0.4 Radicals and Rational Exponents 1

Tuesday, October 17, 2023 8:26 AM

↳ $\sqrt[n]{a}$ is radical with index n

radicaned a

الجذر التربيعي

Exp $\sqrt{4}$ square root of 4

is radical with index $n = 2$

$\sqrt{4} = \sqrt[2]{4}$

radicaned $a = 4$

$\sqrt{4} = 2$

$\sqrt{4} = \sqrt[2]{2 \times 2} = 2$

$\sqrt[3]{8}$ is radical with $n = 3$ (index)

$a = 8$ (radicaned)

$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$

$\sqrt[3]{-27}$ is radical with $n = 3$

$a = -27$

$\sqrt[3]{-27} = \sqrt[3]{-3 \times -3 \times -3} = -3$

$$\sqrt[3]{-27} = \sqrt[3]{(-3)\times(-3)\times(-3)} = -3$$

$$\sqrt[3]{16} = \sqrt[3]{2\times2\times2\times2} = 2\sqrt[3]{2}$$

Remark when **no index** is indicated $\Rightarrow$

This means $n = 2$

Exp $\sqrt{16}$ is radical with $n = 2$

$a = 16$

$$\sqrt{16} = \sqrt[2]{16} = \sqrt[2]{4\times4} = 4$$

Q: Is $\sqrt[n]{a}$ always defined ?

1) If the **index n is odd** (فردي), then

$\sqrt[n]{a}$ is always defined

$\sqrt[n]{a}$ is real number

Exp $\sqrt[3]{8} = \sqrt[3]{2\times2\times2}$

$= 2$ ✓

$n = 3$ odd $\Rightarrow$

$\sqrt[3]{8}$ is defined

($\sqrt[3]{8}$ is real number)

✓ ($\sqrt[3]{8}$ is real number)

Exp $\sqrt[3]{-8} = \sqrt[3]{-2 \times -2 \times -2}$

$= -2$

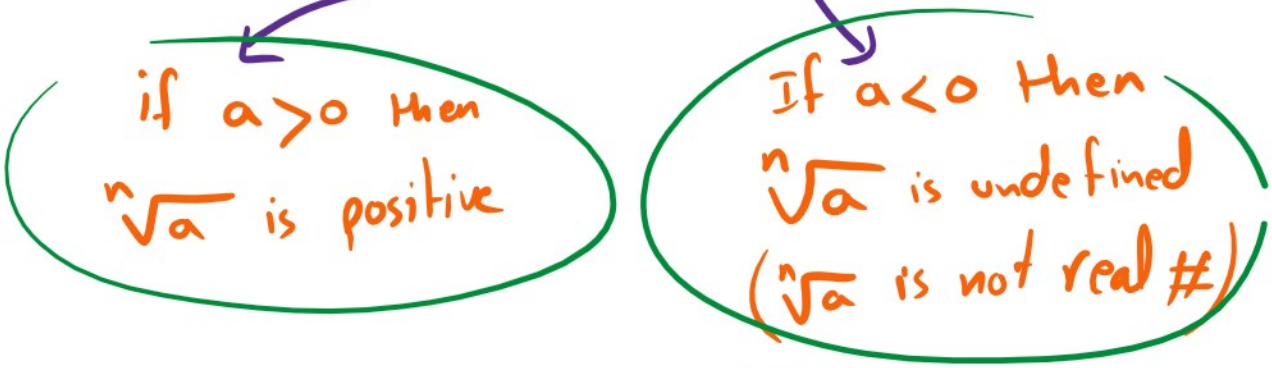
$n = 3$ odd $\Rightarrow$

$\sqrt[3]{-8}$ is defined

$\sqrt[3]{-8}$ is real number

$\sqrt[n]{a}$

(2) If **n is even** زوجي



Exp ① $\sqrt{4} = \sqrt[2]{4}$

$= \sqrt[2]{2 \times 2}$

$= 2$

$n = 2$ even

↓

$a = 4$

↓

$\sqrt[2]{4}$ is defined

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② $\sqrt{-4} = \sqrt[2]{-4}$

↓

undefined

or it is not real number

$n = 2$ even

↓

$a = -4 < 0$

↓

$\sqrt{-4}$ undefined

$\sqrt{-4}$ undefined

③ $\sqrt[4]{-16}$

$= \sqrt[4]{-2 \times -2 \times -2 \times -2}$

$= -2$

$n = 4$ even

$a = -16 < 0$

$\sqrt[4]{-16}$ undefined

(not real number)

Remark If $a = 0 \Rightarrow \sqrt[n]{a} = 0$ for all n

Exp $\sqrt[2]{0} = 0$ → since $a = 0$

$\sqrt[3]{0} = 0$

$\sqrt[4]{\frac{0}{5}} = \sqrt[4]{0} = 0$

Radical   rational Exponent

نسبي   الأس / القوة

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$\frac{m}{n} \in Q = \{Z, \frac{1}{2}, \frac{-3}{2}, \dots\}$

$m, n \in Z = \{0, \pm 1, \pm 2, \dots\}$

$n \neq 0$

Exp ① $16^{\frac{3}{4}} = \sqrt[4]{16^3} = \sqrt[4]{16 \times 16 \times 16}$

$\sqrt[4]{16^3}$

Write the following expressions in radical form and simplify

$= \sqrt[4]{2\times2\times2\times2 \times 2\times2\times2\times2 \times 2\times2\times2\times2}$

$= 2\sqrt[4]{2\times2\times2\times2 \times 2\times2\times2\times2}$

$= 2\times2\sqrt[4]{2\times2\times2\times2}$

$= 2\times2\times2$

$= 8$

Or

$16^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}} = 2^{4\times\frac{3}{4}} = 2^3 = 2\times2\times2 = 8$

m

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Exp ① $a^{\frac{1}{n}} = \sqrt[n]{a^1}$

② $\left(a^{\frac{1}{n}}\right)^n = a^{\frac{1}{n} \times n} = a^1 = a$

$a^{\frac{n}{n}} = \sqrt[n]{a^n} = a$

③ $\sqrt[3]{5^3} = 5^{\frac{3}{3}} = 5^1 = 5$

④ $\sqrt[3]{-1} = \sqrt[3]{(-1)^3} = (-1)^{\frac{3}{3}} = -1$

or $= \sqrt[3]{-1 \times -1 \times -1}$

$= -1$

Exp Wrik the following expressions in radical form and simply

① $x^{-\frac{5}{7}} = \frac{1}{x^{\frac{5}{7}}} = \frac{1}{\sqrt[7]{x^5}}$

② $(2y)^{-\frac{5}{4}} = \frac{1}{(2y)^{\frac{5}{4}}} = \frac{1}{\sqrt[4]{(2y)^5}}$

$= \frac{1}{\sqrt[4]{2y \times 2y \times 2y \times 2y \times 2y}}$

$= \frac{1}{2y\sqrt[4]{2y}}$

Exp write the following expressions without radical signs

(1) $\sqrt{z^7} = z^{\frac{7}{2}}$ $\quad \sqrt{z^7} = \sqrt[2]{z^7}$

(2) $\dfrac{3}{\sqrt[5]{f^6}} = \dfrac{3}{f^{\frac{6}{5}}} = 3 f^{-\frac{6}{5}}$

(3) $\sqrt[8]{(xy)^8} = (xy)^{\frac{8}{8}} = xy$

Exp Simplify the following expression

(1) $3^{\frac{1}{3}} \times 3^{\frac{1}{2}} = 3^{\frac{1}{3}\times\frac{2}{2} + \frac{1}{2}\times\frac{3}{3}} = 3^{\frac{1}{5}}$

1) $3 \times 3 = 3$

$= 3$

$\frac{2}{6} + \frac{3}{6}$

$= 3$

$= 3^{\frac{5}{6}} = \sqrt[6]{3^5}$

2) $\frac{2^{\frac{4}{5}}}{2^{\frac{5}{4}}} \div = 1$

a) 1

b) 0

c) -1

$\frac{4}{5} - \frac{5}{4}$

$= 2$ $= 2$ 9

$= 2^{0} = 1$

$\frac{4-5}{5-4}$ $\frac{-1}{1}$ -1

$= 2$ $= 2$ $= 2$

$= 2$ $= 2^{1}$

$\frac{4}{5} - \frac{5}{4}$

$\frac{4 \times 4}{5 \times 4} - \frac{5 \times 5}{4 \times 5}$

$= 2$

-9

$= 2$

$$\frac{16}{20} - \frac{25}{20}$$

$$= 2$$

$$= 2^{\frac{16-25}{20}} = 2^{\frac{-9}{20}} = \frac{1}{2^{\frac{9}{20}}} = \frac{1}{\sqrt[20]{2^9}}$$