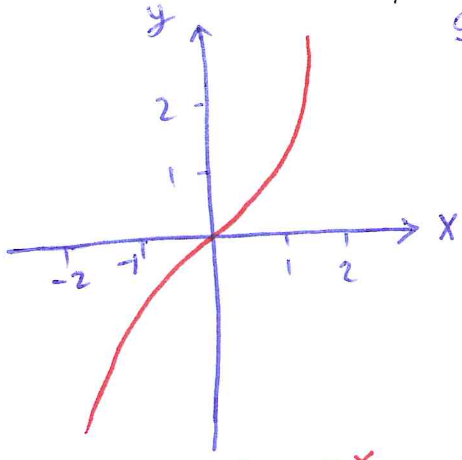
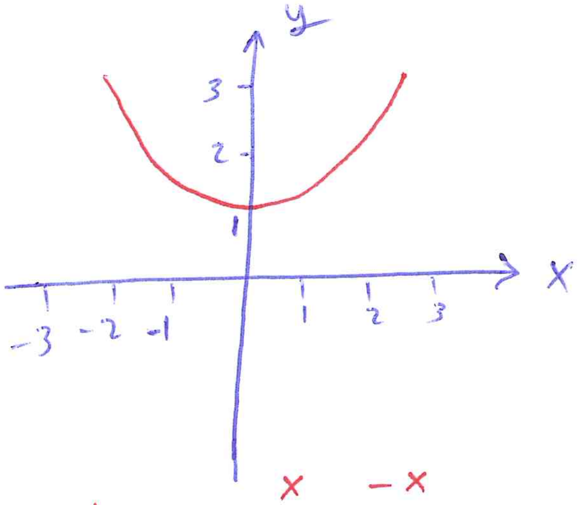
$\boxed{7.7}$ Hyperbolic Functions

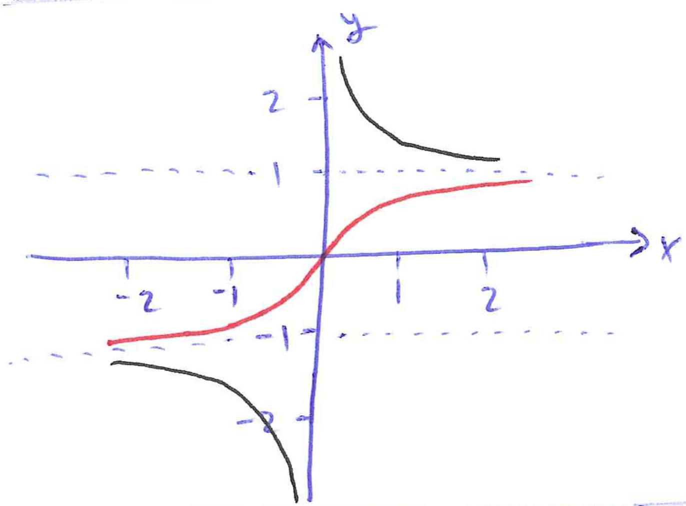
القطعي



$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

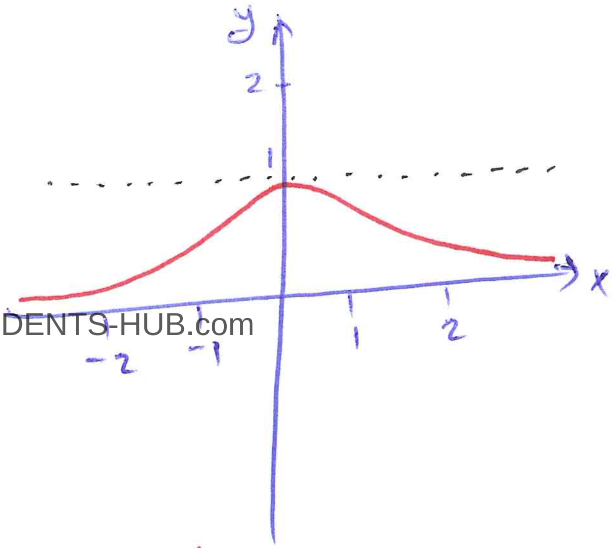


$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

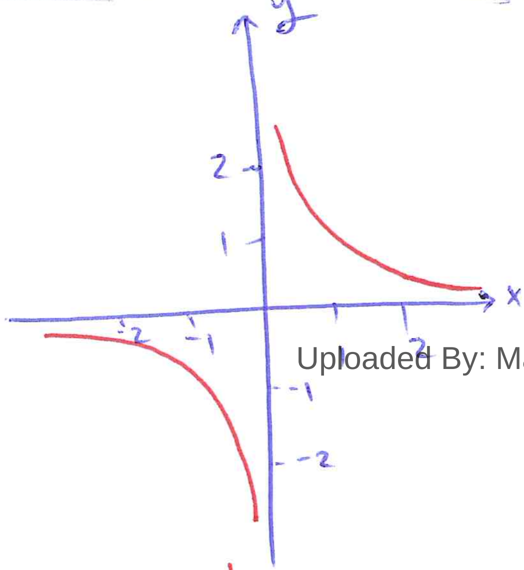


$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$



$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^{x} + e^{-x}}$$



$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^{x} - e^{-x}}$$

* Identities For Hyperbolic Functions

- $\cosh^2 x - \sinh^2 x = 1$ $\Rightarrow$ Proof $\left(\frac{e^{x}+e^{-x}}{2}\right)^2 - \left(\frac{e^{x}-e^{-x}}{2}\right)^2 =$ $\frac{e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}}{4} =$ $\frac{4}{4} = 1$
- $\sinh 2x = 2\sinh x \cosh x$
- $\cosh 2x = \cosh^2 x + \sinh^2 x$
  $= 2\cosh^2 x - 1$
  $= 2\sinh^2 x + 1$
- $\tanh^2 x + \operatorname{sech}^2 x = 1$
- $\coth^2 x - \operatorname{csch}^2 x = 1$

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* Derivatives of Hyperbolic Functions

- $\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$
- $\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$ $\Rightarrow$ Proof

  $\cosh u = \frac{e^{u}+e^{-u}}{2} \Rightarrow$

  $\frac{d}{dx}(\cosh u) = \left(\frac{e^{u}-e^{-u}}{2}\right)\frac{du}{dx}$

  $= \sinh u \frac{du}{dx}$
- $\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$
- $\frac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \frac{du}{dx}$ $\quad (\cot u)' = -\csc^2 u$
- $\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx}$
- $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} u \coth u \frac{du}{dx}$

Exp. Find y' for ① $y = \ln(\sinh x)$ $\Rightarrow y' = \frac{\cosh x}{\sinh x} = \coth x$

② $y = 4\cosh\frac{x}{2}$ $\Rightarrow y' = 2\sinh\frac{x}{2}$

* Integrals of Hyperbolic Functions

- $\int \sinh u \, du = \cosh u + c$
- $\int \cosh u \, du = \sinh u + C$
- $\int \operatorname{sech}^2 u \, du = \tanh u + c$
- $\int \operatorname{csch}^2 u \, du = -\coth u + c$
- $\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$
- $\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + c$

Exp: ① $\int \sinh 2x \, dx = \frac{1}{2}\int \sinh u \, du = \frac{1}{2}\cosh 2x + c$

$u = 2x$
$du = 2\,dx$

② $\int \coth x \, dx = \int \frac{\cosh x}{\sinh x} \, dx$

$= \int \frac{du}{u} = \ln|u| + c = \ln|\sinh x| + c$

$u = \sinh x$
$du = \cosh x \, dx$

* Inverse Hyperbolic Functions: