Leen Sabri

Q6 $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ • $a_n = \frac{1}{n(\ln(n))^2}$ is positive terms $\forall\, n = 2,3,4,\dots$

• $f(x) = \frac{1}{x(\ln x)^2}$ is $+$, cont, $\downarrow$ on $[2,\infty)$

$\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty} \int_2^{b} \frac{dx}{x(\ln x)^2} \Rightarrow$ let $\ln x = u \Rightarrow \frac{1}{x}dx = du \Rightarrow dx = x\,du \Rightarrow \lim_{b\to\infty} \int_{\ln 2}^{b} \frac{x\,du}{x u^2} = \lim_{b\to\infty} \int_{\ln 2}^{b} \frac{du}{u^2} = \lim_{b\to\infty} \int_{2}^{b} u^{-2}\,du = \lim_{b\to\infty} \frac{u^{-1}}{-1}\Big|_{\ln 2}^{b} = -\lim_{b\to\infty} \frac{1}{u}\Big|_{\ln 2}^{b} = -\lim_{b\to\infty} \frac{1}{\ln x}\Big|_{2}^{b} = -\lim_{b\to\infty}\left[\frac{1}{\ln b} - \frac{1}{\ln 2}\right] = +\frac{1}{\ln 2}$ $\therefore$ conv by IT

Q13 $\sum_{n=1}^{\infty} \frac{n}{n+1} \Rightarrow \lim_{n\to\infty} \frac{n}{n+1} = 1 \Rightarrow 1 \neq 0 \therefore$ diverge by n^{th} term test.

Q20 $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}} \Rightarrow \lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \frac{\infty}{\infty} \Rightarrow \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}} \rightarrow$ The nth term test fail

• $a_n = \frac{\ln n}{\sqrt{n}}$ is positive terms $\forall n = 2,3,4,\dots$ $\quad f(x) = \frac{\ln x}{\sqrt{x}}$ is $+$, cont, $\downarrow$ on $[2,\infty)$

$\int_2^{\infty} \frac{\ln x}{\sqrt{x}}\,dx = \lim_{b\to\infty} \int_2^{b} \frac{\ln x}{\sqrt{x}}\,dx \Rightarrow \underset{x=2 \to u=\ln 2}{u = \ln x} \rightarrow du = \frac{1}{x}dx \rightarrow dx = x\,du \rightarrow \boxed{dx = e^u du} \Rightarrow \lim_{b\to\infty} \int_{\ln 2}^{b} \frac{u e^u}{(e^u)^{1/2}}\,du = \lim \int u(e^u)^{1/2}\,du = \lim \int u e^{u/2}\,du$

let $z = u \quad dw = e^{u/2} \Rightarrow \lim_{b\to\infty} \left(2ue^{u/2} - \int_{\ln 2}^{b} 2e^{u/2}\right) = \lim_{b\to\infty} \left(2ue^{u/2} - 2\int_{\ln 2}^{b} e^{u/2}\right) = \lim_{b\to\infty} \left(ue^{u/2} - 4e^{u/2}\Big|_{\ln 2}^{b}\right) = \lim_{b\to\infty} \left(2ue^{u/2} - (4e^{b/2} - 4e^{\ln 2/2})\right) = \lim_{b\to\infty} \left(2ue^{u/2} - 4e^{b/2} + 4e^{\ln 2/2}\right)$

$dz = 1 \longrightarrow v = 2e^{u/2}$

~~الريفة الأولى~~

$\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}} \Rightarrow$ by LMT $\Rightarrow \lim \frac{\frac{\ln n}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim \ln n = \infty \therefore$ div by LMT

$\Rightarrow \frac{\ln n}{\sqrt{n}} > \frac{1}{\sqrt{n}}$

Q22 $\sum_{n=1}^{\infty} \frac{5^n}{4^n+3} \Rightarrow \lim_{n\to\infty} \frac{5^n}{4^n+3} \xrightarrow{\text{Lhop}} \lim_{n\to\infty} \frac{5^n \ln 5}{4^n \ln 4} = \frac{\ln 5}{\ln 4} \lim_{n\to\infty} \left(\frac{5}{4}\right)^n = \infty \to \infty \neq 0 \therefore$ diverge by n^{th} term test

---

Q28 $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n \rightarrow \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \rightarrow e \neq 0 \therefore$ diverge by n^{th} terms test

---

Q32 $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)} = \lim_{b\to\infty} \int_1^{b} \frac{1}{n(1+\ln^2 n)} \rightarrow$ let $\ln n = u \Rightarrow \frac{1}{n}dn = du \Rightarrow dn = n\,du \Rightarrow \lim_{b\to\infty} \int_0^{\infty} \frac{n\,du}{n(1+u^2)} = \lim_{b\to\infty} \int_0^{\infty} \frac{du}{1+u^2} = \lim_{b\to\infty} \tan^{-1} u\Big|_0^{\infty} = \lim_{b\to\infty} \left[\overset{\frac{\pi}{2}}{\tan^{-1}\infty} - \overset{0}{\tan^{-1} 0}\right] = \frac{\pi}{2} \therefore$ converge by Integral test

$n=1 \Rightarrow u = \ln 1 = 0$

Q38 $\sum_{n=1}^{\infty} \frac{n}{n^2+1} \rightarrow \lim \frac{n}{n^2+1} = 0 \therefore$ the nth Trm test fail

$\lim_{b\to\infty} \int_1^{b} \frac{n}{n^2+1} \Rightarrow$ let $u = n^2+1 \Rightarrow du = 2n\,dn \rightarrow dn = \frac{du}{2n} \Rightarrow \lim_{b\to\infty} \int_2^{\infty} \frac{n\,du}{2nu} = \frac{1}{2} \lim_{b\to\infty} \int_2^{\infty} \frac{du}{u} = \frac{1}{2} \lim_{b\to\infty} \ln u\Big|_2^{\infty} = \frac{1}{2} \lim_{b\to\infty} [\ln b - \ln 2] = \infty \therefore$ diverge by Integral test

when $n=1 \rightarrow u = 2$

Q40 $\sum_{n=1}^{\infty} \operatorname{sech}^2 n \rightarrow n^{th}$ fail

$\lim_{b\to\infty} \int_1^{b} \operatorname{sech}^2 n = \lim_{b\to\infty} \tanh n\Big|_1^{b} = \lim_{b\to\infty} [\tanh b - \tanh 1]$

$= 1 - \tanh 1$

$\therefore$ converge by Integral test

Q42 $\sum_{n=3}^{\infty} \left(\frac{1}{n-1} - \frac{2a}{n+1}\right) = \lim_{b\to\infty} \int_3^{b} \left(\frac{1}{n-1} - \frac{2a}{n+1}\right) = \lim_{b\to\infty} \left(\ln(n-1) - 2a\ln(n+1)\right)\Big|_3^{b} = \lim_{b\to\infty} \ln(n-1) - \ln(n+1)^{2a} = \lim_{b\to\infty} \ln\left(\frac{(n-1)}{(n+1)^{2a}}\right)\Big|_3^{b} = \ln\frac{(b-1)}{(b+1)^{2a}} - \frac{2}{4^{2a}}$; $\Rightarrow \lim_{n\to\infty} \frac{b-1}{(b+1)^{2a}}$

$= \lim_{b\to\infty} \frac{1}{2a(b+1)^{2a-1}} =$

STUDENTS-HUB.com