} \rightarrow x = \pm \frac{1}{\sqrt{3}}$

sign of y'': $\quad + + + \quad \big| \quad - - - \quad \big| \quad + + +$

$\qquad\qquad \cup \quad -\frac{1}{\sqrt{3}} \quad \cap \quad \frac{1}{\sqrt{3}} \quad \cup$

y has inflection points at $x = -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

inflection points are:-

$f\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{1}{\sqrt{3}}\right)^4 - 2\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{9} - \frac{2}{3} = \frac{1}{9} - \frac{6}{9} = \frac{-5}{9}$

$\left(\frac{1}{\sqrt{3}}, \frac{-5}{9}\right)$

$f\left(-\frac{1}{\sqrt{3}}\right) = \left(\frac{-1}{\sqrt{3}}\right)^4 - 2\left(\frac{-1}{\sqrt{3}}\right)^2 = \frac{-5}{9} \qquad \left(-\frac{1}{\sqrt{3}}, \frac{-5}{9}\right)$

\- concave up intervals: $\left(-\infty, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, \infty\right)$

concave down intervals: $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

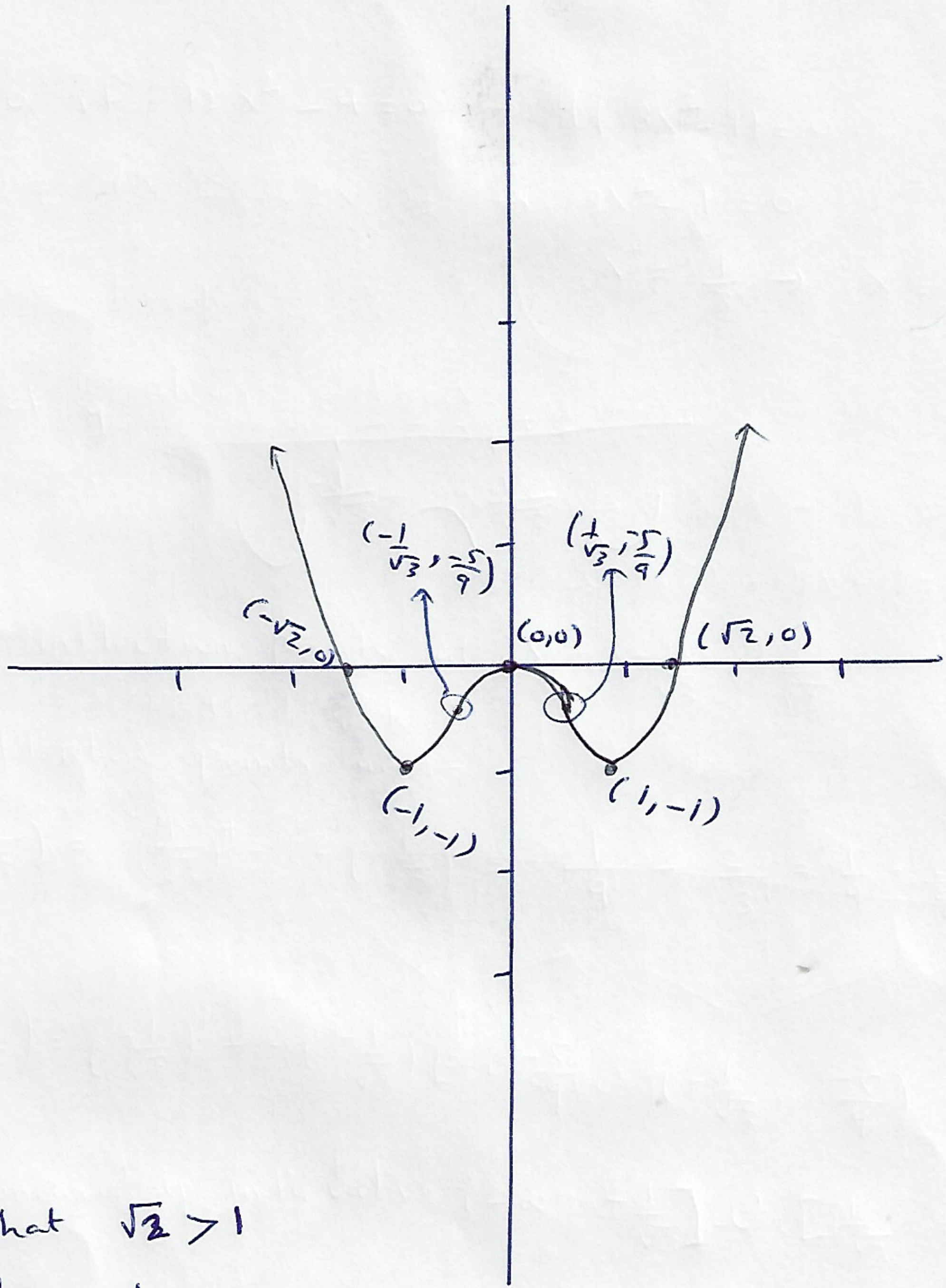
\- y-intercept:- at $x = 0 \rightarrow y = (0)^4 - 2(0)^2 = 0 \rightarrow (0,0)$

\- x-intercept: at $y = 0 \rightarrow 0 = x^4 - 2x^2$

$0 = x^2(x^2 - 2)$

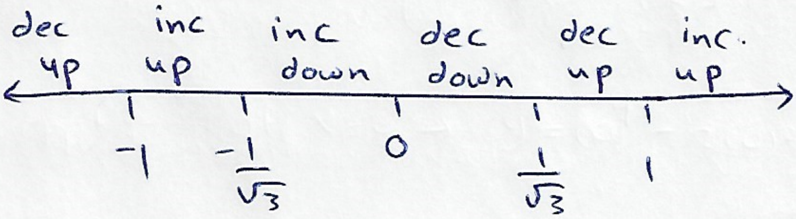
at $x = 0, \; x = \pm\sqrt{2}$

$(0,0), \; (\sqrt{2}, 0) \quad (-\sqrt{2}, 0)$



Note that $\sqrt{2} > 1$

$\sqrt{3} > 1$ so $\frac{1}{\sqrt{3}} < 1$



points: $(-1,-1)$, $(1,-1)$, $(0,0)$, $\left(\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$

$\left(-\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$, $(\sqrt{2}, 0)$, $(-\sqrt{2}, 0)$

d) $y = \dfrac{x^2 - 3}{x - 2}$

- $D: (-\infty, \infty) \setminus \{2\}$

- Horizontal asymptotes:

  no horizontal asymptotes.

(نلاحظ ان درجة البسط اكبر من درجة المقام)

- Vertical asymptotes:

(نبحث بالقرب من اصفار المقام)

$$\lim_{x \to 2^+} \frac{x^2 - 3}{x - 2} = \frac{+}{0^+} = +\infty$$

$$\lim_{x \to 2^-} \frac{x^2 - 3}{x - 2} = \frac{+}{0^-} = -\infty$$

$x = 2$ is a vertical asymptote.

- Oblique asymptote:-

$$\begin{array}{r|l} & x + 2 \\ \hline x - 2 & \cancel{x^2} - 3 \\ & \mp x^2 \pm 2x \\ \hline & 2\cancel{x} - 3 \\ & \mp 2x \pm 4 \\ \hline & +1 \end{array}$$

الناتج

$$\frac{x^2 - 3}{x - 2} = x + 1 + \frac{+1}{x - 2}$$

الباقي

المقسوم عليه

so $y = x + 1$ is an oblique asymptote.

- $y' = \dfrac{(x-2)\cdot(2x) - (x^2 - 3)(1)}{(x-2)^2} = \dfrac{2x^2 - 4x - x^2 + 3}{(x-2)^2} = \dfrac{x^2 - 4x + 3}{(x-2)^2}$

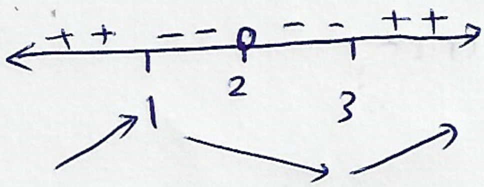
y' undefined at $x = 2$ (but $2 \notin D$)

$y' = 0$ if $x^2 - 4x + 3 = 0 \to (x-3)(x-1) = 0 \to x = 1, 3$

y has critical values at $x = 1, 3$

$f(1) = \dfrac{(1)^2 - 3}{1 - 2} = \dfrac{-2}{-1} = 2 \to (1, 2)$ local max

$f(3) = \dfrac{(3)^2 - 3}{3 - 2} = \dfrac{9 - 3}{1} = 6 \to (3, 6)$ local min sign of y'



- increasing intervals: $(-\infty, 1] \cup [3, \infty)$

- decreasing intervals: $[1, 2) \cup (2, 3]$

(Note that $2 \notin D$)

$$-\,y'' = \frac{(x-2)^2 \cdot (2x-4) - (x^2-4x+3)\cdot(2)(x-2)}{(x-2)^4}$$

$$= \frac{(x^2-4x+4)(2x-4) - (x^2-4x+3)(2x-4)}{(x-2)^4}$$

$$= \frac{(2x-4)\left[(x^2-4x+4)-(x^2-4x+3)\right]}{(x-2)^4}$$

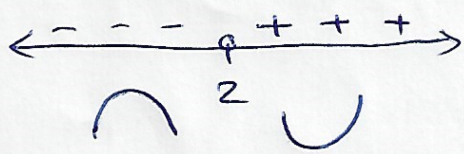
$$= \frac{2\cancel{(x-2)}\left[\cancel{x^2} - \cancel{4x} + 4 - \cancel{x^2} + \cancel{4x} - 3\right]}{(x-2)^{\cancel{4}\,3}} = \frac{2(1)}{(x-2)^3}$$

$$= \frac{2}{(x-2)^3}$$

- y'' undefined at $x=2$ (but $2 \notin D$)

$y'' = 0$ if $\frac{2}{(x-2)^3} = 0 \rightarrow 2 = 0$ which is impossible.

sign of y''



no inflection points.

concave up intervals: $(2, \infty)$

concave down intervals: $(-\infty, 2)$

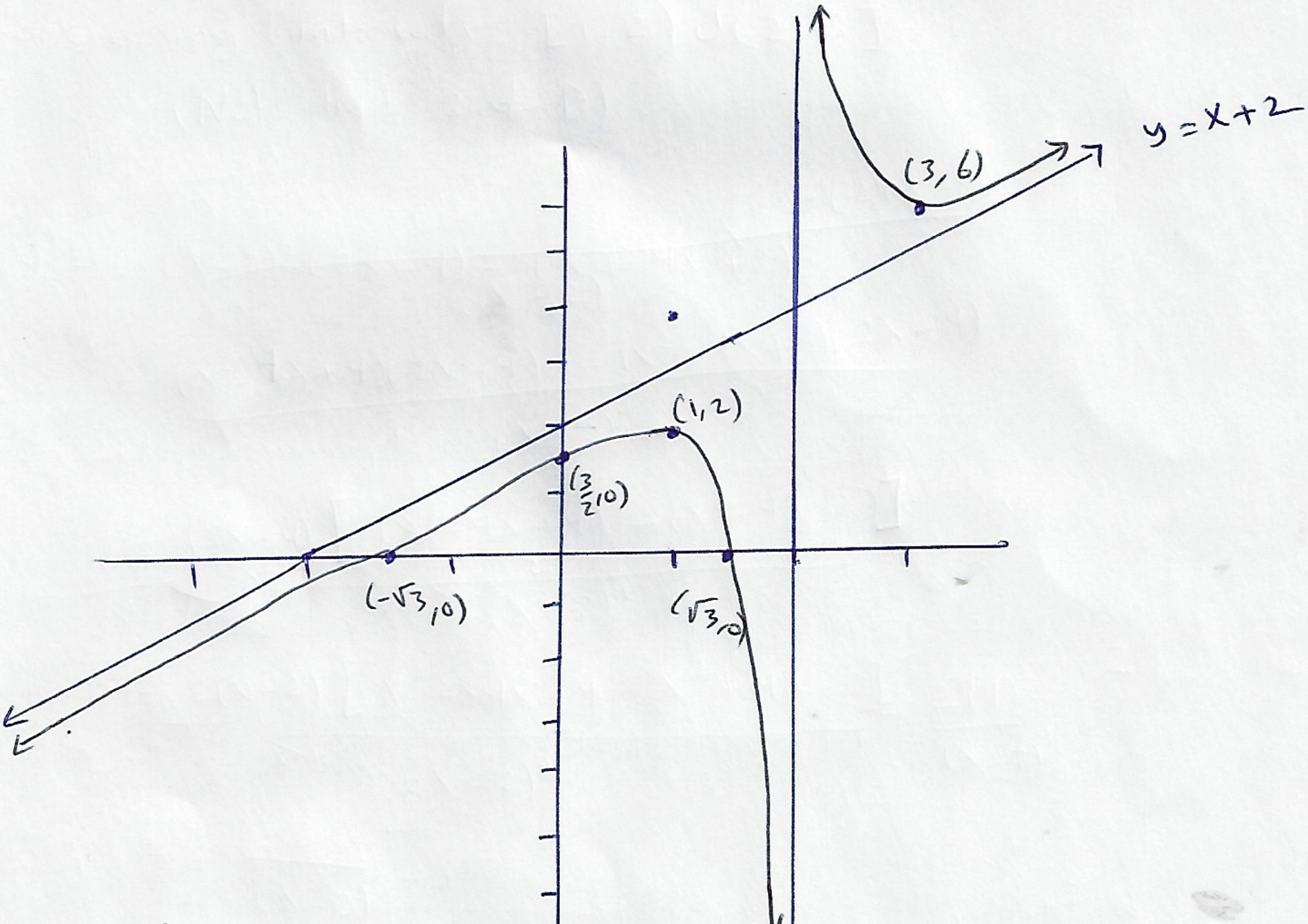
- y-intercept: at $x=0 \rightarrow y = \frac{0^2-3}{0-2} = \frac{-3}{-2} = \frac{3}{2} \rightarrow (0, \frac{3}{2})$

- x-intercept: at $y=0$

$$0 = \frac{x^2-3}{x-2} \rightarrow 0 = x^2-3 \rightarrow x = \pm\sqrt{3}$$

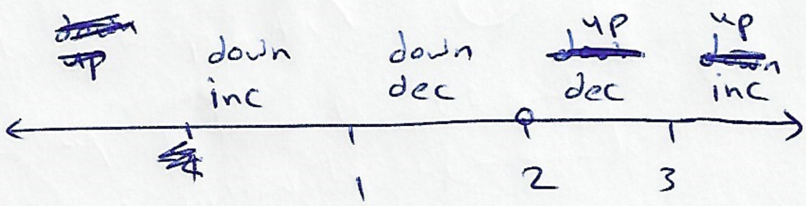
$(\sqrt{3}, 0), (-\sqrt{3}, 0)$

(Note that $\sqrt{3} > 1$)



Note: Range: $(-\infty, 2) \cup (6, \infty)$

points: $(1,2)$, $(3,6)$, $(\sqrt{3}, 0)$, $(-\sqrt{3}, 0)$, $(0, \frac{3}{2})$



e) $y = \sqrt[3]{x^3+1} = (x^3+1)^{\frac{1}{3}}$

D: $(-\infty, \infty)$

$- \ y' = \frac{1}{3}(x^3+1)^{\frac{-2}{3}} \cdot (3x^2) = \dfrac{x^2}{\sqrt[3]{(x^3+1)^2}}$

Sign of y' $\quad$ + + + $\underset{-1}{|}$ + + $\underset{0}{|}$ + + +

y' undefined ~~at~~ when $\sqrt[3]{(x^3+1)^2} = 0 \rightarrow x^3+1=0 \rightarrow x=-1$

$y' = 0$ when $\dfrac{x^2}{\sqrt[3]{(x^3+1)^2}} = 0 \rightarrow x^2 = 0 \rightarrow x = 0$

y has critical values at $x = 0, -1$

increasing intervals : $(-\infty, \infty)$

$$- \ y'' = \frac{(x^3+1)^{2/3} \cdot 2x - x^2 \cdot \frac{2}{3} \cdot (x^3+1)^{-1/3} \cdot 3x^2}{\left[(x^3+1)^{2/3}\right]^2}$$

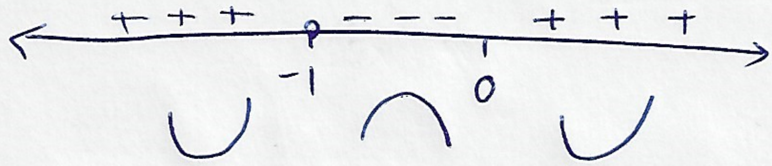
$$y'' = \frac{2x(x^3+1)^{2/3} - \dfrac{2x^4}{\sqrt[3]{x^3+1}}}{(x^3+1)^{1/3}} = \frac{\dfrac{2x(x^3+1) - 2x^4}{(x^3+1)^{1/3}}}{(x^3+1)^{4/3}}$$

$$= \frac{\cancel{2x^4} + 2x - \cancel{2x^4}}{(x^3+1)^{5/3}} = \frac{2x}{(x^3+1)^{5/3}}$$

y'' undefined at $x = -1$

$y'' = 0$ at $x = 0$

sign of y''



Concave up intervals: $(-\infty, -1] \cup [0, \infty)$

Concave down intervals: $[-1, 0]$

$\star$ inflection point $(0, f(0)) = (0,1)$

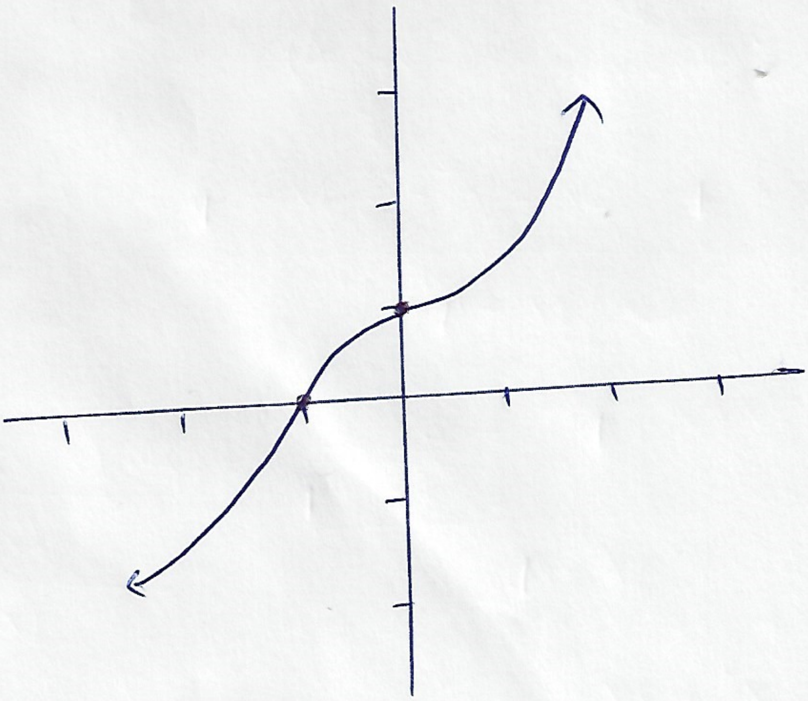
points:-

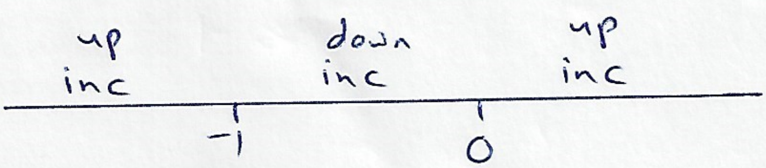
$f(0) = \sqrt[3]{0^3+1} = \sqrt[3]{1} = 1 \rightarrow (0,1)$

$\cancel{f(0)} = f(-1) = \sqrt[3]{(-1)^3+1} = \sqrt[3]{0} = 0 \rightarrow (-1,0)$

x-intercept:- $y=0 \rightarrow \sqrt[3]{x^3+1} = 0 \rightarrow x^3+1=0 \rightarrow x=-1$

y-intercept: $x=0 \rightarrow \sqrt[3]{0+1} = 1$





f) $y = \dfrac{x}{x^2-1}$

- $D: (-\infty, \infty) \setminus \{-1, 1\}$

- Horizonta asymptotes:- (نلاحظ درجة البسط اقل من درجة المقام)

$$\lim_{x\to\infty} \frac{x}{x^2-1} = \lim_{x\to\infty} \frac{1}{2x} = 0$$

$$\lim_{x\to-\infty} \frac{x}{x^2-1} = \lim_{x\to-\infty} \frac{1}{2x} = 0$$

$y=0$ is a horizontal asymptote.

- Vertical asymptotes: نبحث عند اصفار المقام

$$\lim_{x\to 1^+} \frac{x}{x^2-1} = \frac{+}{0^+} = \infty$$

$$\lim_{x\to 1^-} \frac{x}{x^2-1} = \frac{+}{0^-} = -\infty$$

$x=1$ is a vertical asymptote

$$\lim_{x\to -1^+} \frac{x}{x^2-1} = \frac{-}{0^-} = \infty$$

$$\lim_{x\to -1^-} \frac{x}{x^2-1} = \frac{-}{0^+} = -\infty$$

$x=-1$ is a vertical asymptote.

- Oblique asymptotes:

no oblique asymptotes (لان درجة البسط اقل من درجة المقام)

$$y' = \frac{(x^2-1)\cdot 1 - x\cdot 2x}{(x^2-1)^2} = \frac{x^2-1-2x^2}{(x^2-1)^2} = \frac{-1-x^2}{(x^2-1)^2}$$

sign of y': $- - - - \underset{-1}{\circ} - - - \underset{1}{\circ} - - - -$

y' undefined when $(x^2-1)^2=0 \to x = \pm 1$

(but $-1 \notin D$, $1 \notin D$)

$y' = 0$ if $\frac{-1-x^2}{(x^2-1)^2} = 0 \to -1-x^2=0 \to x^2=-1$ impossible

so we have no critical values.

also we have no extreme values.

decreasing intervals : $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$

(note that $1 \notin D$, $-1 \notin D$)

- $y'' = \dfrac{(x^2-1)^2 \cdot (-2x) - (-1-x^2) \cdot 2(x^2-1)(2x)}{(x^2-1)^4}$

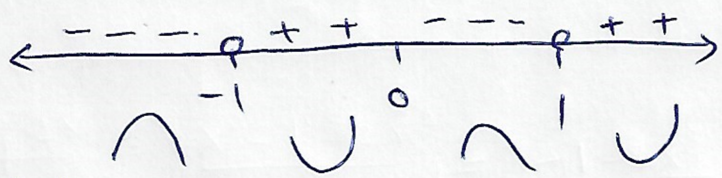
$= \dfrac{\cancel{(x^2-1)}\left[-2x(x^2-1) - 4x(-1-x^2)\right]}{(x^2-1)^{\cancel{4}\,3}}$

$= \dfrac{-2x^3+2x+4x+4x^3}{(x^2-1)^3} = \dfrac{2x^3+6x}{(x^2-1)^3}$

y'' undefined at $x = 1, -1$

$y'' = 0$ if $2x^3+6x=0 \to 2x(x^2+3)=0 \to x=0$

sign of y''

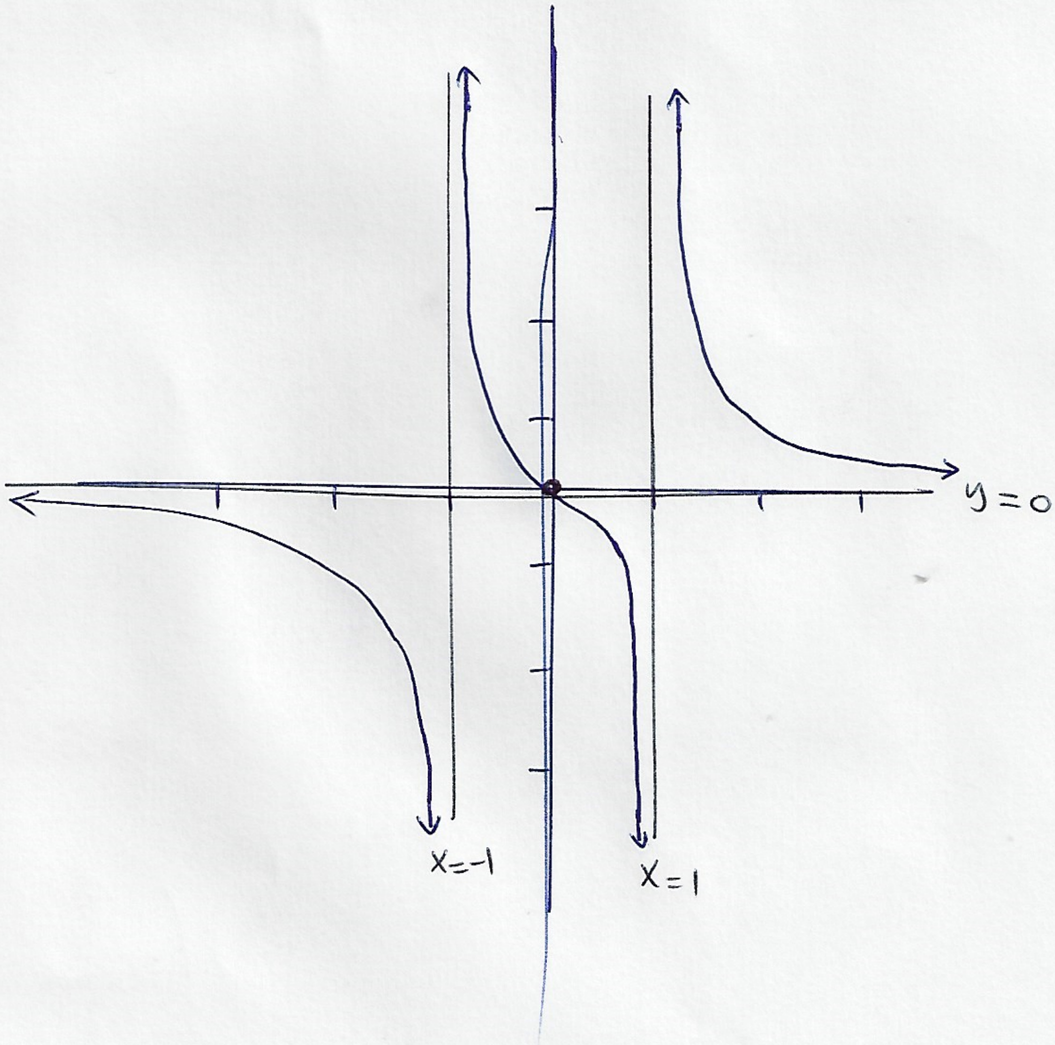


concave up intervals : $(-1,0] \cup [1,\infty)$

Concave down intervals : $(-\infty,-1) \cup [0,1)$

inflection points:- $(0, f(0))$

$f(0) = \frac{0}{0^2-1} = 0 \rightarrow (0,0)$



points : $(0,0)$

dec down		dec up		dec down		dec up
	-1		0		1	

9) $y = x\sqrt{8-x^2}$

D: $8-x^2 \geq 0 \rightarrow 8 \geq x^2 \rightarrow -\sqrt{8} \leq x \leq \sqrt{8}$

D: $[-\sqrt{8}, \sqrt{8}]$ (Note that $\sqrt{4}<\sqrt{8}<\sqrt{9} \rightarrow 2<\sqrt{8}<3$)

$$y' = x \cdot \frac{1}{\cancel{2}}(8-x^2)^{-\frac{1}{2}} \cdot (-\cancel{2}x) + \sqrt{8-x^2} \cdot 1$$

$$= -x^2(8-x^2)^{-\frac{1}{2}} + (8-x^2)^{\frac{1}{2}}$$

$$= \frac{-x^2}{(8-x^2)^{\frac{1}{2}}} + (8-x^2)^{\frac{1}{2}}$$

$$= \frac{-x^2 + (8-x^2)}{(8-x^2)^{\frac{1}{2}}} = \frac{8-2x^2}{(8-x^2)^{\frac{1}{2}}}$$

y' undefined when $(8-x^2)^{\frac{1}{2}} = 0 \rightarrow 8-x^2=0 \rightarrow x = \pm\sqrt{8}$

$y' = 0$ when $\frac{8-2x^2}{(8-x^2)^{\frac{1}{2}}} = 0 \rightarrow 8-2x^2=0 \rightarrow x^2=4 \rightarrow x=\pm 2$

critical values at $x = -2, 2$

sign of y'

$-\sqrt{8}$		-2		2		$\sqrt{8}$
	$- -$		$+ + +$		$- -$	
	$\searrow$		$\nearrow$		$\searrow$	

increasing intervals: $[-2, 2]$

decreasing intervals: $[-\sqrt{8}, -2] \cup [2, \sqrt{8}]$

y has local max at $x = 2$, $f(2) = 2\sqrt{8-(2)^2} = 4$

$(2, 4)$

y has local min at $x = -2$, $f(-2) = -2\sqrt{8-(-2)^2}$

$= -4$ $(-2, -4)$

y has local max at $x = -\sqrt{8}$

$f(-\sqrt{8}) = -\sqrt{8}\sqrt{8-(-\sqrt{8})^2} = -\sqrt{8}\cdot\sqrt{8-8} = 0 \rightarrow (-\sqrt{8}, 0)$

y has local min at $x = \sqrt{8}$

$f(\sqrt{8}) = \sqrt{8}\sqrt{8-(\sqrt{8})^2} = \sqrt{8}\cdot\sqrt{8-8} = 0 \rightarrow (\sqrt{8}, 0)$

y has absolute max at $x = 2$

y has absolute min at $x = -2$

$$y'' = \frac{(8-x^2)^{\frac{1}{2}}\cdot(-4x) - (8-2x^2)\cdot\frac{1}{\cancel{2}}(8-x^2)^{-\frac{1}{2}}\cdot(-\cancel{2}x)}{[(8-x^2)^{\frac{1}{\cancel{2}}}]^{\cancel{2}}}$$

$$= \frac{-4x(8-x^2)^{\frac{1}{2}} + \dfrac{x(8-2x^2)}{(8-x^2)^{\frac{1}{2}}}}{(8-x^2)}$$

$$= \frac{-4x(8-x^2) + x(8-2x^2)}{(8-x^2)^{3/2}} = \frac{-32x + 4x^3 + 8x - 2x^3}{(8-x^2)^{3/2}}$$

$$= \frac{2x^3 - 24x}{(8-x^2)^{3/2}} = \frac{2x(x^2-12)}{(8-x^2)^{3/2}}$$

y'' undefined when $(8-x^2)^{3/2} = 0 \rightarrow x^2 = 8 \rightarrow x = \pm\sqrt{8}$

$y'' = 0$ if $2x(x^2-12) = 0 \rightarrow x = 0,\ x = \pm\sqrt{12}$

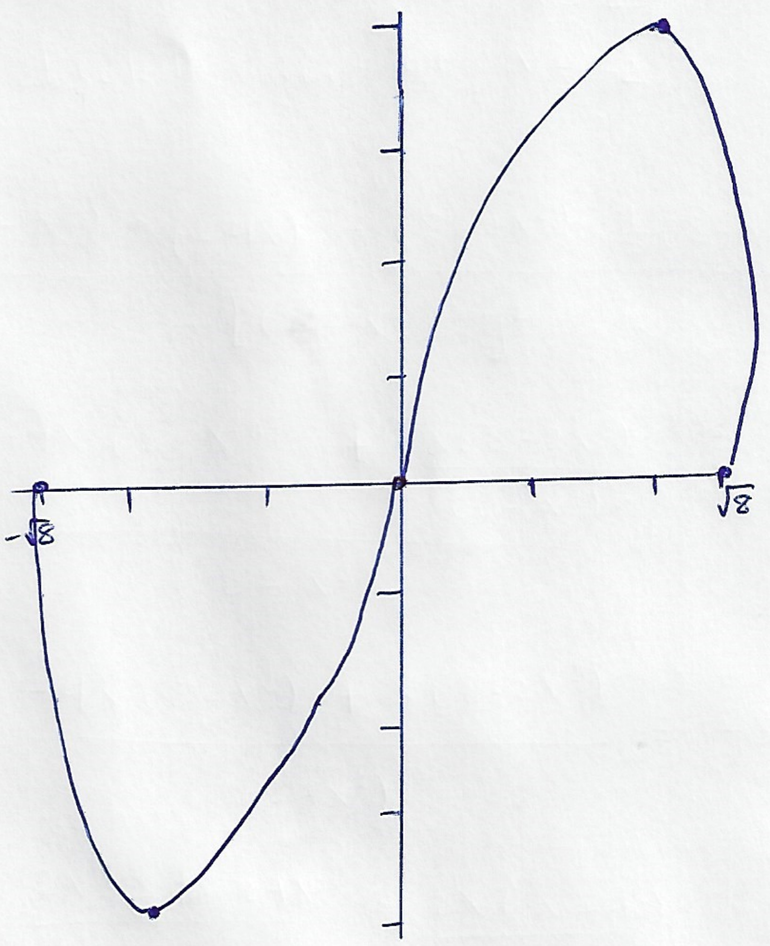
(Note that $\pm\sqrt{12} \notin D$)

sign of y''

$-\sqrt{8}$		0		$\sqrt{8}$
	$+ + + + +$	0	$- - - - -$	
	$\cup$		$\cap$	

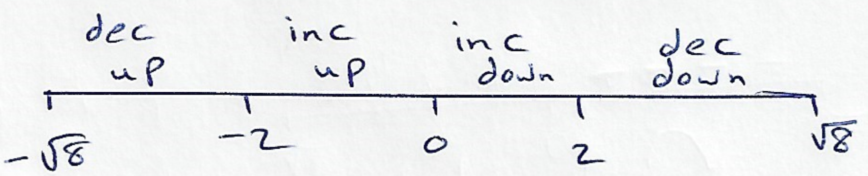
inflection points $(0, f(0))$

$f(0) = 0\sqrt{8-0^2} = 0 \rightarrow (0,0)$



Note ~~f~~ range: $[-4, 4]$

Points: $(0,0)$, $(2,4)$, $(-2,-4)$, $(-\sqrt{8},0)$, $(\sqrt{8},0)$



2) Find the value of c in the conclusion of the mean value theorem for the function $f(x) = \sqrt{x}$ on the interval $[a, b]$, $a > 0$.

$f(x) = \sqrt{x}$ continuous on $[a,b]$ and differentiable on (a,b)

so by the mean value theorem there is $c \in (a,b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

(note that $f'(x) = \dfrac{1}{2\sqrt{x}}$, $f'(c) = \dfrac{1}{2\sqrt{c}}$)

so $\dfrac{1}{2\sqrt{c}} = \dfrac{\sqrt{b} - \sqrt{a}}{b - a} = \dfrac{\cancel{\sqrt{b} - \sqrt{a}}}{\cancel{(\sqrt{b} - \sqrt{a})}(\sqrt{b} + \sqrt{a})}$

$$\frac{1}{2\sqrt{c}} = \frac{1}{\sqrt{b} + \sqrt{a}}$$

$$\sqrt{b} + \sqrt{a} = 2\sqrt{c}$$

$$\frac{\sqrt{b} + \sqrt{a}}{2} = \sqrt{c}$$

$$c = \left(\frac{\sqrt{b} + \sqrt{a}}{2}\right)^2 = \frac{(\sqrt{b} + \sqrt{a})^2}{4} = \frac{b + 2\sqrt{ab} + a}{4}$$

3) For what values of a, m and b does the function

$$f(x) = \begin{cases} 3 & , \quad x=0 \\ -x^2+3x+a & , \quad 0<x<1 \\ mx+b & , \quad 1 \leq x \leq 2 \end{cases}$$

satisfy the hypotheses of the mean value theorem on $[0,2]$

$f(x)$ satisfy the mean value theorem implies that $f(x)$ continuous on $[0,2]$ and differentiable on $(0,2)$

- $f(x)$ is continuous on $[0,2]$:-

$f(x)$ is cont at $x=0$ from right:-

$$\lim_{x\to 0^+} f(x) = f(0)$$

$$\lim_{x\to 0^+} -x^2+3x+a = 3$$

$$0^2+3(0)+a = 3$$

$$\boxed{a = 3}$$

$f(x)$ is continuous at $x=1$

$$\lim_{x\to 1^-} f(x) = f(1)$$

$$\lim_{x\to 1^-} (-x^2+3x+a) = f(1)$$

$$-(1)^2+3(1)+a = m(1)+b$$

$2 + a = m + b$ but $a = 3$

$2 + 3 = \cancel{4 + m} \; m + b$

$$\boxed{5 = m + b}$$

f is diff at $x = 1$ implies $f'(1)^+ = f'(1)^-$

$$f'(x) = \begin{cases} -2x + 3 & 0 < x < 1 \\ m & 1 < x < 2 \end{cases}$$

$f'(1)^+ = f'(1)^-$

$m = -2(1) + 3$

$$\boxed{m = 1}$$

but $m + b = 5$

$1 + b = 5$

$$\boxed{b = 4}$$