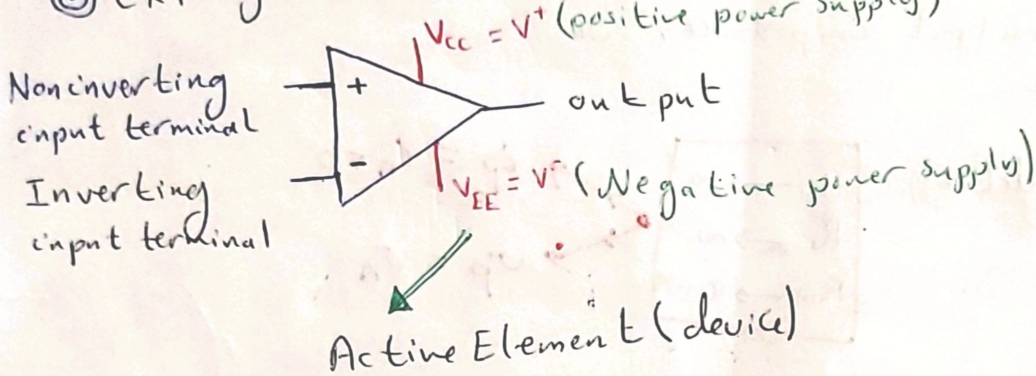
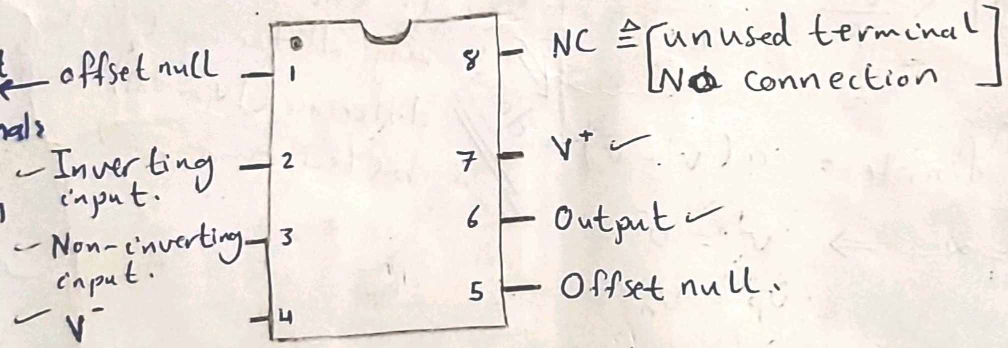
The Operational Amplifier ((Op-amp)):

* It was referred to as operational because it was used to implement the mathematical operations of integration, differentiation, addition, sign changing, and scaling.

CKT symbol for an op-amp.

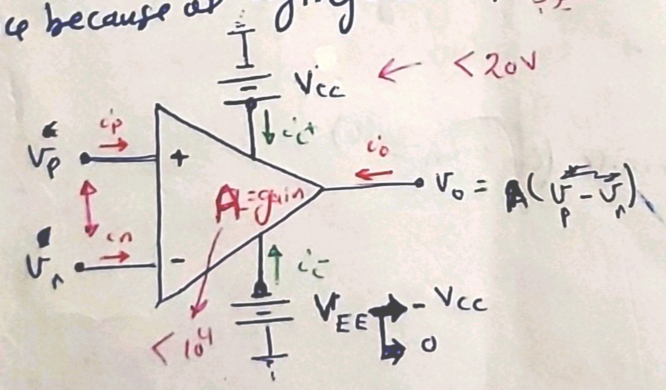


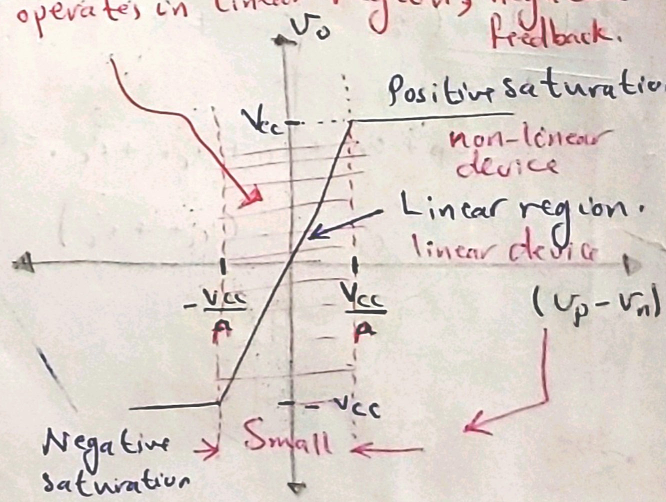
Commercially available device (741)



The offset null terminals may be used in an auxiliary circuit to compensate for a degradation in performance because of aging and imperfections.

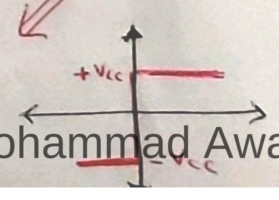
to operates in linear region, negative feedback.





$$v_o = \begin{cases} +V_{cc} & \underbrace{A(v_p - v_n)}_{v_o} > +V_{cc} \\ -V_{cc} & \underbrace{A(v_p - v_n)}_{v_o} < -V_{cc} \\ A(v_p - v_n) & -V_{cc} \leq A(v_p - v_n) \leq +V_{cc} \end{cases}$$

• The voltage transfer characteristic of an op amp.



$i_p + i_n + i_o + i_{c^+} + i_{c^-} = 0$

OP-Amp Properties :-

1) Very Large gain [Ratio between v_o, v_i] , a
   10,000

2) Very Large input impedance. $M\Omega$

3) Very small output impedance. Ω

Ideal Op-amp Model :-

$$\boxed{\begin{array}{l} a \longrightarrow \infty \\ R_i \longrightarrow \infty \\ R_o \longrightarrow 0 \end{array}} \rightarrow \begin{cases} i_p = i_n = 0 \\ v_p = v_n \end{cases}$$

$\boxed{1}$ $i_p = i_n = 0$

$\boxed{2}$ $|v_o| < + V_{cc}$

$|A(v_p - v_n)| < V_{cc}$

$|(v_p - v_n)| < \frac{V_{cc}}{A} = \frac{20V}{10^4} \cong 2mV$ $\Downarrow$ ideal

in the linear region, the mag. of the input voltage difference ($|v_p - v_n|$) must be less than 2 mV

$\boxed{2}$ v_o = finite = $a(v^+ - v^-)$

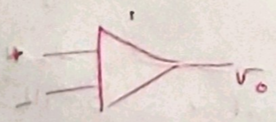
$\frac{v_o}{a} = (v^+ - v^-)$, $a \cong \infty \Rightarrow v^+ = v^-$

$v_p = v_n$

<u>note that :</u> $i_o \neq 0$

$i_p + i_n + i_o + i_{c^+} + i_{c^-} = 0$

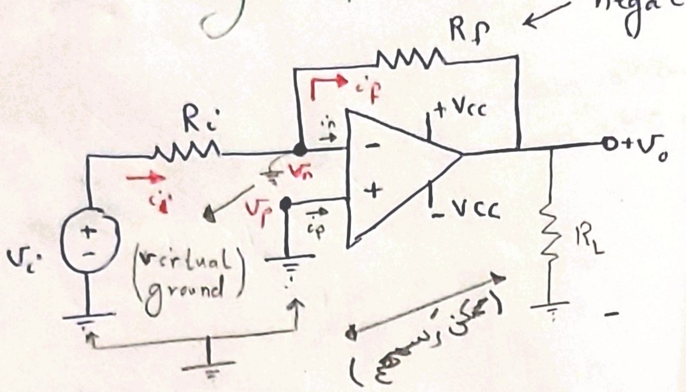
$i_o = -(i_{c^+} + i_{c^-})$ since ($i_p = i_n = 0$)



Some Important Op-Amp Circuits :

[1] Inverting Amplifier :

R_f ← negative feedback.



→ $v_p = v_n$

→ $i_p = i_n = 0$

[a] Since $v_p = 0$, $v_n = v_p = 0$

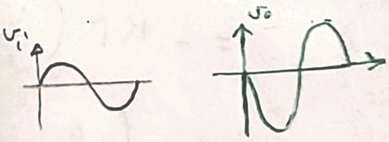
$$i_i = \frac{v_i}{R_i}$$

[b] Since $i_n = i_p = 0 \Rightarrow i_f = i_i = \frac{v_i}{R_i}$

$$\therefore \ v_o = -R_f \, i_f = -R_f\left[\frac{v_i}{R_i}\right] = -\frac{R_f}{R_i} v_i$$

$$\boxed{\frac{v_o}{v_i} = \text{gain} = -\frac{R_f}{R_i}}$$

→ inverting, input from inv terminal



* » The upper limit on the gain $\frac{R_f}{R_i}$, is determined by the power supply voltages and the value at the signal voltage v_i.

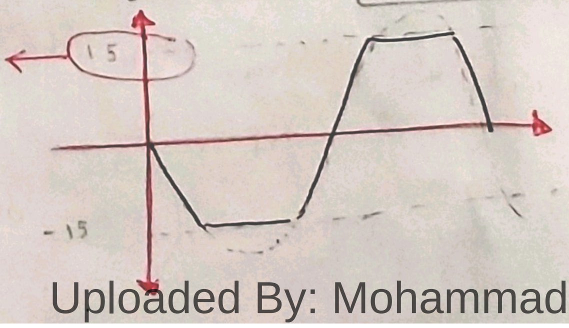
» if $V^+ = V^- \Rightarrow V_{cc} = V_{EE} = V_{cc}$

$$|v_o| \leq V_{cc}$$

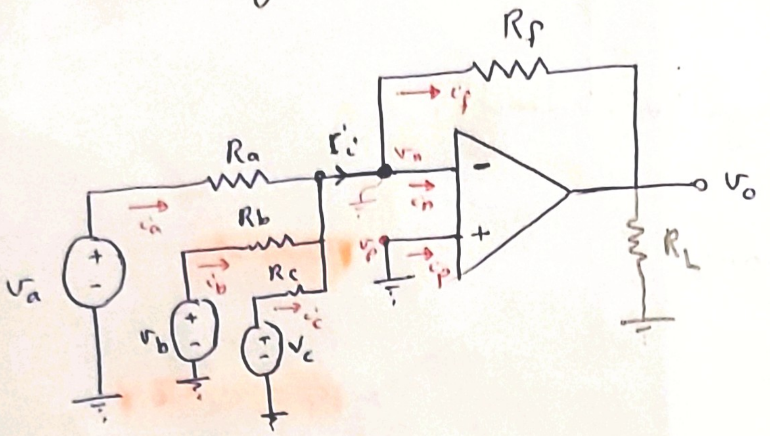
$$\left|\frac{R_f}{R_i} v_s\right| \leq V_{cc}$$

$$\frac{R_f}{R_i} \leq \left|\frac{V_{cc}}{v_s}\right| \quad , \text{ if } V_{cc} = 15\,V , \ v_s = 10\,mV \ , \ \boxed{\frac{R_f}{R_i} \leq 1500}$$

practically not $15-\epsilon$



[7] Inverting Adder ((Summing Amplifier)) ((Mixer)) (مزج / خلط)



(a) Since $v_p = 0 \Longrightarrow v_p = v_n = 0$

$$i_a = \frac{v_a}{R_a} \quad , \quad i_b = \frac{v_b}{R_b} \quad , \quad i_c = \frac{v_c}{R_c}$$

$$i_i = i_a + i_b + i_c$$

(b) Since $i_n = i_p = 0 \Longrightarrow i_i = i_f$

$$v_o = -R_f \, i_f = -R_f \left(i_a + i_b + i_c \right)$$

$$= -R_f \left(\frac{v_a}{R_a} + \frac{v_b}{R_b} + \frac{v_c}{R_c} \right)$$

$$= -\left(\frac{R_f}{R_a} v_a + \frac{R_f}{R_b} v_b + \frac{R_f}{R_c} v_c \right)$$

Special Cases

i) if $R_a = R_b = R_c = R_i$

$$v_o = -\frac{R_f}{R_i} \left(v_a + v_b + v_c \right)$$

ii) if $R_a = R_b = R_c = R_i = R_f$

$v_o = -(v_a + v_b + v_c)$ op-amp as an adder

iii) if $R_a = R_b = R_c = R_i$, and

$$R_f = \frac{R_i}{3}$$

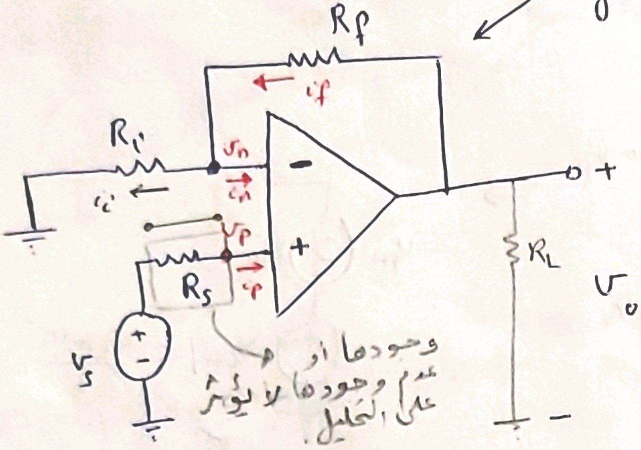
$v_o = -\left(\frac{v_a + v_b + v_c}{3} \right)$ ((average))

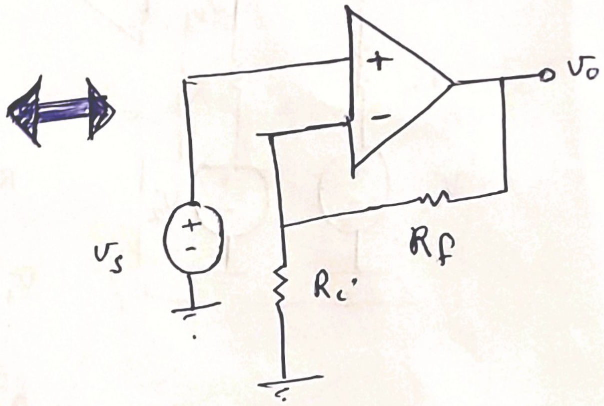
[3] Non-inverting Amplifier:-

→ non-inverting input terminal
→ output = (gain)(v_i)

non-inv.

negative feedback.

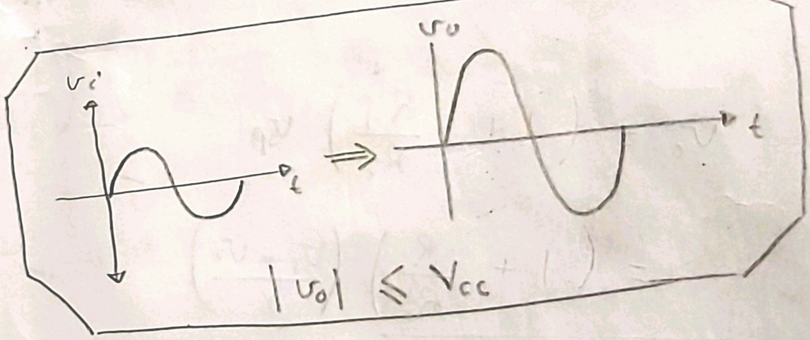




* since $v_p = v_n \Rightarrow v_p = v_s$ because ($i_n = i_p = 0$)

$$i_i = \frac{v_s - 0}{R_i} = \frac{v_s}{R_i}$$

* since $i_n = 0 \Rightarrow i_i = i_f$



$$v_o = R_f i_f + v_s$$

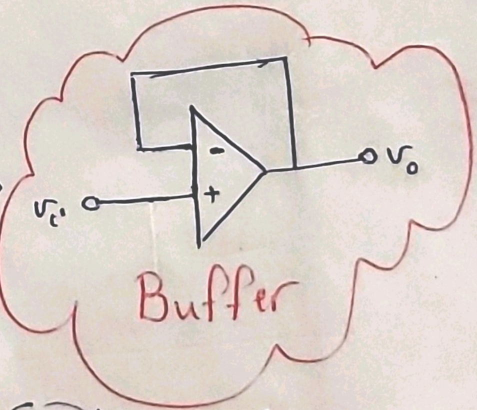
$$= R_f \left(\frac{v_s}{R_i}\right) + v_s$$

$$v_o = \left(1 + \frac{R_f}{R_i}\right) v_s$$

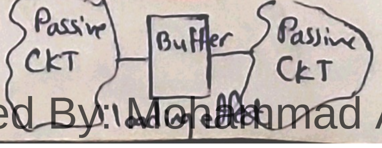
$$\boxed{\frac{v_o}{v_i} = \text{gain} = 1 + \frac{R_f}{R_i}}$$

Special Case:-

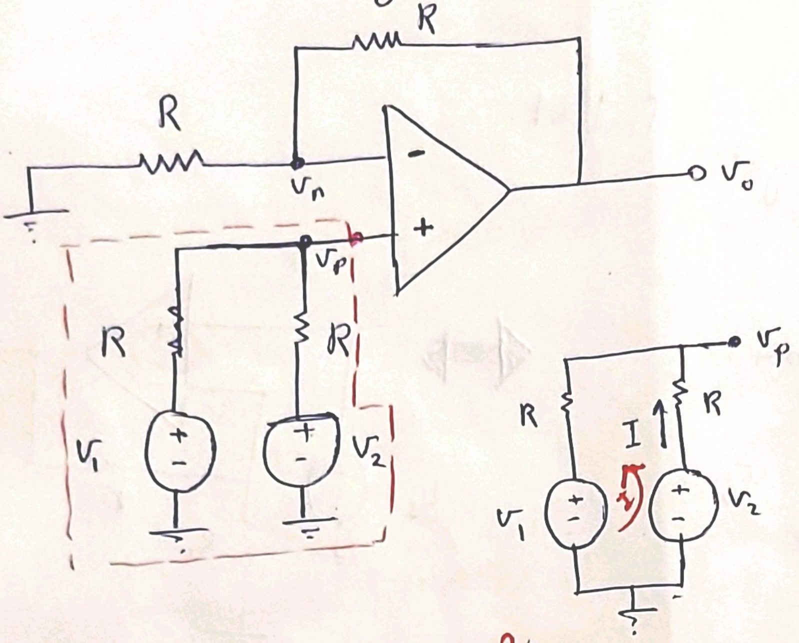
if $R_i = \infty$ (open CKT) مش موجوده
$R_f = 0$ (shor CKT) سلك



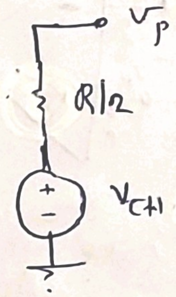
$\frac{v_o}{v_i} = 1 = A \triangleq$ unity gain Amplifier
$\triangleq$ Buffer



[4] Non-inverting Adder (Summing Amp) (Mixer)



$\Rightarrow$



$v_{TH} = \frac{v_2 + v_1}{2}$

Loop

$v_2 = RI + RI + v_1 =$

$I = \frac{v_2 - v_1}{2R}$

$= \frac{v_2}{2R} - \frac{v_1}{2R}$

$R_{TH} = R_{oc} = \frac{R}{2}$

$v_{TH} = -RI + v_2$

$= -\frac{v_2}{2} + \frac{v_1}{2} + v_2$

$= \frac{v_2}{2} + \frac{v_1}{2}$

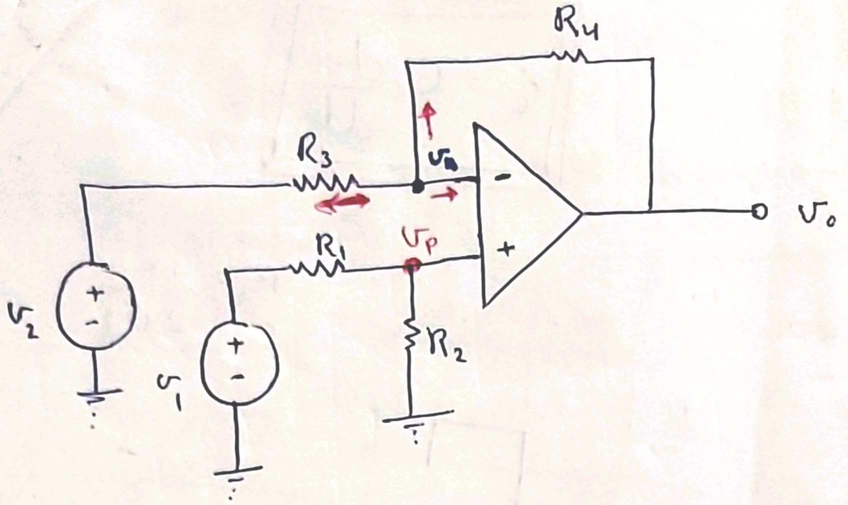
$= \frac{v_2 + v_1}{2}$

$v_o = \left(1 + \frac{R_f}{R_i}\right) v_s$

$= \left(1 + \frac{R}{R}\right)\left(\frac{v_1 + v_2}{2}\right)$

$\boxed{v_o = v_1 + v_2}$

5) Difference Amplifier ((Voltage Subtraction))



$$\frac{v_n - v_2}{R_3} + \frac{v_n - v_o}{R_4} + i_n = 0$$

$* \; i_p = i_n = 0$

$* \; v_p = v_n = \frac{R_2}{R_2 + R_1} v_1$

using superposition :-

$$v_o = \underbrace{v_{o1}}_{\text{kill } v_1} + \underbrace{v_{o2}}_{\text{kill } v_2}$$

$$= v_2\left(-\frac{R_4}{R_3}\right) + \left(1 + \frac{R_4}{R_3}\right) v_p \leftarrow \text{voltage divider}$$

$$= v_2\left(-\underbrace{\frac{R_4}{R_3}}_{b}\right) + \underbrace{\left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right)}_{a} v_1$$

$$= a v_1 - b v_2$$

if $R_4 = \infty$
$R_3 = 0$
$R_2 = \infty$
$R_1 = 0$

$v_o = v_2(-\infty) + (1+\infty)(\infty) v$

Special Case :-

$R_2 = R_4 = aR \Rightarrow A = \left(\frac{R_2}{R_1}\right) = \left(\frac{R_4}{R_3}\right)$

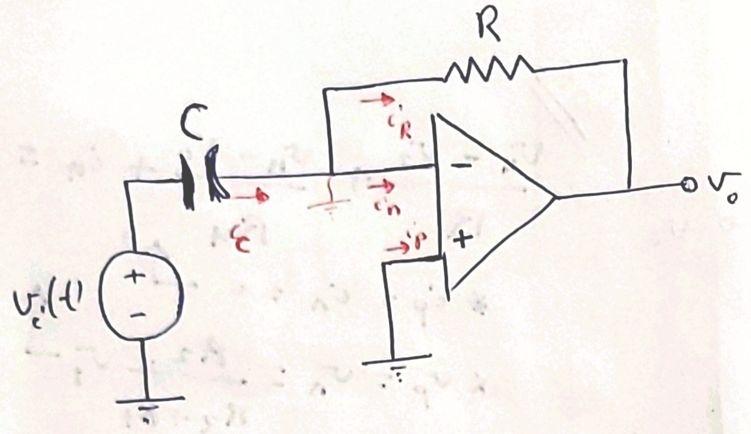
$R_1 = R_3 = R$

$$\boxed{v_o = A(v_1 - v_2)}$$

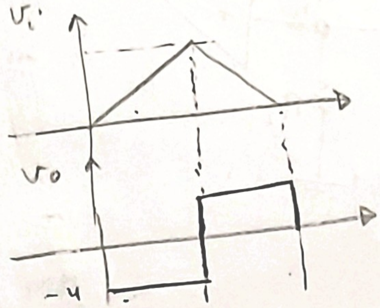
$$= -v_2\left(\frac{aR}{R}\right) + \left(1 + \frac{aR}{R}\right)\left(\frac{aR}{R + aR}\right) v_1$$

$$= -a v_2 + (1+a)\left(\frac{a}{1+a}\right) v_1$$

[6] Differentiator



$$i_c = C\frac{dv}{dt}$$



$i_C = i_R$

$$C\frac{dv_c}{dt} = i_R = C\frac{d}{dt}(v_i(t) - 0) = C\frac{dv_i}{dt}$$

$$v_o = -R\,i_R = -R\left[C\frac{dv_i}{dt}\right] = -RC\frac{dv_i(t)}{dt}$$

$$\boxed{v_o(t) = (-)RC\frac{dv_i(t)}{dt}}$$

* C, $+\ v(t)\ -$, $\rightarrow i(t)$

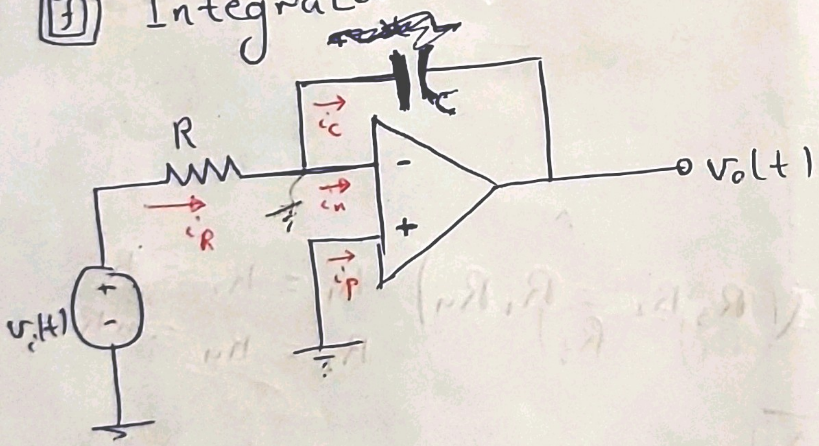
$i(t) = C\frac{dv}{dt}$

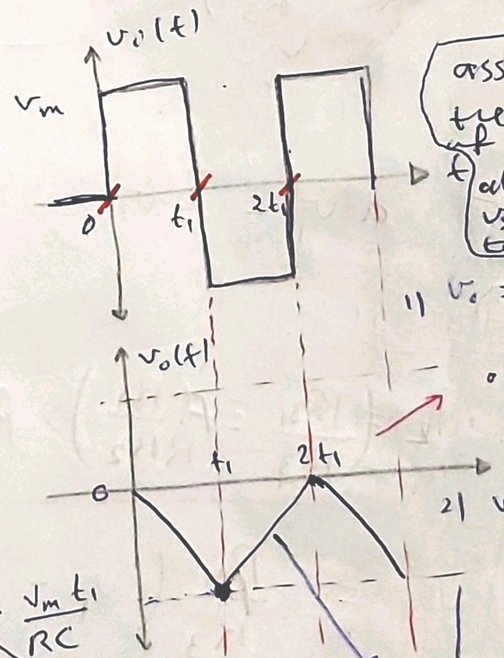
$v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$

in many practical app. the initial time is zero, $t_0 = 0$

$v(t) = \frac{1}{C}\int_0^t i\,d\tau + v(0)$

[7] Integrator





assume that the initial value of $v_o(t)$ is zero at the instant v_i steps from 0 to v_m

1) $v_o = -\frac{1}{RC}v_m t + 0$, $0 \le t \le t_1$

2) $v_o = -\frac{1}{RC}\int_{t_1}^{t} -v_m\,dt - \frac{1}{RC}v_m t_1$

$= \frac{v_m}{RC}t - \frac{2v_m}{RC}t_1$, $t_1 \le t \le 2t_1$

$$i_R = \frac{v_i(t)}{R}$$

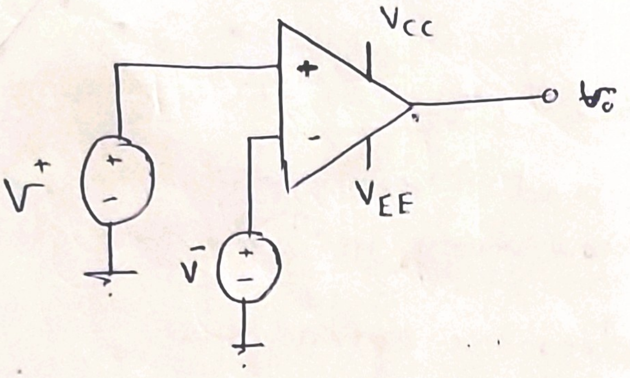
$$v_o(t) = -v_c(t) = -\frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v_o(t_0)$$

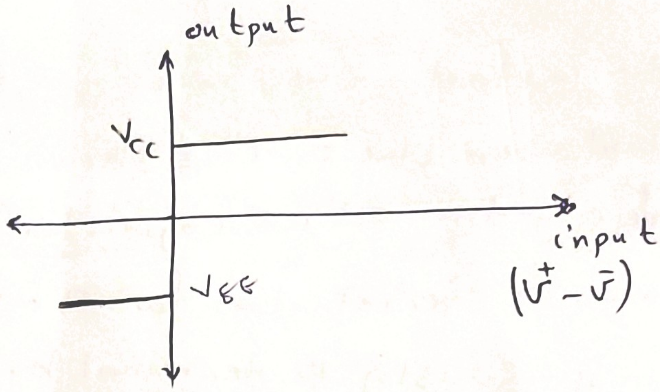
$$\boxed{v_o(t) = -\frac{1}{CR}\int_{t_0}^{t} v_i(t)\,dt + v_o(t_0)}$$

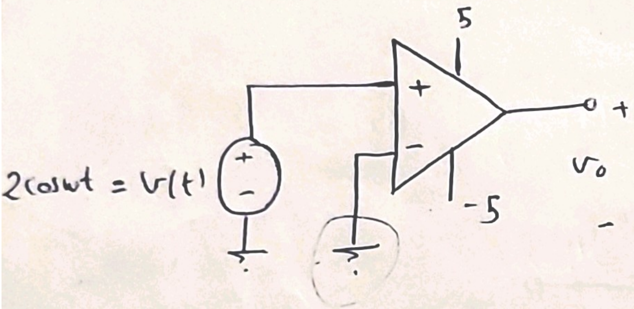
$\boxed{\text{if } t_0 = 0}$ (Capacitor initially discharged)

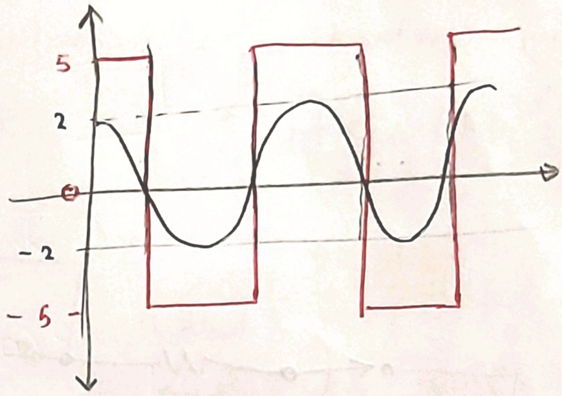
$$= -\frac{1}{RC}\int_0^t v_i(t)\,dt + v_o(0)$$

8 Comparators









9 Other Op-Amp CKTs

Uses:-

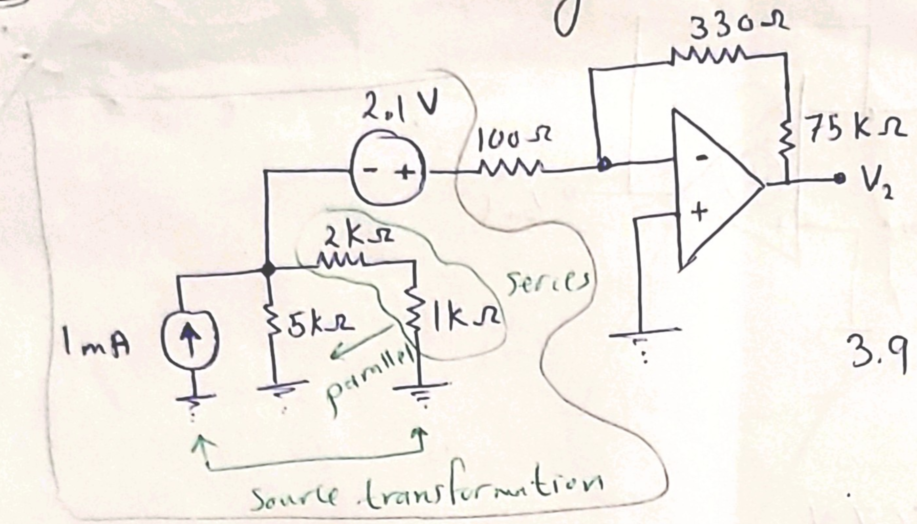
(a) $i_p = i_n = 0$, $v_p = v_n$

(b) Apply nodal analysis to the resulting CKT

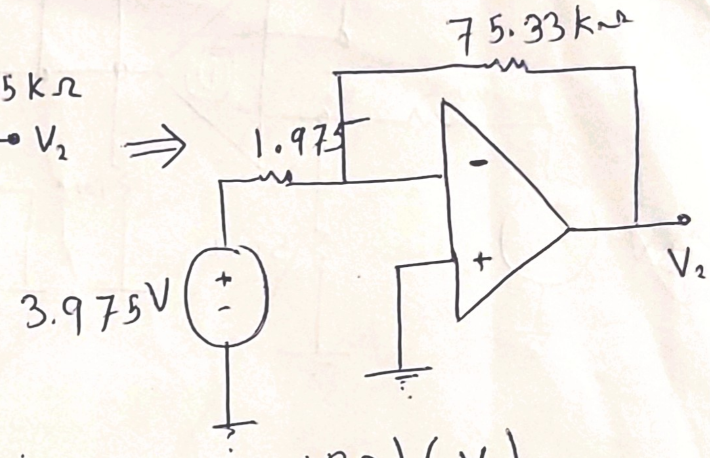
(c) Solve nodal equations to express the output voltage in terms of the op-amp input signals.

Examples

[1] For the circuit of Figure E1, Calculate the Voltage V_2.



$\Rightarrow$



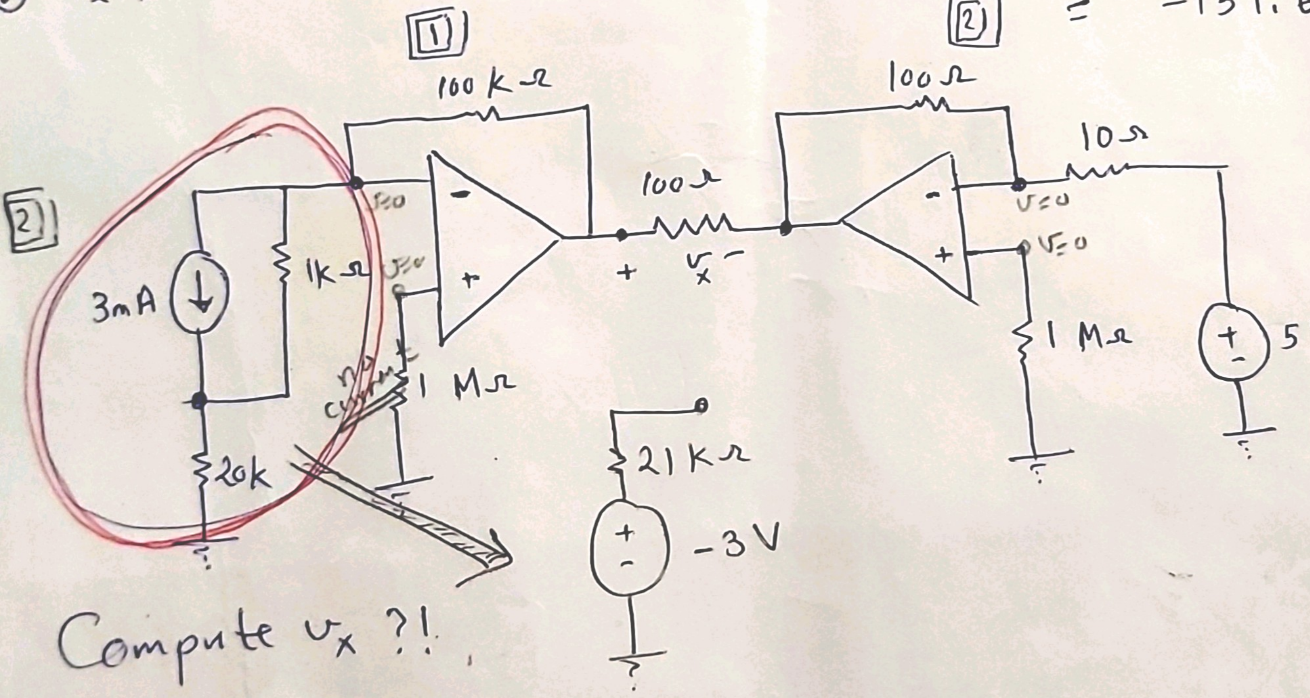
- $(5k \parallel 3k) = 1.875\ k\Omega$
- $(1\,mA)(1.875\,k\Omega) = 1.875\ V$
- $2.1\ V + 1.875\ V = 3.975\ V$

$$V_2 = -\left(\frac{R_f}{R_i}\right)(V_1)$$

$$= -\left(\frac{75.33k}{1.975k}\right)(3.975)$$

$$= -151.6\ V$$

[2]



Compute v_x ?!

$$v_{out_1} = -\left(\frac{100k}{21k}\right)(-3) = 14.29$$

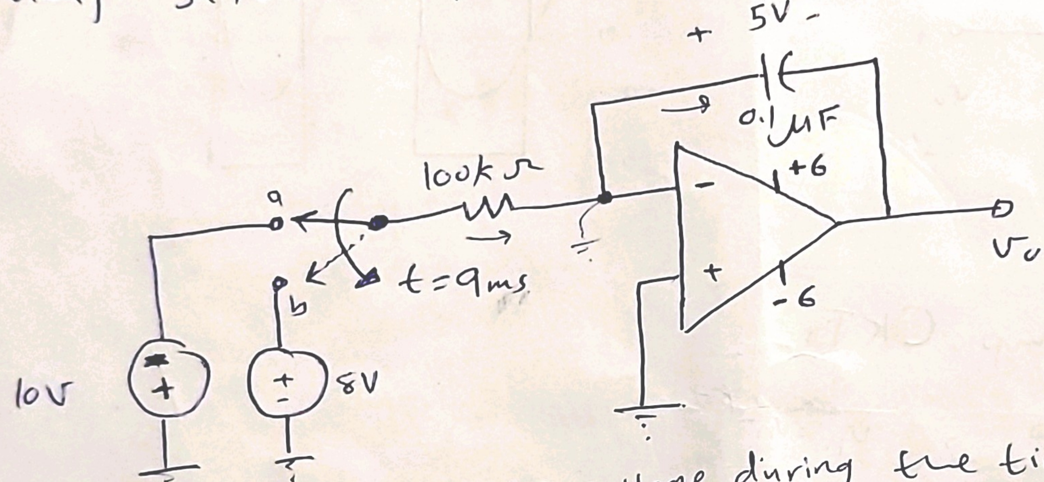
$$v_{out_2} = -\left(\frac{100}{10}\right)(5) = -50$$

$$v_x = v_{out_1} - v_{out_2} = 14.29 + 50 = 64.29\ V$$

ok

example

At the instant the switch makes contact with terminal a in the circuit shown in Figure (1), the voltage on the 0.1 μF Capacitor is 5 V. The switch remains at terminal a for 9 ms and then moves instantaneously to terminal b. How many ms after making contact with terminal b does the operational amp. saturate.?



The expression for the output voltage during the time the switch is at ~~at position (a)~~ terminal a is

$$v_o = -\frac{1}{RC}\int_{t_0}^{t} v_s\,dt + v_o(t_0)$$

$$= -\frac{1}{10^{-2}}\int_{0}^{t} (10)\,dt - 5$$

$$= (-5 + 1000t)\ V$$

at $t = 9$ ms

$$\boxed{v_o = -5 + 9 = 4\ V}$$

The expression for the output voltage after the switch moves to terminal b ~~at position (b)~~ is

$$v_o = 4 - \frac{1}{10^{-2}}\int_{9\times10^{-3}}^{t} 8\,dt = 4 - 800(t - 9\times10^{-3})$$

$$= (11.2 - 800t)\ V$$

$\Rightarrow 11.2 - 800t_s = -6$

$t_s = 21.5$ ms