STAT2361

ملاحظات المحاضرات

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BZU-HUB

Chapter 8

ملاحظة: المحاضرات من شرح الدكتور محمد مضية

الشابتر مهم جدًا

Chapter 8: Confidence Intervals

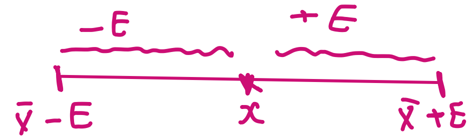
Interval Estimation ← Single Value بدل ال

Range of Values

* Interval Estimation for μ → Sample, → probability

point estimate ± Margin of error

$\bar{x} \pm E$



* Each Interval should be construct with a known probability (given)

$P(\bar{X} \pm E) = \text{known}$

* The known probability is called confidence level

* Confidence level $(100-\alpha)100\%$, $\alpha \equiv$ Type of error (ch.9)

الهامش

* Common confidence levels:

90% ⟶ $\alpha = 10\%$

95% ⟶ $\alpha = 5\%$

99% ⟶ $\alpha = 1\%$

* Interval for μ:

Case 1: σ- Known Case (Z-table)

case 2: σ- Unknown Case (t-table)

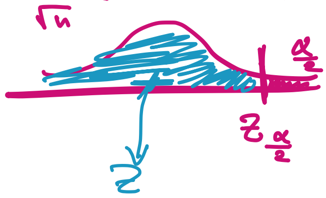
* Case 1: σ- Known Case (population S.D is known)

The $(100-\alpha)100\%$ Confidence Interval for μ is

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

$E = Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$

$\frac{\sigma}{\sqrt{n}}$ = standard error of the mean



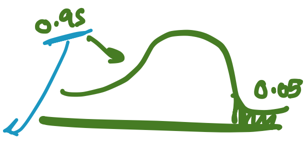
$Z_{0.05}$

* Construct 90% Confidence Interval for μ

1. $\alpha = 10\%$
2. $\frac{\alpha}{2} = 5\% \longrightarrow Z_{0.05}$
3. $P(Z > 0.05)$
4. $Z_{0.05} = 1.645$

$Z_{0.025} = 1.96$

$Z_{0.005} = 2.576$



* Example: $n = 50$, $\sigma = 6$ (σ → Known), $\bar{x} = 32$

Construct 90%, 95%, 99% (Intervals estimate, Confidence Level)

$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \rightarrow 32 \pm Z_{\frac{\alpha}{2}} \frac{6}{\sqrt{50}}$

$= 32 \pm (0.85)\, Z_{\frac{\alpha}{2}}$

① 90% : $32 \pm 0.85\,(Z_{0.05})$

$= 32 \pm (0.85)(1.645)$

$= 32 \pm 1.4 \rightarrow E$

$30.6 \;,\; 33.4$

There is a probability (confidence) of (0.9) that the population mean is between 30.6 and 33.4

② 95% : 32 ± 1.67

$30.33 \;,\; 33.67$

③ 99% : 32 ± 2.2

$29.8 \;,\; 34.2$

As confidence level increase error increasing

* إذا بدنا نقلل ال E بنزيد ال n

* Case 2: σ-Unknown case : No information about σ

⇓

S (sample S.D) is a point estimate for σ → t-Distribution

$\bar{X} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$ 　　 t: bell-shaped

degrees of freedom = $n-1$

*Example: $n = 36$, $\bar{x} = 100$, $S.D = 12$ (for a sample)

$$\bar{X} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

Construct 95% and 99%

$100 \pm t_{\frac{\alpha}{2}} (2)$ $df = 36 - 1 = 35$

من الجدول

$95\% \Rightarrow \frac{\alpha}{2} = \frac{5}{2} \Rightarrow t_{0.025} = 2.030$

$99 \Rightarrow t_{0.005} = 2.724$

$100 \pm (2)(2.030) = 100 \pm 4.06$

$100 \pm (2)(2.724) = 100 \pm 5.45$

$df \uparrow \rightarrow t \approx$ zero

8.4: Proportion (Percentage)

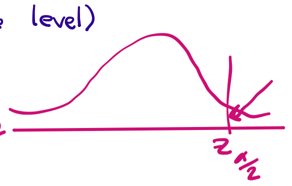
- $\bar{p} \equiv$ Sample proportion $\Rightarrow$ Statistic
- $p \equiv$ population proportion $\Rightarrow$ parameter
- $\bar{p} = \frac{x}{n}$, $p = \frac{x}{N}$
- $\bar{p}$ sample propotion is a point estimate for p = population proportion
- $\bar{p} \rightarrow p$

Confidence interval for the population proportion (confidence level)

- point estimate $\pm$ margin of error

$\bar{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ → Standard error of the population $\alpha/2$

90%
95%
99%



- $E = t_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

Example 1

A simple random sample of 400 individuals provides 100 Yes responses. x

Yes
No
wo pi

1. What is the point estimate of the proportion of the population that would provide **Yes** responses?

$\bar{p}$ is a point estimate of p.

$\bar{p} = \frac{x}{n} = \frac{100}{400} = 0.25$

2. What is your estimate of the standard error of the proportion?

S.E of the proportion

$= \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{(0.25)(0.75)}{400}} = 0.0216$

$= 0.02$

3. Compute the 95% confidence interval for the population proportion.

$\bar{p} \pm z\,(S.E) = 0.25 \pm (1.96)(0.02$ → Error

$= 0.25 \pm 0.04$

Interval : 0.21 to 0.29 ??

There is a probability (confidence) of 0.95 that the proportion of all yes responses is between 21% to 29%.

Example 2

A simple random sample of 900 elements generates a sample proportion $\bar{p} = 0.45$.

n

1. Provide a 95% confidence interval for the population proportion.

0.01

$S.E = \sqrt{\dfrac{(0.45)(0.55)}{900}} = 0.02$

$\bar{p} \pm z_{\alpha/2}\,(S.E)$

$0.45 \pm (1.96)(0.02) = 0.45 \pm 0.04$

interval 0.41 to 0.49

2. Provide a 99% confidence interval for the population proportion.

$0.45 \pm (2.576)(0.02) = 0.45 \pm 0.05$

Interval: 0.40 to 0.50

Example

A company is planning to market a new offer. However, before marketing this offer, the company wants to find what percentage of customers will like it. The company's research department select a random of 500 customers and asked them to evaluate this offer. Of these 500 customers, 290 said they liked it. Find with a 99% confidence level, what percentage of customers will like this offer?

Solution

$$\bar{p} = \frac{x}{n} = \frac{290}{500} = 0.58$$

$$\bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\Rightarrow 0.58 \pm (2.575)(0.0221)$$

$$\Rightarrow 0.58 \pm .057$$

$$\Rightarrow 0.523 \textbf{ to } 0.637$$

Thus, we can state with 99% confidence that 52.3% to 63.7% all customers will like this offer.

* Planning Value for proportion:

$$p: \left(E = z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right)^2$$

$$E^2 n = z^2 * \bar{p}(1-\bar{p})$$

$$\Rightarrow n = \frac{z^2 * \bar{p}^*(1-\bar{p}^*)}{E^2}$$ → Planning value for p^*

z, E: Given

p^*: ??

SAMPLE SIZE FOR AN INTERVAL ESTIMATE OF A POPULATION PROPORTION

$$n = \frac{(z_{\alpha/2})^2 p^*(1-p^*)}{E^2} \quad \textbf{(8.7)}$$

In practice, the planning value p^* can be chosen by one of the following procedures.

1. Use the sample proportion from a previous sample of the same or similar units.
2. Use a pilot study to select a preliminary sample. The sample proportion from this sample can be used as the planning value, p^*.
3. Use judgment or a "best guess" for the value of p^*.
4. If none of the preceding alternatives apply, use a planning value of $p^* = .50$.

p^*
1) History
2) Pilot study
3) Judgment
4) $p^* = \frac{1}{2}$

Example

In a survey, the planning value for the population proportion is $p^* = 0.55$. How large a sample should be taken to provide a 90% confidence interval with a margin of error of 0.02? → E

$n = ?$

$z = 1.645$

$$n = \frac{z^2 (p^*(1-p^*))}{E^2} = \frac{(1.645)^2 (0.55)(0.45)}{(0.02)^2}$$

$$= 1674.35 \Rightarrow 1675$$

Example

At 95% confidence, how large a sample should be taken to obtain a margin of error of 0.03 for the estimation of a population proportion? Assume that past data are not available for developing a planning value for p*.

$E = 0.03$

$z = 1.96$

Let $p^* = \frac{1}{2}$

$$n = \frac{(1.96)^2 (0.5)^2}{(0.03)^2}$$

$$= \boxed{1068}$$