Chapter 7.1, Problem 14E

Problem

Let *J*5 = {0, 1, 2, 3, 4}, and define functions *h: J*5→*J*5 and *k: J*5→ *J*5 as follows: For each $x \in J5$,

h(*x*) = (*x* + 3)3 *mod* 5 and *k*(*x*) = (*x*3 + 4*x*2 + 2*x* + 2) *mod* 5.

Is *h* = *k*? Explain.

Step-by-step solution

Step 1 of 3

Consider the functions, $h: J_5 \to J_5$ and $k: J_5 \to J_5$.

Here, $J_5 = \{0,1,2,3,4\}$.

Define $h(x)$ and $k(x)$ for each $x \in J_5$ as follows:

$$h(x) = (x+3)^3 \bmod 5$$
$$k(x) = (x^3 + 4x^2 + 2x + 2) \bmod 5.$$

The objective is to check whether $h = k$ or not.

Step 2 of 3

x	$h(x) = (x+3)^3 \bmod 5$	$k(x) = (x^3 + 4x^2 + 2x + 2) \bmod 5$
0	$h(0) = 3^3 \bmod 5$ $= 2$	$k(0) = 2 \bmod 5$ $= 2$
1	$h(1) = 64 \bmod 5$ $= 4$	$k(1) = 9 \bmod 5$ $= 4$
2	$h(2) = 125 \bmod 5$ $= 0$	$k(2) = (8+16+4+2) \bmod 5$ $= 30 \bmod 5$ $= 0$
3	$h(3) = 216 \bmod 5$ $= 1$	$k(3) = (27+36+6+2) \bmod 5$ $= 71 \bmod 5$ $= 1$
4	$h(4) = 343 \bmod 5$ $= 3$	$k(4) = (64+64+8+2) \bmod 5$ $= 138 \bmod 5$ $= 3$

Step 3 of 3

The above table shows that,

$\boxed{h(x)=k(x)}$ for every $x \in \{0,1,2,3,4\}$.