

Power series

Ex: $\sum_{n=0}^{\infty} X^n = 1 + X + X^2 + \dots$

- General form of power series with center a :

$$\sum_{n=0}^{\infty} C_n (X-a)^n$$

* a is the center

- Convergence of this series has 3 cases :- R : Radius of convergence

1) exist R such that $\sum C_n (X-a)^n$ converges abs for

$$|X-a| < R \Rightarrow [-a-R < X-a < +a+R] \Rightarrow [a-R < X < a+R]$$

and diverges for $|X-a| > R \Rightarrow [X-a > R \text{ or } X-a < -R]$

The series may or may not conv at End points

End Points are :- $X=a+R, X=a-R$

or 2) The series converges abs for all X : $R=\infty, -\infty < X < \infty$

or 3) The series converges only at $X=a$

Note: It's always conv at $X=a$

How to solve Problems like this?

1- you find the series of Abs. val / Then you test if it converges or not

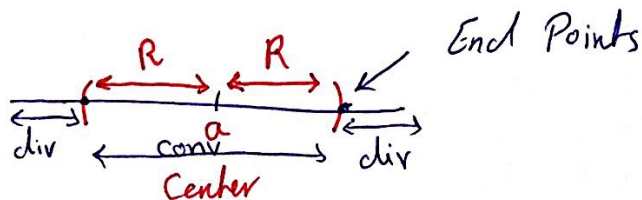
2- if it conv then The series conv. abs for an interval such that

3- you check the End Points by putting

them in the series

instead of X and finding whether the series converges or div at them

4- you summarize the intervals of convergence & divergence



Power series (Completion of 10.7)

Operations on power series

→ Multiplication

- When you multiply two series the Radius of Convergence stays the same

→ Differentiation

- If you have a series (such as $\sum x^n$) converges for a specific interval (here at $-1 < x < 1$) Then

$$\sum x^n = 1 + x + \dots = \frac{1}{1-x} \approx f(x) \text{ at } -1 < x < 1$$

and if you differentiate it you get a new series and a new function and the Radius of conv stays the same

$$f'(x) = \frac{-1}{(1-x)^2} = 1 + 2x + 3x^2 + \dots \text{ at } -1 < x < 1$$

→ integration

The only difference here is that you have to find C (The integration constant) :-

$$\text{Ex } \int \frac{1}{1-x} = \int 1 + x + x^2 + \dots$$

$$= -\ln|1-x| = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^{n+1}}{n+1} + C$$

you find a value of x to find C

here we use $x=0$

$$\begin{aligned} 0 &= 0 + C \\ C &= 0 \end{aligned}$$

$$\text{So } \Rightarrow -\ln|1-x| = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \text{ at } -1 < x < 1$$

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