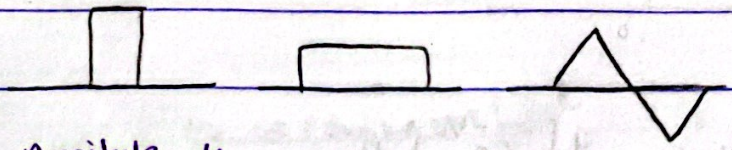
Intro to Signal and system:-

→ How do we measure signal size?



Ampitute
time
Area

} not good enough

⇒ power and energy

$$\to \int_{-\infty}^{\infty} |g(t)|^2 dt$$

$$\to P_{avg} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 dt .$$

Speedral case "periodic"

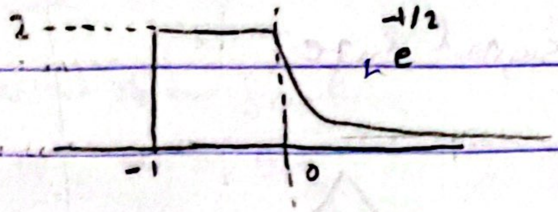
$$= \lim_{T\to\infty} \frac{1}{T_0} \int^{T} |g(t)|^2 dt .$$

- P_{avg} :- time averge of the Energy.
  the main square" $\sqrt{P_{avg}}$ = rms"

- periodic
  random
  ] power.
  limited + pandeel
  deterministic
  } Energye

Energy . "في الطبيعية Signal"

Ex:-

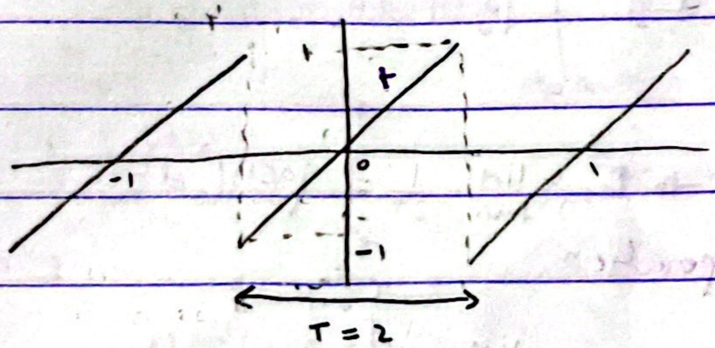


$$\int_{-\infty}^{-1} 0\,dt + \int_{-1}^{0} 4\,dt + 4\int_{0}^{\infty} (e^{-t/2})^2\,dt$$

$$0 + 4(0--1) + 4\left[-e^{-t}\Big|_0^{\infty}\right]$$

$$4 + 4 = 8\,J$$

Ex:-



$$P_{avg} = \frac{1}{T}\int_0^T |g(t)|^2 dt$$

$$= \frac{1}{2}\int_{-1}^{1} t^2\,dt$$

$$\frac{1}{2}\left[\frac{1}{3}t^3\Big|_{-1}^{1}\right] = \frac{1}{2}\cdot\frac{1}{3}[1+1]$$

$$= \frac{1}{3}\text{ watt}.$$

Ex:- $g(t) = c\cos(\omega_o t + \theta)$, $-\infty < t < \infty$

periodic with period $T_o = \frac{2\pi}{\omega_o}$, $\omega_o = 2\pi f$

$$P_{av} = \frac{1}{T}\int_0^T c^2\cos^2(\omega_o t + \theta)\,dt$$

, $\omega_o = \frac{2\pi}{T_o}$

so $T_o = \frac{2\pi}{\omega_o}$.

$$= \frac{c^2}{2T}\int_0^T \left[1 + \cos(2\omega_o t + 2\theta)\right] dt$$

$$= \frac{c^2}{2T}\left[\int_0^T dt + \cancel{\int_0^T \cos(2\omega_o t + 2\theta)\,dt}^{\,0}\right.$$. "Altirnaligu.

$$= \frac{c^2}{2\cancel{T}} \cdot \cancel{T} = \frac{c^2}{2}$$

$rms = \frac{c}{\sqrt{2}}$.

proofing.

$\Rightarrow$

$$\frac{c^2}{2T}\int_0^T \cos(2\omega_o t + 2\theta)\,dt .$$

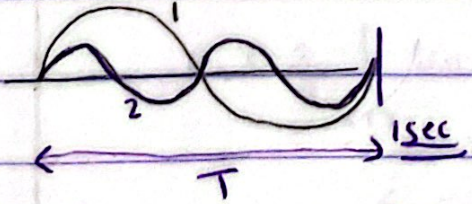
$\Rightarrow$ $\sin(2\pi t) \rightarrow f = 1,\ T = 1$

$\sin(2 \cdot 2\pi t) \rightarrow f = 2,\ T = \frac{1}{2}$

$$\frac{c^2}{2T} \cdot \frac{1}{2\omega_o}\left[\sin(2\omega_o t + 2\theta)\right]_0^T .$$

$$\overset{0}{\sin(\cancel{2\omega_o T}^{2\pi} + 2\theta)} - \sin(2\theta) .$$

$$\sin(2\theta) - \sin(2\theta) = 0 .$$



• يوجد دورتين من تاني موجة داخل الاولى وبالتالي تكاملها

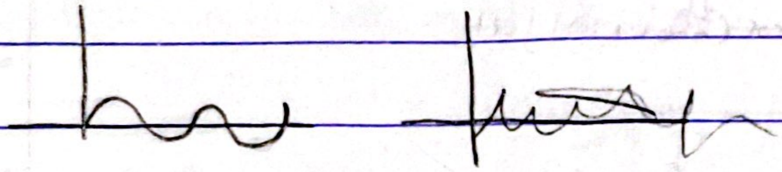
يساوي 0 .

classification ad signal

1) Analog (continuos.) vs Digital → discrite.

2) periodis vs aperiodic $f(t)$ is periodic iff $f(t+T) = f(t)$.

3) determinitic vs Random

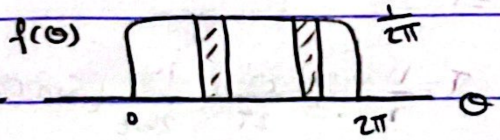


$\Rightarrow$ $x(t) = A\cos(2\pi f_c t + \theta)$

$\{A, f_c, \theta$ are constants$\} \Rightarrow$ deterministc.

$x(t)$ is Random $\left\{ \begin{array}{l} A, f_c \text{ are constant} \\ \theta : \text{random variabel, uniform } [0, 2\pi] \end{array} \right\}$

pdf:-



$y(t) = \underset{\substack{\downarrow \\ \text{deteminsite}}}{x(t)} + n(t) \rightarrow$ random

so, $y(t)$ is Random.

4) Energy vs power signal.

- if E is Bonded ($E < \infty$) and ($P_{avg} = 0$)
  then, the signal is called "Energy signal".

- if P_{avg} is Bonded ($P_{avg} < \infty$) and ($E = \infty$)
  then, we have "power signal".

- Some signal are neither energy singnal nor power signal

Fourier series:-

- For periodic signal f(t), with freq $w_0 = 2\pi f_0 = \frac{2\pi}{T}$
- three Forms:-

[I] Trigometric Form.

$$f(t) = a_0 + \sum_{n=1}^{\infty} \textcircled{a_n} \cos(\underbrace{nw_0t}) + \textcircled{b_n} \sin(\underbrace{nw_0t})$$

integer multiple.

$$a_n = \frac{2}{T} \int_{\langle T \rangle} f(t) \cos(nw_0t)\, dt.$$

$$b_n = \frac{2}{T} \int_{\langle T \rangle} f(t) \sin(nw_0t)\, dt.$$

$$a_0 = \frac{1}{T} \int_{\langle T \rangle} f(t)\, dt.$$

2 complex FS (polar FS)

based on $e^{j\alpha} = \cos\alpha + j\sin\alpha$

$$\Rightarrow f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} .$$

when C_n is complex!

$$\boxed{C_n = \frac{1}{T}\int_{<T>} f(t) e^{-jn\omega_0 t} dt}$$

$$= \frac{1}{2}a_n - j\frac{1}{2}b_n .$$

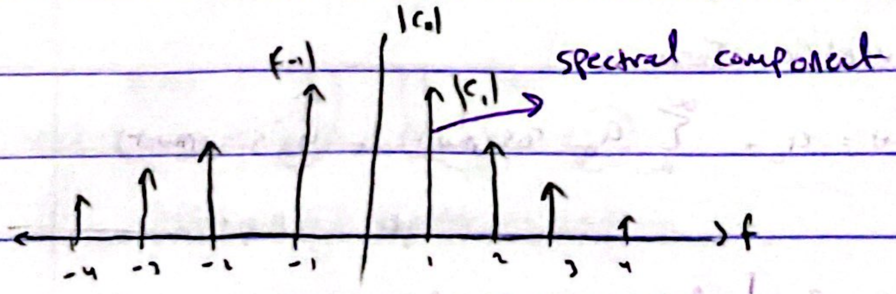
if f(t) is real signal then $C_{-n} = C_n^*$

$C_n = |C_n| \angle \theta_n$

for real signal: $C_{-n} = C_n^*$

$|C_{-n}| = |C_n|$ even symm.

we obtain Double-Sided Amp. spectrum.



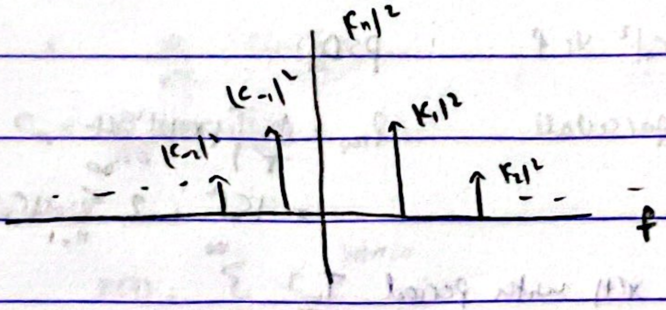
$\theta_{-n} = -\theta_n$ odd symm

3 Compact FS.

$$f(t) = c_0 + \sum_{n=1}^{\infty} 2\boxed{|c_n|}\cos(n\omega_0 t + \boxed{\theta_n})$$

power spectral Density (PSD).

* the plot of $|C_n|^2$ vs f is called PSD.
* The PSD displays the power content of each spectral component $|C_n|$ of a signal.



Parsevals theorem:

$$P_{avg} = \frac{1}{T}\int_{\langle T\rangle} |g(t)|^2\, dt = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$= |C_0|^2 + 2\sum_{n=1}^{\infty} |C_n|^2.$$

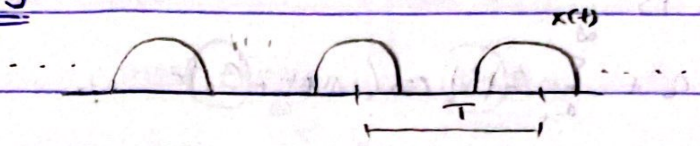
- $\frac{1}{T}\int g(t)\, g^*(t)\, dt.$

$\frac{1}{T}\int \sum c_n^* e^{-jn\omega_0 t} g(t)$

$\sum c_n^* \frac{1}{T}\int g(t) e^{-jn\omega_0 t}\, dt.$

$\sum c_n^* c_n c_n = \sum |C_n|^2.$

FS



$x(t) = \sum c_n e^{jn2\pi t}$

complex coefficent $|c_n| \angle \theta_n$

$|c_n|$ vs f : Ampetud spectrum.

$|\theta_n|$ vs f : phase "

$|c_n|^2$ vs f : PSD

parsevall : $P_{av} = \frac{1}{T}\int |x(t)|^2 dt$

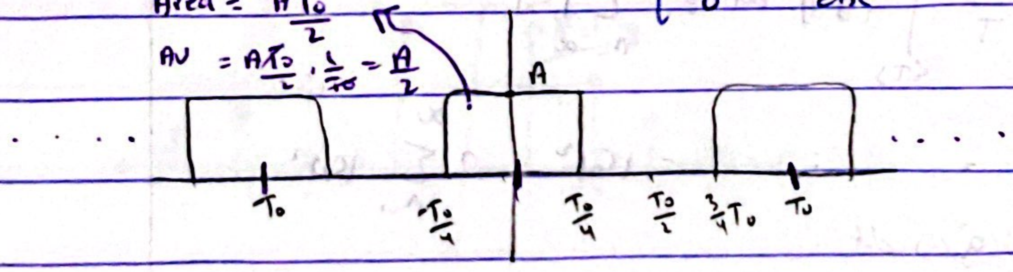
$= |c_0|^2 + 2\sum_{n=1}^{\infty} |c_n|^2.$

Example:- Periodic Signal x(t) with period T_0

x(t) for one period : $\begin{cases} A & -\frac{T_0}{4} < t < \frac{T_0}{4} \\ 0 & \text{else} \end{cases}$

Area $= \frac{AT_0}{2}$

AV $= \frac{AT_0}{2} \cdot \frac{1}{T_0} = \frac{A}{2}$



1) Find DC component "Averge value".

$$C_0 = \frac{1}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} A \, dt = \frac{A}{T_0}\left[\frac{T_0}{4} - -\frac{T_0}{4}\right]$$

$$= \frac{A}{T_0} \cdot \frac{T_0}{2} = \underline{\underline{\frac{A}{2}}}.$$

2) Find c_n

$$c_n = \frac{1}{T_0}\int_{\langle T_0\rangle} x(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} A e^{-jn\omega_0 t}\, dt = \frac{A}{T_0}\,\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Bigg]_{-T_0/4}^{T_0/4}$$

$$= \frac{\ominus A}{jn\underset{2\pi}{\underline{\underline{\omega_0 T_0}}}}\left[e^{-jn\omega_0\frac{T_0}{4}} - e^{+jn\omega_0\frac{T_0}{4}}\right]$$

$$= \frac{+A}{jn2\pi}\left[\frac{e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}}}{2j}\right]$$

$$= \frac{A}{n\pi}\sin\left(\frac{\pi}{2}n\right)$$

$$\Longrightarrow c_n = \frac{A}{n\pi}\sin\left(\frac{\pi}{2}n\right)$$

remember:

$$x(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$\frac{A}{2} + \frac{A}{\pi}\sum_{\substack{-\infty \\ n\neq 0}}^{\infty}\frac{1}{n}\sin\left(\frac{\pi}{2}n\right)e^{jn\omega_0 t}$$

$c_0 = A/2$

$c_1 = \frac{A}{\pi}$

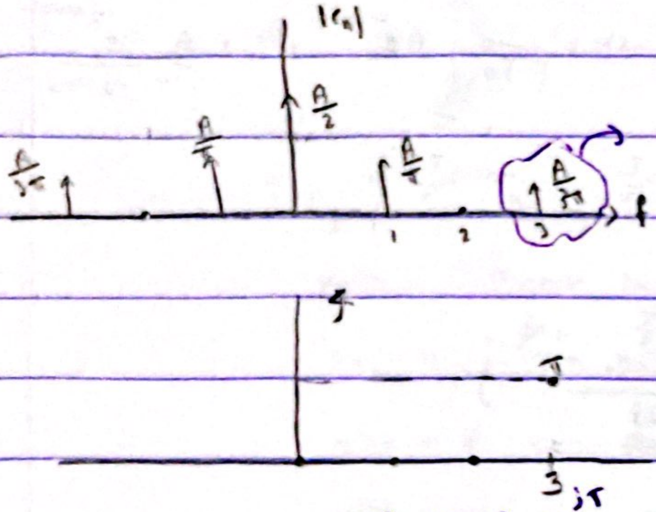
$c_2 = c_4 = c_6 = \dots = 0$

$c_3 = \frac{-A}{3\pi}$

$c_5 = \frac{A}{5\pi}$

$$c_n = \begin{cases} \frac{A}{2}, & n=0 \\ 0, & n \text{ even} \\ \frac{A}{n\pi}, & 1, 5, \dots \\ \frac{-A}{n\pi}, & 3, 7, \dots \end{cases}$$

3) plot $|C_n|$ vs f .

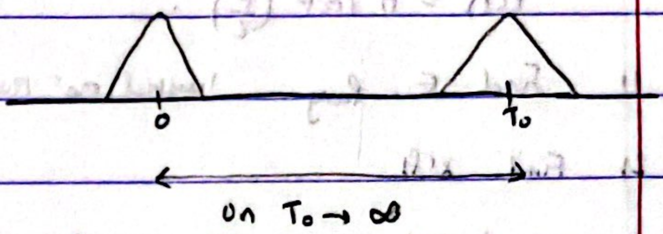


هي بالأصل $-\frac{A}{3\pi}$ ، ودكتورنا
ومع ال نفسها + وتكون
بجهة الـ $\underline{\underline{\pi}}$ ، بس حكى
للأرقام الموجبة ال دكتورنا
رسمها $\underline{\underline{\theta}}$.

$$\frac{-A}{3\pi} = \frac{A}{3\pi}e^{j\pi} = \frac{A}{3\pi}\angle\pi .$$

Fourier Transform:

for non periodic signal



x(t) =>

$$x(t) = \int_{-\infty}^{\infty} x(f)\, e^{j2\pi ft}\, df$$ "Inverse FT."

$$x(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$ "FT"

Remark:-

- x(f) is complex

  $x(f) = |x(f)| \angle \theta(f)$

- energy signal $E = \int |x(t)|^2 dt < \infty$

  have FT.

- $|x(f)|$ is continuous Amplitude spectrum.

  $\theta(f)$ is " phase "

- Energy Spectral Density (ESD)

  $|X(f)|^2$ vs f.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(f)|^2 df.$$ Rayleigh relation "parsevals"

Example: Rectangular pulse.

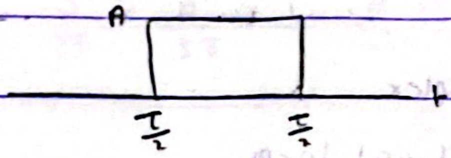
$x(t) = A \, \text{rect}\left(\frac{t}{\tau}\right)$.

1) Find E, P_{avg}. limitied and Bunded so its E, $P_{avg} = 0$

2) Find $X(f)$

3) plot $|X(f)|$

$$x(t) = \begin{cases} A, & -\frac{1}{2} < \frac{t}{\tau} < \frac{1}{2} \\ 0, & \text{else} \end{cases} = \begin{cases} A, & \frac{-\tau}{2} < t < \frac{\tau}{2} \\ 0, & \text{else} \end{cases}$$



2) $$X(f) = \int_{-\infty}^{\infty} A e^{-j2\pi f t} \, dt = A \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j2\pi f t} \, dt.$$

$$= A \left. \frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-\tau/2}^{\tau/2}$$

$$= \frac{-A}{j2\pi f} \left[e^{-j\pi f \tau} - e^{j\pi f \tau} \right].$$

$$= \frac{A}{\pi f} \left[\frac{e^{j\pi f \tau} - e^{-j\pi f \tau}}{2j} \right]$$

$$= \frac{\textcircled{\tau} A}{\textcircled{\tau} \pi f} (\sin \pi f \tau)$$

$$= A\tau \, \text{sinc}(f\tau) \qquad \text{sinc } x = \frac{\sin x\pi}{x\pi}$$

3) $|X(f)| = A\tau \frac{\sin \pi f \tau}{\pi f \tau}$

$|X(f=0)| = A\tau$

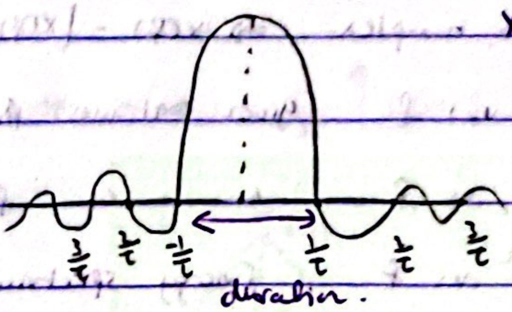
$|X(f)| = 0 ?$

$\sin(\pi f \tau) = 0.$

$\pi f \tau = k\pi$, $k = \pm 1, \pm 2, \ldots$

$f = \frac{k}{\tau}$

X(f) : not | |



AS $\tau \uparrow$

x(t) duration become wide

x(f) " " narrowed.

$$\begin{tikzcd} x(t) \arrow[rr, bend left, "X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt"] & & X(f) \arrow[ll, bend left, "x(t) = \int_{-\infty} X(f) e^{j2\pi ft} dt"] \end{tikzcd}$$

Remark:-

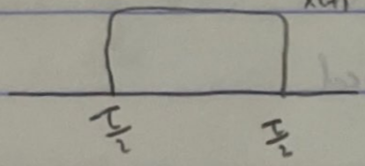
- Energy signals are FT
- $X(f)$ is complex $\Rightarrow X(f) = |X(f)| \angle \theta(f)$

$X(f)$ vs f given continuous Amplitude spectrum.

$\theta(f)$ vs f " " phase " .

$|X(f)|^2$ vs f Energy spectrum Density.

Example:-



$x(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)$

$X(f) = A\tau \ \text{sinc}(f\tau)$

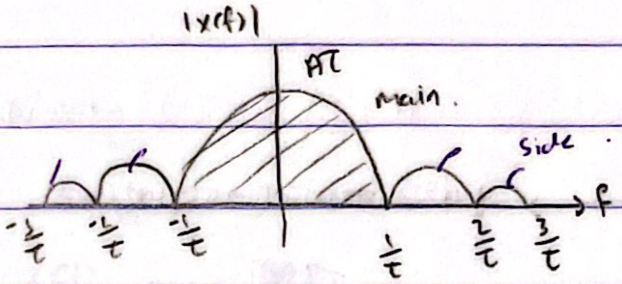
plot $|X(f)|$

$|X(f=0)| = \frac{0}{0} \Rightarrow$ lopetal.

$A\tau \ \cancel{\tau} \frac{\cos(\pi f \tau)}{\cancel{t}\pi}\Big|_{f=0} = A\tau$.

zero crossing:

$f = \frac{k}{\tau}$, $k = \pm 1, \pm 2 \ldots$



• Magnitude spectrum.

← تتركز ال energy بال main loop ال Main ويقل تركيزها بال Side.

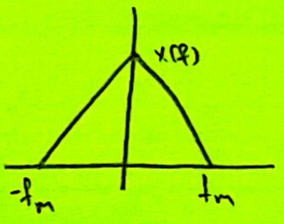
← نتحكم بال T من أجل التعريف ال band width.

إذا صغر T صغر ↑ يقل ال BW ويزداد ال BW بال ES.

Properties of FT:-

1) Linearity.

$$F[ax_1(t) + bx_2(t)]$$
$$= aX_1(f) + bX_2(t)$$



Bandwidth (B.W) = max positive freq. component in the signal spectrum

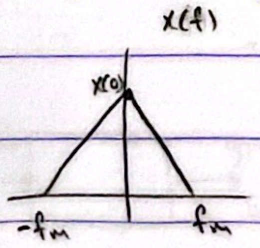
$\Rightarrow$ B.W = f_m for base Band signal.

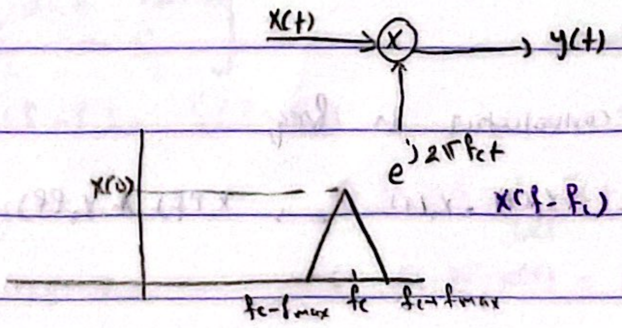
2) time scaling.

$$x(t) \longleftrightarrow X(f).$$
$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right).$$

3) Freq. shifting.

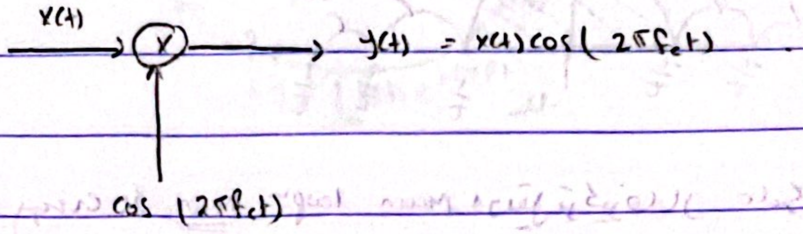
$X(f)$ $\qquad Y(f) = X(f - f_c)$.





$y(t) = x(t)e^{j2\pi ft}$

4) Modulation.

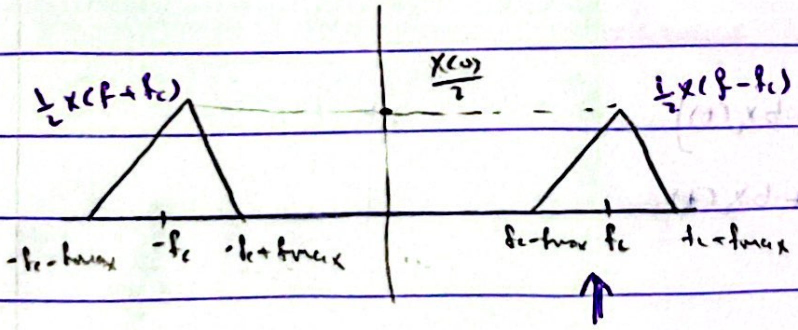


$$y(t) = x(t)\cos(2\pi f_c t)$$

$$= \frac{x(t)}{2} e^{\oplus j2\pi f_c t} + \frac{x(t)}{2} e^{\ominus j2\pi f_c t}$$

$$Y(f) = F[y(t)]$$ عكس بعض

$$= \frac{1}{2}X(f \ominus f_c) + \frac{1}{2}X(f \oplus f_c)$$



passband signal

$B.W = 2f_m$.

5) convolution in time.

$$x_1(t) * x_2(t) \longleftrightarrow X_1(f) \cdot X_2(f)$$

6) convolution in freq

$$x_1(t) \cdot x_2(t) \longleftrightarrow X_1(f) * X_2(f)$$

7) Duality.

$\text{rect}\left(\frac{t}{\tau}\right) \quad \tau \,\text{sinc}(\tau f)$

$x(t) \longleftrightarrow X(f)$

$X(t) \longleftrightarrow x(-f)$

$\tau \,\text{sinc}(\tau t) \qquad \text{rect}\left(\frac{-f}{\tau}\right) = \text{rect}\left(\frac{f}{\tau}\right)$ ∵ rect, even^n.

8) FT for periodic signal.

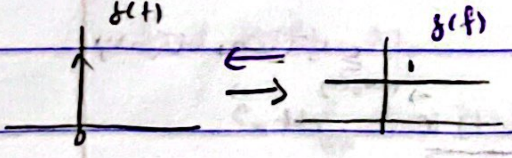
$x(t)$ is periodic, then $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$

$x(f) = F\left[\sum c_n e^{jn\omega_0 t}\right] = \sum c_n F\left[e^{j2\pi n f_0 t}\right]$

$\Rightarrow \; = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_0)$

7)

$F[\delta(t)] = 1$

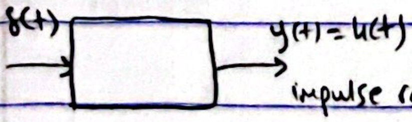


ex:- 1) $x(t) = 1 \longleftrightarrow x(f) = \delta(f)$.

2) $1 . e^{j2\pi f_c t} \longleftrightarrow \delta(f - f_c)$.

3) $F\{\cos 2\pi f_c t\} = F\left[\frac{e^{j2\pi f_c t}}{2} + \frac{e^{-j2\pi f_c t}}{2}\right]$

$= \frac{1}{2}\delta(f - f_c) + \frac{1}{2}\delta(f + f_c)$.



impulse response.

delta function:-

1) $\delta(t) = \begin{cases} \infty & t=0 \\ 0 & \text{else} \end{cases}$

it's even func

$\delta(t) = \delta(-t)$ 2) $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

3) $g(t)\,\delta(t - t_0) = g(t_0)\,\delta(t - t_0)$

$\int g(t)\,\delta(t - t_0)\, dt = \int g(t_0)\,\delta(t - t_0)\, dt$

$= g(t_0) \int \delta(t - t_0)\, dt$

$= g(t_0)$

4) $\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$.

5) $\delta(t) \circledast x(t) = x(t)$

$\delta(t - t_0) \circledast x(t) = x(t - t_0)$

6) $\delta(t) = \frac{d}{dt} u(t)$

$u(t) = \int_{-\infty}^{\infty} \delta(t)$

→ $s(t) = m(t)\cos(2\pi f_c t) \Rightarrow M(f) * F[\cos(2\pi f_c t)] = S(f)$

$$S(f) = M(f) * \left[\frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)\right]$$

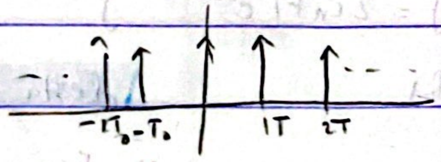
$$= \frac{1}{2}M(f)*\delta(f-f_c) + \frac{1}{2}M(f)*\delta(f+f_c)$$

$$= \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c).$$

ex:- consider train of impulse $g(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_0)$.

Find $G(f)$??

$g(t)$ is periodic with period $T_0 \Rightarrow f_0 = \frac{1}{T_0}$



$$C_n = \frac{1}{T_0}\int_{(T_0)} g(t)\, e^{-jn\omega_0 t}\, dt$$

عادي أي مضروب δ(t) بحسب عند

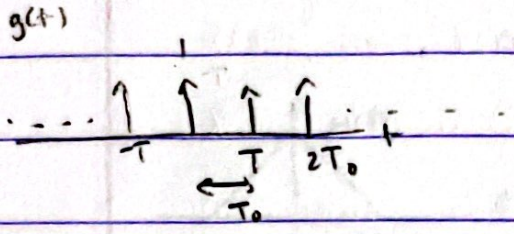
الازاحة، بوصلي ع

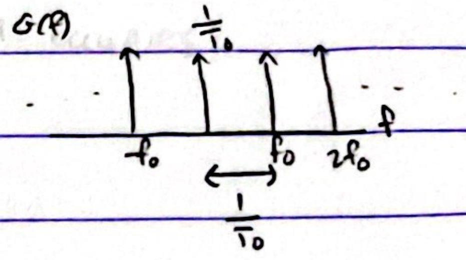
$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \underline{\delta(t)\, e}^{-jn\omega_0(0)}\, dt$$

$$= \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t)\, dt = \frac{1}{T_0} = f_0$$

$$\Rightarrow G(f) = \sum c_n\, \delta(f-nf_0) = \frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta(f-nf_0).$$

$g(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$ $\qquad$ $G(f) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$.





=>periodic $\qquad\qquad$ =>periodic

T_0 $\qquad\qquad$ $\frac{1}{T_0}$

ex:-

g(t):-

Find $G(f) = ??$ $\qquad$ $C_n = ??$

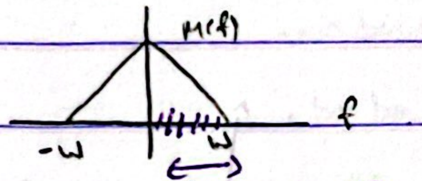
$C_n = \frac{1}{\pi n} \sin(\frac{\pi}{2} n)$. $\qquad$ "from previos exampel".

$G(f) = \sum C_n \delta(f - nf_0)$.

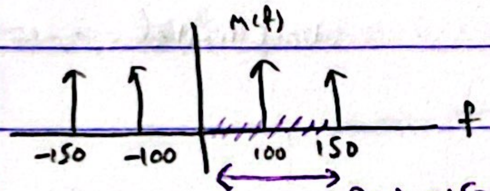
$= \frac{1}{\pi} \sum \frac{1}{n} \sin(\frac{\pi}{2} n) \delta(f - nf_0)$.

Bandwidth of Signals
and Systems

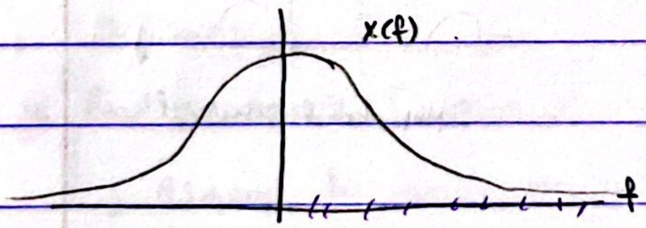
def. Bandwidth (B.W): Amount of positive freq. spectrum that a signal occupies.



B.W = W Hz



B.W = 150 Hz.

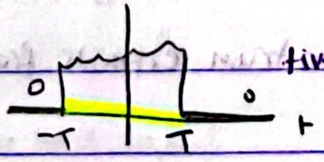


B.W = ∞ "theoritical"

1 Band limited signal.

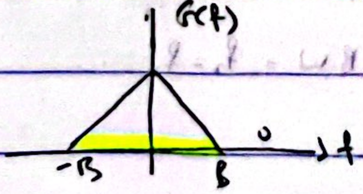
2 time limited signal.

→ if $g(t)=0$ for $|t|>T$



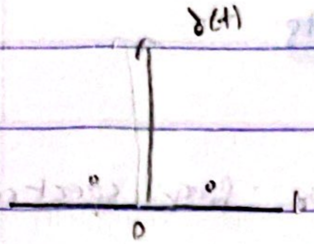
time limited.

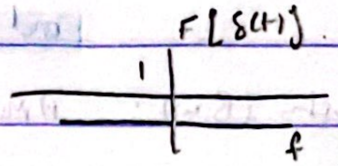
→ if $G(f)=0$ for $|f|>B$.



x يمكن ان تكون ال signal
band + time
limited.

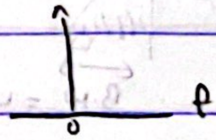
duration.



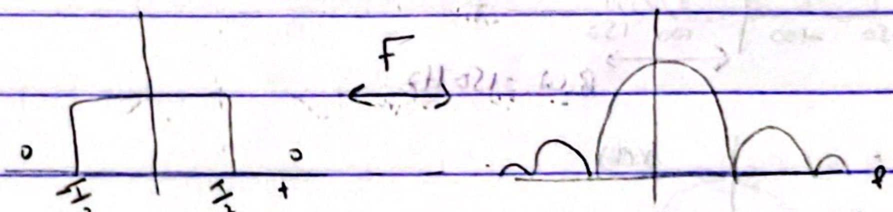


• time limited • not Band limited.

duration = 0



• time unlimited. • band limited.



• time limited • Not band limited.

Remarks:-

1) B.W provides a measure of extent of significant freq. content.

2) B.W of a signal is the width of the positive freq. band.

3) For baseband signals, the spectrum extends from $-B$ to B

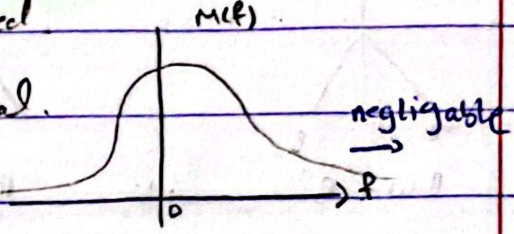
$\Rightarrow$ the B.W = B HZ.

4) For band pass signals, the spectrum extends between (f_1, f_2) and $(-f_2, -f_1)$ $\Rightarrow$ the B.W = $f_2 - f_1$.

3 basband signal "low-pas, messag".

↳ for which most of the energy is contained within a band centered around the zero freq. "it's a signal".

- non band limited.
- basband signal.

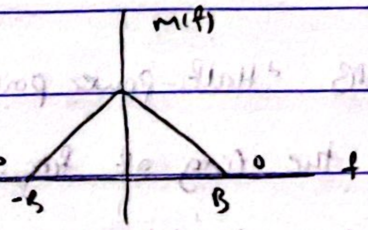


قليلة

عند توزيع M(f) تقع القيم القريبة

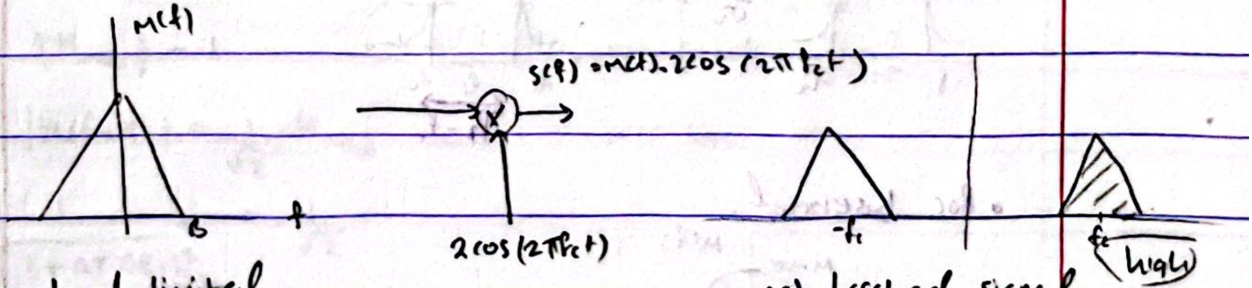
من 0 كبيرة ، والابعد صفر تقريباً لذلك مهملة.

- band limited
- bas band signal.



4 Band pass signals (passband, modulated signal)

↳ A signal for which the energy is centerd "concentrated" around some high freq. carrier f_c, otherwise negligible.



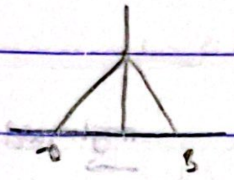
- band limited
- base band signal
- B.W = B

- not baseband signal.
- band pass signal.
- B.W = 2 B " $(f_c+B)-(f_c-B)$

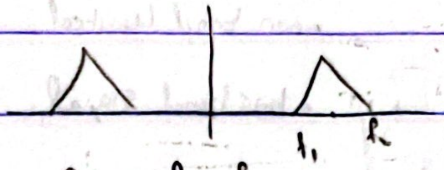
Some Definition of B.W

$\boxed{1}$ Absolute B.W

its defined for band limited.



B.W = B

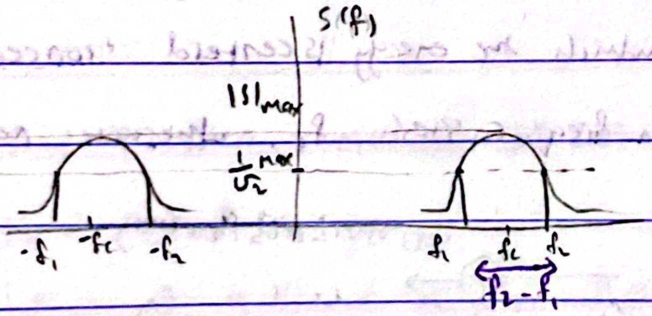


B.W = $f_2 - f_1$

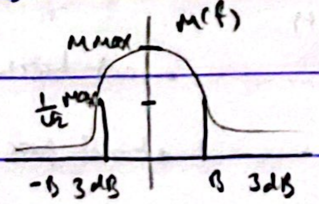
$\boxed{2}$ 3dB "Half-power point" B.W

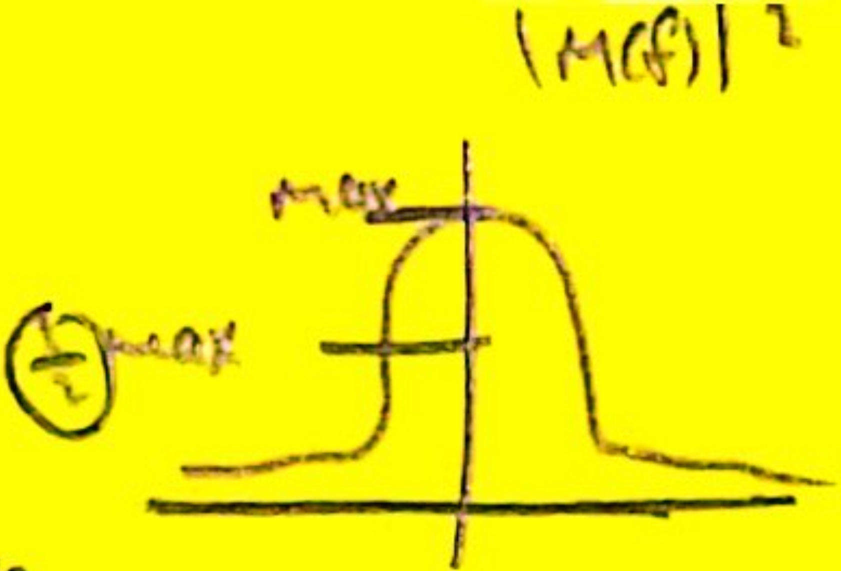
applicable for baseband { the rang of freq. from (0) to some freq. (B) at which $|M(f)|$ drops to $\frac{1}{\sqrt{2}}$ of its max. value $|M_{max}|$

- As for band pass.



- for baseband.



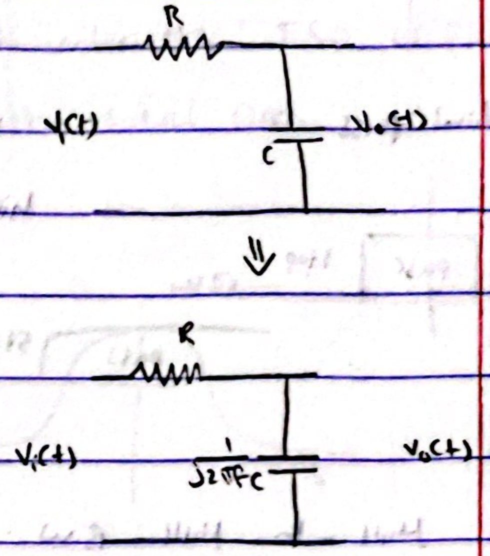


- half-power.

$$10 \log \frac{1}{2} \quad = 3dB .$$

Example:- RC circuit " First order " $\boxed{\text{low pass filter}}$.

Find 3dB B.W ??



Eq, $V_o(f) = V_i(f) * \frac{\frac{1}{j2\pi fC}}{\frac{1}{j2\pi fC} + R}$

complex

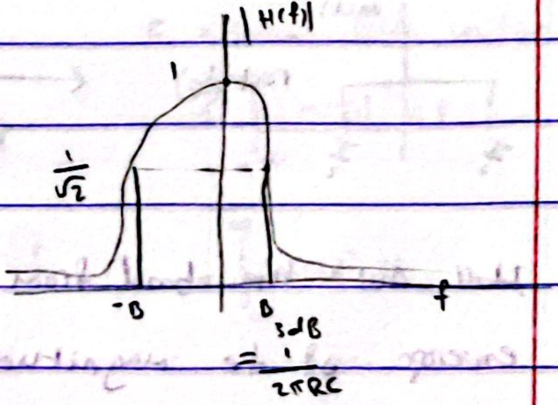
$$\frac{V_o(f)}{V_i} = \boxed{H(f) = \frac{1}{1+j2\pi fRC}}$$

$$H(f) = \frac{1}{\sqrt{(1)^2 + (2\pi fRC)^2}}$$

$$|H(f=0)| = \frac{1}{\sqrt{1+0}} = 1$$

- as $f \uparrow \Rightarrow |H(f)| \downarrow$

as $f \to \infty \Rightarrow H(f) \to 0$



⇒ to find B.W 3dB:

- not band limited.
- base band

$$|H_{max}| = 1$$

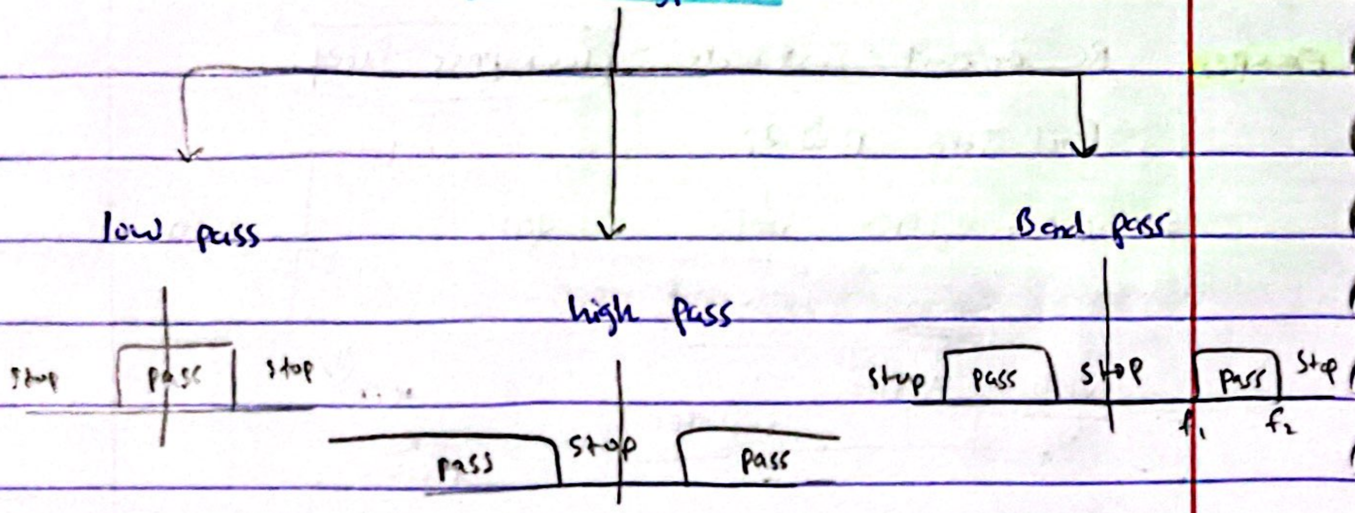
$$|H(f=B)| = \frac{1}{\sqrt{2}} H_{max}$$

$$\frac{1}{\sqrt{1+(2\pi BRC)^2}} = \frac{1}{\sqrt{2}} \cdot 1$$

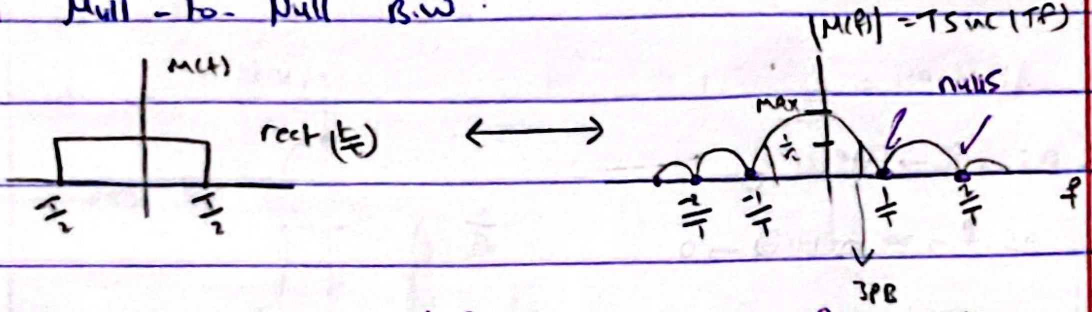
$$\therefore 2\pi BRC = 1$$

$$\boxed{\therefore B = \frac{1}{2\pi RC}}$$

Filter types :-



3) Null - to - Null B.W.

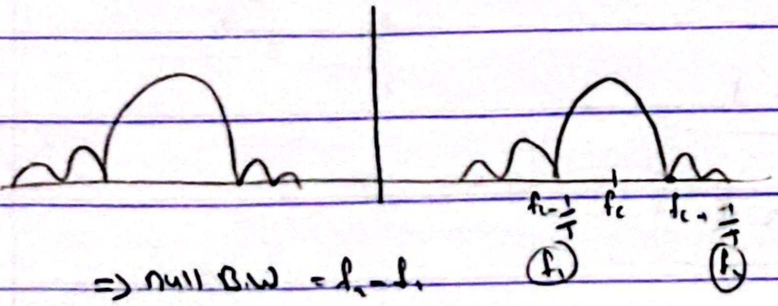


Null - BW : the band from zero to the first null in the envelope of the magnitude spectrum. $\boxed{+ve}$

$\Rightarrow B.W = \frac{1}{T}$

″ ″ ″ ″ to the second null

$\Rightarrow B.W = \frac{2}{T}$



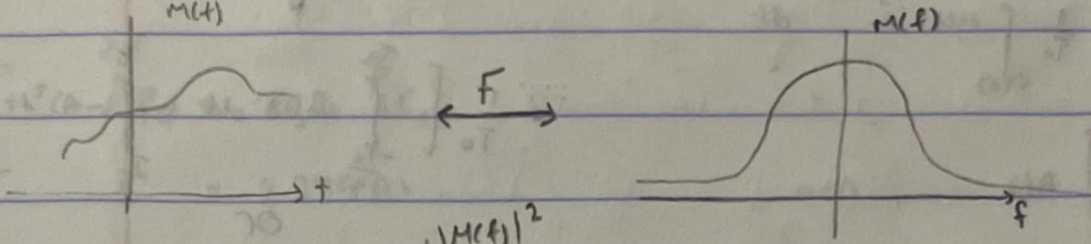
$\Rightarrow$ null B.W $= f_2 - f_1$

$= \frac{2}{T}$

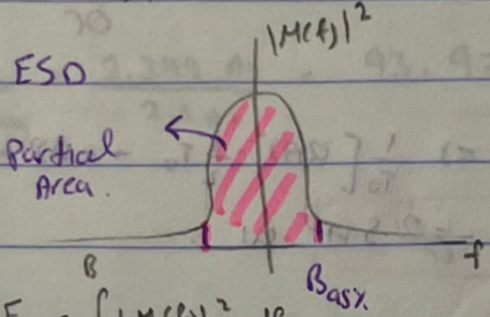
[4] 95% energy or power B.W.

Band of freq. where Area under the "ESD or PSD" is at least "95% or 99%" of the total Area.

A) Energy signal :-



=> ESD



$$E_B = \int_{-B}^{B} |M(f)|^2 \, df$$

$$\frac{E_B}{E} \times 100\% \geq 95\%$$

E = total Area under ESD :-

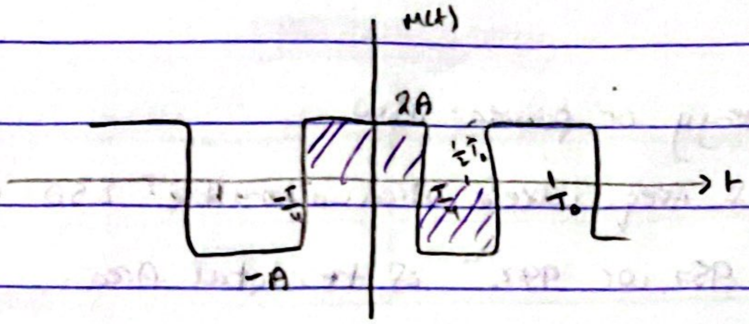
$$= \int_{-\infty}^{\infty} |M(f)|^2 \, df$$

$$= \int_{-\infty}^{\infty} |m(t)|^2 \, dt$$

B) power signal:

periodic signal $m(t) = \begin{cases} 2A & -\frac{T}{4} < t < \frac{T}{4} \\ -A & \text{o.w} \end{cases}$

Find 95% B.W?



2) PSD : $|C_n|^2$ Vs. f

$$C_n = \frac{1}{T_0}\int_{<T_0>} m(t)\, e^{-jn\omega_0 t}\, dt$$

$$|C_n| = \begin{cases} A/2 & , n=0 \\ 3A/|n|\pi & , n=\pm1, \pm5, \dots \\ -3A/|n|\pi & , n=\pm3, \pm7, \dots \\ 0 & , \text{even} \end{cases}$$

1) $P_{avg} = \frac{1}{T_0}\int_0^{T_0} |m(t)|^2\, dt$

$$= \frac{1}{T_0}\left[\int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} (2A)^2 dt + \int_{\frac{T}{4}}^{\frac{3}{4}T_0} (-A)^2 dt\right]$$

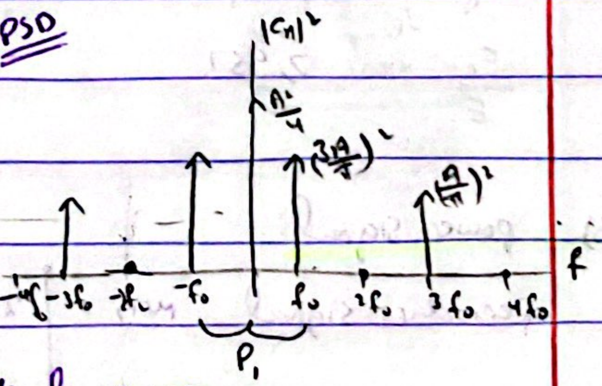
or

$$\Rightarrow \frac{1}{T_0}\left[(2A)^2 \cdot \frac{1}{2}T_0 + (-A)^2 \cdot \frac{1}{2}T_0\right]$$

$= 2.5\, A^2$ W.

$$|C_n|^2 = \begin{cases} (A/2)^2 & , n=0 \\ |3A/|n|\pi)^2 & , n=\text{odd} \\ 0 & , n=\text{even} \end{cases}$$

PSD



$\Rightarrow P_1 = |C_0|^2 + 2\sum_{n=1}^{1} |C_n|^2$ partial power.

$P_1 = |C_0|^2 + 2(C_1)^2$

$= \frac{A^2}{4} + 2\left(\frac{3A}{\pi}\right)^2 = 2.073\, A^2$

$\frac{P_1}{P_{av}} = \frac{2.073}{2.5}\frac{A^2}{A^2} \times 100\% = 82.95\%$ $\Rightarrow$ B.W $= 1 f_0$ No X

$$P_3 = |C_0|^2 + 2\left[C_1^2 + C_3^2\right]$$

$$= P_1 + 2C_3^2 = P_1 + \frac{A^2}{\pi^2} = 2.276\,A^2$$

$$\frac{P_3}{P_{avg}} = \frac{2.276\,A^2}{2.5\,A^2} \times 100\% = 91.05\% \quad \text{NO } \times$$

$$P_6 = P_3 + 2C_5^2$$

$$= 2.349\,A^2$$

$$\frac{P_5}{P_{avg}} = \frac{2.349\,A^2}{2.5\,A^2} = 93.97\% > 93\% \quad \checkmark$$

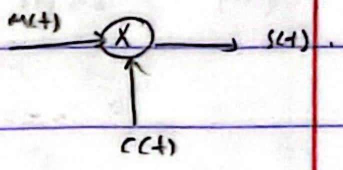
$$\boxed{\therefore B_{93\%} = 5 f_0}$$

$\Rightarrow \frac{2}{2} \cos (400-200)\pi t + \frac{2}{2} \cos(400+100)\pi t$

Example:- $m(t) = 2 \cos (400 \pi t) \cos (200 \pi t)$

$c(t) = 4\cos (6000 \pi t)$

let $s(t) = m(t) \cdot c(t)$



$\boxed{1}$ P_{avg} for $m(t)$ and $c(t)$:-

for $c(t) \Rightarrow P_{avg} = \frac{Ac^2}{2} = \frac{(4)^2}{2} = 8\,w$

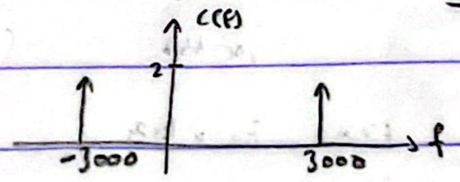
for $m(t) = 1\cos (200\pi t) + 1\cos(600\pi t)$

$\Rightarrow P_{avg} = \frac{1}{2} + \frac{1}{2} = 1\,w$

$\boxed{2}$ find and plot $c(f)$

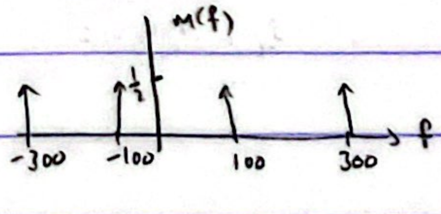
$c(t) = 4 \cos (2\pi \cdot 3000\,t)$

$c(f) = 2 \left[\delta(f-3000) + \delta(f+3000) \right]$



$\boxed{3}$ find and plot $M(f)$

$m(t) = \cos (2\pi \cdot 100t) + \cos(2\pi \cdot 300t)$



note :-

$\cos(2\pi f_0 t) \longleftrightarrow \frac{1}{2}\delta(f-f_0) +$
$\frac{1}{2}\delta(f+f_0)$.

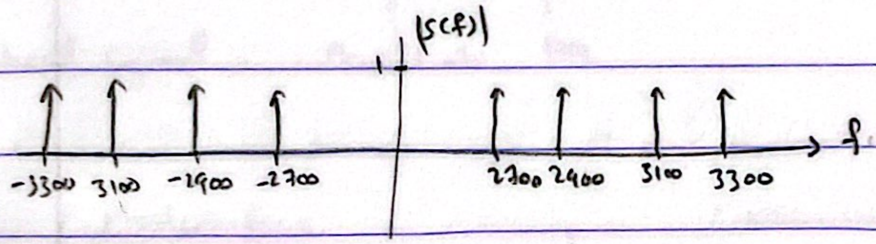
$\boxed{4}$ Find and plot $S(f)$.

$S(t) = M(t)\,C(t)$

$S(f) = M(f) * C(f)$.

$= M(f) * \{2\delta(f-3000) + 2\delta(f+3000)\}$.

$= 2M(f-3000) + 2M(f+3000)$



$\boxed{5}$ Find B.W for :-

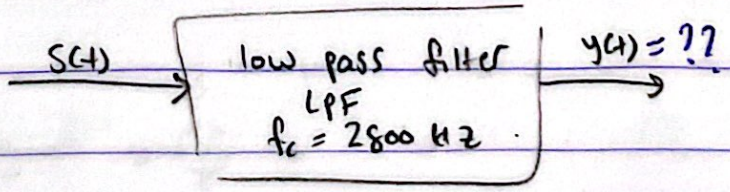
$M(t) \Rightarrow$ B.W = 300 HZ "from $\boxed{1}$"

$C(t) \Rightarrow$ B.W = 3000 HZ .

$S(f) \Rightarrow$ B.W $= f_1 - f_1 = 3300 - 2700$

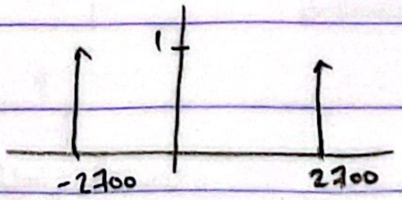
$= 600$ HZ .

$\boxed{6}$



$Y(f) = \delta(f-2700) + \delta(f+2700)$

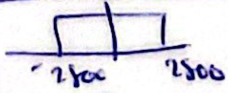
$Y(f) =$



$y(t) = F^{-1}\{Y(f)\}$

$= 2\cos(2\pi \cdot 2700t)$

$\boxed{7}$ B.W for $\underline{6}$ "filter"

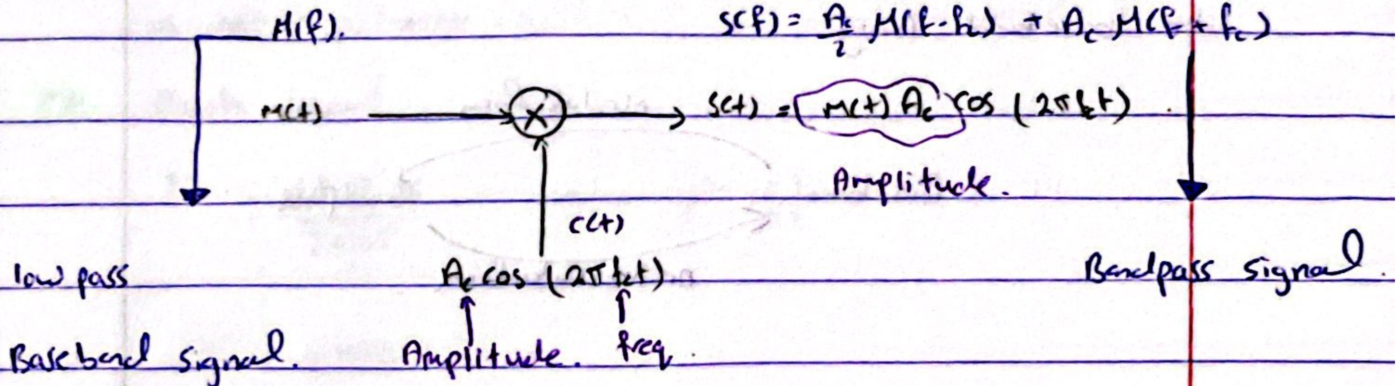


B.W = 2800 HZ

Modulation:

Rmrk:-

Fourier transform ⟶ Moduliation property.



$S(f) = \frac{A_c}{2} M(f-f_c) + A_c M(f+f_c)$

$s(t) = \boxed{m(t) A_c} \cos(2\pi f_c t)$

$A_c \cos(2\pi f_c t)$

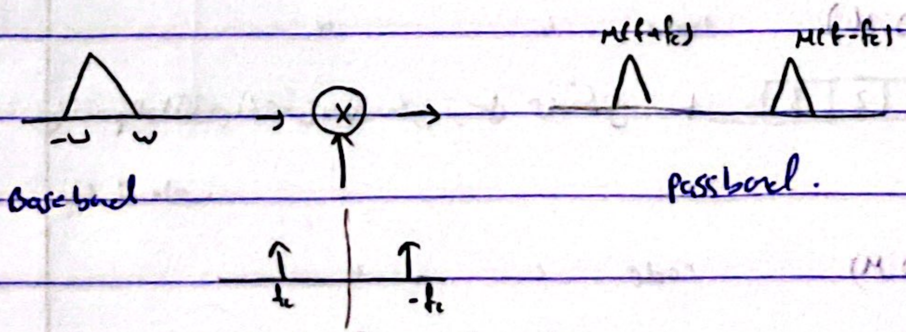
diff Modulation:- "transmitter "عملية تتم عند المرسل"

time domain:-

Is the process by which some characteristic of a carrier wave is varied in accordance with the message.

freq. domain:-

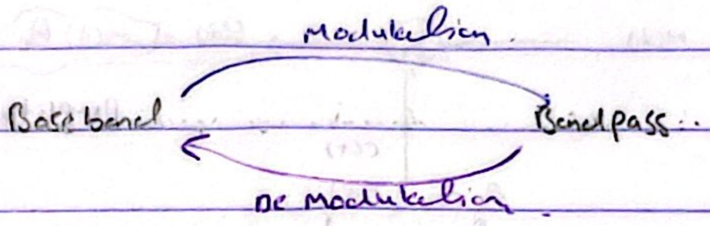
the process of shifting the baseband signal "message" to the passband range.



carier in Freq. domain.

Demodulation:- "at Receiver"

the process of shifting the passband signal into the baseband range.

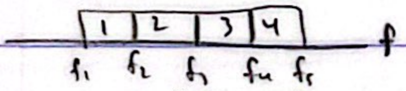


A signal maybe sent in its Baseband format when a dedicated خاصة wired chanil is avilable, otherwise it must be converted into passband format.

Q:- why do we need modulation?

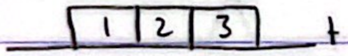
A:- [1] Simiultanes transmission of several signal

(FDM) Frequancy divition multiplexion.



← تقسيم نفس ال Band بين اكثر من مستخدم .

(TDM) time " "



← تقسم الداتا لاكثر من داتا ويكن ترسل مش بشكل بعث واحد .

(CDM) code " "

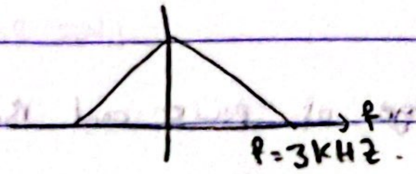
$\boxed{2}$ practical Antenna Design. "antenna length $\propto \frac{1}{f}$"

for efficient transmission :-

⇒ antenna length $= \lambda/4$, λ = wave length.

ex:- Audio signal

$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^3}$



$= 100KM$.

∴ antenna length $= \frac{100km}{4} = 25\ km$ "impossipel"

cellular freq. (GSM)

$f = 1000\ M\ HZ$.

$\lambda = \frac{c}{f} = \frac{3\times10^8}{1000\times10^6} = 0.3M$

∴ antenna length $= \frac{0.3M}{4} = 7.5\ cm$. "possipel"

⇐ تم تغيير ال freq حتى نقلل ال antenna.le وبالتالي امكانية افضل في تصميم عملي ومنطقي.

الانتشار/البث

3) propagation characteristics are differant at differant frequacies . "low freq. penetraten walls"

تمر من خلال الجدار بشكل أفضل .

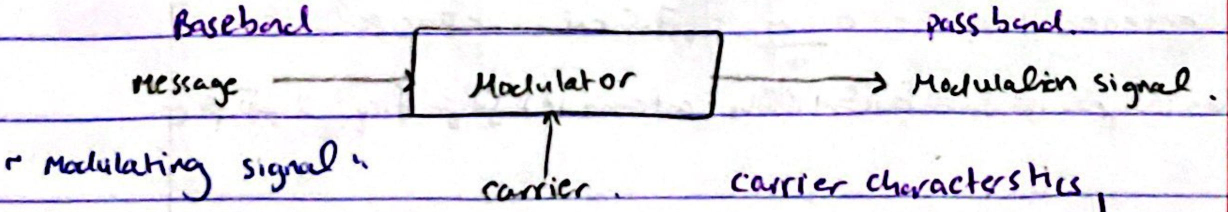
4) Exchange of power and Bandwidth "clear when we study FM".

Two types of Analog communication:-

1) Amp ilitude Modulation (AM).

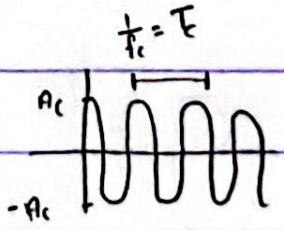
2) Angle Modulation (Freq. modulation FM).

Amplitude Modulation:- (AM) .



carrier characterstics
→ phase (freq.)
→ Amplitud.

$A_c \cos(2\pi f_c t)$



AM: is the process in which the amplitude of the carrier $c(t)$ is varied linearly with the message.

common form of carrier is : $c(t) = A_c \cos(2\pi f_c t + \Phi)$

Four types of AM:-

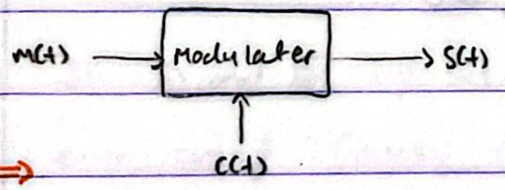
1. Normal AM.
2. Double sideband suppressed carrier (DSB-SC).
3. Single sideband (SSB).
4. Vestigial sideband (VSB). . بث ال TV.

Normal AM:-

is defined $s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$.

k_a: Amplitude transmitter sensitivity $(1 + k_a m(t)) c(t)$

1/volt. $= c(t) + k_a m(t) c(t)$.



$\Rightarrow$

$s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$.

$= A(t) \cos(2\pi f_c t)$.

where $A(t) = \underline{A_c} + \underline{A_c k_a}\, m(t)$. , $y = a + bx$

It's clear that the relationship between $A(t)$ with $m(t)$ is linear.

define <u>Envelop</u> of $s(t)$ as :- $|A(t)| = A_c |1 + k_a m(t)|$

Example: let $m(t) = \cos(2\pi f_m t)$.

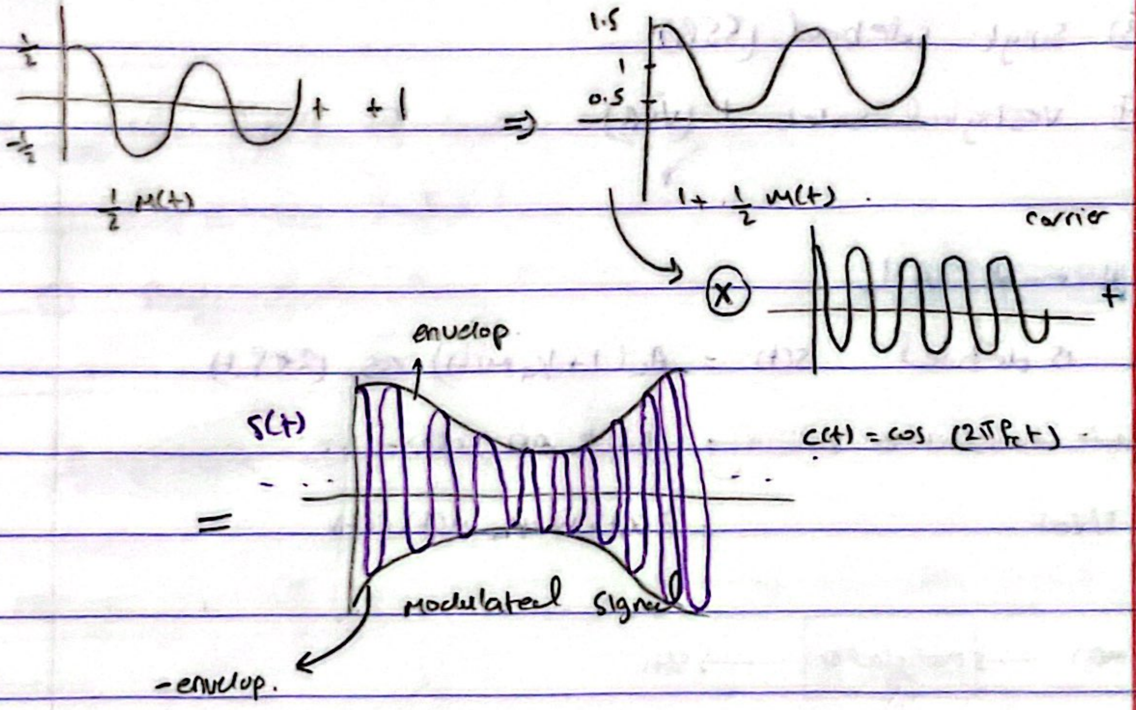
$c(t) = \cos(2\pi f_c t)$

$k_a = \frac{1}{2}\ \text{volt}^{-1}$

- note:- $f_c \gg f_m$.

$s(t) = A_c\,(1 + k_a m(t))\cos(2\pi f_c t)$

$= (1 + \frac{1}{2}\cos(2\pi f_m t))\cos(2\pi f_c t)$



=> The envelop of $s(t)$ has the same shape as $m(t)$ provided that:-

[1] $|k_a m(t)| \leq 1$ for all t.

if $|k_a m(t)| > 1$ then we have overmodulation

"phase reversal"

=> envelop Distortion.

[2] $f_c >> W$

W = Bandwidth of $m(t)$.

"highst freq. component of the message".

Recommended f_c at least 10 times of W.

ex:- let $m(t) = 5\cos(2\pi 10t) + 7\cos(2\pi 30t)$

B.W for $m(t)$ $W = 30Hz$.

f_c recommended : $f_c > 10 \cdot 30$

$f_c > 300 Hz$.

If [1] and [2] are satisfied, then we can use a single Receiver called "Envelop Detector" to demodulate the message $m(t)$ from $s(t)$.

Example:- let $s(t) = A_c(1+k_a m(t)) \cos(2\pi f_c t)$, $A_c = 1$,

$m(t) = A_m \cos(2\pi f_m t)$.

$s(t) = \left[1+k_a A_m \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$

- The envelop $|A(t)| = \left|1+k_a A_m \cos(2\pi f_m t)\right|$
- Define $\mu = k_a A_m$ "later will be called modulation index"

$s(t) = \left[1+\mu \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$

$= A(t) \cos(2\pi f_c t)$

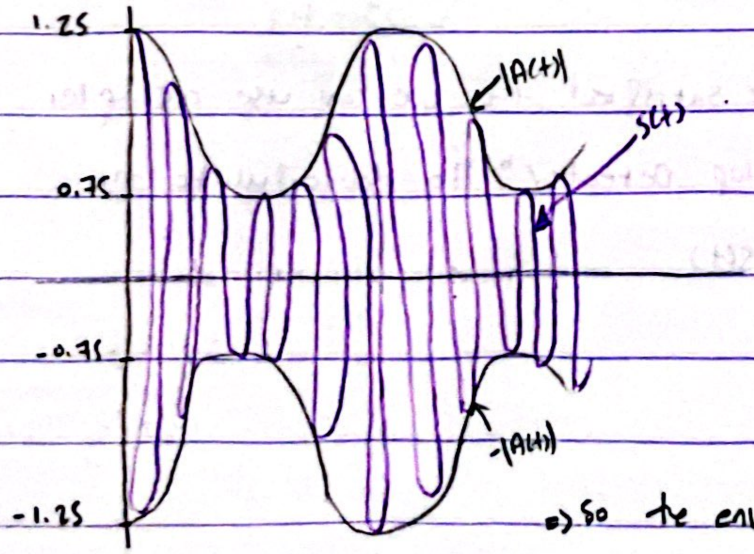
Case I:-

$$\left.\begin{matrix} k_a = 0.25 \\ A_m = 1 \end{matrix}\right\} \mu = 0.25 .$$

$|A(t)| = |1+0.25 \cos(2\pi f_m t)|$

$-1 < \cos(2\pi f_m t) < 1$

$.75 < 1+.25 \cos(2\pi f_m t) < 1.25$



- $f_c >> f_m$.

Cuz $T_m > T_c$.

- $|k_a A_m \cos(2\pi f_m t)|$

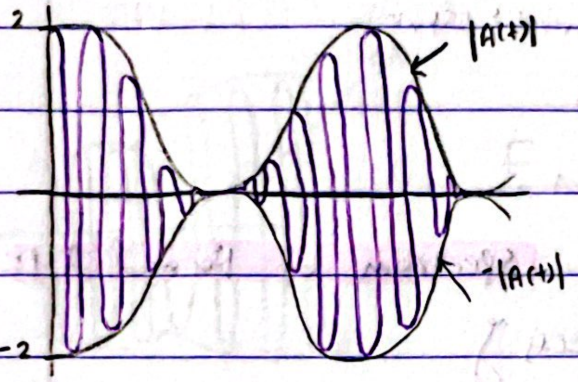
$= k_a A_m = 0.25 < 1$

⇒ so the envelop symmeler to the message.

case II :-

$K_a = 0.25$
$A_M = 4$ $\Big\}$ $\mu = 1$

$|A(t)| = |1 + 1\cos(2\pi f_m t)|$



$\underline{\underline{f_c \gg f_m}}$

$|k_a m(t)| = |\mu \cos(2\pi f_m t)|$

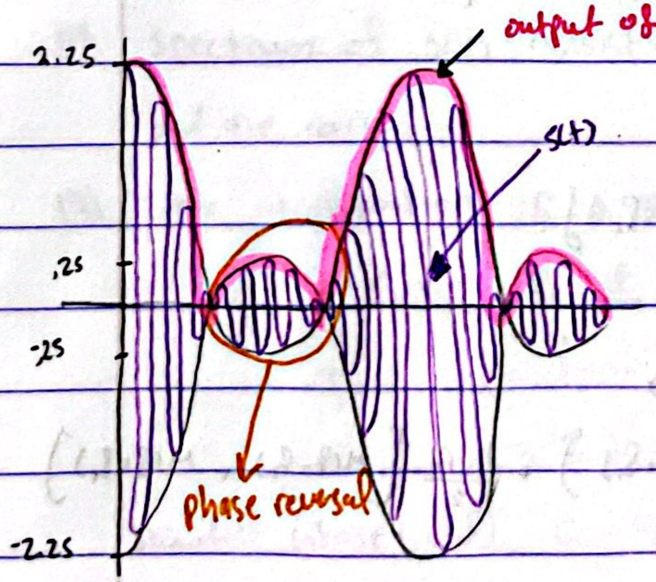
$\underline{\underline{\mu = 1 \leq 1}}$.

- we don't have modulation distortion
  'No over modulation'.

case III :-

$k_a = 0.25$
$A_M = 5$ $\Big\}$ $\mu = 1.25$

$|A(t)| = |1 + 1.25\cos(2\pi f_m t)|$



$\underline{\underline{f_c \gg f_m}}$

$|k_a m(t)| = |\mu \cos(2\pi f_m t)|$
$= 1.25 > 1$

- so we have over modulation
  Modulation distortion
- output of Envelop Detector $\neq \underline{\underline{m(t)}}$.
  sinosoid

For Normal AM:-

- in t-domain:-

$$s(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t)$$

$$= [1 + k_a m(t)]\, c(t)$$

$$= c(t) + k_a m(t)\, c(t)$$

- in F-domain:-

$$s(t) \xrightarrow{F} s(f) .$$

Spectrum of Normal AM:-

$$s(f) = F[c(t) + k_a m(t) c(t)]$$

$$= \underbrace{F\{c(t)\}}_{①} + k_a \underbrace{F[m(t) c(t)]}_{②}$$

1) But $c(t) = A_c \cos(2\pi f_c t)$.

$$c(f) = F\{c(t)\} = \frac{A_c}{2}\delta(f - f_c) + \frac{A_c}{2}\delta(f + f_c)$$

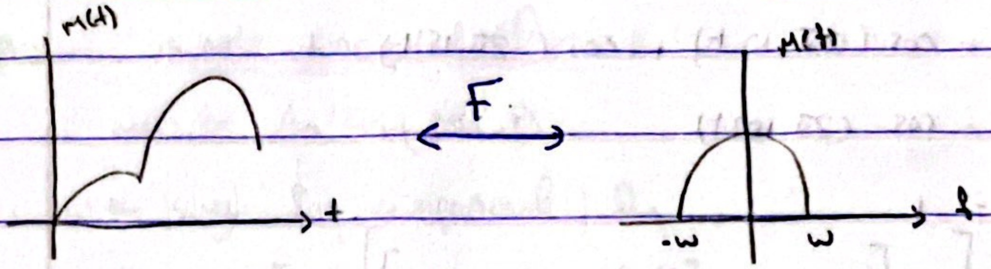
2) $F[m(t) . c(t)] = M(f) \circledast c(f)$

$$= M(f) \underset{\substack{\uparrow \\ \text{convolution}}}{\circledast} \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] .$$

$$= \frac{A_c k_a}{2} M(f - f_c) + \frac{A_c}{2} M(f + f_c)$$

$$\Rightarrow s(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

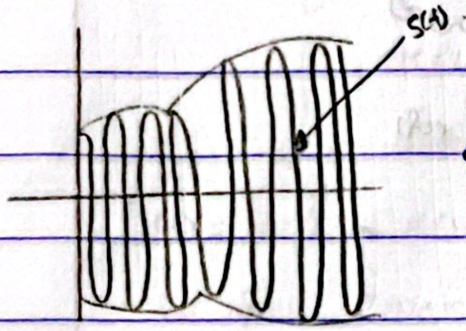
t- domain   f- domain

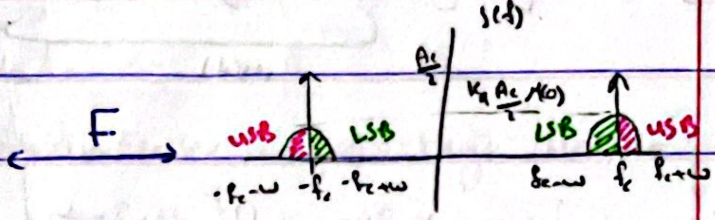


Baseband

B.W = W

Bandlimited





LSB: Lower SideBand.

USB: upper sideband.

From the spectrum of S(t).

[1] Baseband spectrum M(f) has been shifted into the Bandpass region.

[2] Spectrum of S(t) consists of two sidebands (USB, LSB) and the carrier.

[3] the transmition B.W of S(t) is

$$B.W = f_2 - f_1$$

$$= f_c - W - (f_c - W)$$

$$= 2W \quad = \text{twice the message B.W}$$

Audio (Baseband) W = 4 KHZ

- Normal AM

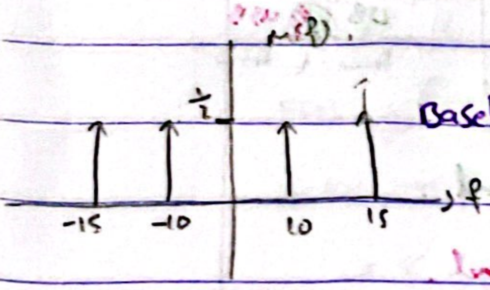
B.W = 8 KHZ required.

Example:- $m(t) = \cos(2\pi.10.t) + \cos(2\pi.15.t)$.

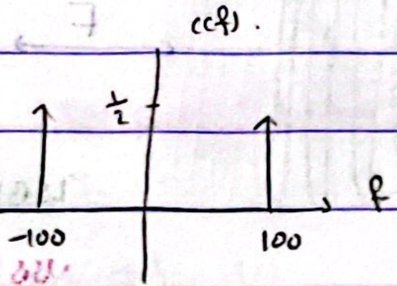
$c(t) = \cos(2\pi.100.t)$

$k_a = 1$

$$\Rightarrow s(t) = \left[1 + \underbrace{\left[\cos(2\pi.10.t) + \cos(2\pi.15.t)\right]}_{m(t)}\right] \cos(2\pi.100t).$$



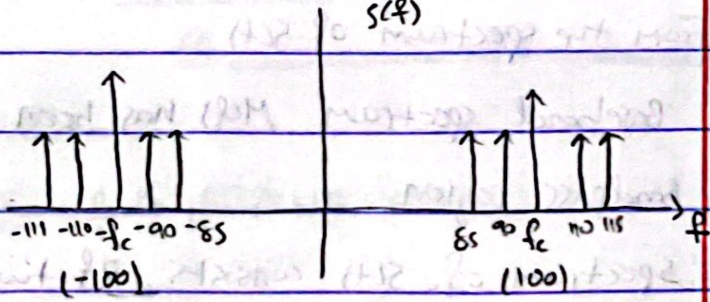
Baseband signal.



B.W for m(t):

B.W = 15

Bandpass signal



B.W = 115 - 85 = 30.

Example: single - tone Modulation.

$m(t) = A_m \cos(2\pi f_m t)$

with single freq. component f_m.

$\Rightarrow s(t) = A_c[1 + \underbrace{k_a A_m}\cos(2\pi f_m t)]\cos(2\pi f_c t)$.

$\mu = k_a A_m$

modulation index or percentage Modulation.

$s(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$.

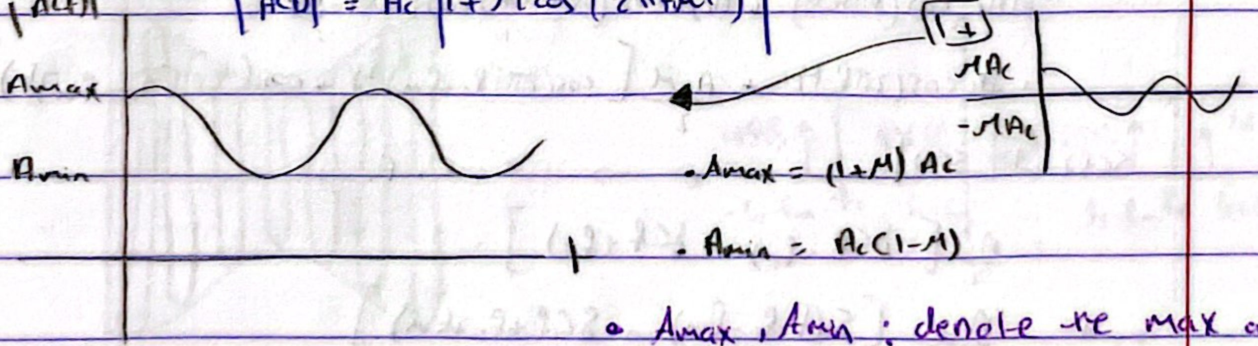
To avoid envelop distortion. "due to overmodulation".

$|k_a m(t)| \leq 1$

$\mu = k_a A_m \leq 1$

$\Rightarrow$ Envelop:-

$|A(t)| = A_c|1 + \mu\cos(2\pi f_m t)|$



- $A_{max} = (1+\mu)A_c$
- $A_{min} = A_c(1-\mu)$
- Amax, Amin: denote the max and min value of the envelop of s(t).

$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$

$A_{max}(1-\mu) = A_{min}(1+\mu)$.

$\mu(A_{max} + A_{min}) = A_{max} - A_{min}$.

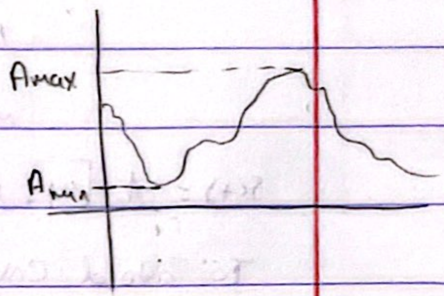
$\Rightarrow \boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$

→ when we defined $M = k_a A_m$ "special for single tone sinisodel modulation $m(t) = A_m \cos(2\pi f_m t)$ "

→ when $m(t)$ is in general not siniusodel

$$\boxed{M.I = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

general.



let us find the spectrum for single tone modulation interms M.

$$s(t) = A_c\left[1 + M\cos(2\pi f_m t)\right]\cos(2\pi f_c t).$$

$$= A_c \cos(2\pi f_c t) + A_c M \cos(2\pi f_m t)\cos(2\pi f_c t).$$

$$= A_c \cos(2\pi f_c t) + \frac{A_c M}{2}\left[\cos(2\pi[f_c + f_m]t) + \cos(2\pi[f_c - f_m]t)\right]$$

$$S(f) = F[s(t)]$$

$$= \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right].$$

$$+ \frac{A_c M}{4}\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right]$$

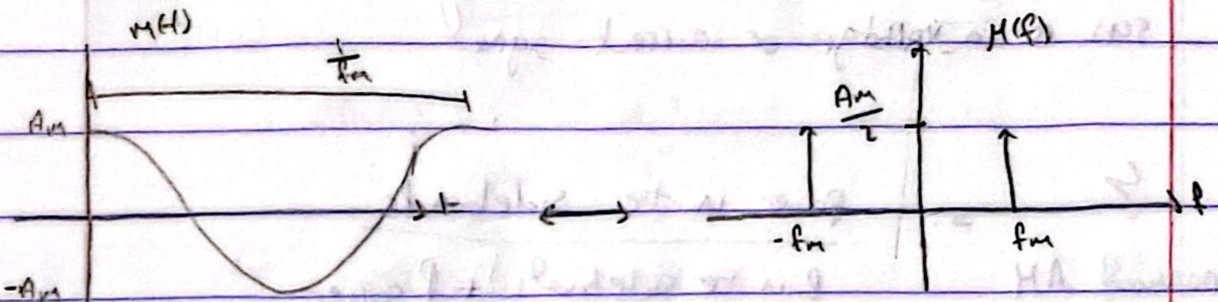
$$+ \frac{A_c M}{4}\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right]$$

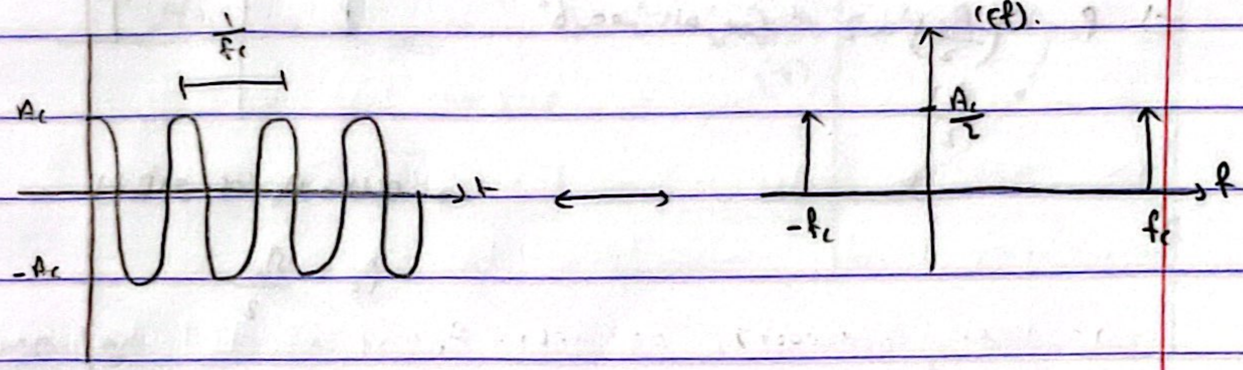
$$s(t) = A_c(1 + k\,m(t))\cos(2\pi f_c t)$$

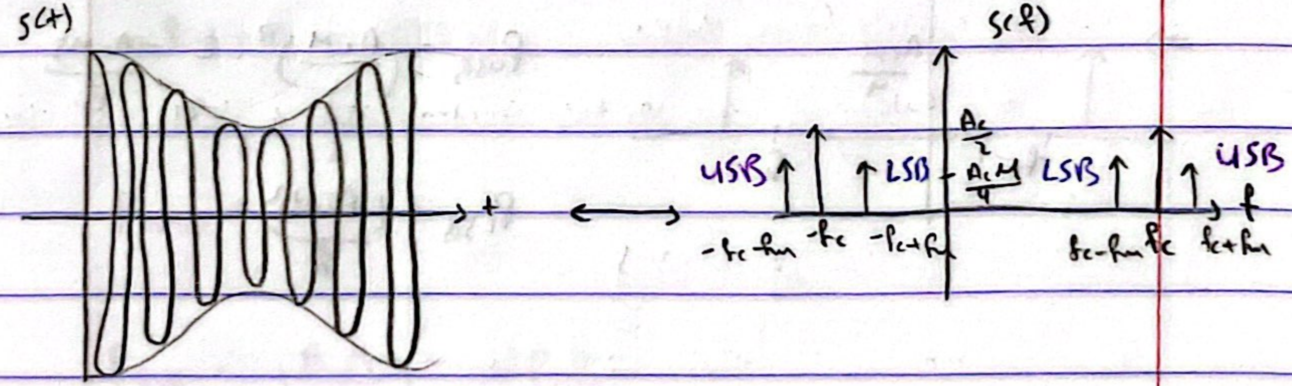
$$s(t) = A(t)\cos(2\pi f_c t).$$

envelop $|A(t)| = A_c|1 + k_a m(t)|$

t-domain **f-domain.**



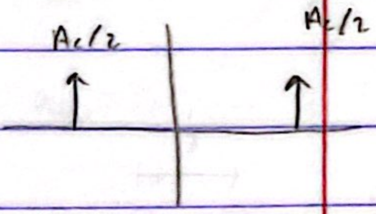




power effeciency:

s(t) is a voltage or current signal.

$$\underset{\text{normal AM}}{\eta} = \frac{\text{Power in the sidebands.}}{P \text{ in the sidebands} + P_{\text{carrier}}}$$



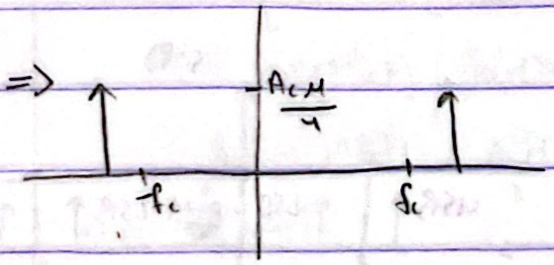
$\Rightarrow P_c = \left(\frac{A_c}{2}\right)^2 \times 2$ "لأنه من الطرفين"

$= \frac{A_c^2}{2}$

$c(t) = A_c \cos(2\pi f_c t)$

$P_c = \frac{A_c^2}{2}$

$\Rightarrow$



$P_{USB} = \left(\frac{A_c M}{4}\right)^2 \cdot 2 = \frac{(A_c M)^2}{8}$

$P_{LSB} = \frac{(A_c M)^2}{8}$

$$\xi = \frac{(A_c M)^2}{8} \boxed{\times 2} \Rightarrow$$ cuz its counts of USB and LSB.

$$\frac{(A_c M)^2}{8} \times 2 + \frac{A_c^2}{2}$$

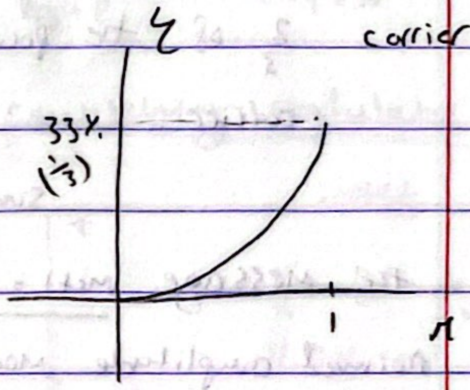
$$\xi = \frac{M^2}{M^2+2} \qquad 0 \le M \le 1$$

. $\frac{2}{3}$ losses in the carrier.

$M = 1 \Rightarrow \xi_{max} = \frac{1}{3}$

$= 33\%$

$M = 0 \Rightarrow \xi = 0$



Remember:

$f(t) = A \cos(2\pi f_c t)$

$P_{avg} = \frac{A^2}{2}$

$P_{carrier} = \frac{A_c^2}{2}$

$P_{USB} = \frac{\left(\frac{A_c M}{2}\right)^2}{2} = P_{LSB}$

$P_{sidebands} = P_{USB} + P_{LSB} = 2 \cdot P_{USB} = \frac{A_c^2 M^2}{4}$

Remark:- $\boxed{1}$ Normal AM is not BW effecient.

message B.w = w

transmmition B.w = 2w.

$\boxed{2}$ Normal AM is not Power effecient

$\frac{2}{3}$ of the power is wasted in the carrier.

singel tone, cuz its just one freq.

Ex:- the message $\underline{m(t) = 0.3\cos(2\pi\boxed{500}t)}$ is applied to a normal amplitude modulation, $k_a = 0.2$ and a $c(t) = 10\cos(2\pi 10000t)$ and $s(t) = A_c\cos(2\pi f_c t)(1 + k_a m(t))$.

a) Find the modulation index.

b) Find the averge power in the carrier and in each of the side bands.

c) Find the power effeciency.

$f_m = 500$ HZ $\quad A_c = 10$

$A_m = 0.3 \quad f_c = 10^4$ HZ.

$k_a = 0.2$

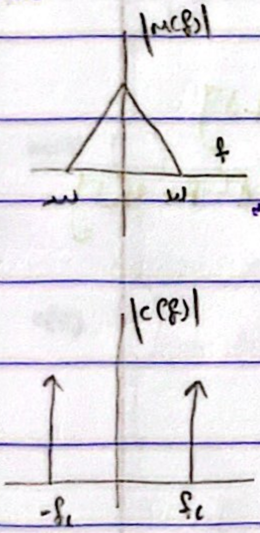
a) $M = k_a A_m = 0.2 \cdot 0.3 = 0.06$

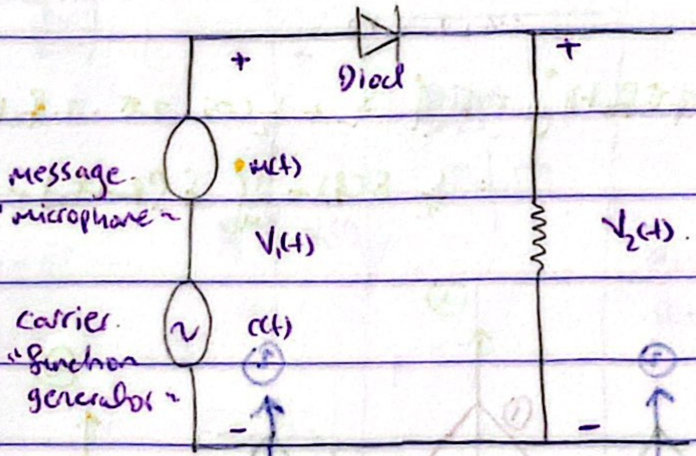
b) $P_c = \frac{A_c^2}{2} = 50$

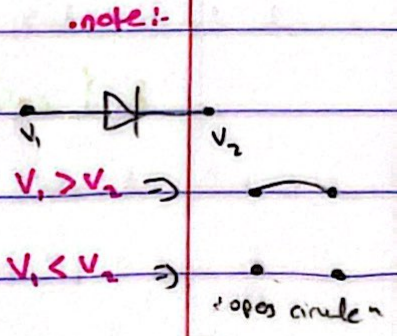
$P_{USB} = \frac{A_c^2 M^2}{4} = 0.09 = P_{side\,bands}$

c) • $\frac{M^2}{M^2+2} = 1.79\,m$ • $\frac{P_{sb}}{P_{sb}+P_c} = \frac{0.09}{0.09+50} = 1.79\,m$

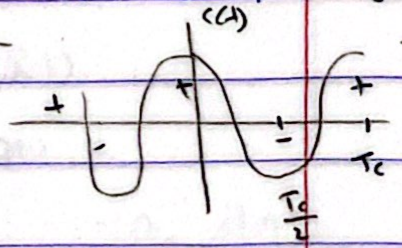
transmitter side ← Generation of Normal AM "switching Modulator".







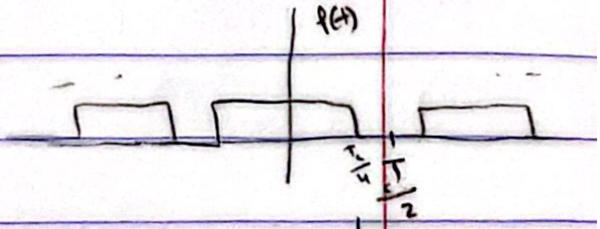
let $|c(t)| > |m(t)|$ => diod acts like an ideal switch. • $T_c = \frac{1}{f_c}$

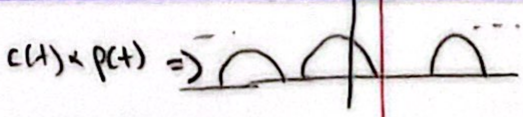


$$V_2(t) = \begin{cases} m(t) + c(t) & , c(t) > 0 \\ 0 & , c(t) < 0 \end{cases}$$

$$V_2(t) = [m(t) + c(t)]\, p(t)$$

• p(t) is periodic squear function.





using Fs expansion.

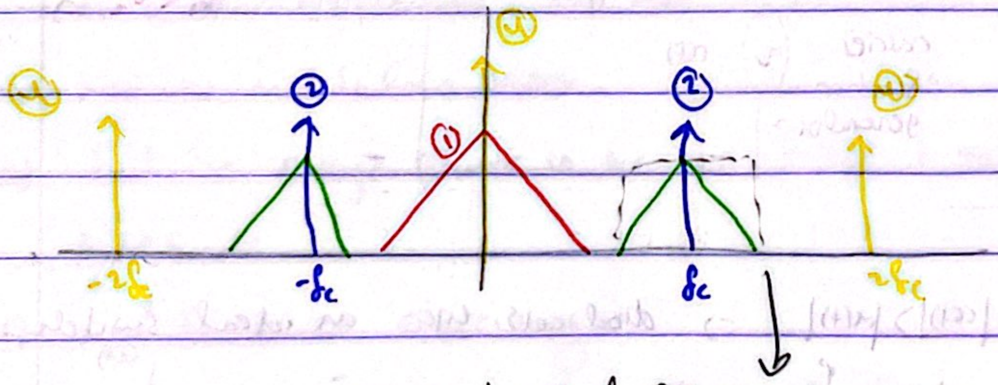
- $p(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos(\omega_c t) - \frac{1}{3}\cos(3\omega_c t) + \frac{1}{5}\cos(5\omega_c t) + \cdots\right]$
- $v_2(t) = [m(t) + c(t)]\, p(t)$

$$= \frac{1}{2}(\overset{1}{\textcircled{M}} + \textcircled{C})^{2} + \frac{2}{\pi}\underbrace{\overbrace{(M+C)\cos(\omega_c t)}^{③}}_{④} - \frac{2}{3\pi}(M+C)\cos(3\omega_c t)$$

$+ \cdots$

$$F\{m(t)\cos(\omega_c t)\} = M(f) \circledast \left[\frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)\right]$$
$$= \frac{1}{2}M(f+f_c) + \frac{1}{2}M(f-f_c).$$

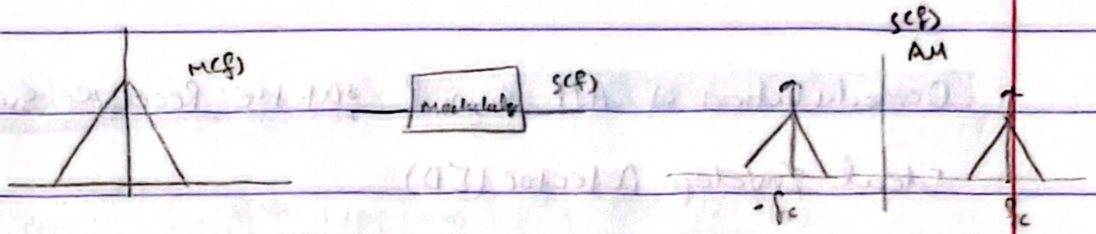
$$F\{\cos^2(2\pi f_c t)\} = F\left[\frac{1}{2} + \frac{1}{2}\cos(2\pi \cdot 2f_c t)\right]$$
$$= \frac{1}{2}\delta(f) + \frac{1}{4}[\delta(f-2f_c) + \delta(f+2f_c)]$$



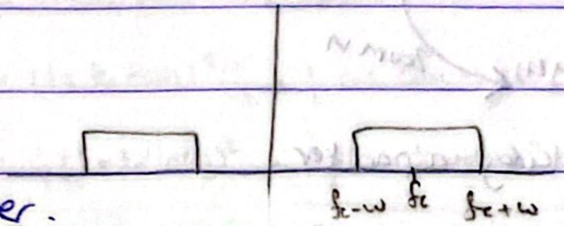
we use basepand filter to here

signal like this :- Λ | Λ

which is Λ after modulation.

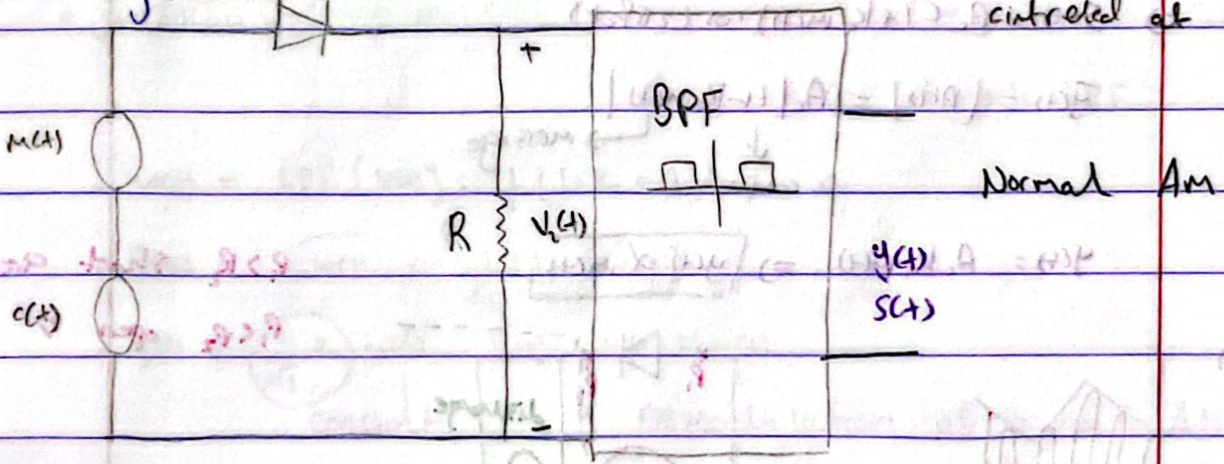


$v_2(t) \rightarrow \boxed{\begin{matrix}\text{BPF}\\ H(f)\end{matrix}} \rightarrow$ Normal AM.



• BPF with:
Bw = 2w
centreled at fc.

Switching modulater.



• من نتيجة الأزرق، والأخضر في الصفحة السابقة .

$$\Rightarrow y(t) = \frac{2}{\pi} m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)$$

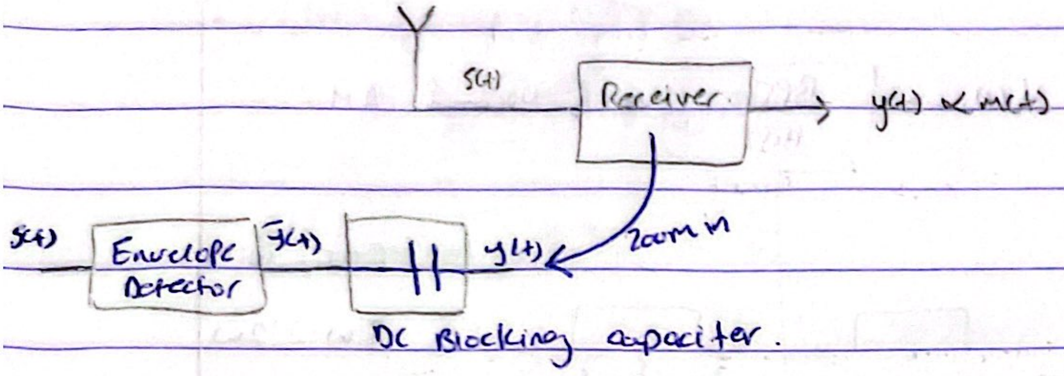
$$= \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos(2\pi f_c t)$$

$$\overset{?}{=} A_c\left[1 + k_a\, m(t)\right]\cos(2\pi f_c t).$$

yes, $k_a = \frac{4}{\pi A_c}$

Demodulation of AM signal "at the Receiver side"

Ideal Envelop Detector (ED).



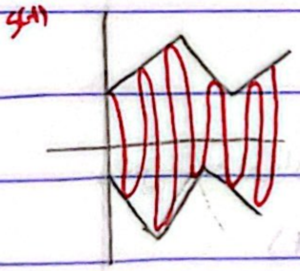
$$\Rightarrow s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$$

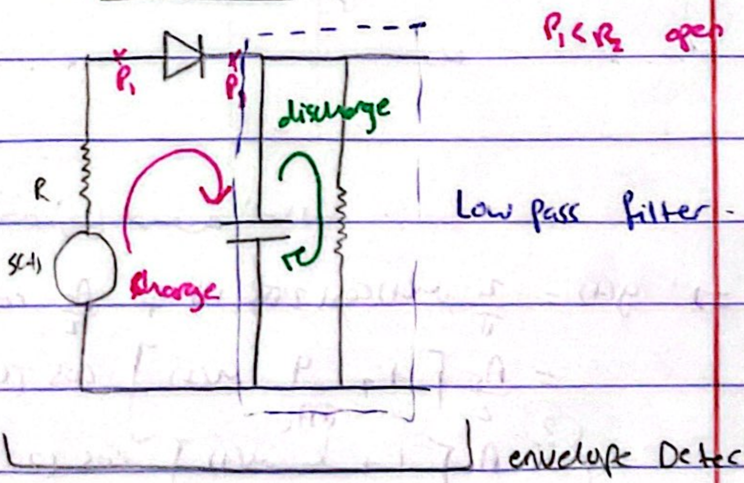
$$\bar{y}(t) = |A(t)| = A_c|\underbrace{1}_{\text{DC component}} + \underbrace{k_a m(t)}_{\text{message}}|$$

$$y(t) = A_c k_a m(t) \Rightarrow \boxed{y(t) \propto m(t)}$$

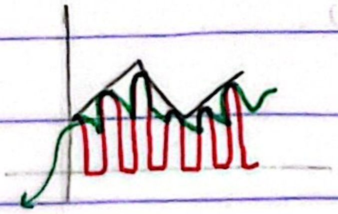
$P_1 > P_2$ short circuit

$P_1 < P_2$ open "





after the diod.

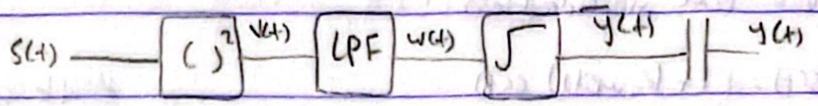


charging and discharging.

to follow the envelope of s(t), the circuit time constant ($R_L C$) should be chosen such that:

$$\underset{\text{carrier freq}}{\frac{1}{f_c}} \ll R_L C \ll \underset{\text{BW of the message}}{\frac{1}{W}}$$

Example: let $s(t) = (1+k_a m(t))\cos(w_c t)$



Find $v(t), w(t), \bar{y}(t), y(t)$?

- $v(t) = s^2(t) = (1+k_a m(t))^2 \cos^2(w_c t)$

$$= (1+k_a m(t))^2\left(\frac{1}{2} + \frac{1}{2}\cos(2w_c t)\right)$$

$$= \underbrace{\frac{1}{2}(1+k_a m(t))^2}_{\text{Base band (LP)}} + \underbrace{\frac{1}{2}(1+k_a m(t))^2 \cos(2\pi \boxed{2f_c} t)}_{\text{Pass band}}$$

" حول نقطة الاصل " Base band (LP) . Pass band " بعيد عن نقطة الأصل "

"will not pass through LPF".

- $w(t) = LPF\{v(t)\} = \frac{1}{2}(1+k_a m(t))^2$
- $\bar{y}(t) = \sqrt{w(t)} = \frac{1}{\sqrt{2}}(1+k_a m(t))$
- $y(t) = \underbrace{\frac{1}{\sqrt{2}}k_a}_{\text{constant}} m(t) \Rightarrow y(t) \propto m(t)$.

" DeModulation of Normal AM "

Conclusion and Remark about Normal AM:

virtues
* Modulation is simple "simple nonlinear device"
* Demodulation is simple "envelope Detector"

limitation
* AM is wasteful of power "power losses due to carrier".
* Transmission B.W = twice the message B.W = 2W

Solution:

1) DSB_SC Modulation " solve the problem of effeciency 100% "

2) SSB Modulation "solve Power and B.W"

Name from spectrum.

cancelled
↑

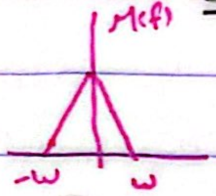
Double sideband - suppressed carrier (DSB-SC).

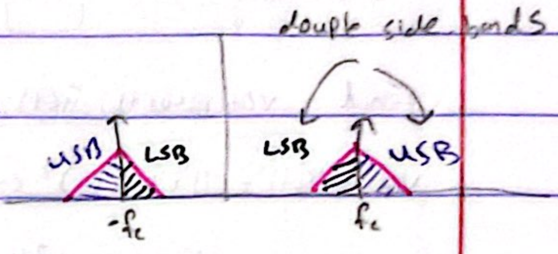
in Normal AM modulation :

$$s(t) = (1 + k_a m(t))\, c(t)$$

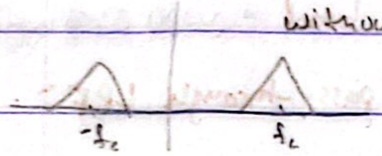
$$= \underline{\underline{c(t)}} + \underline{k_a m(t)\, c(t)}$$

if



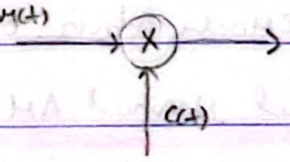


⇒ without the carrier.



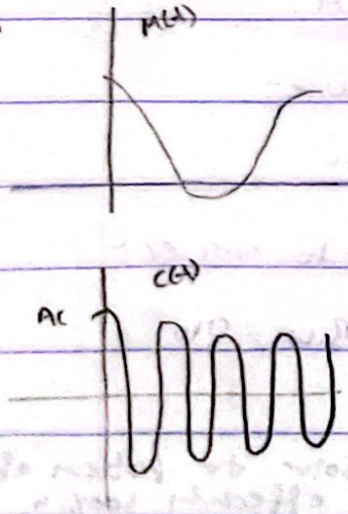
$$\eta = \frac{P_{\text{side band}}}{P_{sb} + \underset{0}{P_c}} = 100\%$$

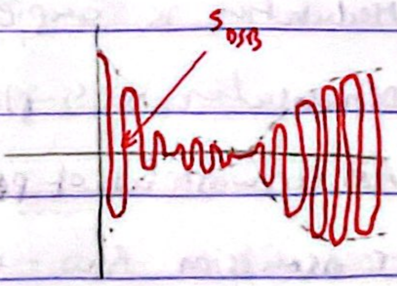
DSB-SC "consists of the product modulation"



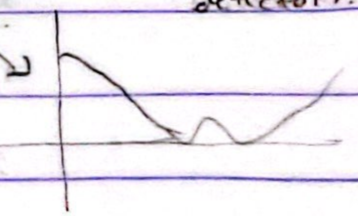
$$s(t)_{DSB} = m(t)\, c(t)$$

$$= A_c\, m(t) \cos(2\pi f_c t).$$





↓ pass over envelope detector.



⇒ so ED cannot use.

1) in Normal AM $s_{AM}(t) = (1 + k_a m(t))\, c(t)$

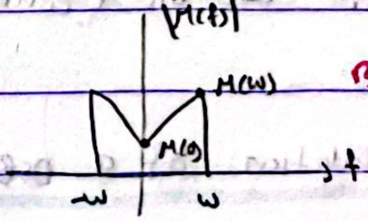
if $m(t) = 0$ then $s_{AM}(t) = c(t)$.

in DSB-SC $s_{DSB}(t) = m(t)\, c(t)$

if $m(t) = 0$ then $s_{DSB}(t) = 0$

2) in DSB-SC we can't use the simple demodulator "envelope detector"

The spectrum of $s_{DSB}(t)$



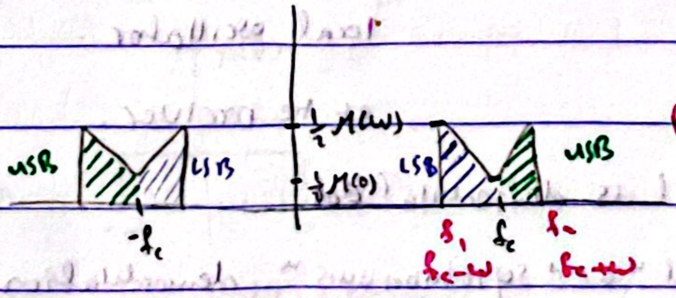
Baseband signal.

from $s_{DSB}(t) = m(t) \cdot c(t)$, let

then $S(f) = M(f) \circledast C(f)$ ↓ convolution

$$= M(f) \circledast \left(\tfrac{1}{2}\delta(f-f_c) + \tfrac{1}{2}\delta(f+f_c)\right)$$

$$= \tfrac{1}{2} M(f-f_c) + \tfrac{1}{2} M(f+f_c).$$



passband signal

$B.W = f_2 - f_1 = 2W$

From the spectrum:

Double sideband (LSB+USB)

Suppresed carrier "no carrier is transmited"

Remarks:-

* No impulses are present at $\pm f_c$ "suppressed carrier"

* the transmission B.W = 2W "similar to the Normal AM"

* power effeciency = $\frac{P_{\text{in the sideband}}}{P_{\text{total}}} = \frac{P_{sb}}{P_{sb} + \cancel{P_c}} = 100\%$

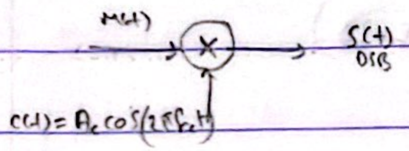
" Remember in Normal AM 33% when $M=1$"

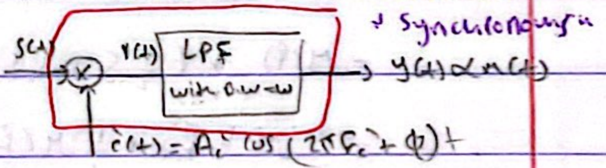
thus is a power effcient modulation schene

* Envelope Detector "can't be used for demodulation instead we use ("coherent demodulation)"

Demodulation of a DSB-SC signal "at reciever side"

transmitter "modulater" demodulater "coherent" + synchronous"





local oscillator.

at the receiver.

DSB-SC signal is demodulated using "coherent" or "sychronous" demodulation.

source and mise synchornization:-

1) amplitude, $A_c' \neq A_c$

2) phase shift, $\phi \neq 0$

3) $f_c' \neq f_c$

CASE I:- perfect coherent demodulation.

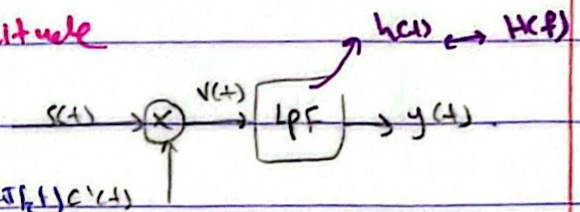
$\xrightarrow{\text{receiver}}$ $s(t) = A_c\, m(t) \cos(2\pi f_c t)$.

$\xrightarrow{\text{local carrier}}$ $c'(t) = A_c' \cos(2\pi f_c t)$

phase is perfect

carrier " "

difference in the Amplitude

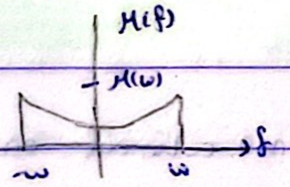


$v(t) = A_c A_c' m(t) \cos^2(2\pi f_c t)$

$$= \underbrace{\overset{\text{pass}}{\frac{A_c A_c'}{2} m(t)}}_{\text{Base band}} + \underbrace{\overset{\text{rejected}}{\frac{A_c A_c'}{2} m(t) \cos(2\pi f_c t)\, c'(t)}}_{\text{Pass band}}$$

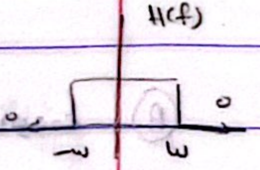
" DSB_SC signal centered at $2f_c$ ".

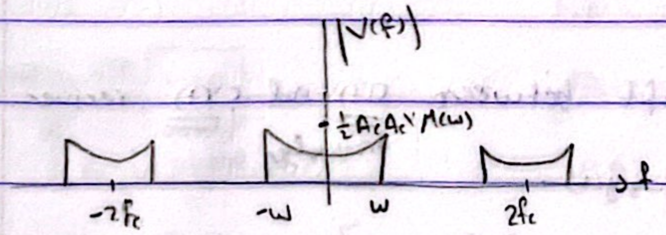
let

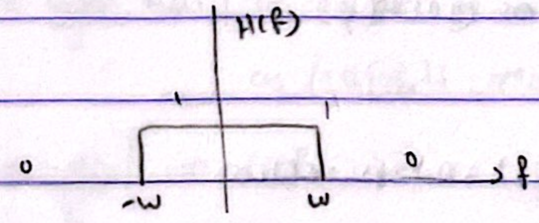


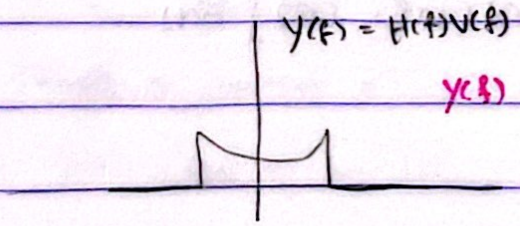
$\Rightarrow y(t) = v(t) * h(t)$

$Y(f) = V(f) H(f)$.







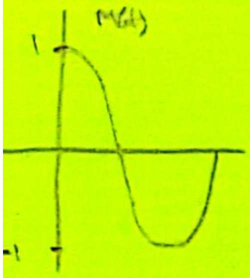


$Y(f) \propto M(f)$ "demodulation spectrum".

$y(t) = \text{LPF}\{v(t)\} = \frac{A_c A_c'}{2} m(t) \Rightarrow y(t) \propto m(t)$. m(t) is recovered from s(t) without distortion.

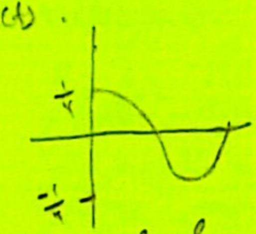
note:-

$$y(t) = \frac{A_c A_c'}{2} m(t)$$



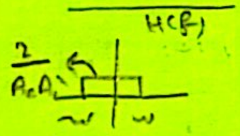
let $A_c = 1$, $A_c' = \frac{1}{2}$

$\Rightarrow y(t) = \frac{1}{4} m(t)$.



attenuated.

we can compensate the attenuation either by amplifier or using the filter gain



Summery:-

CASE I:-

$c'(t) = A_c' \cos(2\pi f_c t)$

$$y(t) = \frac{A_c A_c' m(t)}{2}$$

$= K m(t)$.

$y(t) \propto m(t)$

$\Rightarrow$ No distortion.

CASE II:-

$\boxed{A}$

$c'(t) = A_c' \cos(2\pi f_c t + \theta)$

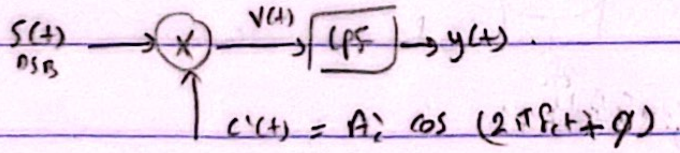
$$y(t) = \frac{A_c A_c'}{2} m(t) \cos\theta$$

$= K m(t)$

$\Rightarrow y(t)$ suffers from attenuation due to θ

CASE II:- Effect of carrier non-coherent on demodulation signal.

Ⓐ constant phase shift between $\underline{\underline{c(t)}}$ and $\underline{\underline{c'(t)}}$ receiver.
transmitter



$\Rightarrow V(t) = S(t)\, c'(t)$

$= A_c m(t) \cos(2\pi f_c t) \cdot A_c' \cos(2\pi f_c t + \phi)$

$$= \frac{A_c A_c'}{2} \left[\cos\phi + \cos(2\pi f_c t + \phi) \right] m(t)$$

$$v(t) = \underbrace{\frac{A_c A_c'}{2} m(t) \cos\theta}_{\text{Base band}} + \underbrace{\frac{A_c A_c'}{2} m(t) \cos(2\pi f_c t + \theta)}_{\text{pass band "DSB centered at } 2f_c\text{"}}.$$

pass by the LPF   rejected by the LPF

$$y(t) = \text{LPF}\{v(t)\}$$

$$= \frac{A_c A_c'}{2} \cos\theta \; m(t).$$

$\Rightarrow$

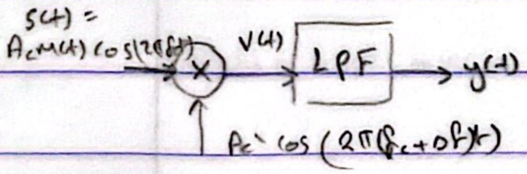
$\theta = 0 \Rightarrow y(t) = \frac{A_c A_c'}{2} m(t).$ } attenuation.

$\theta = \frac{\pi}{4} \Rightarrow y(t) = \frac{1}{2\sqrt{2}} A_c A_c' m(t).$

$\theta = \frac{\pi}{2} \Rightarrow y(t) = 0$ } critical phase shift.

"the message dissappers".

Ⓑ constant freq. between c(t) and c'(t):



$c(t) = A_c \cos(2\pi f_c t)$

$c'(t) = A_c' \cos(2\pi(f_c + \Delta f)t)$

e.g:- $f_c = 1000\,Hz$, $\Delta f = 50\,Hz$

$\Delta f << f_c$.

$$\Rightarrow v(t) = \frac{A_c A_c'}{2} m(t) \left[\cos(2\pi \Delta f t) + \cos(2\pi[2f_c]t + 2\pi \Delta f t) \right]$$

$$v(t) = \frac{A_c A_c'}{2} \left[\underbrace{m(t) \cos(2\pi \Delta f t)}_{\text{Baseband Signal}} + \underbrace{m(t) \cos(2\pi[2f_c]t + 2\pi \Delta f t)}_{\text{pass band signal}} \right]$$

"DSB-SC centered at Δf"   "DSB-SC centered at $2f_c + \Delta f$"

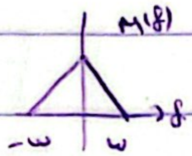
Pass by the LPF   rejected by the LPF

$y(t) = LPF\,[v(t)]$.

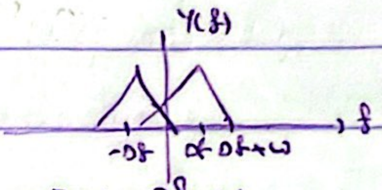
$$\boxed{y(t) = \frac{A_c A_c'}{2}\, m(t) \cos(2\pi \Delta f t)}$$ Distortion.

$y(t) \neq k\,m(t)$.

Eg:-



- B.W = W



- B.W = Δf + W

Example: let $m(t) = \cos(2\pi \cdot 1000\,t)$

$c(t) = \cos(2\pi \cdot 10^5 t)$ at transmitter.

$c'(t) = \cos(2\pi \cdot (10^5 + 100)\,t)$ at Receiver.

B.W for $m(t)$ is 1000 HZ.

$f_c = 10^5$ HZ.

$f_c' = 10^5 + 100$ HZ

$\Delta f = 100$ HZ.

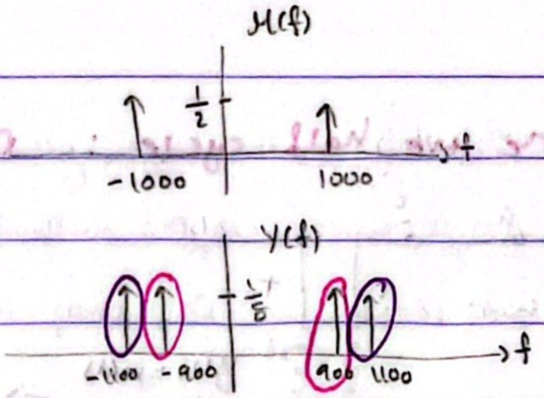
$$y(t) = \frac{A_c A_c'}{2}\, m(t) \cos(2\pi \Delta f t)$$

$$= \frac{1}{2}\, m(t) \cos(2\pi \cdot 100 t) \neq m(t).$$

$$Y(f) = \frac{1}{2} \cdot \frac{1}{2}\left[\delta(f-1000) + \delta(f+1000)\right] * \frac{1}{2}\left[\delta(f-100) + \delta(f+100)\right]$$

$$= \frac{1}{8}\Big[\delta(f-1000-100) + \delta(f-1000+100) +$$

$$\delta(f+1000-100) + \delta(f+1000-100)\Big]$$



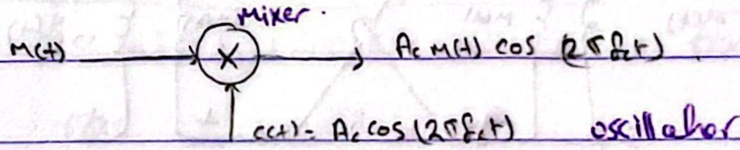
Four inverse:-

$$y(t) = \frac{1}{4}\cos(2\pi . 900) + \frac{1}{4}\cos(\pi . 1100) \neq m(t)$$ "distortion"

Generation of DSB_SC:-

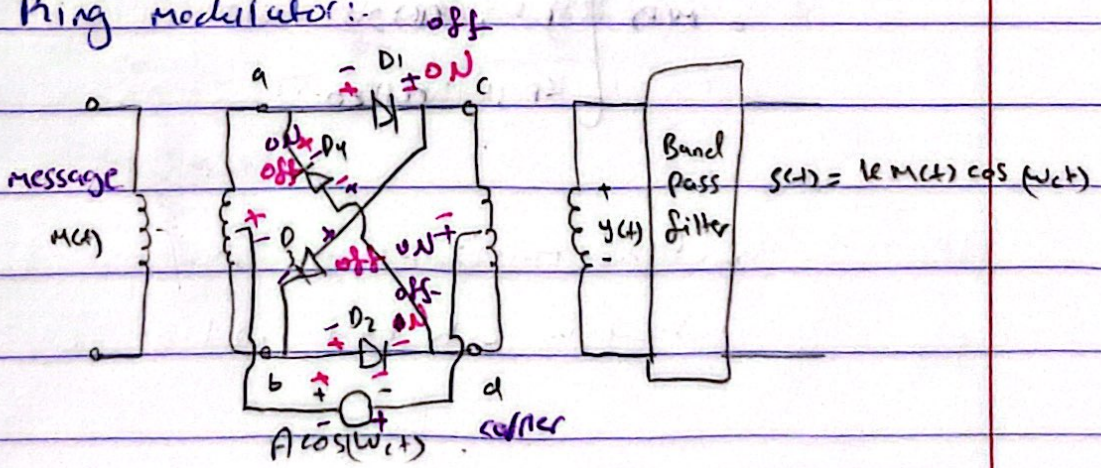
[1] product modulator:-

Such modulator can be obtained from a variable gain Amplifier.



this technique is usually applicable when low power levels are possible and over a limitted carrier freq. • using in software

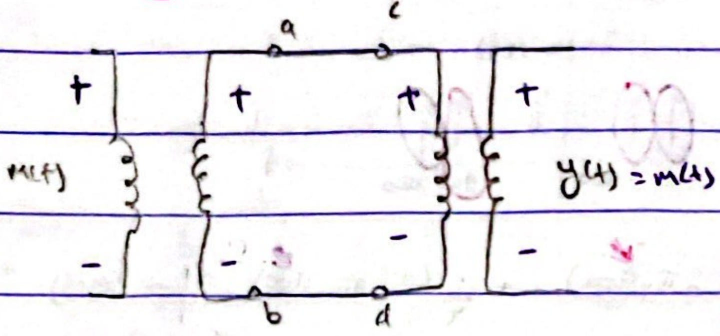
[2] Ring modulator:-



let $c(t) \gg m(t)$, here the carrier controls the diodes behaviour.

* During the +ve half cycle : D_1 and D_2 are on

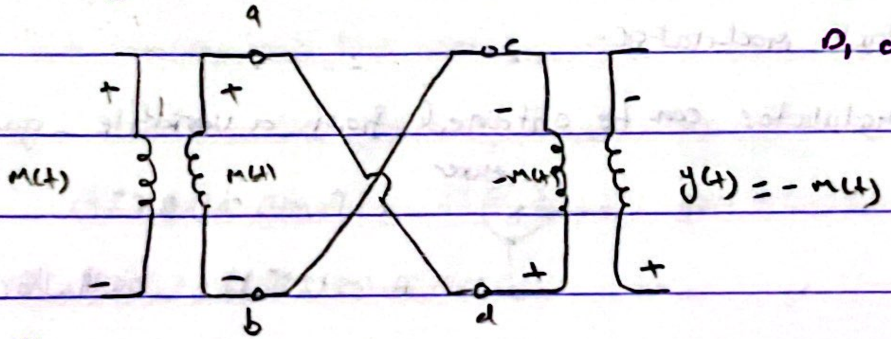
D_3 and D_4 are off



$\Rightarrow m(t) = y(t)$

* During the -ve half cycle $(c(t)<0)$: D_3 and D_4 are on

D_1 and D_2 are off

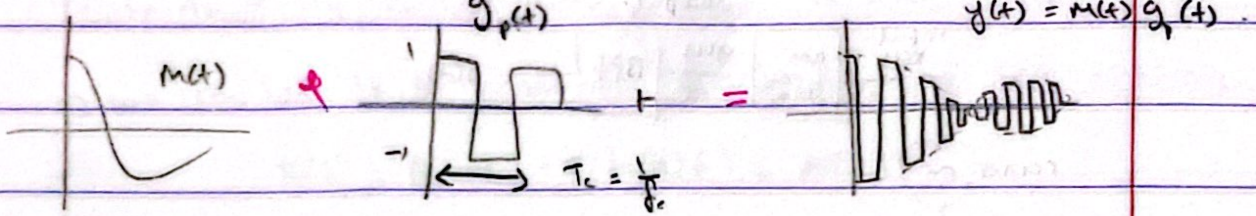


Here, $y(t) = -m(t)$.

$$y(t) = \begin{cases} m(t) & , c(t) > 0 \\ -m(t) & , c(t) < 0 \end{cases}$$

$$= m(t) \begin{cases} 1 & , c(t) > 0 \\ -1 & , c(t) < 0 \end{cases}$$

Mathematically, y(t) behaves as if multiplied by the switching function $g_p(t)$. Where $g_p(t)$ is the square periodic function with period $T_c = \frac{1}{f_c}$, f_c is the carrier frequency.



by expanding $g_p(t)$ in FS, we get:-

$$y(t) = m(t) \cdot \left[\frac{4}{\pi} \cos(2\pi f_c t) - \frac{4}{3\pi} \cos(3 \cdot 2\pi f_c t) + \frac{4}{5\pi} \cos(5 \cdot 2\pi f_c t) - \cdots \right.$$

$$= \underbrace{m(t) \frac{4}{\pi} \cos(2\pi f_c t)}_{\text{DSB-SC at } 1f_c} - \underbrace{m(t) \frac{4}{3\pi} \cos(3 \cdot 2\pi f_c t)}_{\text{DSB-SC at } 3f_c} + \underbrace{m(t) \frac{4}{5\pi} \cos(5 \cdot 2\pi f_c}_{\text{DSB-SC at } 5f_c}$$

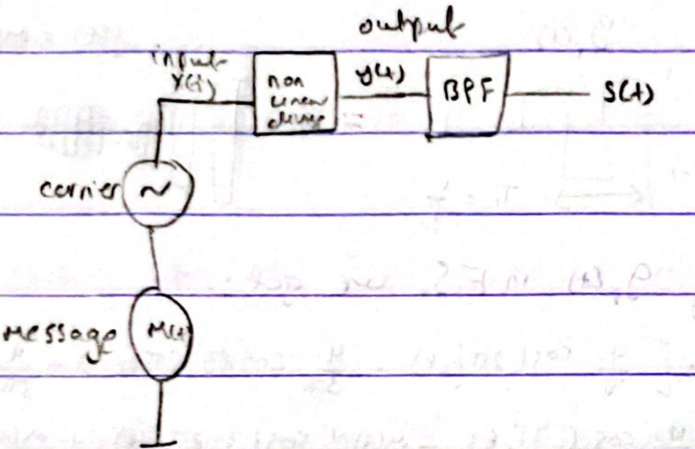
Pass Rejected Rejected

* when y(t) passes through the BPF with center frequency f_c and Bandwidth = 2W, the only component that appears at the output is the desired DSB-SC signal, which is :-

$$\boxed{s(t) = \frac{4}{\pi} m(t) \cos(2\pi f_c t)}$$

3) Nonlinear Modulation:

Modulation can also be acheired by using nonlinear devices, such as diode or transistor.



Let the nonlinear characterstic be of the form:-

$$y(t) = a_0 x(t) + a_1 x^3(t)$$

Let $x(t) = A\cos(\overset{\pi f_c/2}{2\pi f_c t}) + m(t)$, $m \Rightarrow$ message

then :- $y(t) = a_0 (A\cos(2\pi f_c) + m(t)) +$

$$a_1 (A\cos(2\pi f_c) + m(t))^3 .$$

$$= \overset{\alpha}{a_0} \cos(2\pi f_c t) + \overset{\alpha}{a_0 m(t)} + \overset{\alpha}{a_1 A^3} \cos^3(2\pi f_c t)$$

$$+ \overset{\alpha}{a_1 m^3(t)} + \boxed{3a_1 A^2 m(t) \cos^2(2\pi f_c t)} + \overset{\alpha}{3Aa} \cos(2\pi f_c t)$$

expand this

After some algebraic manipulations, a DSB-SC term appears in x(t) along with other undesirable terms.

the BPF will admit the desired DSB-SC signal

$$\boxed{s(t) = \frac{3A^2a}{2} m(t) \cos(2\pi \cdot 2f_c t)}$$

DSB-SC centerd at $2f_c$.

note that the carrier freq. $= 2f_c$ in this case:-

=> we can let it be centerd at f_c if we let $x(t)$ =>

$$x(t) = A\cos(2\pi(f_c/2)t) + m(t)$$

Example:-

$m(t) = 2\cos(2\pi \overset{f_1}{\underline{\underline{40}}}t) + 4\cos(2\pi \overset{f_2}{\underline{\underline{80}}}t)$.

$c(t) = 4\cos(2\pi 1000t)$. $A_c = 4$, $f_c = 1\text{KHZ}$.

they applied to a modulator that generate the double sideband suppressed carrier signal $s(t)$.

a) Find the average power of $m(t)$.

$$\frac{A_c^2}{2} \Rightarrow \frac{4}{2} + \frac{16}{2} = 10 \text{ w}$$

b) Find the time domain expression of the modulation signal $s(t)$.

$$S_{DSB-SC}(t) = m(t) \cdot c(t)$$

$$= 8\cos(2\pi \cdot 40t)\cos(2\pi \cdot 1000t) + 16\cos(2\pi \cdot 80t)\cos(2\pi \cdot 1000t) .$$

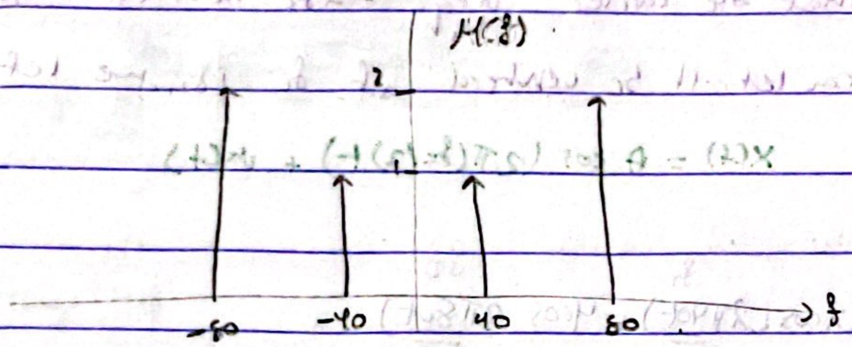
$$= \frac{8}{2}\cos(2\pi \cdot 1040t) + \frac{8}{2}\cos(2\pi \cdot 960) +$$

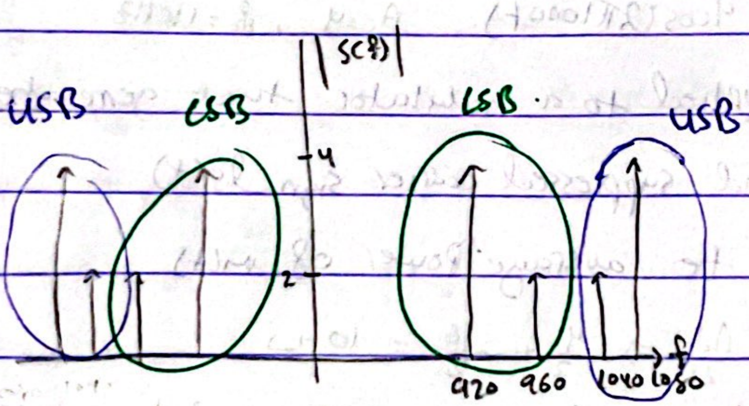
$$\frac{16}{2}\cos(2\pi \cdot 1080t) + \frac{16}{2}\cos(2\pi \cdot 920t).$$

c) Find and plot the spectrum $S(f)$.

$$S(f) = \frac{4}{2}\left[\delta(f-1040) + \delta(f+1040)\right] + \frac{4}{2}\left[\delta(f-960) + \delta(f+960)\right]$$

$$+ \frac{8}{2}\left[\delta(f-1080) + \delta(f+1080)\right] + \frac{8}{2}\left[\delta(f+920) + \delta(f-920)\right].$$





d) Find the B.w of $\underset{DSB}{S(t)}$ $\Rightarrow$ $1080 - 920 = 160$ HZ.

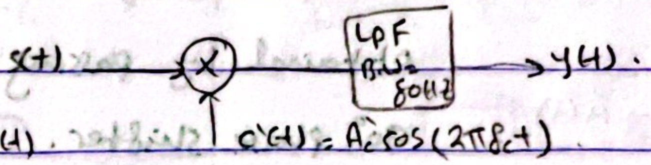
or

$B.w = 2w$

$= 2 * B.w$ of $m(t)$

$= 2 * 80 = 160$ HZ.

e) Draw the block diagram of the demodulator used to recover m(t) from s(t) without distortion specifying the details of each block. "perfect demodulation"?



$c'(t) = A_c' \cos(2\pi f_c t)$

$= 1 . \cos(2\pi . 1000t)$.

$\Rightarrow y(t) = \frac{A_c A_c'}{2} m(t)$.

$= \frac{4 . 1}{2} m(t)$

$\boxed{= 2 m(t)}$

AM modulation :-

[1] Normal AM
- Power efficiency η low.
- $B.W = 2 * m(t)$ BW.

[2] DSB-SC
- $\eta = 100\%$
- $B.W = 2 * m(t)$ BW.

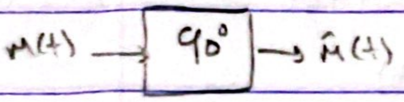
[3] SSB — $B.W = m(t) B.W$.

Single sideband Modulation (SSB).

* t- domain expression:-

$$\underset{SSB}{S(t)} = A_c m(t) \cos(2\pi f_c t) \pm A_c \hat{m}(t) \sin(2\pi f_c t)$$

$\hat{m}(t) \Rightarrow$ Hilbert transform of m(t)



obtained by passing m(t) through 90° phase shifter.

+ve sign used to retain lower sideband (LSB).

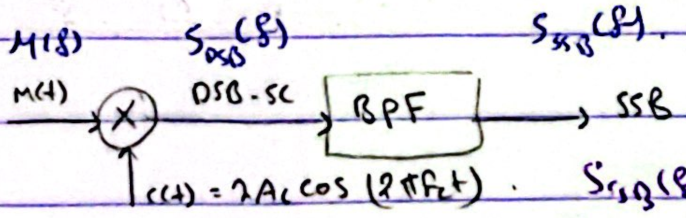
-ve sign used to retain upper sideband (USB).

* from freq domain :-
point of view we can generate SSB by:

I Frequency discrimination :

step 1 :- DSB-SC signal.

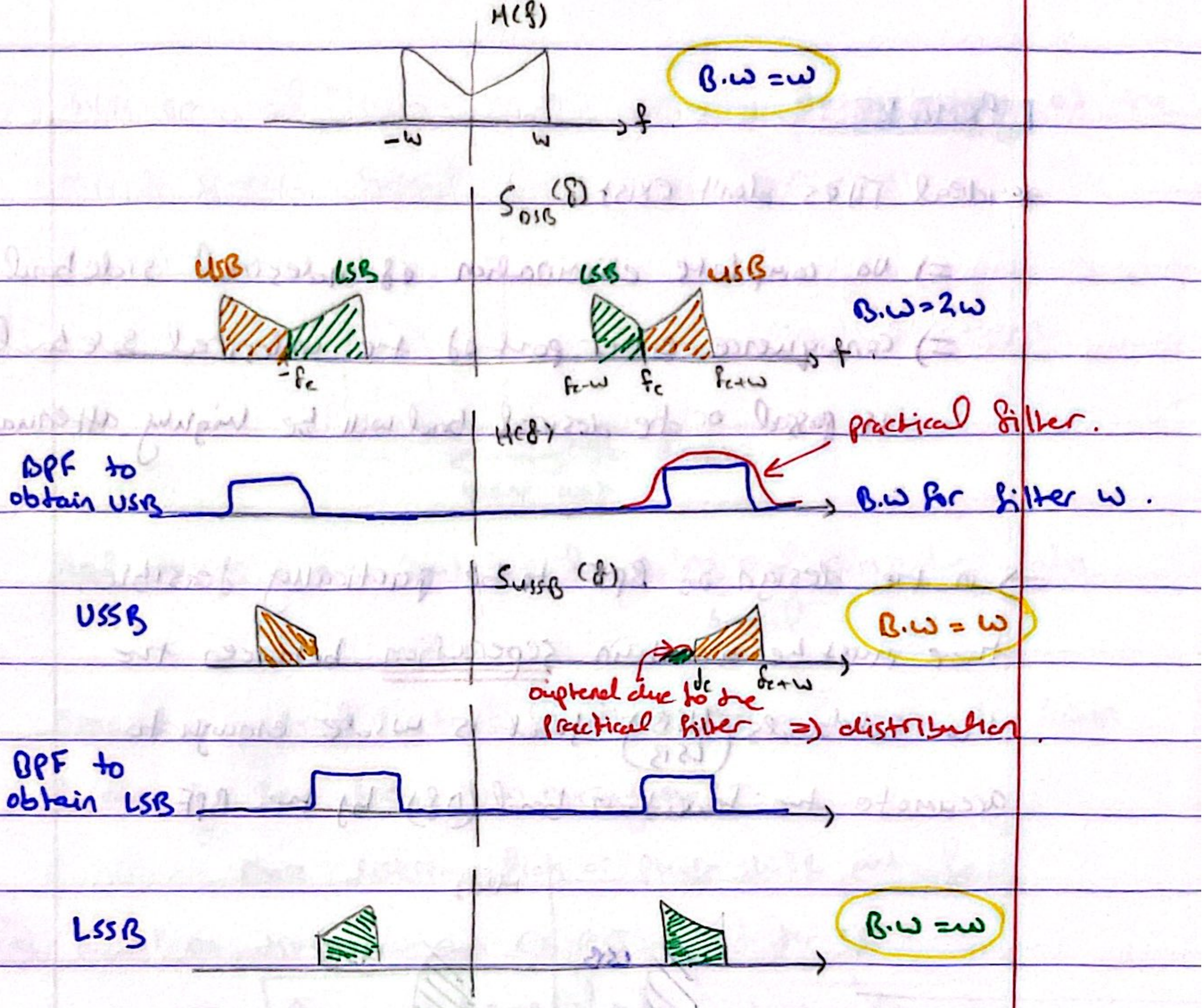
step 2 :- Bandpass Filter "BPF".



$S_{SSB}(f) = \underset{DSB}{S(f)} \cdot H(f)$

in time domain : $y(t) = S_{DSB}(t) * h(t)$.

in freq domain : $Y(f) = \underset{DSB}{S(f)} \cdot H(f)$.



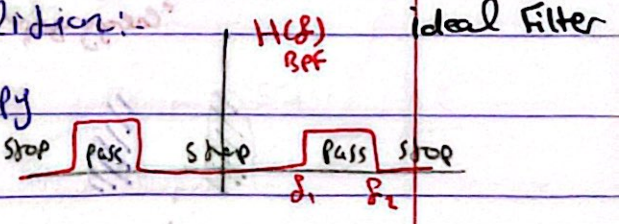
The BPF must satisfy two condition:

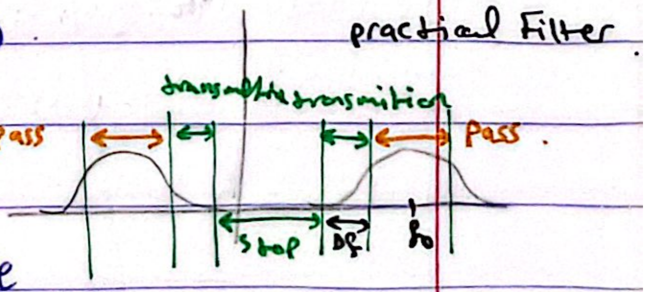
[1] pass band of the filter must occupy the same freq. rang an the desired sideband (USB or LSB).

[2] Δf must be at least 1% f0

=> $\boxed{\Delta f \geq 0.01 f_0}$

Rule of thumb for relizable filter.





Δf: width of the transmition band

f0: center freq of the filter.

Remarks:

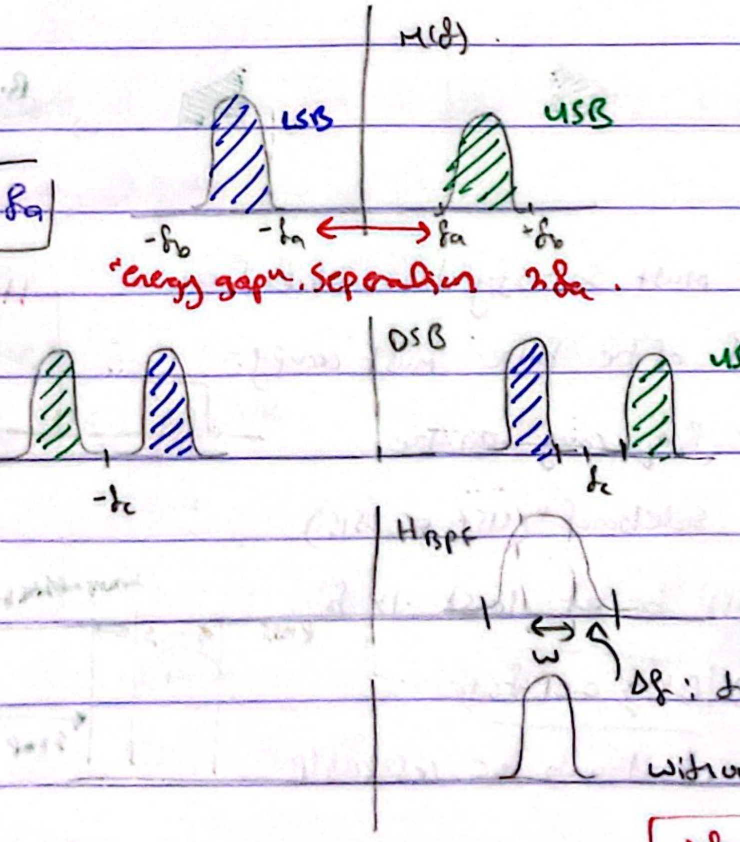
* ideal Filters don't exist.

=) No complete elimination of undesired sideband

=) consequence either part of the undesired Side band is passed or the desired band will be highly attenuated.

=> in the design of BPF to be practically feasible, there must be a certain seperation between the two Sidebands $\binom{USB}{LSB}$ that is wide enough to accumote the transition band (Δf) by the BPF.

$M(f) = 0, \ -f_a < f < f_a$

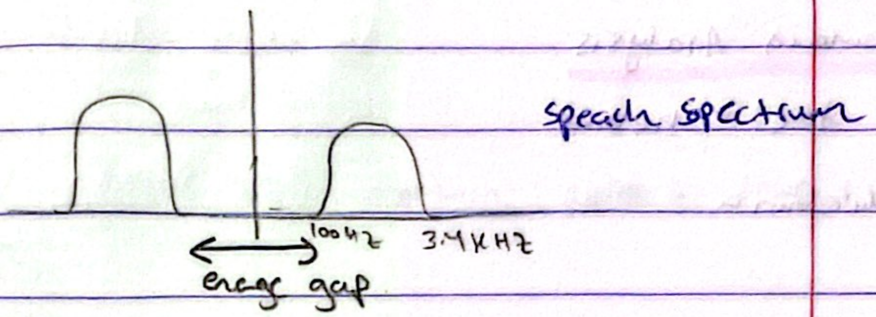


"energy gap". Seperation $2f_a$.

Δf : transmition rate without distortion

$\boxed{\Delta f \le 2f_a}$

Such required "Energy gap" limits the applicability of the SSB to speech signal for which $f_a \simeq 100HZ$.



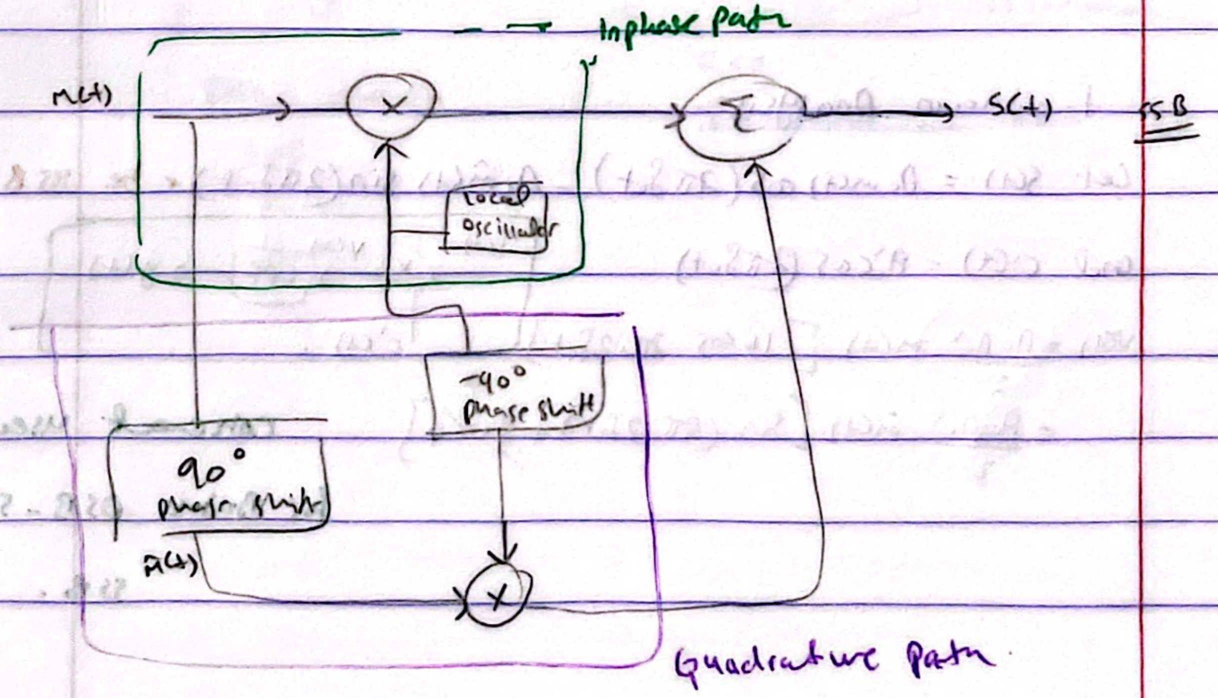
and makes it not suitable for video, and computer data signal

Since the spectral content in those signal extends down to almost zero freq. "No energy gap".

phase discrimination or phase shift method

[2] Based on time domain expression of the SSB:-

$$s(t) = \boxed{A_c m(t) \cos(2\pi f_c t)} \pm \boxed{A_c \hat{m}(t) \sin(2\pi f_c t)}$$



Demodulation of SSB

"at the receiver side"

In t-domain Analysis:-

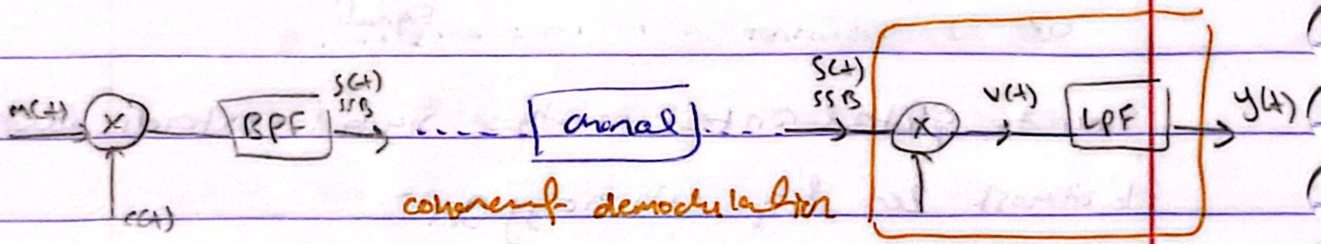
we use the "coherent demodulation".

How to generate SSB?

① Freq. discriminator "Filtering Method"

② Phase discrimination "phase shift" Method.

two Blocks "input path + Quarture Path".



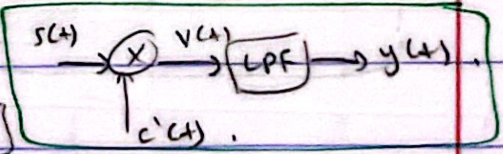
$c'(t) = A_c' \cos(2\pi f_c t)$.

oscillator at the receiver should have the same freq and phase as those of the transmitter carrier.

t-domain Analysis:-

let $s(t) = A_c m(t) \cos(2\pi f_c t) - A_c \hat{m}(t) \sin(2\pi f_c t)$ "be USSB"

and $c'(t) = A_c' \cos(2\pi f_c t)$.



$$v(t) = \frac{A_c A_c'}{2} m(t) \left[1 + \cos 2\pi \cdot 2 f_c t \right]$$
$$- \frac{A_c A_c'}{2} \hat{m}(t) \left[\sin(2\pi \cdot 2 f_c t) + \overset{0}{\cancel{\sin 0}} \right]$$

coherent used for both DSB-SC SSB.

$$v(t) = \underbrace{\frac{A_c A_c'}{2} m(t)}_{\text{Baseband.}} + \underbrace{\frac{A_c A_c'}{2} m(t) \cos(2\pi \cdot 2f_c t)}_{\text{passband at } 2f_c} - \underbrace{\frac{A A_c'}{2} \hat{m}(t) \sin(2\pi \cdot 2f_c t)}_{\text{passband at } 2f_c.}$$

$$y(t) = LPF\{v(t)\} = \frac{A_c A_c'}{2} m(t).$$

LPF admits only the 1st term.

output $y(t) \propto m(t)$ " No distortion (perfect demodulation)"

explation:-

$$s(t) = A_c m(t) \cos(w_c t) - A_c \hat{m}(t) \sin(w_c t).$$

$$c(t) = A_c' \cos(w_c t).$$

$$v(t) = s(t)\, c(t)$$

sin(جمع) - sin(طرح)

$$= A_c A_c' m(t) \cos^2(w_c t) - A_c A_c' \hat{m}(t) \underbrace{\sin(w_c t) \cos(w_c t)}$$

$$= A_c A_c' m(t) \left[\frac{1}{2} + \frac{1}{2} \cos(2w_c t) \right] - A_c A_c' \hat{m}(t) \frac{1}{2} \left[\sin(2w_c t) + \cancel{\sin(0)} \right]$$

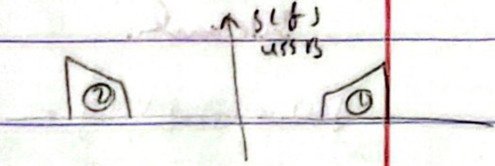
$$= \frac{A_c A_c'}{2} m(t) + \underbrace{\cancel{\frac{A_c A_c'}{2} m(t) \cos(2w_c t)} - \cancel{\frac{A_c A_c'}{2} \sin(2w_c t)}}_{\text{BSB Pass}}.$$

rejected

Phass through LPF.

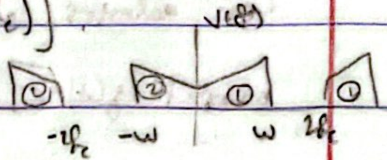
Freq domain Analysis for the Demodulation of USSB.

Step(1):-



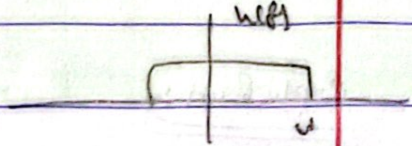
$V(f) = S_{USSB}(f) * \left[\frac{A_c}{2} \delta(f - f_c) + \frac{A_c}{2} \delta(f + f_c) \right]$.

$= \frac{A_c}{2} S(f - f_c) + \frac{A_c}{2} S(f + f_c)$.



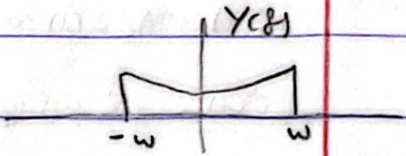
Step(2):-

given LPF $h(t) \longleftrightarrow H(f)$.



$y(t) = h(t) * v(t)$.

$Y(f) = H(f) V(f)$.



$\Rightarrow Y(f) \propto M(f)$.

$y(t) \propto m(t)$

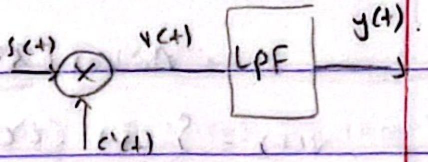
$\Rightarrow$ the message is reccored without distortion.

⇒ coherent Demodulation SSB :-

[1] $c'(t) = A_c' \cos(2\pi f_c t)$.

we have perfect demodulation.

⋮



$$y(t) = \frac{A_c A_c'}{2} m(t).$$

⇒ no destortion.

[2] constant phase shift between c(t) and c'(t).

$c'(t) = A_c' \cos(\omega_c t + \emptyset)$.

let $s(t)_{USSB} = A_c m(t) \cos(\omega_c t) - A_c \hat{m}(t) \sin(\omega_c t)$.

$$v(t) = \frac{A_c A_c'}{2} m(t) \left[\cos(2\omega_c t + \emptyset) + \cos(\emptyset)\right] - \frac{A_c A_c'}{2} \hat{m}(t) \left[\sin(2\omega_c t + \emptyset) + \sin(\emptyset)\right]$$

$$v(t) = \underbrace{\frac{A_c A_c'}{2} m(t) \cos(2\omega_c t + \emptyset) - \frac{A_c A_c'}{2} \hat{m}(t) \sin(2\omega_c t + \emptyset)}_{\text{Pass band signal}} + \underbrace{\frac{A_c A_c'}{2} m(t) \cos\emptyset - \frac{A_c A_c'}{2} \hat{m}(t) \sin\emptyset}_{\text{Low pass signal + Baseband}}.$$

$$y(t) = LPF\{v(t)\}$$
$$= \frac{A_c A_c'}{2}\left[m(t)\cos\emptyset - \hat{m}(t)\sin\emptyset\right] \neq k\,m(t).$$

⇒ there is destortion.

[3] constant freq. shift

$c(t) = A_c \cos(w_c t)$.

$c'(t) = A_c' \cos((w_c + \Delta w)t)$

$= A_c' \cos(2\pi(f_c + \Delta f)t)$.

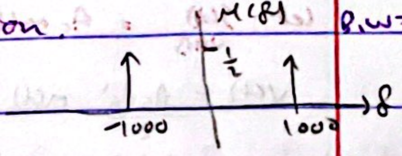
$v(t) = \underset{USSB}{S}(t) \cdot c'(t)$.

$y(t) = LPF[v(t)]$.

$y(t) = \frac{A_cA_c'}{2} m(t) \cos(2\pi\Delta f t) + \frac{A_cA_c'}{2} \hat{m}(t) \sin(2\pi\Delta f t) \neq K m(t)$

it's lower SSB modulated signal on a carrier freq. $= \Delta f$.

=) once again we have distortion.



B.W = 1000

Ex:- let $m(t) = \cos(2\pi \cdot 1000t)$

$c(t) = \cos(2\pi \cdot 10000t)$

$c'(t) = \cos(2\pi(\overset{f_c}{10000} + \overset{\Delta f}{100})t)$

$s(t)$ be USSB, then:-

$y(t) = \frac{A_cA_c'}{2} m(t) \cos(2\pi\Delta f t) + \frac{A_cA_c'}{2} \hat{m}(t) \sin(2\pi\Delta f t)$

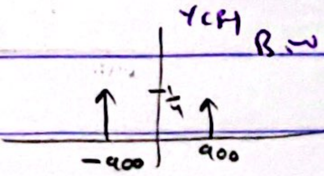
$= \frac{1}{2}\cos(2\pi \cdot 1000t)\cos(2\pi 100t) + \frac{1}{2}\sin(2\pi \cdot 1000t)\sin(2\pi \cdot 100t)$.

note that:- $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$.

$\alpha = 2\pi \cdot 1000$, $\beta = 2\pi \cdot 900$.

$y(t) = \frac{1}{2}\cos(2\pi \cdot 1000 - 2\pi \cdot 100)t$

$= \frac{1}{2}\cos(2\pi 900t)$.



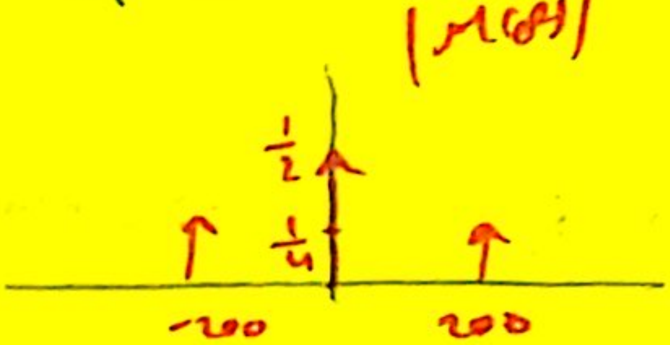
B.W = 900.

$$\Rightarrow \quad m(t) = \cos^2(2\pi \cdot 100t).$$

$$= \frac{1}{2} + \frac{1}{2}\cos(2\pi \cdot 200t).$$

$$M(f) = \frac{1}{2}\delta(f) + \frac{1}{4}\delta(f-200)$$

$$+ \frac{1}{4}\delta(f+200).$$



Example:- $m(t) = \cos(2\pi \cdot 100t) + \cos(2\pi \cdot 200t) \rightarrow B.W = 200$.

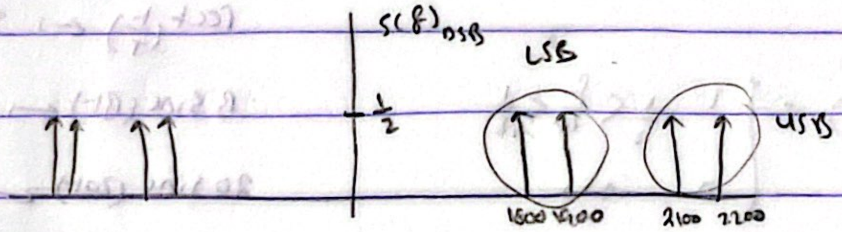
$c(t) = 2\cos(2\pi \cdot 2000t)$

① write t-domain expression for DSB-SC, LSSB?

DSB-SC

$s_{DSB}(t) = m(t) \cdot c(t)$.

$$= \boxed{2 \cdot \frac{1}{2}\cos(2\pi \cdot 2100t) + \cos(2\pi \cdot 2200t)} \boxed{+ 2 \cdot \frac{1}{2}\cos(2\pi \cdot 1900t) + \cos(2\pi \cdot 1800t)}$$



LSSB

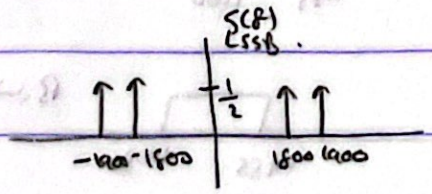
$s_{LSSB}(t) = A_c\, m(t) \cos(2\pi f_c t) + A_c\, \hat{m}(t) \sin(2\pi f_c t)$.

$= 2m(t)\cos(2\pi \cdot 200t) + 2\hat{m}(t)\sin(2\pi \cdot 2002t)$.

$m(t) =$ given

$\hat{m}(t) = \sin(2\pi \cdot 100t) + \sin(2\pi \cdot 200t)$.

$s(t) = \cos(2\pi \cdot 1900t) + \cos(2\pi \cdot 1800t)$.



Example:- $M(t) = 30 \, \text{sinc}(20t) + \cos(2\pi \cdot 15t)$.

$c(t) = 2\cos(2\pi \cdot 1000t)$.

$\boxed{1}$ $S_{DSB}(t) = M(t) \cdot c(t)$.

$= 40 \, \text{sinc}(20t)\cos(2\pi \cdot 1000t) + 2\cos(2\pi \cdot 15t)\cos(2\pi \cdot 1000t)$

$\boxed{2}$ $S_{DSB}(f) \Rightarrow$

Remember:-

$\text{rect}\left(\frac{t}{T}\right) \leftrightarrow T \, \text{sinc}(Tf)$.

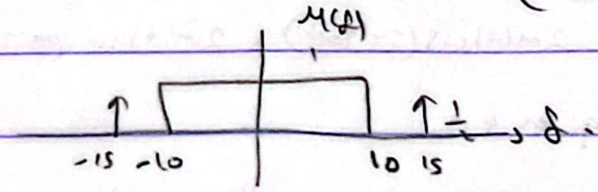
$B \, \text{sinc}(Bt) \leftrightarrow \text{rect}\left(\frac{+f}{B}\right)$.

$20 \, \text{sinc}(20t) \leftrightarrow \text{rect}\left(\frac{f}{20}\right)$

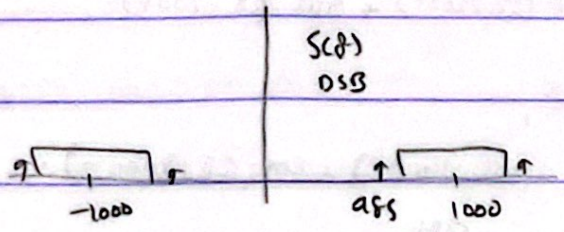
$$\text{rect}\left(\frac{f}{20}\right) = \begin{cases} 1 , & -\frac{1}{2} < \frac{f}{20} < \frac{1}{2} \\ 0 , & o.w \end{cases}$$

$$= \begin{cases} 1 , & -10 < f < 10 \\ 0 , & o.w \end{cases}$$

$2 \times 20 \, \text{sinc}(20t)\cos(2\pi \cdot 1000t) \leftrightarrow 2 \cdot \frac{1}{2} \, \text{rect}\left(\frac{f-1000}{20}\right) + 2 \cdot \frac{1}{2} \, \text{rect}\left(\frac{f+1000}{20}\right)$

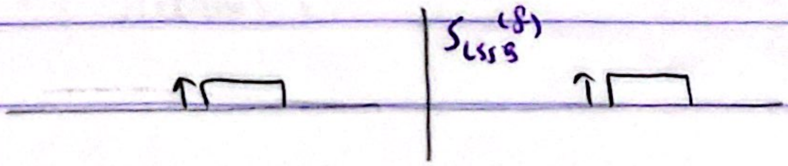


$B_W = 15$



$B.W = 30$

$B.W = 15$.



chapter (4):- Angle Modulation

→ Freq Modulation (FM).

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\alpha)d\alpha\right].$$

→ note:-

Modem: Modulator/Demodulator

→ phase Modulation (PM).

$$S_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right].$$

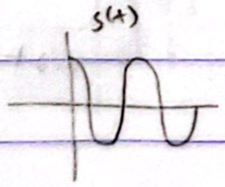
* in angle modulation, the angel of the carrier wave is varied according the message while the amplitude is maintained constant $s(t) = A_c \cos(\theta(t))$

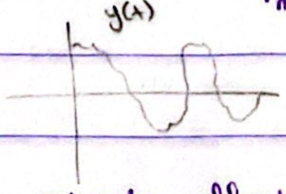
$\uparrow$ constant $\uparrow$ instant phase varies with the message.

* important feature in Angle modulation over AM is that it provides better discriminat against noise and interferense. the cost is that Angle Modulation require more transmission BW and the modulator / demodulator complexity increase.

modulated signal $s(t)$ → channel "cable.." → "at the receiver" $y(t) = s(t) + n(t)$

$\uparrow$ noise





amplitude affected by the noise not the phase "Freq"..

- * to generate an angle Modulated signal, either the phase or the time derivative of the phase phase is varied linearly with the message.

$$s(t) = A_c \cos(\overset{\text{instant phase}}{\theta_i(t)})$$

$$= A_c \cos(\underbrace{2\pi f_c t}_{\text{carrier freq.}} + \overset{\text{phase}}{\theta(t)})$$

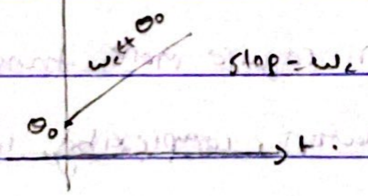
let us introduce the concept of instantaneous freq. : $f_i(t)$.

$x(t) = \cos(\omega_c t + \theta_0)$. otherwise " if you don't have

freq = constant = ω_c. linear"

$\theta_i(t) = \omega_c t + \theta_0$ instant freq.

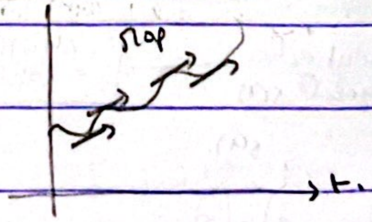
Plot:-



$$\omega_i(t) = \frac{d\,\theta_i(t)}{dt}$$

$$2\pi f_i(t) = \frac{d\,\theta_i(t)}{dt}$$

$$\boxed{f_i(t) = \frac{1}{2\pi}\frac{d\,\theta_i(t)}{dt}}$$



$$s(t) = A_c \cos(\underbrace{2\pi f_c t + \theta(t)}_{\theta_i(t)})$$

the instantaneon freq:-

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \frac{1}{2\pi} \frac{d}{dt}\left[2\pi f_c t + \theta_i\right]$$

$$= \frac{1}{2\pi}\left[2\pi f_c + \frac{d}{dt}\theta_i(t)\right].$$

$$= f_c + \frac{1}{2\pi}\frac{d}{dt}\theta(t).$$

$\boxed{II}$ phase modulation (PM).

It is a form of angle modulation in which the instant angle $\theta_i(t)$ is varried linearly with the message $m(t)$.

$$s_{PM}(t) = A_c \cos\left[2\pi f_c t + K_p m(t)\right]$$

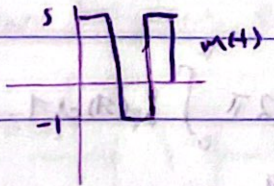
where $\theta_i(t) = \underbrace{2\pi f_c t}_{\text{angle of unmodulated carrier}} + K_p m(t).$

K_p → phase sensitivity "constant" "rad/volt".

peak-phase devition. ← عالجبرية مضمنة ال

$$\Delta\theta_{max} = (\theta_i(t) - 2\pi f_c t)_{max} = K_p |m(t)|_{max}$$

e.g:-



$\Rightarrow |m(t)|_{max} = 5$ volt.

let $K_p = 10$ rad/volt.

$\therefore \Delta\theta_i = 50$ rad.

instant phase for PM:-

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

but $\theta(t) = k_p m(t)$.

$$\Rightarrow \boxed{f_i(t) = f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t)}$$

Peak freq. deviation $\Delta f_{max} = (f_i(t) - f_c)_{max}$

$$= \frac{k_p}{2\pi}\left|\frac{d}{dt}m(t)\right|_{max}.$$

[2] Freq. Modulation (FM).

it's a form of Angle Modulation in which the instantaneous frequency $f_i(t)$ is varied linearily with the message $m(t)$.

$$f_i(t) = \underbrace{f_c}_{\text{freq. of unmodulated carrier}} + \underbrace{k_f}_{\text{freq. sensitivity factor [Hz/volt]}} m(t)$$

remember:-

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) \Rightarrow \theta_i(t) = 2\pi\int_0^t f_i(\tau)\,d\tau.$$

$$\theta_i(t) = 2\pi\int_0^t [f_c + k_f m(\tau)]\,d\tau$$

$$= 2\pi\int_0^t f_c\,d\tau + 2\pi k_f\int_0^t m(\tau)\,d\tau.$$

$$= 2\pi f_c t + 2\pi k_f\int_0^t m(\tau)\,d\tau.$$

$\Rightarrow$ $S_{FM}(t) = A_c \cos(\Theta_i(t))$

where $\Theta_i(t) = 2\pi f_c t + 2\pi k_f \int^t m(\tau)d\tau$

- peak phase deviation :-

$$\Delta\Theta_{max} = (\Theta_i(t) - 2\pi f_c t)_{max} = 2\pi k_f \left(\int^t m(\tau)d\tau\right)_{max}$$

- peak freq. deviation:-

$$f_i(t) = f_c + k_f m(t).$$

$$= \Delta f_{max} = k_f |m(t)|_{max}.$$

*** Summary:-**

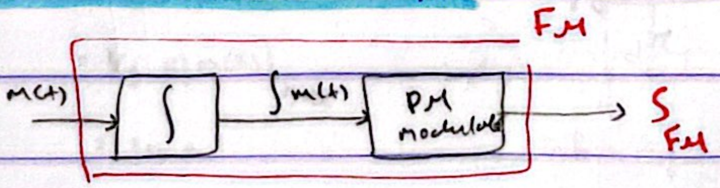
Angle modulation: $s(t) = A_c \cos(\Theta_i(t))$.

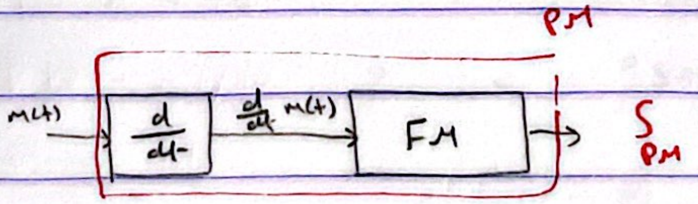
→ PM, $s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$.

→ FM, $s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int^t m(\tau)d\tau]$.

* P_{avg} for PM or FM signal = constant = $\frac{A_c^2}{2}$

Relation between PM and FM:





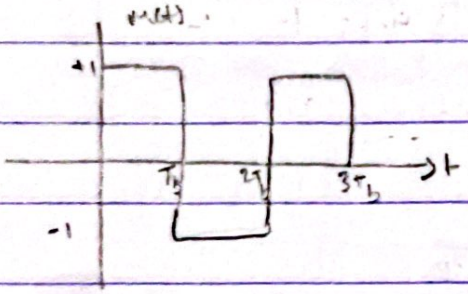
Example:- Binary freq. shift keying (BFSK)

consider the periodic square message m(t) modulates

the carrier : $c(t) = A_c \cos(2\pi.100t)$

to produce FM Signal : $s_{FM}(t) = A_c \cos(\overset{\theta_i(t)}{\boxed{2\pi.100t + 2\pi k_f \int^{t} m(\tau)d\tau}})$

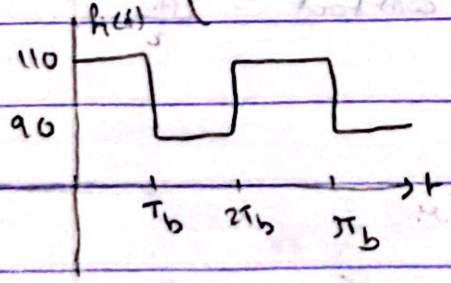
$k_f = 10$ HZ/Volt.



(a) Find and plot the instant freq. $f_i(t)$!

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) = \ldots = f_c + k_f m(t)$$

$$= \begin{cases} 100 + 10(+1) = 110 & 0 < t < T_b \\ 100 + 10(-1) = 90 & T_b < t < 2T_b \end{cases}$$



- "the freq. hops between two values"

(b) Find the time domain expression for $S_{FM}(t)$.

$$S_{FM}(t) = A_c \cos\left(2\pi\, 100t + 2\pi K_f \int_0^t m(\lambda)\,d\lambda\right)$$

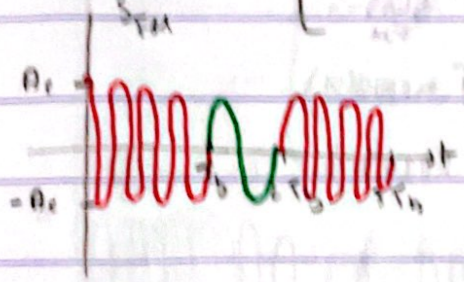
$\Rightarrow$ for $0 \le t < T_b$, $m(t) = +1$, $S_{FM}(t) = A_c \cos\left(2\pi\, 100t + 2\pi\, 10 \int_0^t (1)\,dt\right)$

$$= A_c \cos(2\pi\, 100t + 2\pi\, 10t)$$

$$= A_c \cos(2\pi\, 110t).$$

$\Rightarrow$ for $T_b \le t < 2T_b$, $m(t) = -1$, $S_{FM}(t) = A_c \cos(2\pi\, 90t)$

$$\therefore S_{FM}(t) = \begin{cases} \underline{A_c \cos(2\pi\, 110t)} , & 0 \le t < T_b \\ \underline{A_c \cos(2\pi\, 90t)} , & T_b \le t < 2T_b \end{cases}$$



كل ما زادت ال freq بزداد عدد اللفات

وكل ما قلت بقل عدد اللفات.

(c) Find the peak freq. deviation.

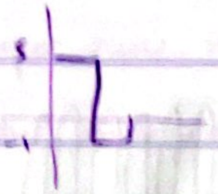
$$\Delta f = f_i(t) - f_c$$

$$= K_f \times |m(t)|_{max}$$

$$= 10 \times 1$$

$$= 10 \text{ Hz}.$$

extra example $\Rightarrow$



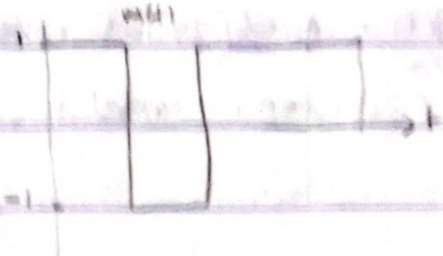
$K_f = 10$ Hz/Volt

$$\Delta f = K_f \times (m(t))_{max}$$

$$= 10 \times 5$$

$$= 50 \text{ Hz}.$$

Example: Sketch FM, PM wave for the digital modulating signal m(t).



consider $k_p = \frac{\pi}{2}$

$k_f = 10^5$

$f_c = 100$ MHz

II) FM Signal

$f_i(t) = f_c + k_f\, m(t)$

$$= \begin{cases} 10^8 + 10^5(+1) = 100.1\,MHz, & m(t) = +1 \\ 10^8 + 10^5(-1) = 99.9\,MHz, & m(t) = -1 \end{cases}$$

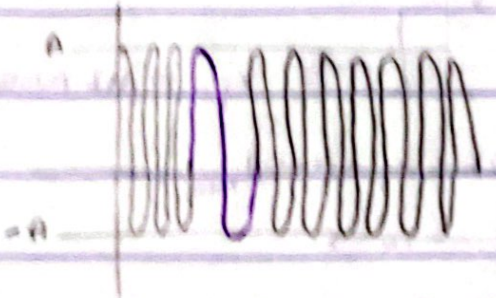
Write t-domain expression for $s(t)_{FM}$.

$$s(t)_{FM} = A_c \cos\left(2\pi \cdot 100t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

⇒ for $m(t) = 1$, $A_c \cos(2\pi \cdot 100t + 2\pi 10^5 t)$

⇒ for $m(t) = -1$, $A_c \cos(2\pi \cdot 100t - 2\pi \cdot 10^5 t)$.

ج كلما زاد التردد يزداد عدد الـ C :-

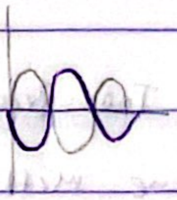


$s(t)_{FM}$ =

Note: this scheme of carrier freq. modulation is called Freq. Shift keying (FSK).

cuz information digits are transmitted by keying different freq.

$\boxed{2}$ PM signal.



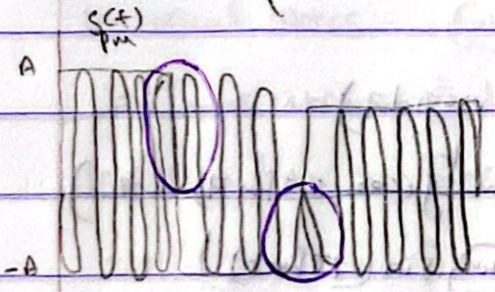
$$s(t) = A\cos(\Theta_i(t))$$

$$\Theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$= \begin{cases} 2\pi f_c t + \frac{\pi}{2}(+1) & , m = +1 \\ 2\pi f_c t + \frac{\pi}{2}(-1) & , m = -1 \end{cases}$$

$$\Rightarrow s(t)_{PM} = \begin{cases} A\cos\left(2\pi f_c t + \frac{\pi}{2}\right) = -A\sin(2\pi f_c t) & , m = +1 \\ A\cos\left(2\pi f_c t - \frac{\pi}{2}\right) = A\sin(2\pi f_c t) & , m = -1. \end{cases}$$

$$= \begin{cases} -A\sin(2\pi f_c t) & , m = +1 \\ A\sin(2\pi f_c t) & , m = -1 \end{cases}$$



notes -

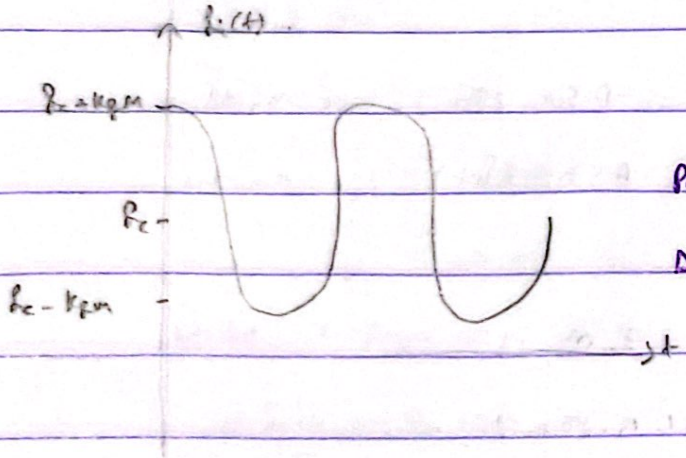
* carrier freq. is fixed &.
* carrier phase is shifted 180°.
* this modulation is called phase shift keying (PSK). "will covered later".

Single Tone freq. Modulation:-

- Assume $m(t) = A_m \cos(2\pi f_m t)$
- instant. freq for FM modulated signal.

$$f_i(t) = f_c + k_f m(t).$$

$$\boxed{f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)}$$ plot.



Peak freq. deviation:-

$$\Delta f = k_f m_p = k_f A_m.$$

- the FM wave :-

$$s_{FM}(t) = A\cos\left(\omega_c t + 2\pi k_f \int_0^t m(\lambda)\, d\lambda\right)$$

$$= A\cos\left(\omega_c t + 2\pi k_f A_m \int_0^t (\cos 2\pi f_m \lambda)\, d\lambda\right)$$

$$= A\cos\left(\omega_c t + \frac{2\pi \, k_f A_m}{2\pi f_m} \sin(2\pi f_m t)\right)$$

$\Rightarrow$ β: modulation index.

$$s_{FM}(t) = A\cos(\omega_c t + \beta \sin(2\pi f_m t))$$

where $\beta = \frac{\boxed{k_f A_m}}{f_m} = \frac{\text{Peak freq. deviation}}{\text{message B.W}} = \frac{\Delta f}{f_m}$

$$\boxed{\beta = \frac{\Delta f}{f_m}}$$ "this impact of β will be studied later (FM spectrum)".

Example:- let $m(t) = 3\cos(2\pi \cdot 100t)$.

$c(t) = 2\cos(2\pi \cdot 1500t)$

$k_f = 10\,HZ/V$.

Ⅰ) Find Δf, BW of the message, β for the FM wave?

$\Delta f = k_f \cdot m_p = 10 \times 3 = 30\,HZ$.

$B.W = 100\,HZ$

$\beta = \frac{30}{100} = 0.3$

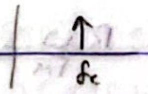
Ⅱ) write the FM modulation wave in t-domain.

$s(t) = A_c\cos(2\pi f_c t + \beta\sin(2\pi f_m t))$

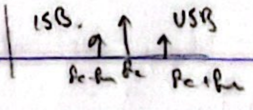
$= 2\cos[2\pi \cdot 1500t + 0.3\sin(2\pi \cdot 100t)]$

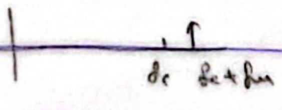
Historical Notes:- False Start)

* FM was introduced to reduce side band noise "noise power is proportional to the modulated Signal B.W".

$c(t) \xrightarrow{F}$ 

$m(t) = \sin(2\pi f_m) \xrightarrow{F}$  , $B.W = f_m$".

Am signal $\xrightarrow{F}$  , $B.W = 2f_m$".

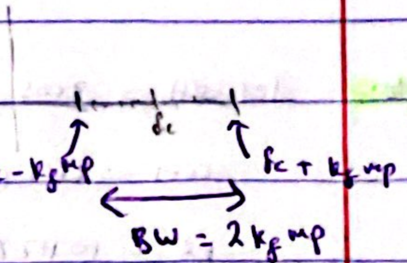
SSB $\xrightarrow{F}$  , $B.W = f_m$"

$s_{FM}(t) = A_c \cos(\theta_i(t))$

$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(\tau) d\tau$

$f_i(t) = f_c + k_f m(t)$.

$k_f \downarrow \rightarrow BW \downarrow$



$BW = 2 k_f m_p$

wrong!!

- it was thought the spectral componants of $S_{FM}(t)$ would remain within the band $f_c - k_f m_p \le f_c \le f_c + k_f m_p$. with the $B.W = 2\Delta f$ centered at f_c.
- the understanding was controlling k_f can control the modulated signal B.W.

$\Rightarrow$ * unfortunately, experimental results showed that the underlying reasoning was wrong!

- $B.W_{FM} \geq B.W_{AM}$

  equal at the best case $B.W_{FM} = B.W_{AM}$.

  we shall soon find out $BW_{FM} = ??$

  $B.W \neq 2 k_f m_p$.

Spectrum of a single tone FM signal.

- objective "to find a meaningful definition for the B.W".

Single tone $\Rightarrow$ $m(t) = A_m \cos(2\pi f_m t)$

$$s_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$ "modulation index"

"control the B.W"

so $\Rightarrow$ we need to find the spectrum $S_{FM}(f) = F\{s(t)\}$

$S_{FM}(f)$!! صعبة.

$\Rightarrow$ we need to find alternative expression $s_{FM}(t)$

then we find the spectrum.

Hint:- $e^{j\theta} = \cos\theta + j\sin\theta$.

$Re\{e^{j\theta}\} = \cos\theta$.

$Im\{e^{j\theta}\} = \sin\theta$

Assume $f_c >> f_m$:-.

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

$$= Re\{A_c e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\} = R\{\underbrace{A_c e^{j\beta \sin 2\pi f_m t}}_{\tilde{s}(t)} e^{j2\pi f_c t}\}.$$

$$\tilde{s}(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

"complex envelope of FM signal"

$\tilde{s}(t)$ is $\boxed{\text{periodic}}$ function with fundamental freq. $f_m = \frac{1}{T_m}$.

"to test periodicity $\tilde{s}(t + \frac{1}{f_m}) = \tilde{s}(t)$"

Since $\mathfrak{J}(t)$ is periodic with period $T_m = \frac{1}{f_m}$, we can expand it using complex FS.

$$g(t) = \sum_{-\infty}^{\infty} c_n e^{jn\omega_c t}, \quad c_n = \frac{1}{T_m}\int^{T_m} g(t)\, e^{-jn\omega_c t},$$

$$\omega_m = \frac{2\pi}{T_m} = 2\pi f_m.$$

$$\Rightarrow c_n = \frac{1}{T_m}\int_0^{T_m} e^{j\beta \sin 2\pi f_m t}\, e^{-jn\omega_c t}\, dt$$

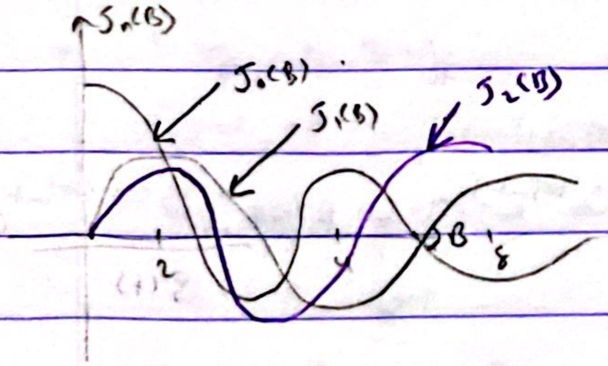
"hard to evalue and get a closed Form expression"

$\boxed{C_n = J_n(\beta)}$ where $J_n(\beta)$ is the Bessel function of the first kind of order (n).

the integral can evaluated numerically " software matlab "

available → table
└→ plots.

More about Bessel function $J_n(\beta)$.



property $J_n(\beta)$:-

$$J_n(x) = (-1)^n J_{-n}(x) \Rightarrow \boxed{|J_n(x)| = |J_{-n}(x)|}.$$

$$\tilde{s}(t) = A_c e^{j\beta \sin(2\pi f_m t)}$$

$$\tilde{s}(t) = A_c \sum c_n e^{jn\omega_m t}, \quad c_n = J_n\beta.$$

$$\boxed{\tilde{s}(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta)\, e^{j2\pi n f_m t}}$$

Back to eq ①

$$s(t) = Re\left[A_c \sum J_n(\beta) e^{j2\pi n f_m t} \cdot e^{j2\pi f_c t} \right]$$

$$= Re\left[A_c \sum J_n(\beta) e^{j2\pi (f_c + n f_m) t} \right].$$

$$\therefore s(t) = A_c \sum J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

Form I:- $s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$] equivalent

Form II:- $s(t) = A_c \sum J_n\beta \cos\left[2\pi(f_c + n f_m)t\right]$] representation.

to get the spectrum (use form II) :

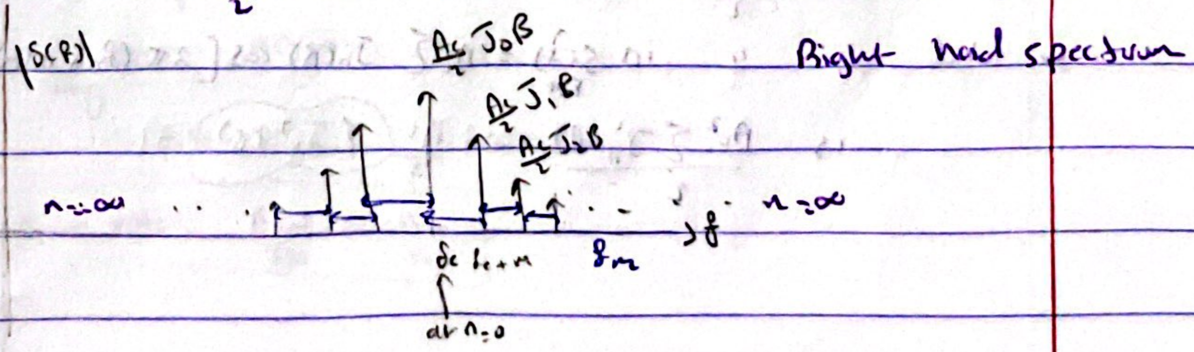
$$S(f) = F\{s(t)\}$$

$$= A_c \sum J_n\beta\, F\{\cos[2\pi(f_c + n f_m)t]\}$$

$$= A_c \sum J_n\beta \left[\tfrac{1}{2}\delta(f - f_c - n f_m) + \tfrac{1}{2}\delta(f + f_c + f_m)\right].$$

∴ the spectrum:-

$$S(f) = \frac{A_c}{2} \sum J_n\beta \left[\delta(f - f_c - n f_m) + \delta(f + f_c + f_m)\right].$$



- the spectrum of s(t) consist of an infinite number of delta functions spaced at $f = f_c \pm n f_m$ $n = 0, \pm 1, \pm 2, \cdots$

- therefor, B.W (theoritically) is ∞

- it's not practical $B.W = \infty$, since the freq. spectrum is shared by many users "interferse"

- Instead, we truncate the spectrum so that say 99% of total power is contained within a certain B.W

Power consideration:-

- any term in s(t) takes the form $A_c J_n(\beta) \cos 2\pi (f_c + n f_m) t$.
- the average power in every term is $\frac{[A_c J_n(\beta)]^2}{2}$.

- Total average power in $s_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$ is $\frac{A_c^2}{2}$

- total " " in $s_{FM}(t) = A_c \sum J_n(\beta) \cos[2\pi (f_c + n f_m)t]$

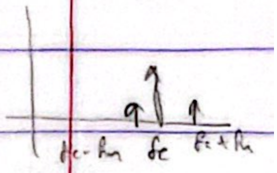
  is $\frac{A_c^2 \sum J_n^2(\beta)}{2} = \frac{A_c^2}{2}$ $\boxed{\sum J_n^2(\beta)} = 1$

Example:- Find the (99%) power B.W of an FM signal when $\beta = 1$ and $\beta = 0.2$

$$s(t) = A_c \sum_{-\infty}^{\infty} J_n(\beta)\left[\cos 2\pi(f_c + nf_m)t\right]$$

$\boxed{\text{Start } \beta = 1}$

try to find P_{avg} in the first three terms.



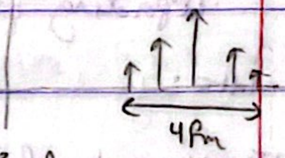
- $P_3 = \sum_{-1}^{1} \frac{A_c^2 J_n^2(\beta)}{2} = \frac{A_c^2}{2}\left[J_0^2(1) + J_1^2(1) + J_{-1}^2(1)\right]$

$$= \frac{A_c^2}{2}\left[J_0^2 + 2J_1^2\right]$$

$$= \frac{A_c^2}{2}\left[(0.7652)^2 + 2(0.4401)^2\right] = 0.9729 \frac{A_c^2}{2}$$

$\Rightarrow \frac{P_3}{P_{avg}} \times 100\% = 97.29\%$

"Still, add more components".



- $P_5 = \frac{A_c^2}{2} \sum_{-2}^{2} J_n^2(\beta)$

$$= \frac{A_c^2}{2}\left[J_0^2 + 2J_1^2 + 2J_2^2\right]$$

$$= \frac{A_c^2}{2}\left[0.9729 + 2\cdot(0.1149)^2\right] = 0.9993 \frac{A_c}{2}$$

$\therefore 99.93\% > 99\%$

$\therefore BW = 4f_m$. "wide band FM signal".

$\boxed{\beta = 0.2}$

- $P_3 = \frac{A_c^2}{2}\left[J_0^2(0.2) + 2J_1^2(0.2)\right] = 0.999 \frac{A_c^2}{2}$

$\frac{P_3}{avg} \times 100\% = 99.9\% > 99\%$

$\therefore B.W = 2f_m$. "Narrow band".

$\boxed{\text{Note that } B.W_{FM} \geq B.W_{AM}}$

Carson's Rule "carson used experimental validation".

"A 98% power B.W of an FM/PM signal can be estimated using carson's Rules".

Bandwidth transmition $(B_T) = 2(\beta+1) f_m$

$= 2(\Delta f + f_m)$

Δf = peak freq. deviation.

$\Delta f_{FM} = k_f m_p$.

$\Delta f_{PM} = \frac{k_p}{2\pi} \dot{m}_p$.

Summary:-

Depending on the B.W, FM/PM signal can be classified into:-

1. Narrowband "when $\beta \ll 1$" $\Rightarrow$ B.W $= 2 f_m$. "similar to the AM"
   ↳ used in telephone exchange tenden.
2. wideband "else" $\Rightarrow$ B.W $= 2(\beta+1) f_m$
   ↳ used in FM broadcast service

For the earilier example:-

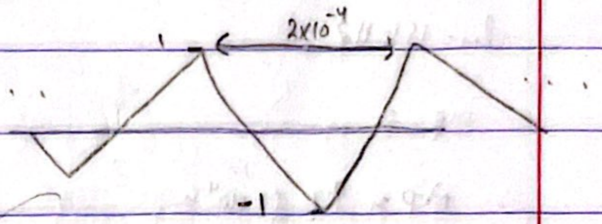
$\beta = (0.2)$ $\beta \ll 1$ $\Rightarrow$ narrowband B.W $= 2 f_m$.

$\beta = 1$ else $\Rightarrow$ B.W $= 2(\beta+1) f_m$

$= 4 f_m$.

Example:- estimate B_{FM}, B_{PM} for the modulating signal $m(t)$ if

$k_f = 10^5$, $k_p = 5\pi$.



$B.W_{FM/PM} = 2(\beta+1) f_m$

$\beta = \frac{\Delta f}{f_m}$

$\Rightarrow \Delta f$:-

[1] PM signal :-

$\Delta f = \frac{k_p}{2\pi} \dot{m}_p$

$\therefore \dot{m}_p = 10^4$

$\Rightarrow \Delta f = \frac{5\pi}{2\pi} \times 10^4 = 25KHz$.

$\beta = \frac{\Delta f}{f_m} = \frac{25KHZ}{15KHZ}$

$= \frac{5}{3} \Rightarrow$ it's wide

$\therefore B.W = 2(\frac{5}{3}+1) \times 15k$

$= \boxed{80KHZ}$

$\Rightarrow \Delta f$:-

[2] FM signal :-

$\Delta f = k_f m_p = 10^5(1) = 100k$ Hz .

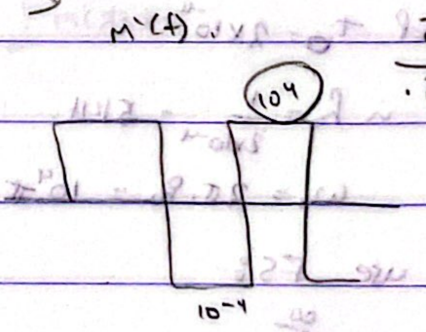
f_m = message B.W .

assume ($f_m = 15$ kHZ)

$\Rightarrow \beta = \frac{100k}{15k} = \frac{20}{3} \Rightarrow$ it's wideband FM sign

$B.W = 2(\beta+1) f_m$

$= 2(\frac{20}{3}+1) 15k = \boxed{230k\ HZ}$



فرق الصادات

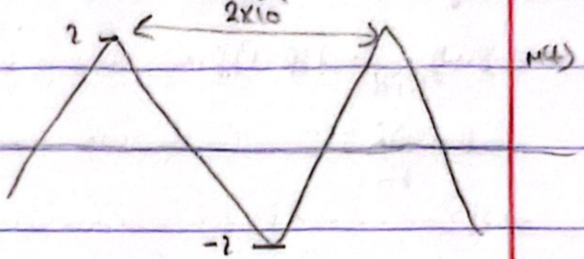
ف السينات .

Q:- what if we doubled the amplitude of m(t)?

$f_m = 15\,kHz$,

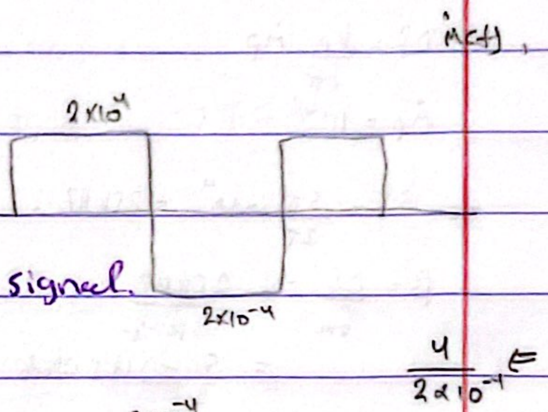
$m_p = 2$

$\dot{m}_p = 2 / 2\times10^{-4}$



⇒

	Double	before
$B.W_{FM}$	430 KHz	250 KHz.
$B.W_{PM}$	130 KHz	80 KHz



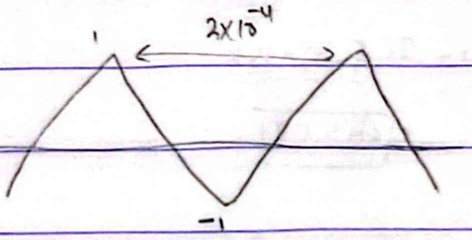
doubling m_p $\boxed{\text{roughly}}$ doubles that B.W of both FM and PM signal.

Q:- why $f_m = 15\,KHZ$.

m(t) is periodic with period $T_0 = 2\times10^{-4}$

$\therefore f_0 = \frac{1}{2\times10^{-4}} = 5\,KH.$

$w_0 = 2\pi . f_0 = 10^4 \pi$.



we use FS:-

$$m(t) = \sum_{n=1}^{\infty} C_n \cos(n w_0 t) \;,\; C_0 = 0 \;,\; C_n = \begin{cases} \frac{8}{n^2\pi^2} \;, & n = \text{odd} \\ 0 \;, & n = \text{even} \end{cases}$$

$$m(t) = \frac{8}{\pi^2}\cos(2\pi . 5kt) + \frac{8}{9\pi^2}\cos(2\pi . 15kt) + \frac{8}{25\pi^2}\cos(2\pi . 25kt) + \ldots$$

we should check which one has 99%.

$$\Rightarrow C_n = \begin{cases} \frac{8}{n^2\pi^2} & n: \text{odd} \\ 0 & n: \text{even} \end{cases}$$

$C_0 = 0$.

$\downarrow$

Harmonic no (n)	0	1	2	3	4	5	
$\frac{C_n}{C_1} = \frac{1}{n^2}$	0	1	0	$\frac{1}{9}$	0	$\frac{1}{25}$	. . .
$\frac{P_n}{P_1} = \frac{1}{n^4}$	0	$\boxed{100\%}$	0	$\frac{1}{81} =$ $\boxed{1.21\%}$	0	$\frac{1}{(25)^2} = 0.16\%$	

$\uparrow$ negligable.

$\therefore$ B.W for the message $= 3f_0$

$= 3 \times 5k = 15\,kHz$.

$\Rightarrow$ Summary:-

Angle Modulation:-

$s(t) = A_c \cos(2\pi f_c t + \theta(t))$

$\rightarrow$ FM : $\theta(t) = 2\pi k_f \int_0^t m(\tau)d\tau$

$\rightarrow$ PM : $\theta(t) = k_p m(t)$

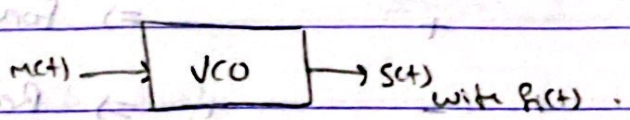
Direct method For Generating FM signal "voltage controlled oscillator VCO".

* In VCO, the freq. is controlled by an external voltage m(t)

* FM signal by using m(t) which controls $f_i(t)$.

$$\boxed{f_i(t) = f_c + k_f m(t)}$$

* Schematic diagram.

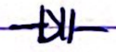


* Relization of VCO

→ Schmitt- trigger circuit "opAmp + comparator".

→ Hartley oscillator "varactor: voltage variable capacitor"

diod operated in reverse bias region

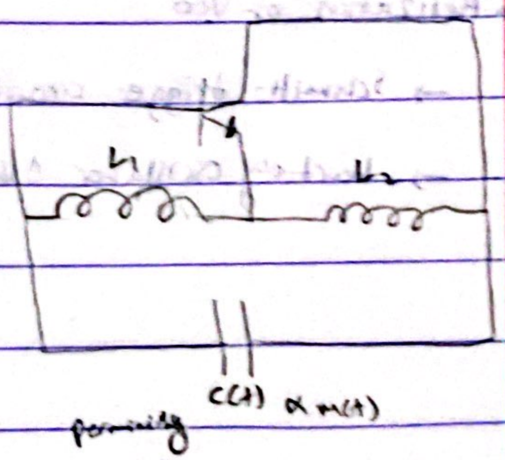


⇒ Hartley oscillator :-

- the capacitance of the varactor varies in response to m(t).
- Varactor "diod operation in the Reverse bias Region can act like a variable capacitor".

\+ − ⇒ Forward bias

− + ⇒ Reverse bias.



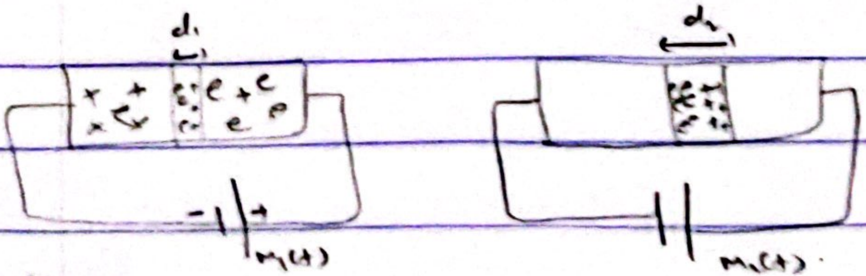
permitivity

$$c = \frac{\varepsilon A}{d}$$

A ← area of PN junction.

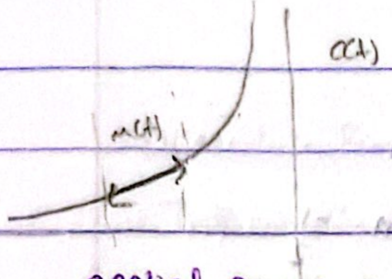
d width.

$$c \propto \frac{1}{d}.$$



$d_2 > d_1$

$|m_2| > |m_1|$



$$C_T = \frac{\epsilon A}{d}$$

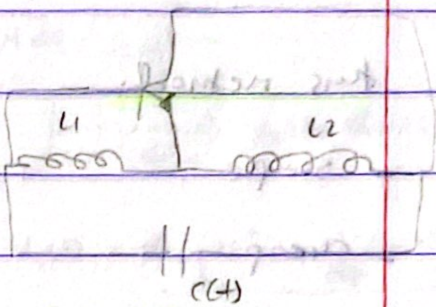
$$C_T \propto \frac{1}{d} \quad , d \propto |m(t)|$$

$$m\uparrow \rightarrow d\uparrow \rightarrow \boxed{c(t)}\downarrow$$

applied reverse voltage.

$$m\downarrow \rightarrow d\downarrow \rightarrow \boxed{c(t)}\uparrow \quad \underline{\underline{C_T}}.$$

For Hartely Oscillator :-



* the freq. of the oscillator is given by:

$$f_i = \frac{1}{2\pi\sqrt{LC}} \quad , L = L_1 + L_2$$

* if c(t) varies with m(t) Varactor

$$c(t) = C_0 - k\,m(t) \qquad c(t) = C_0 - k\,m(t).$$

$$\Rightarrow f_i(t) = \frac{1}{2\pi\sqrt{L(C_0 - km(t))}}$$

→ if $m(t) = 0$

$$\Rightarrow f_i(t) = \frac{1}{2\pi\sqrt{LC_0}} = f_c.$$

called unmodulated carrier.

→ if $m(t) \neq 0$

$$\Rightarrow f_i(t) = \frac{1}{2\pi\sqrt{LC_0(1 - \frac{k}{C_0} m(t))}} = \frac{1}{\boxed{2\pi\sqrt{LC_0}}\sqrt{1 - \frac{k}{C_0} m(t)}}$$

$$f_i(t) = f_c\left(1 - \frac{k}{C_0} m(t)\right)^{\frac{-1}{2}}.$$

if $\frac{k}{C_0} m(t) << 1$, we can use Taylor series approx.

$$(1+x)^n \approx 1+nx \quad , \quad |x| \ll 1$$

$$\Rightarrow f_i(t) = f_c\left(1+\frac{k}{2C_0}m(t)\right) \quad , \text{with } n=\frac{-1}{2}$$

$$f_i(t) = f_c\left(1+k_f m(t)\right) \quad , k_f = \frac{k}{2C_0}$$

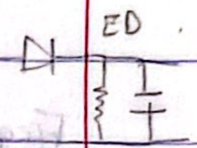
$f_i(t) \Rightarrow$ varien linearly with $m(t)$.

this method:

- simple
- cheap
- it can't be used in broadcast application

  " LC oscillaer is not very stable "

  insted we use inderict method.

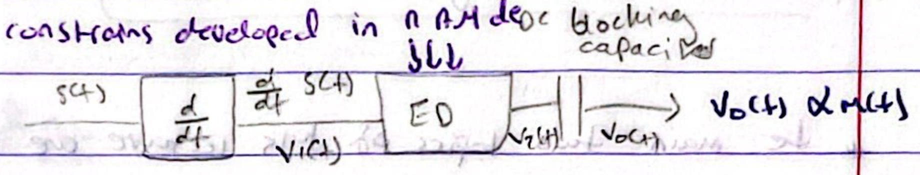


Demodulation of FM Signal:

Two method :- → used before for normal AM Dem.

[1] Discrimination " Diff followed by envelope Detector "

[2] phase locked loop . " in the lab " .



$s(t) = A_c \cos(2\pi f_c t + \theta(t))$

message hidden in $\theta(t)$.

- $\underset{FM}{\theta(t)} = 2\pi k_f \int m(\lambda)\, d\lambda$.
- $\underset{PM}{\theta}(t) = k_p m(t)$

• $V_1(t) = -\underbrace{A_c\left(2\pi f_c + \frac{d}{dt}\theta(t)\right)}_{A(t)} \cos(2\pi f_c t + \theta$

• $V_2(t) = |A(t)|$

$= A_c \left| \underset{\overset{\uparrow}{dc}}{2\pi f_c} + \frac{d}{dt}\theta(t) \right|$

• $V_o(t) = A_c \frac{d}{dt}\theta(t)$.

→ FM ⇒ $V_o(t) = 2\pi k_f A_c m(t)$

⇒ $V_o(t) \propto m(t)$.

→ PM ⇒ $V_o(t) = A_c k_p \frac{d}{dt} m(t)$

" we need integrator "

⇒ $V_o(t) \propto m(t)$

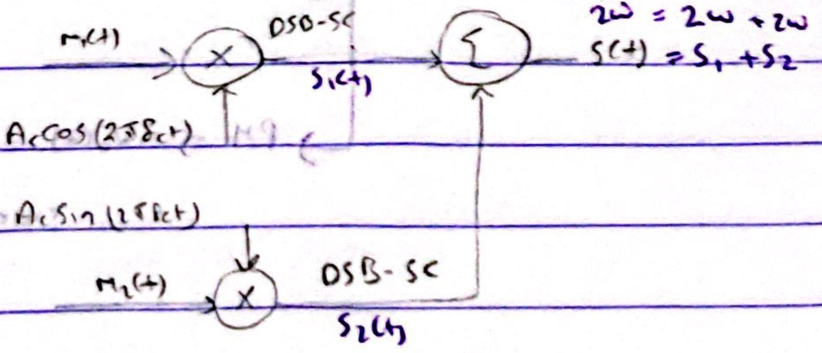
Frequency Division Multiplexing.

- Multiplexing:- A technique which allows multiple users to use the same channel at the same time by assigning each user a portion of the available bandwidth without interfering with other users.

- the main two topics of this lecture are quadrature carrier modulation and frequency division multiplexing.

- Quadrature carrier multiplexing:-

this scheme enable two AM DSB-SC modulated signals to occupy the same transmission B.W and yet allows for the separation of the message signals at the reciver.

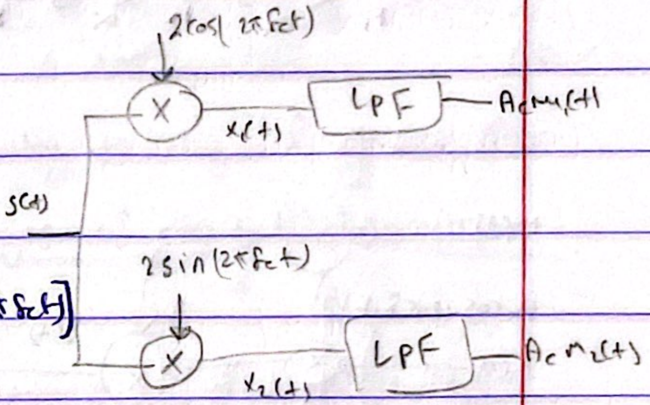


⇒ the method provides bandwidth conservation, that is, two DSB-SC signals are transmitted within the bandwidth of one DSB-SC signal. there for, this multiplexing technique provides bandwidth reduction by one half.

instead of $2w + 2w = 4w$

it's just $2w$.

Demodulation:-



$$x_1(t) = 2\cos(2\pi f_c t)\, s(t)$$

$$= 2\cos(2\pi f_c t)\big[A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)\big]$$

$$= 2A_c m_1(t)\cos^2(2\pi f_c t) + 2A_c m_2(t)\sin\omega_c t\cos\omega_c t.$$

$$= 2A_c m_1(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) + A_c m_2(t)\sin 2\omega_c t.$$

$$= \underline{A_c m_1(t)} + \underset{0}{\cancel{A_c m_1(t)\cos(2\omega_c t)}} + \underset{0}{\cancel{A_c m_2(t)\sin(2\omega_c t)}} \quad \text{after the LPF.}$$

$$\therefore y_1(t) = A_c m_1(t)$$

$$y_2(t) = A_c m_2(t)$$

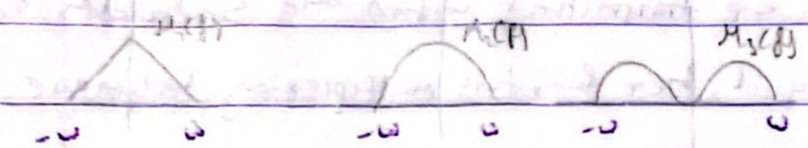
⇒ Synchronization is a problem:-

same f_c and $\theta = 0$ ✓

otherwize "interference type of distortion".

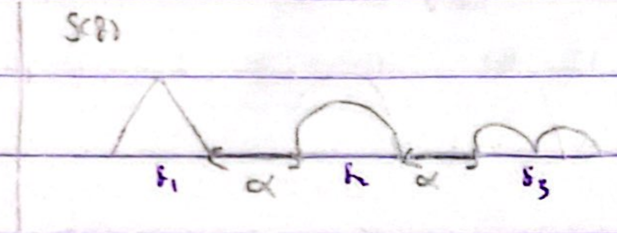
تداخل بين ال signal

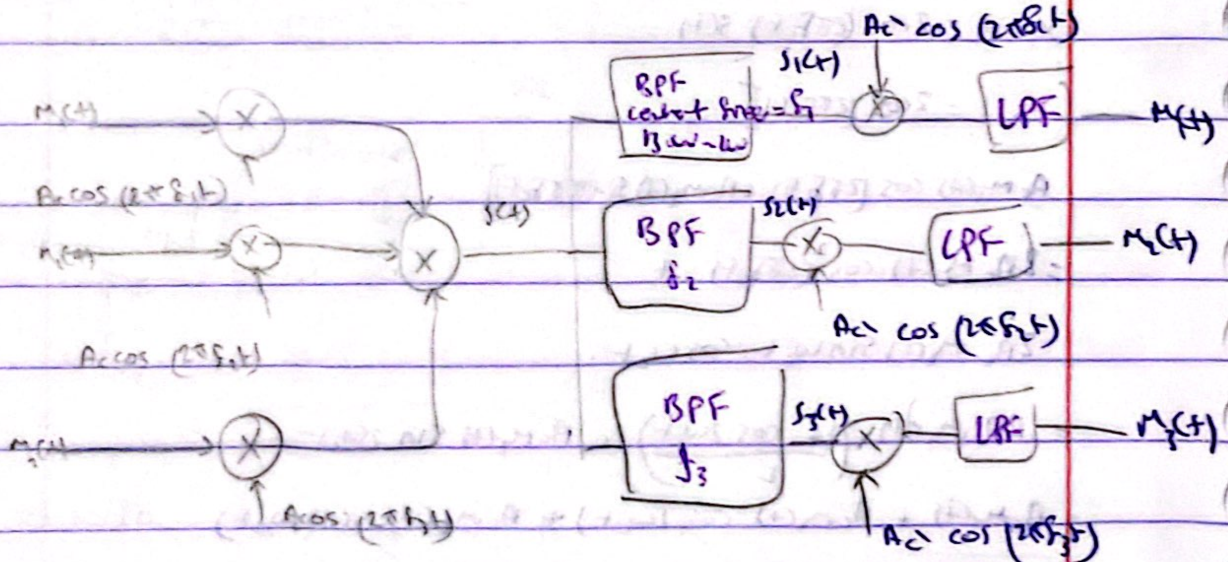
→ another way:-



$f_2 - W \geq f_1 + W$ or $f_2 - f_1 \geq 2W$

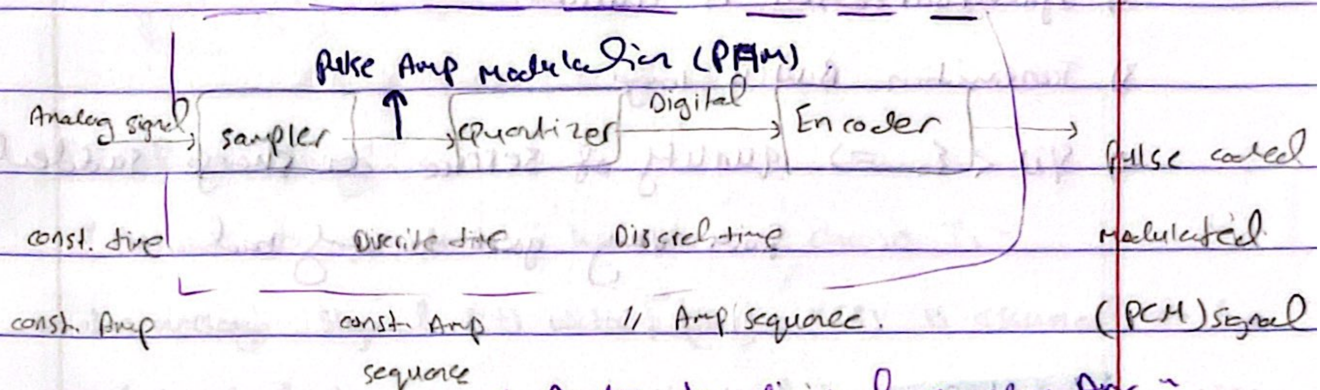
$f_3 - W \geq f_2 + W$ or $f_3 - f_2 \geq 2W$





chapter(5):- pulse modulation.

"transition from Analog to Digital modulation".



" Analog to digital conveter ADC "

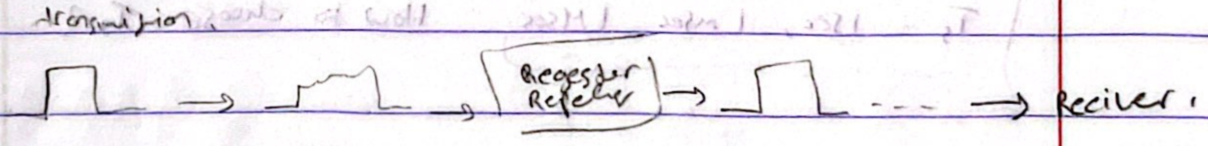
Advantages and Disadvantages of digital transmision:-

Advantages:-

1) Digital signal are more immune to channel noise.

2) " " belongs to finite set of possible waveform

=) we can use Repeater along the transmition path.



3) Digital signal are process by using microprocessor or VLSI.

4) " " make use of DSP. "encryption. error control coding, ... ".

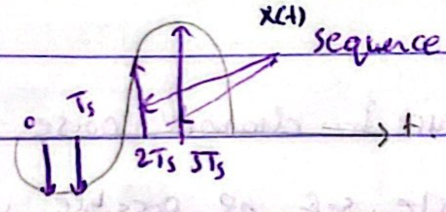
Disadvantages:-

1) Heavy signal processing
2) Synchronization is crucial.
3) Transmition B.W is large
4) $S/N < \epsilon \Rightarrow$ quality of service can change suddenly from very good to very bad.

Sampling:-

Sampling:- process of converting cont. time cont Amp, into discrete time, cont. Amp sequence.

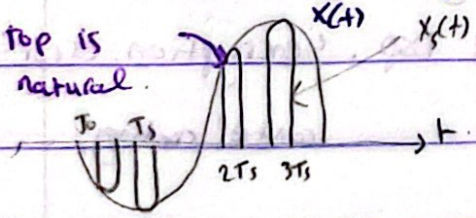
① Idel Sampling:-



T_s = Sampling period.

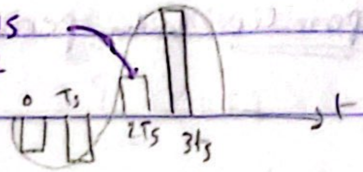
$\boxed{T_s = 1sec, 1msec, 1Msec}$ How to choose T_s ?.

② Natural sampling:-



③ flat-top sampling:- "sample and hold"

top is flat



Ideal sampling:-

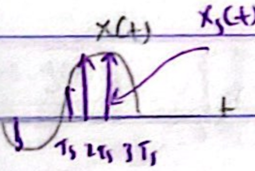
"Answer the question of how do we choose T_s"

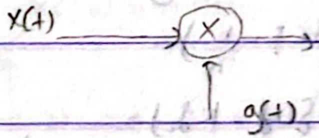
* the message signal $x(t)$ with $\mathcal{F}[x(t)] = X(f)$ is assumed to be band-limited to (W) Hz is multiplied by a periodic sequence of ideal impulse with period (T_s) to produce the sampled signal $x_s(t)$.

$$\therefore x_s(t) = \sum_k x(kT_s)\,\delta(t - kT_s)$$

$$= \sum_k x(t)\,\delta(t - kT_s)$$

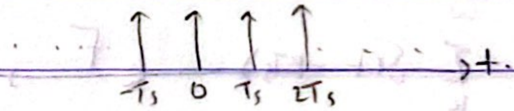
in t- domain





$$x_s(t) = x(t) \sum_k \delta(t - kT_s)$$

$$g(t) = \sum_{-\infty}^{\infty} \delta(t - kT_s).$$



Remember:-

$x(t) . \delta(t) = x(0)\,\delta(t)$.

$x(t) . \delta(t-1) = x(1)\,\delta(t-1)$

$x(t) \ast \delta(t) = x(t)$

$x(t) \ast \delta(t-1) = x(t-1)$

* Find F{g(t)} ?

$\boxed{g(t) = \sum_k \delta(t - kT_s)}$ is <u>periodic</u> with period

$T_s = \frac{1}{f_c}$

we an expand it using complex FS:-

$$g(t) = \sum_k C_k e^{j2\pi f_c t k}$$

$$C_k = \frac{1}{T_s} \int_0^{T_s} \delta(t)\, e^{-j2\pi f_c t k}\, dt.$$

$$= \frac{1}{T_s} \int_0 \delta(t)\, dt = \frac{1}{T_s}$$

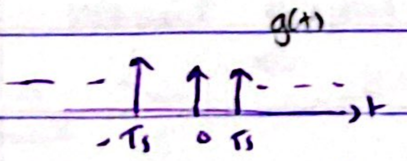
$$\Rightarrow \boxed{g(t) = \frac{1}{T_s} \sum_k e^{j2\pi k \frac{t}{T_s}}}$$

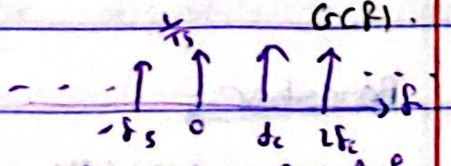
$$G(f) = F\, \frac{1}{T_s}\left[\sum e^{j2\pi k \frac{t}{T_s}}\right\}$$

$$G(f) = \frac{1}{T_s} \sum \delta(f - \frac{k}{T_s})$$

$$G(f) = f_c \sum \delta(f - kf_c)$$

$$g(t) = \sum_k \delta(t - kT_s) \xleftrightarrow{F} G(f) = \frac{1}{T_s} \sum_k \delta\left(f - \frac{k}{T_s}\right)$$





periodic with period T_s , periodic with period f_s

in t domain

$x_s(t) = g(t) \cdot x(t)$ (Mult)

$= \sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t-kT_s)$

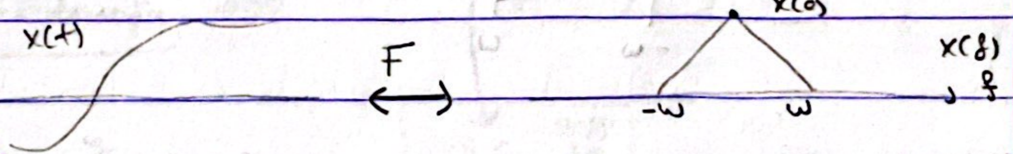
in f domain

$X_s(f) = G(f) * X(f)$ (convolution)

$= f_s \sum \delta(f-kf_s) * X(f)$.

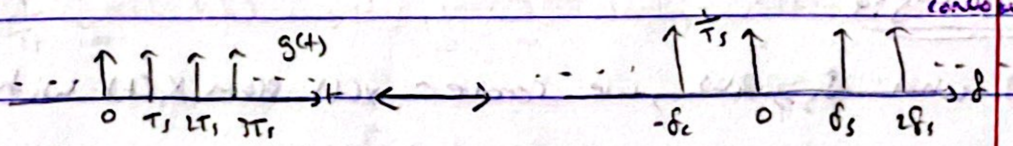
$= f_s \sum \delta(f-kf_s) * X(f)$.

$\therefore X_s(t) = f_s \sum_{k=-\infty}^{\infty} X(f-kf_s)$.

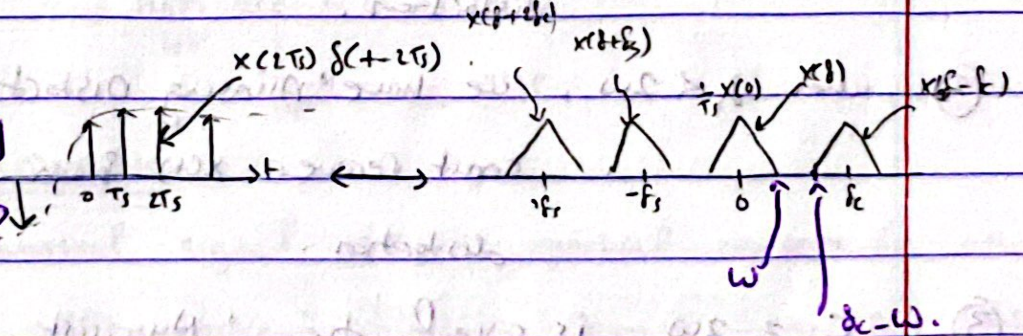


Multi

convolution.



$\boxed{x_s(t) = x(t) \cdot g(t)}$



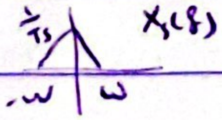
$x(-T_s)\,\delta(t+T_s)$

to recover X(f) from $X_s(f)$ without distortion

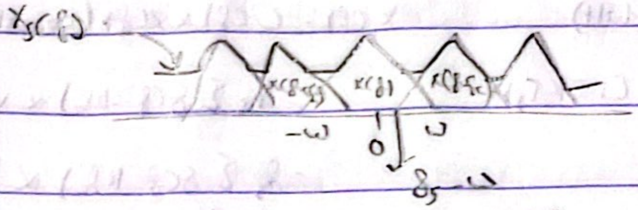
$f_s - W \geq W \Rightarrow \boxed{f_s \geq 2W}$

$f_s = \frac{1}{T_s}$

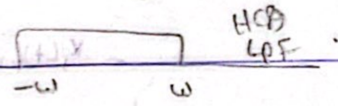
we can recover x(t) from $x_s(t)$ by passing $x_s(t)$ through a low pass filter with B.W=W, without distortion.

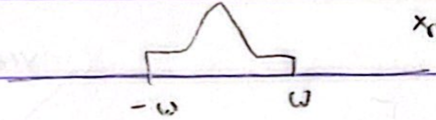


Now if $f_s < 2W$.



$X_s = \frac{1}{T_s} \sum$ replicas.





$x_r(f) \neq X(f)$.

Notes:-

① when $f_s \geq 2W$, we recover $x(t)$ from $X_s(t)$ without distortion.

② when $f_s < 2W$, we have "Aliasing Distortion", we can't recover $x(t)$ from $X_s(t)$ without distortion.

③ $F_s = 2W$, is called the "Nyquist sampling Rate" it's the min. rate at which a signal with B.W = W must be sampled in order to reconstruct it from it's samples without distortion

Reconstruction of the sampled signal:-

in t-domain.

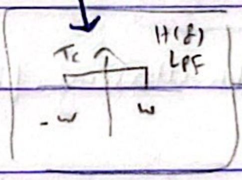
LPF

$$x_s(t) = \sum_{-\infty}^{\infty} x(kT_s)\,\delta(t-kT_s) \longrightarrow \boxed{\begin{matrix} h(t) \\ \text{impulse response} \end{matrix}} \longrightarrow x_r(t) = \sum_{k} x(kT_s)\,h(t-kT_s).$$

F

in f-domain

$$X_s(f) = \frac{1}{T_s}\sum_{-\infty}^{\infty} X(f-kf_s) \longrightarrow$$



$$\longrightarrow x_r(t) = x(t), \quad f_s \geq 2W$$

$$x_r(t) \neq x(t), \quad f_s < 2W.$$

$$\underset{\text{LPF}}{H(f)} = \begin{cases} T_s, & |f| < w \\ 0, & \text{else} \end{cases}$$

Sampling theorem:-

A band limited signal with no spectral components above (w) HZ can be recovered uniquely from it's samples taken every Ts, provided that:

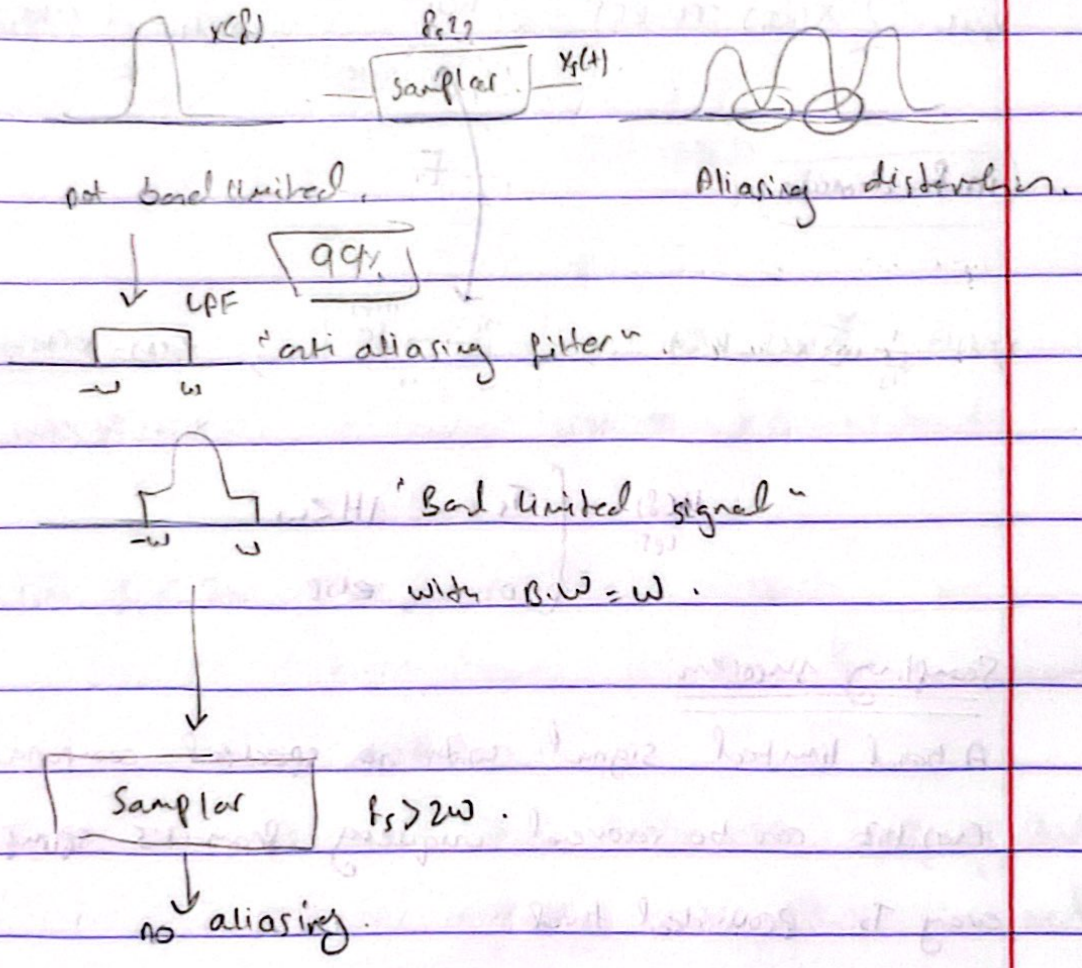
$$f_s \geq 2W \Rightarrow \frac{1}{T_s} \geq 2W \Rightarrow \boxed{T_s \leq \frac{1}{2W}}.$$

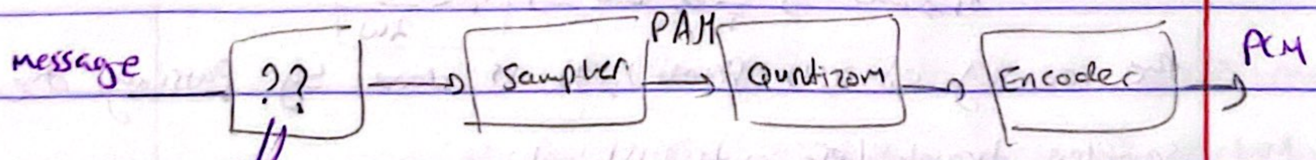
the recovery of $x(t)$ from $x_s(t)$ is done by passing the samplen trough LPF with B.W = W.

practical consideration "Aliasing phenomenon"

if the signal is not Bandlimited "In practice this is the case of physical signals".

⇒ we have always Aliasing. How to avoid that??

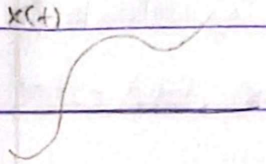


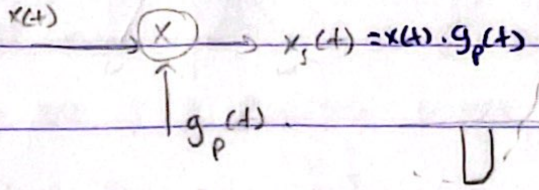


is called anti-Aliasing LPF with B.W = W.

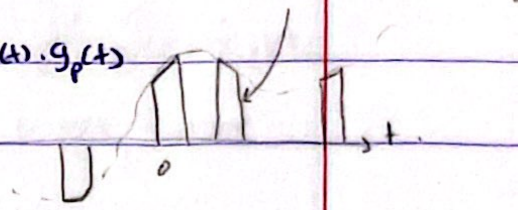
Natural Sampling:-

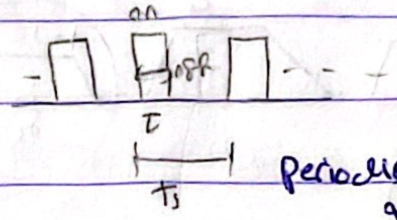
in t domain:





$x_s(t)$ natural sampling





$\tau = 1$, $T_s = 5$

$\frac{1}{5} = 20\%$ "on"

" $\frac{\tau}{T_s}$: duty cycle "

periodic signal.

$$g_p(t) = \sum_{-\infty}^{\infty} c_n e^{j2\pi n f_s t}$$

$$c_n = \frac{1}{T_s}\int_0^{T_s} A e^{-j2\pi n f_s t} dt .$$

$$c_n = \frac{A}{\pi n} \sin\left(n\pi \frac{\tau}{T_s}\right)$$ n↑ ⇒ $|C_n|$↓

$$C_0 = \frac{A\tau}{T_s}$$

$$\Rightarrow g_p(t) = C_0 + \sum_{n=1}^{\infty} 2|C_n| \cos(2\pi n f_s t) .$$

$$X_s(t) = x(t) g_p(t) .$$

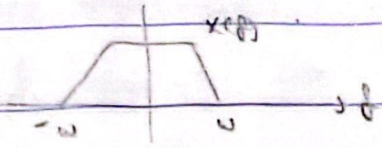
$$= x(t) . \left[C_0 + \sum_{n=-\infty}^{\infty} 2|C_n| \cos(2\pi n f_s t)\right] .$$

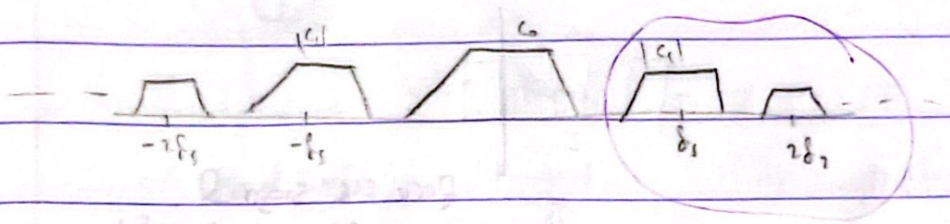
$$X_s(f) = F[X_s(t)]$$

$$= F[C_0 x(t)] + \sum_{n=1}^{\infty} F\{2|C_n| x(t) \cos(2\pi f_s t)\} .$$

$$\therefore X_s(f) = C_0 X(f) + \sum_{n=1}^{\infty} |C_n| [X(f - nf_s) + X(f + nf_s)] .$$

$$X(f) = C_0 X(f) + \sum_{n=1}^{\infty} |C_n| \left[X(f-nf_s) + X(f+nf_s) \right].$$





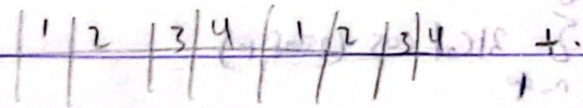
Amplitud of the repeted of X(f) is decaying.

due to $|C_n| \propto \frac{1}{\pi n} \sin(\ )$

$|C_n| \propto \frac{1}{n}$.

⇒ Time Divition multiplexing:-

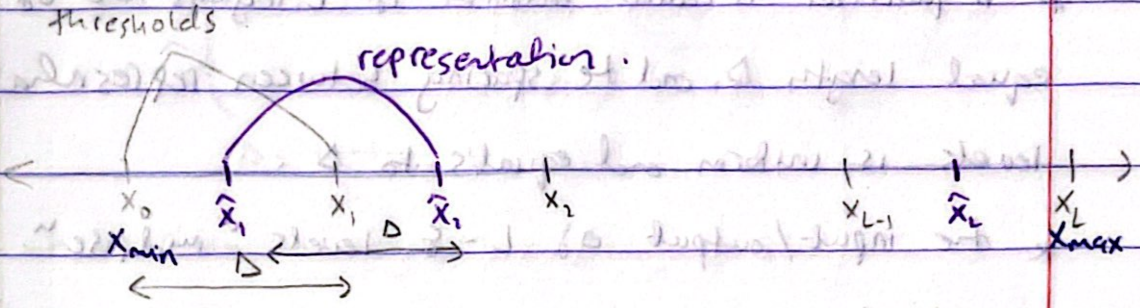
"synchronization".



Quantization process:-

as the process of converting the cont. sample amplitude $x(kT_s)$ of a message signal into a discrete amplitude $\hat{x}(kT_s)$ taken from a finite set of L possible levels / values

$$\hat{x} = \{\hat{x}, \hat{x}_2, \dots, \hat{x}_L\}$$



Dynamic Range = $X_{max} - X_{min}$.

Step size $\Delta = \frac{X_{max} - X_{min}}{L}$, Δ: spacing between Representat
" " thresholds.

L: # of levels, $L = 2^n$, n: # of bits / sample.

The uniform Quntizer "input/output characteristics".

$$X(KT_s) \longrightarrow \boxed{\text{Quntizer}} \longrightarrow y = Q(X(KT_s))$$

$$= \hat{X}(KT_s)$$

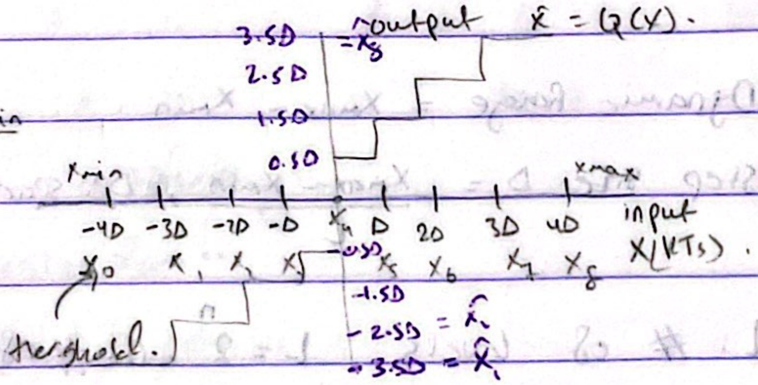
$$\in [\hat{X}_1, \hat{X}_2, \dots, \hat{X}_L]$$

- * A quantizer is called uniform if L regions are of equal length Δ, and the spacing between represntation levels is uniform and equals to Δ.
- * the input/output of L = 8 levels "midrise" Quantizer:-

$$\Delta = \frac{X_{max} - X_{min}}{L}$$

$$\Delta = \frac{2X_{max}}{8}$$

$$4\Delta = X_{max}$$



in Quantization, we loss information,

Quantization error, $e = x - \hat{x}$

$$= x - Q(x)$$

$$x(KT_s) \longrightarrow \boxed{Q} \longrightarrow y = \hat{x}$$

$$\Delta = \frac{X_{max} - X_{min}}{L}$$

⇒ $x(t) = \cos(2\pi f_c t)$, 8 level Quantization.

$X_{max} = 1$

$X_{min} = -1$

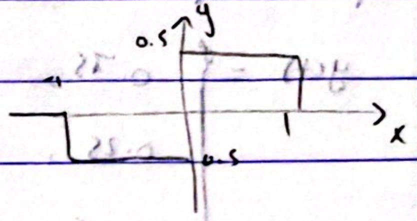
$$\Rightarrow \Delta = \frac{X_{max} - X_{min}}{L} = \frac{2X_{max}}{L} = \frac{2 \times 1}{8} = 0.25.$$

$L = 2^n = 2^1 = 2$ levels.

Example:- the one-bit quantizer:-

the signal $x(t) = \cos(2\pi t)$ is uniformly sampled at a rate of 20 samples per second, the samples are applied to a sign detector, whose input-output characteristic is defined as:-

$$y(t) = \begin{cases} 0.5, & 0 < x < 1 \\ -0.1, & -1 < x < 0 \end{cases}$$



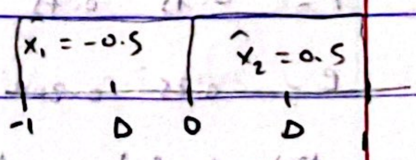
$$D = \frac{1 - -1}{2} = 1$$

we take samples every kT_s, $T_s = \frac{1}{20} = 0.05$ sec.

$t =$	0	0.05,	0.1,	0.15,	0.2,	0.25
$x(t) =$	①	0.9511	0.8090	0.5878	0.3090	0.000
$y =$	$\overset{\frown}{0.5}$	0.5	0.5	0.5	0.5	0.5
$e =$	0.5	0.4511	0.3090	0.0878	-0.191	-0.51

note that:-

$$-\frac{D}{2} < e < \frac{D}{2} \Rightarrow -0.5 < e < 0.5$$

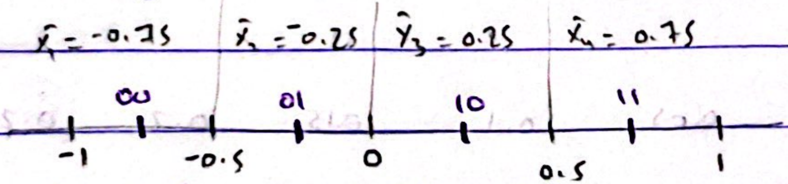


$L = 2^n = 2^2 = 4$ levels

Example:- the two-bit quantizer:-

the signal $x(t) = \cos(2\pi t)$ is sampled uniformly at a rate of 20 samples per second. the samples are applied to a four levels uniform quantizer with input-output characteristic.

$$y(t) = \begin{cases} 0.75, & 0.5 < x < 1 \\ 0.25, & 0 < x < 0.5 \\ -0.25, & -0.5 < x < 0 \\ -0.75, & 1 < x < -0.5 \end{cases}$$



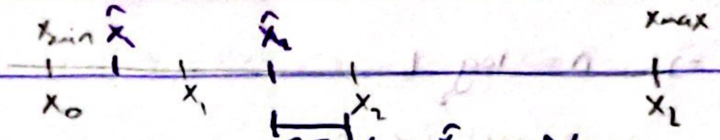
$$\Delta = \frac{X_{max} - X_{min}}{L} = \frac{1 - -1}{4} = 0.5$$

$t =$	0	0.05	0.1	0.15	0.2	0.25
$x(t) =$	1	0.9511	0.8090	0.5878	0.3090	0.0000
$y =$	0.75	0.75	0.75	0.75	0.25	0.25
$e =$	0.25	0.2011	0.059	-0.1622	0.059	-0.25

Note that: $\frac{-\Delta}{2} < e < \frac{\Delta}{2}$, $-0.25 < e < 0.25$.

- In Quantization we loss information, error/sample $= x(KT_s) - \hat{x}(KT_s)$.

$$-\frac{D}{2} < e < \frac{D}{2}$$



$e = x_2 - \hat{x}_n = D/2$.

$$\Rightarrow \quad -\frac{D}{2} < e < \frac{D}{2}$$

$$|e| < \frac{D}{2}$$

Example: A) Design an 8-level uniform Quantizer with dynamic rang (−4, +4) volts.

You need to find "thresholds, representions, step size".

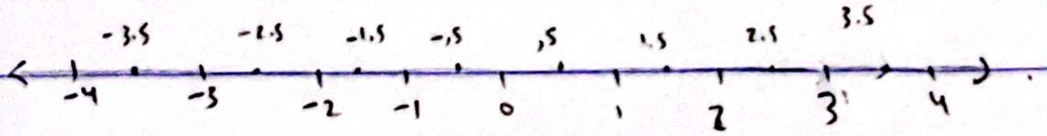
→ $L = 8$ levels.

→ Dynamic Rang $(X_{max}, X_{min}) = (-4, 4)$

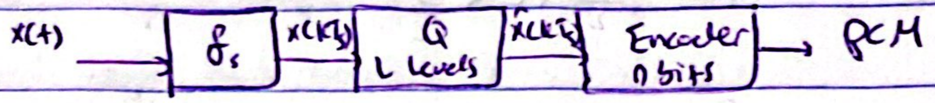
→ $D = \frac{4 - -4}{8} = 1$ Volt.

→ thresholds $= \{\underset{x_{min}}{\underset{\uparrow}{-4}}, -3, -2, -1, 0, 1, 2, 3, \underset{x_{max}}{\underset{\uparrow}{4}}\}$

→ Representations $= \{-3.5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5\}$



B) How many binary digits are needed to represent the samples?



$L = 2^n \Rightarrow n = \log_2 L$

$8 = 2^n \quad \therefore n = 3$ bits / samples

if the message B.W = 4KHZ. find the bit rate R_b "bit/sec".

→ $f_s = f_N = 2 \cdot W = 2 \cdot 4k = 8K\,HZ$

= 8 k samples / sec.

→ bit rate $R_b = f_s \times n$

$= 8K \frac{\text{samples}}{\text{sec}} \times 3 \frac{\text{bits}}{\text{sample}}$

= 24 k bit/sec

c) Find the Representation Value and quantization error when a 1.64 V sample is applied to the 8-level quantization. Find the Binary representation if we use the (NBC)

"Natural Binary coding"

$$2 \leq 1.64 < 2 \Rightarrow \hat{x}(KT_s) = 1.5$$

$$\text{error} = 1.64 - 1.5 = 0.14$$

NBC

-3.5	000	0.5	100
-2.5	001	$\boxed{1.5}$	$\boxed{101}$
-1.5	010	2.5	110
-0.5	011	3.5	111

∴ the Binary seq is 101

	-3.5	-2.5	-1.5	-0.5	0.5	1.5	2.5	3.5
Gray	000	001	011	010	110	111	101	100
FBC	011	010	001	000	100	101	110	111

"Folded Binary coding".

d) Repet (C) when $x(kT_s) = -2.1$ V.

$\hat{x}(kT_s) = -2.5$

$e = -2.1 + 2.5 = 0.4$

NBC 001

Gray 001

FBC 010

Signal to Quantization noise Ratio (SQNR).

* the quantization error / sample

$$e = x - \hat{x}$$

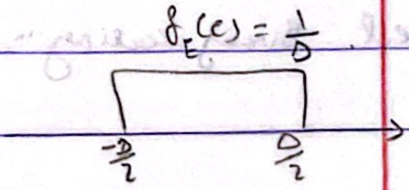
$$x \rightarrow \boxed{Q(x)} \rightarrow \hat{x}$$

* the max error "called resolution" $= \left|\frac{\Delta}{2}\right|$

* when Δ is small, the error e is assumed to be a uniform Random variable over the interval

$$-\frac{\Delta}{2} < e < \frac{\Delta}{2}$$

* the average quantization error "Distortion" over all samples of the signal is:



$$D = E(x-\hat{x})^2 = E(e)^2$$

$$= \int_{-\Delta/2}^{\Delta/2} e^2 f_E(e)\, de$$

$$= \frac{1}{\Delta} \frac{e^3}{3} \Big|_{-\Delta/2}^{\Delta/2} = \frac{\Delta^2}{12}$$

$$\boxed{D = \frac{\Delta^2}{12}} \quad , \quad \Delta = \frac{X_{max} - X_{min}}{L}$$

Statistical Average

note that D depends on the design of the quantizer not on the message signal applied.

Example (1):-

let $x(t) = A\cos(2\pi f_0 t)$ be applied to a uniform quantizer with Dynamic Range $(-A, A)$ Find the SQNR?

$$SQNR = \frac{P_x}{D}$$

$$P_x = \frac{A^2}{2}$$

$$D = \frac{\Delta^2}{12}$$

$$\therefore SQNR = \frac{A^2/2}{\Delta^2/12} = \frac{6A^2}{\Delta^2} , \Delta = \frac{2A}{L}$$

$$\Rightarrow SQNR = \frac{3}{2}L^2 , L = 2^n$$

$$\Rightarrow \quad = \frac{3}{2}2^{2n}$$

- In dB scale, $SQNR_{dB} = 10\log_{10} SQNR$

$$= 10\log_{10}\frac{3}{2}2^{2n}$$

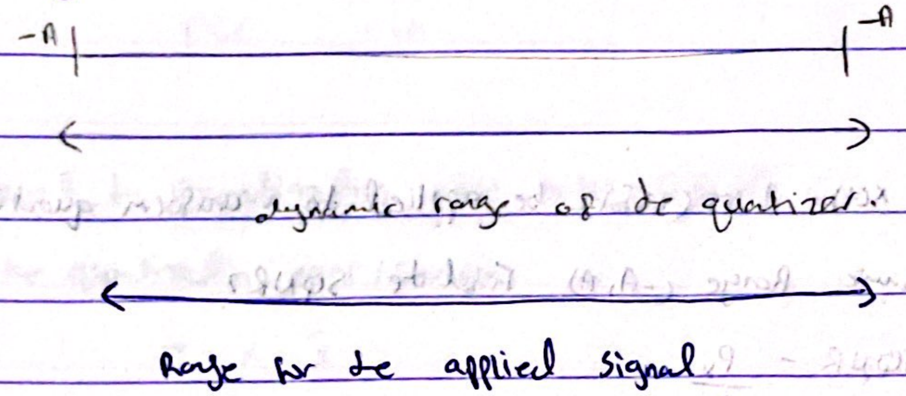
$$= 10\log_{10}\frac{3}{2} + 10\log_{10}2^{\textcircled{2n}}$$

$$= \underbrace{1.76}_{\text{old dB scale}} + 6.02n .$$

Notes:- Strong Signal

* From $SQNR = \frac{3}{2} 2^{2n}$, SQNR increses exponentially with the number of bits sample (n).

* From $SQNR_{dB}$, SQNR improves by 6.02 dB for every bit added.



$x(t) = A \cos (2\pi f_0 t)$.

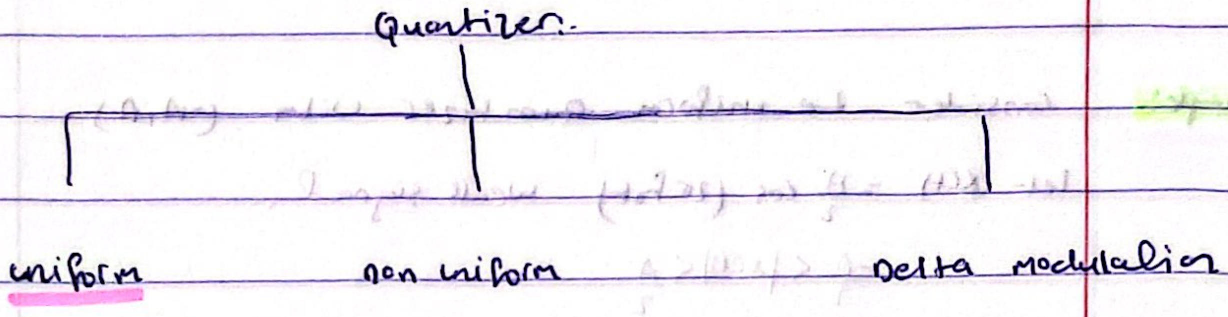
the two Range are equal, this means that the applied signal is strong and utilize the full Dynamic range of the Quantizer.

$$\Rightarrow \boxed{SQNR = \frac{3}{2} 2^{2n}}$$

Example:- let SQNR = 10 dB, when "n=1" increased by 1 bit what is the new SQNR?

$$SQNR_{new} = SQNR_{old} + 6.02\,n$$
$$= 10 + 6.02$$
$$= 16.02 \text{ dB}$$

Always we have 6.02 dB improved for every extra bit.



$SQNR = \frac{P_x}{\Delta}$

Dynamic range:

$x_{min} = -A$

$x_{max} = A$

L = # of levels. Find L, n Such that

$|e| < \frac{\Delta}{2}$, $\boxed{|e| < 0.01}$

$\Rightarrow \Delta \leq 0.02$

$\Delta = \frac{A - -A}{L} = \frac{2A}{L} < 0.02$

$\Rightarrow L > \frac{2A}{0.02}$

$L > \frac{2A}{\frac{2}{100}}$, $L > 100A$.

Note:-

$x(t) = A\cos(w_1 t) \Rightarrow SQNR_1 = \frac{3}{2} L^2$

$x(t) = \frac{A}{2}\cos(w_2 t) \Rightarrow SQNR_2 = \frac{12}{32} L^2$

$x(t) = \frac{A}{4}\cos(w_2 t) \Rightarrow SQNR_3 = \frac{4}{32} L^2$

$\rightarrow \frac{1}{4} SQNR1$

$\rightarrow \frac{1}{16} SQNR1$.

← عندما تقل ال SQNR تهمل ال recover signal

تحتوي على nois .

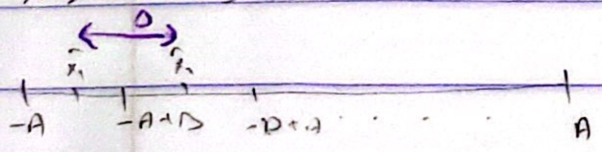
∘ let A = 1V

then L > 100

⇒ min no. of bit's / samples

$L = 2^n$.

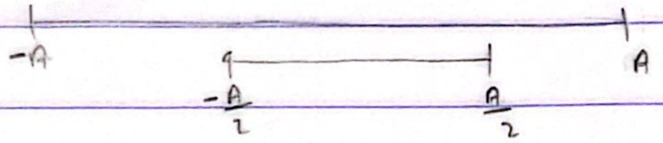
$2^n > 100 \Rightarrow n$ ✓



Example:- Consider the uniform quantizer with (-A, A)

let $x(t) = \frac{A}{2}\cos(2\pi f_0 t)$ weak signal.

$$-\frac{A}{2} < |x(t)| < \frac{A}{2}$$



$$P_x = \frac{(\frac{A}{2})^2}{2} = \frac{A^2}{2}$$

$$D = \frac{\Delta^2}{12}, \quad \Delta = \frac{2A}{L} \Rightarrow SQNR = \frac{P_x}{D} = \frac{12}{32}L^2$$

$$= \frac{12}{32}2^{2n}.$$

$$\therefore SQNR_{dB} = 6.02\,n - 4.77.$$

strong signal $x(t) = A\cos(2\pi f_c t)$

$$SQNR_A = \frac{3}{2}L^2.$$

weak signal $x(t) = \frac{A}{2}\cos(2\pi f_0 t)$.

$$SQNR_{\frac{A}{2}} = \frac{12}{32}L^2$$

Note that $SQNR_A > SQNR_{\frac{A}{2}}$

مع التكملة مع . notes الدكتور .