139. $\frac{1}{\sqrt{\theta}}(\sin\theta)^{\sqrt{\theta}}\left(\frac{\ln\sin\theta}{2}+\theta\cot\theta\right)$

141. **(a)** $\frac{dS}{dt}=(4\pi r+2\pi h)\frac{dr}{dt}$ **(b)** $\frac{dS}{dt}=2\pi r\frac{dh}{dt}$

(c) $\frac{dS}{dt}=(4\pi r+2\pi h)\frac{dr}{dt}+2\pi r\frac{dh}{dt}$

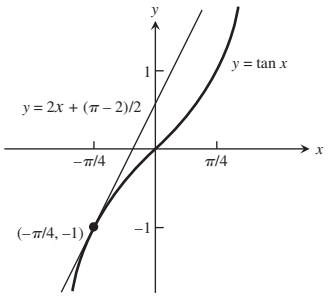
(d) $\frac{dr}{dt}=-\frac{r}{2r+h}\frac{dh}{dt}$

143. $-40\ \text{m}^2/\text{sec}$ **145.** 0.02 ohm/sec **147.** 22 m/sec

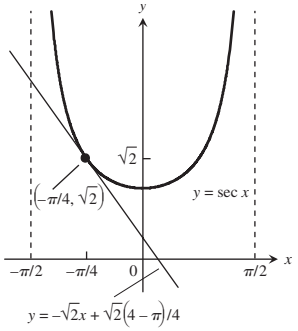
149. **(a)** $r=\frac{2}{5}h$ **(b)** $-\frac{125}{144\pi}$ ft/min

151. **(a)** $\frac{3}{5}$ km/sec or 600 m/sec **(b)** $\frac{18}{\pi}$ rpm

153. **(a)** $L(x)=2x+\frac{\pi-2}{2}$



(b) $L(x)=-\sqrt{2}x+\frac{\sqrt{2}(4-\pi)}{4}$



155. $L(x)=1.5x+0.5$ **157.** $dS=\frac{\pi r h_0}{\sqrt{r^2+{h_0}^2}}dh$

159. **(a)** 4% **(b)** 8% **(c)** 12%

Additional and Advanced Exercises, pp. 218–220

1. **(a)** $\sin 2\theta=2\sin\theta\cos\theta$; $2\cos 2\theta=2\sin\theta(-\sin\theta)+\cos\theta(2\cos\theta)$; $2\cos 2\theta=-2\sin^2\theta+2\cos^2\theta$; $\cos 2\theta=\cos^2\theta-\sin^2\theta$

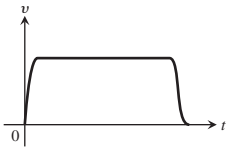
(b) $\cos 2\theta=\cos^2\theta-\sin^2\theta$; $-2\sin 2\theta=2\cos\theta(-\sin\theta)-2\sin\theta(\cos\theta)$; $\sin 2\theta=\cos\theta\sin\theta+\sin\theta\cos\theta$; $\sin 2\theta=2\sin\theta\cos\theta$

3. **(a)** $a=1, b=0, c=-\frac{1}{2}$ **(b)** $b=\cos a, c=\sin a$

5. $h=-4, k=\frac{9}{2}, a=\frac{5\sqrt{5}}{2}$

7. **(a)** $0.09y$ **(b)** Increasing at 1% per year

9. Answers will vary. Here is one possibility.



11. **(a)** 2 sec, 64 ft/sec **(b)** 12.31 sec, 393.85 ft

15. **(a)** $m=-\frac{b}{\pi}$ **(b)** $m=-1, b=\pi$

17. **(a)** $a=\frac{3}{4}, b=\frac{9}{4}$ **19.** f odd $\Rightarrow f'$ is even

23. h' is defined but not continuous at $x=0$; k' is defined *and* continuous at $x=0$.

27. **(a)** 0.8156 ft **(b)** 0.00613 sec
(c) It will lose about 8.83 min/day.

CHAPTER 4

Section 4.1, pp. 227–230

1. Absolute minimum at $x=c_2$; absolute maximum at $x=b$

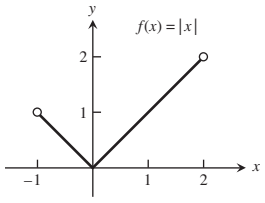
3. Absolute maximum at $x=c$; no absolute minimum

5. Absolute minimum at $x=a$; absolute maximum at $x=c$

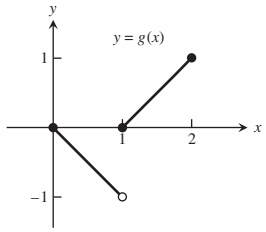
7. No absolute minimum; no absolute maximum

9. Absolute maximum at (0, 5) **11.** (c) **13.** (d)

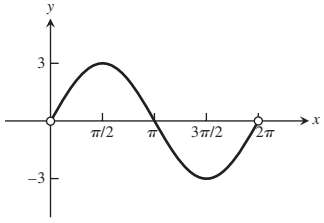
15. Absolute minimum at $x=0$; no absolute maximum



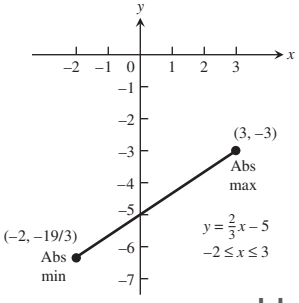
17. Absolute maximum at $x=2$; no absolute minimum



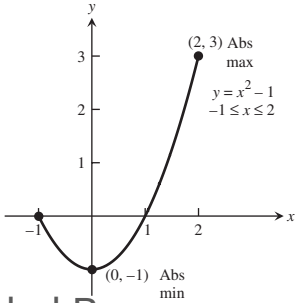
19. Absolute maximum at $x=\pi/2$; absolute minimum at $x=3\pi/2$



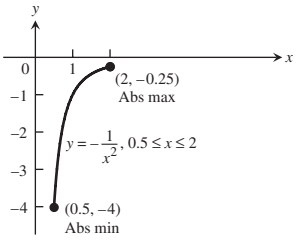
21. Absolute maximum: -3; absolute minimum: $-19/3$



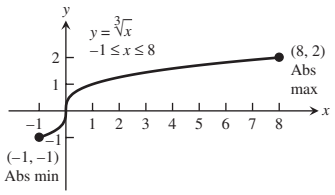
23. Absolute maximum: 3; absolute minimum: -1



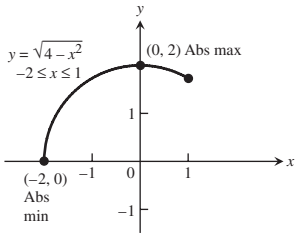
25. Absolute maximum: -0.25; absolute minimum: -4



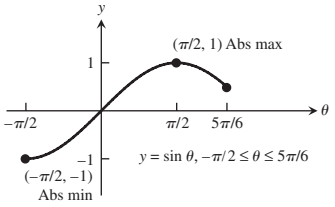
27. Absolute maximum: 2; absolute minimum: -1



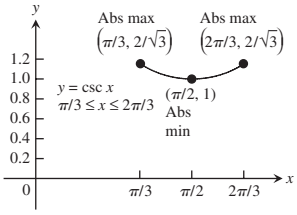
29. Absolute maximum: 2; absolute minimum: 0



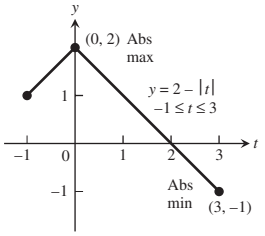
31. Absolute maximum: 1; absolute minimum: -1



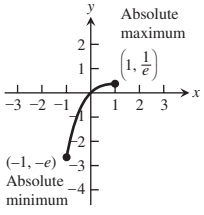
33. Absolute maximum: $2/\sqrt{3}$; absolute minimum: 1



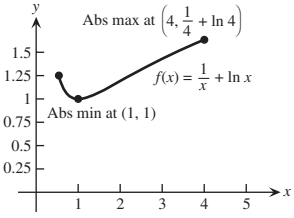
35. Absolute maximum: 2; absolute minimum: -1



37. Absolute maximum is $1/e$ at $x = 1$; absolute minimum is $-e$ at $x = -1$.



39. Absolute maximum value is $(1/4) + \ln 4$ at $x = 4$; absolute minimum value is 1 at $x = 1$; local maximum at $(1/2, 2 - \ln 2)$.



41. Increasing on $(0, 8)$, decreasing on $(-1, 0)$; absolute maximum: 16 at $x = 8$; absolute minimum: 0 at $x = 0$

43. Increasing on $(-32, 1)$; absolute maximum: 1 at $\theta = 1$; absolute minimum: -8 at $\theta = -32$

45. $x = 3$

47. $x = 1, x = 4$

49. $x = 1$

51. $x = 0$ and $x = 4$

53. Minimum value is 1 at $x = 2$.

55. Local maximum at $(-2, 17)$; local minimum at $\left(\frac{4}{3}, -\frac{41}{27}\right)$

57. Minimum value is 0 at $x = -1$ and $x = 1$.

59. There is a local minimum at (0, 1).

61. Maximum value is $\frac{1}{2}$ at $x = 1$; minimum value is $-\frac{1}{2}$ at $x = -1$.

63. The minimum value is 2 at $x = 0$.

65. The minimum value is $-\frac{1}{e}$ at $x = \frac{1}{e}$.

67. The maximum value is $\frac{\pi}{2}$ at $x = 0$; an absolute minimum value is 0 at $x = 1$ and $x = -1$.

69.

Critical point or endpoint	Derivative	Extremum	Value
$x = -\frac{4}{5}$	0	Local max	$\frac{12}{25}10^{1/3} \approx 1.034$
$x = 0$	Undefined	Local min	0

71.

Critical point or endpoint	Derivative	Extremum	Value
$x = -2$	Undefined	Local max	0
$x = -\sqrt{2}$	0	Minimum	-2
$x = \sqrt{2}$	0	Maximum	2
$x = 2$	Undefined	Local min	0

73.

Critical point or endpoint	Derivative	Extremum	Value
$x = 1$	Undefined	Minimum	2

75.

Critical point or endpoint	Derivative	Extremum	Value
$x = -1$	0	Maximum	5
$x = 1$	Undefined	Local min	1
$x = 3$	0	Maximum	5

77. **(a)** No
(b) The derivative is defined and nonzero for $x \neq 2$. Also, $f(2) = 0$ and $f(x) > 0$ for all $x \neq 2$.
(c) No, because $(-\infty, \infty)$ is not a closed interval.
(d) The answers are the same as parts (a) and (b) with 2 replaced by a.

79. Yes **81.** g assumes a local maximum at $-c$.

83. **(a)** Maximum value is 144 at $x = 2$.
(b) The largest volume of the box is 144 cubic units, and it occurs when $x = 2$.

85. $\frac{{v_0}^2}{2g} + s_0$

87. Maximum value is 11 at $x = 5$; minimum value is 5 on the interval $[-3, 2]$; local maximum at $(-5, 9)$.

89. Maximum value is 5 on the interval $[3, \infty)$; minimum value is -5 on the interval $(-\infty, -2]$.

Section 4.2, pp. 236–238

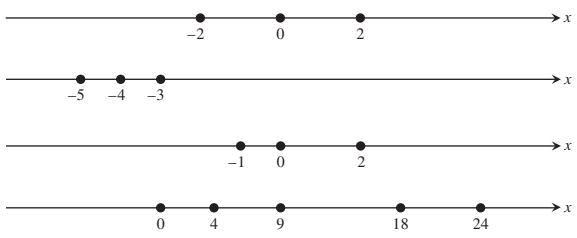
1. $1/2$ **3.** 1 **5.** $\pm\sqrt{1 - \frac{4}{\pi^2}} \approx \pm 0.771$

7. $\frac{1}{3}\left(1 + \sqrt{7}\right) \approx 1.22, \frac{1}{3}\left(1 - \sqrt{7}\right) \approx -0.549$

9. Does not; f is not differentiable at the interior domain point $x = 0$.

11. Does **13.** Does not; f is not differentiable at $x = -1$.

17. **(a)**



29. Yes **31.** **(a)** 4 **(b)** 3 **(c)** 3

33. **(a)** $\frac{x^2}{2} + C$ **(b)** $\frac{x^3}{3} + C$ **(c)** $\frac{x^4}{4} + C$

35. **(a)** $\frac{1}{x} + C$ **(b)** $x + \frac{1}{x} + C$ **(c)** $5x - \frac{1}{x} + C$

37. **(a)** $-\frac{1}{2}\cos 2t + C$ **(b)** $2\sin\frac{t}{2} + C$

(c) $-\frac{1}{2}\cos 2t + 2\sin\frac{t}{2} + C$

39. $f(x) = x^2 - x$ **41.** $f(x) = 1 + \frac{e^{2x}}{2}$

43. $s = 4.9t^2 + 5t + 10$ **45.** $s = \frac{1 - \cos(\pi t)}{\pi}$

47. $s = e^t + 19t + 4$ **49.** $s = \sin(2t) - 3$

51. If $T(t)$ is the temperature of the thermometer at time t, then $T(0) = -19$ °C and $T(14) = 100$ °C. From the Mean Value Theorem, there exists a $0 < t_0 < 14$ such that $\frac{T(14) - T(0)}{14 - 0} = 8.5$ °C/sec $= T'(t_0)$, the rate at which the temperature was changing at $t = t_0$ as measured by the rising mercury on the thermometer.

53. Because its average speed was approximately 7.667 knots, and by the Mean Value Theorem, it must have been going that speed at least once during the trip.

57. The conclusion of the Mean Value Theorem yields

$$\frac{\frac{1}{b} - \frac{1}{a}}{b - a} = -\frac{1}{c^2} \Rightarrow c^2\left(\frac{a - b}{ab}\right) = a - b \Rightarrow c = \sqrt{ab}.$$

61. $f(x)$ must be zero at least once between a and b by the Intermediate Value Theorem. Now suppose that $f(x)$ is zero twice between a and b. Then, by the Mean Value Theorem, $f'(x)$ would have to be zero at least once between the two zeros of $f(x)$, but this can't be true since we are given that $f'(x) \neq 0$ on this interval. Therefore, $f(x)$ is zero once and only once between a and b.

71. $1.09999 \leq f(0.1) \leq 1.1$

Section 4.3, pp. 241–243

1. **(a)** 0, 1
(b) Increasing on $(-\infty, 0)$ and $(1, \infty)$; decreasing on $(0, 1)$
(c) Local maximum at $x = 0$; local minimum at $x = 1$

3. **(a)** -2, 1
(b) Increasing on $(-2, 1)$ and $(1, \infty)$; decreasing on $(-\infty, -2)$
(c) No local maximum; local minimum at $x = -2$

5. **(a)** Critical point at $x = 1$
(b) Decreasing on $(-\infty, 1)$, increasing on $(1, \infty)$
(c) Local (and absolute) minimum at $x = 1$

7. **(a)** 0, 1
(b) Increasing on $(-\infty, -2)$ and $(1, \infty)$; decreasing on $(-2, 0)$ and $(0, 1)$
(c) Local minimum at $x = 1$

9. **(a)** -2, 2
(b) Increasing on $(-\infty, -2)$ and $(2, \infty)$; decreasing on $(-2, 0)$ and $(0, 2)$
(c) Local maximum at $x = -2$; local minimum at $x = 2$

11. **(a)** -2, 0
(b) Increasing on $(-\infty, -2)$ and $(0, \infty)$; decreasing on $(-2, 0)$
(c) Local maximum at $x = -2$; local minimum at $x = 0$

13. **(a)** $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

(b) Increasing on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$; decreasing on $\left(0, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$, and $\left(\frac{4\pi}{3}, 2\pi\right)$

(c) Local maximum at $x = 0$ and $x = \frac{4\pi}{3}$; local minimum at $x = \frac{2\pi}{3}$ and $x = 2\pi$

15. **(a)** Increasing on $(-2, 0)$ and $(2, 4)$; decreasing on $(-4, -2)$ and $(0, 2)$
(b) Absolute maximum at $(-4, 2)$; local maximum at $(0, 1)$ and $(4, -1)$; absolute minimum at $(2, -3)$; local minimum at $(-2, 0)$

17. **(a)** Increasing on $(-4, -1)$, $(1/2, 2)$, and $(2, 4)$; decreasing on $(-1, 1/2)$
(b) Absolute maximum at $(4, 3)$; local maximum at $(-1, 2)$ and $(2, 1)$; no absolute minimum; local minimum at $(-4, -1)$ and $(1/2, -1)$

19. **(a)** Increasing on $(-\infty, -1.5)$; decreasing on $(-1.5, \infty)$
(b) Local maximum: 5.25 at $t = -1.5$; absolute maximum: 5.25 at $t = -1.5$

21. **(a)** Decreasing on $(-\infty, 0)$; increasing on $(0, 4/3)$; decreasing on $(4/3, \infty)$
(b) Local minimum at $x = 0$ $(0, 0)$; local maximum at $x = 4/3$ $(4/3, 32/27)$; no absolute extrema

23. **(a)** Decreasing on $(-\infty, 0)$; increasing on $(0, 1/2)$; decreasing on $(1/2, \infty)$
(b) Local minimum at $\theta = 0$ $(0, 0)$; local maximum at $\theta = 1/2$ $(1/2, 1/4)$; no absolute extrema

25. **(a)** Increasing on $(-\infty, \infty)$; never decreasing
(b) No local extrema; no absolute extrema

27. **(a)** Increasing on $(-2, 0)$ and $(2, \infty)$; decreasing on $(-\infty, -2)$ and $(0, 2)$
(b) Local maximum: 16 at $x = 0$; local minimum: 0 at $x = \pm 2$; no absolute maximum; absolute minimum: 0 at $x = \pm 2$

29. **(a)** Increasing on $(-\infty, -1)$; decreasing on $(-1, 0)$; increasing on $(0, 1)$; decreasing on $(1, \infty)$
(b) Local maximum: 0.5 at $x = \pm 1$; local minimum: 0 at $x = 0$; absolute maximum: $1/2$ at $x = \pm 1$; no absolute minimum

31. **(a)** Increasing on $(10, \infty)$; decreasing on $(1, 10)$
(b) Local maximum: 1 at $x = 1$; local minimum: -8 at $x = 10$; absolute minimum: -8 at $x = 10$

33. **(a)** Decreasing on $(-2\sqrt{2}, -2)$; increasing on $(-2, 2)$; decreasing on $(2, 2\sqrt{2})$
(b) Local minima: $g(-2) = -4$, $g(2\sqrt{2}) = 0$; local maxima: $g(-2\sqrt{2}) = 0$, $g(2) = 4$; absolute maximum: 4 at $x = 2$; absolute minimum: -4 at $x = -2$

35. **(a)** Increasing on $(-\infty, 1)$; decreasing when $1 < x < 2$, decreasing when $2 < x < 3$; discontinuous at $x = 2$; increasing on $(3, \infty)$
(b) Local minimum at $x = 3$ $(3, 6)$; local maximum at $x = 1$ $(1, 2)$; no absolute extrema

37. **(a)** Increasing on $(-2, 0)$ and $(0, \infty)$; decreasing on $(-\infty, -2)$
(b) Local minimum: $-6\sqrt[3]{2}$ at $x = -2$; no absolute maximum; absolute minimum: $-6\sqrt[3]{2}$ at $x = -2$

39. **(a)** Increasing on $(-\infty, -2/\sqrt{7})$ and $(2/\sqrt{7}, \infty)$; decreasing on $(-2/\sqrt{7}, 0)$ and $(0, 2/\sqrt{7})$
(b) Local maximum: $24\sqrt[3]{2}/7^{7/6} \approx 3.12$ at $x = -2/\sqrt{7}$; local minimum: $-24\sqrt[3]{2}/7^{7/6} \approx -3.12$ at $x = 2/\sqrt{7}$; no absolute extrema

41. **(a)** Increasing on $((1/3)\ln(1/2), \infty)$, decreasing on $(-\infty, (1/3)\ln(1/2))$
(b) Local minimum is $\dfrac{3}{2^{2/3}}$ at $x = (1/3)\ln(1/2)$; no local maximum; absolute minimum is $\dfrac{3}{2^{2/3}}$ at $x = (1/3)\ln(1/2)$; no absolute maximum

43. **(a)** Increasing on (e^{-1}, ∞), decreasing on $(0, e^{-1})$
(b) A local minimum is $-e^{-1}$ at $x = e^{-1}$, no local maximum; absolute minimum is $-e^{-1}$ at $x = e^{-1}$, no absolute maximum

45. **(a)** Local maximum: 1 at $x = 1$; local minimum: 0 at $x = 2$
(b) Absolute maximum: 1 at $x = 1$; no absolute minimum

47. **(a)** Local maximum: 1 at $x = 1$; local minimum: 0 at $x = 2$
(b) No absolute maximum; absolute minimum: 0 at $x = 2$

49. **(a)** Local maxima: -9 at $t = -3$ and 16 at $t = 2$; local minimum: -16 at $t = -2$
(b) Absolute maximum: 16 at $t = 2$; no absolute minimum

51. **(a)** Local minimum: 0 at $x = 0$
(b) No absolute maximum; absolute minimum: 0 at $x = 0$

53. **(a)** Local maximum: 5 at $x = 0$; local minimum: 0 at $x = -5$ and $x = 5$
(b) Absolute maximum: 5 at $x = 0$; absolute minimum: 0 at $x = -5$ and $x = 5$

55. **(a)** Local maximum: 2 at $x = 0$; local minimum: $\dfrac{\sqrt{3}}{4\sqrt{3} - 6}$ at $x = 2 - \sqrt{3}$
(b) No absolute maximum; an absolute minimum at $x = 2 - \sqrt{3}$

57. **(a)** Local maximum: 1 at $x = \pi/4$; local maximum: 0 at $x = \pi$; local minimum: 0 at $x = 0$; local minimum: -1 at $x = 3\pi/4$

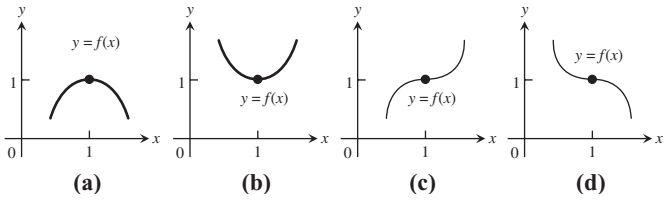
59. Local maximum: 2 at $x = \pi/6$; local maximum: $\sqrt{3}$ at $x = 2\pi$; local minimum: -2 at $x = 7\pi/6$; local minimum: $\sqrt{3}$ at $x = 0$

61. **(a)** Local minimum: $(\pi/3) - \sqrt{3}$ at $x = 2\pi/3$; local maximum: 0 at $x = 0$; local maximum: π at $x = 2\pi$

63. **(a)** Local minimum: 0 at $x = \pi/4$

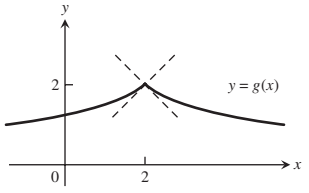
65. Local maximum: 3 at $\theta = 0$; local minimum: -3 at $\theta = 2\pi$

67.

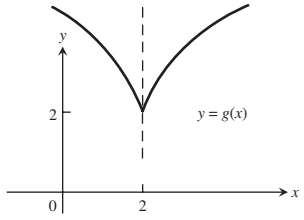


(a) **(b)** **(c)** **(d)**

69. **(a)**



(b)



73. $a = -2$, $b = 4$

75. **(a)** Absolute minimum occurs at $x = \pi/3$ with $f(\pi/3) = -\ln 2$, and the absolute maximum occurs at $x = 0$ with $f(0) = 0$.
(b) Absolute minimum occurs at $x = 1/2$ and $x = 2$ with $f(1/2) = f(2) = \cos(\ln 2)$, and the absolute maximum occurs at $x = 1$ with $f(1) = 1$.

77. Minimum of $2 - 2\ln 2 \approx 0.613706$ at $x = \ln 2$; maximum of 1 at $x = 0$

79. Absolute maximum value of $1/2e$ assumed at $x = 1/\sqrt{e}$

83. Increasing; $\dfrac{df^{-1}}{dx} = \dfrac{1}{9}x^{-2/3}$

85. Decreasing; $\dfrac{df^{-1}}{dx} = -\dfrac{1}{3}x^{-2/3}$

Section 4.4, pp. 251–254

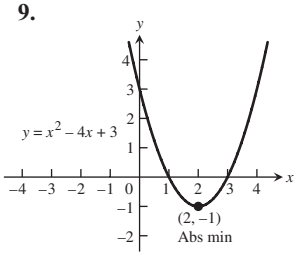
1. Local maximum: $3/2$ at $x = -1$; local minimum: -3 at $x = 2$; point of inflection at $(1/2, -3/4)$; rising on $(-\infty, -1)$ and $(2, \infty)$; falling on $(-1, 2)$; concave up on $(1/2, \infty)$; concave down on $(-\infty, 1/2)$

3. Local maximum: $3/4$ at $x = 0$; local minimum: 0 at $x = \pm 1$; points of inflection at $\left(-\sqrt{3}, \dfrac{3\sqrt[3]{4}}{4}\right)$ and $\left(\sqrt{3}, \dfrac{3\sqrt[3]{4}}{4}\right)$; rising on $(-1, 0)$ and $(1, \infty)$; falling on $(-\infty, -1)$ and $(0, 1)$; concave up on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$; concave down on $(-\sqrt{3}, \sqrt{3})$

5. Local maxima: $\dfrac{-2\pi}{3} + \dfrac{\sqrt{3}}{2}$ at $x = -2\pi/3$, $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{2}$ at $x = \pi/3$; local minima: $-\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}$ at $x = -\pi/3$, $\dfrac{2\pi}{3} - \dfrac{\sqrt{3}}{2}$ at $x = 2\pi/3$; points of inflection at $(-\pi/2, -\pi/2)$, $(0, 0)$, and $(\pi/2, \pi/2)$, rising on $(-\pi/3, \pi/3)$; falling on $(-2\pi/3, -\pi/3)$ and $(\pi/3, 2\pi/3)$, concave up on $(-\pi/2, 0)$ and $(\pi/2, 2\pi/3)$; concave down on $(-2\pi/3, -\pi/2)$ and $(0, \pi/2)$

7. Local maxima: 1 at $x = -\pi/2$ and $x = \pi/2$, 0 at $x = -2\pi$ and $x = 2\pi$; local minima: -1 at $x = -3\pi/2$ and $x = 3\pi/2$, 0 at $x = 0$; points of inflection at $(-\pi, 0)$ and $(\pi, 0)$; rising on $(-3\pi/2, -\pi/2)$, $(0, \pi/2)$, and $(3\pi/2, 2\pi)$; falling on $(-2\pi, -3\pi/2)$, $(-\pi/2, 0)$, and $(\pi/2, 3\pi/2)$; concave up on $(-2\pi, -\pi)$ and $(\pi, 2\pi)$; concave down on $(-\pi, 0)$ and $(0, \pi)$

9.



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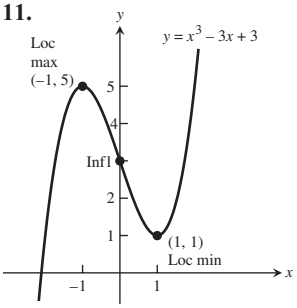

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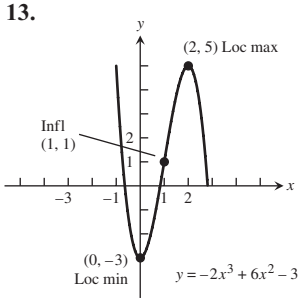

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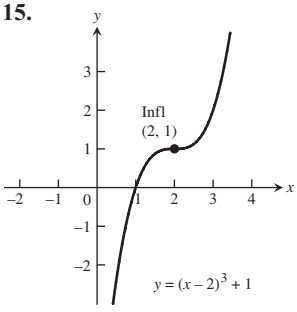

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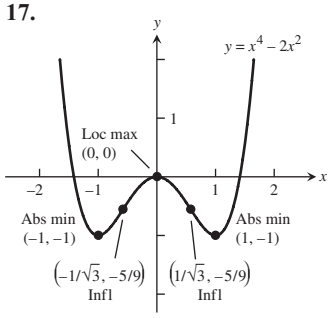

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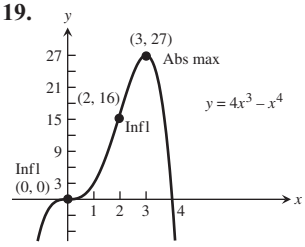

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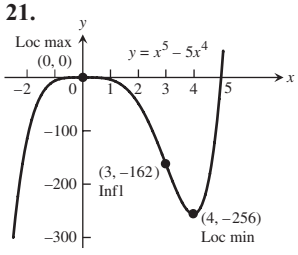

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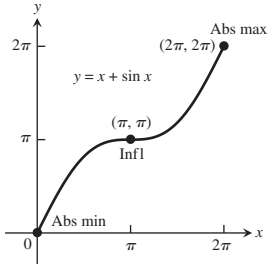

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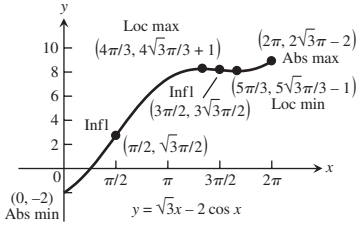

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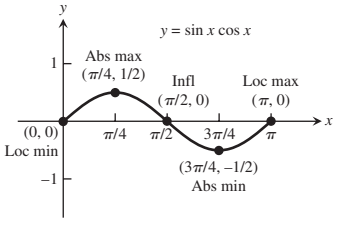

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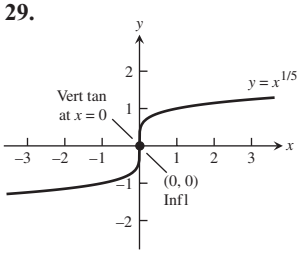

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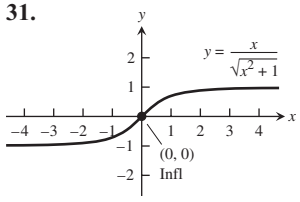

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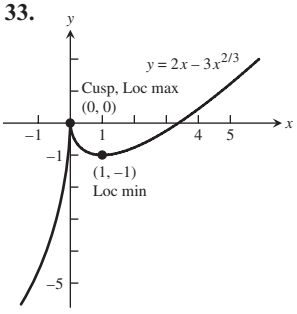

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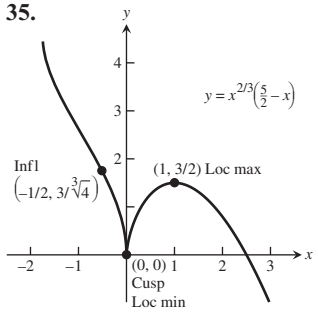

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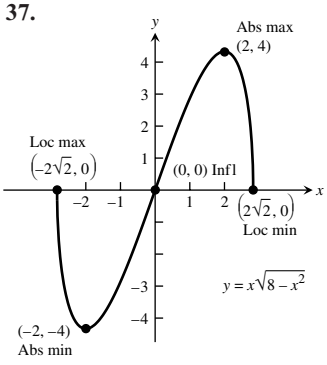

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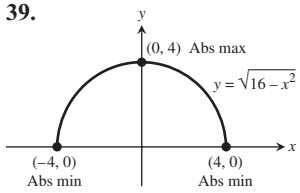

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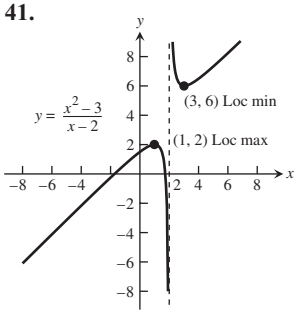

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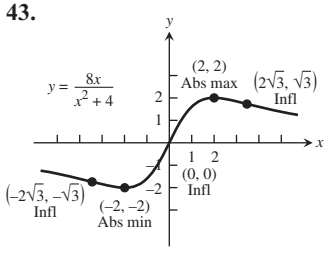

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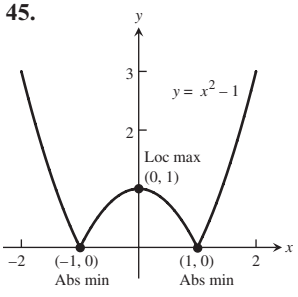

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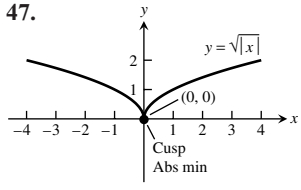

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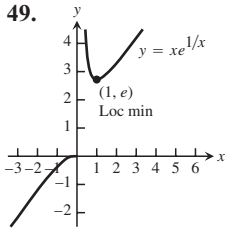

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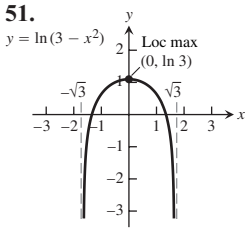

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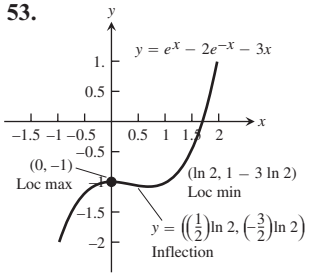

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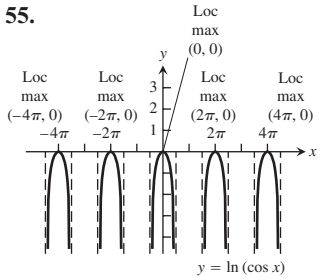
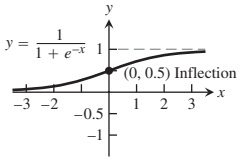
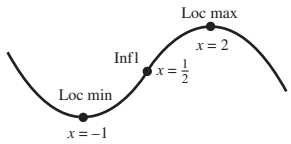


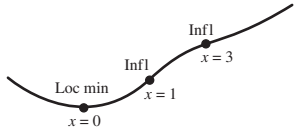
57.



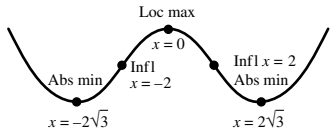
59. $y'' = 1 - 2x$



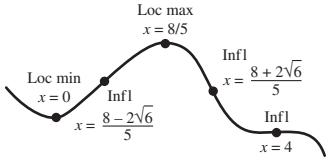
61. $y'' = 3(x - 3)(x - 1)$



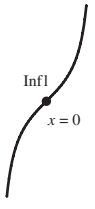
63. $y'' = 3(x - 2)(x + 2)$



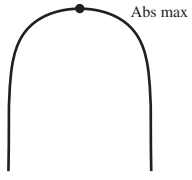
65. $y'' = 4(4 - x)(5x^2 - 16x + 8)$



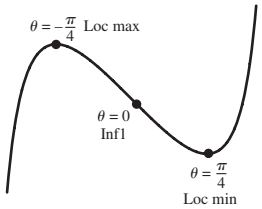
67. $y'' = 2\sec^2 x \tan x$



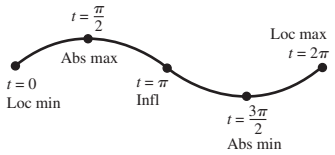
69. $y'' = -\frac{1}{2}\csc^2\frac{\theta}{2}$, $0 < \theta < 2\pi$



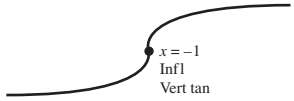
71. $y'' = 2\tan\theta\sec^2\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



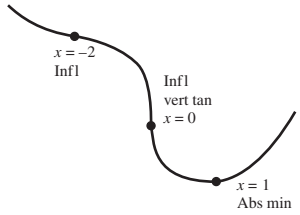
73. $y'' = -\sin t$, $0 \le t \le 2\pi$



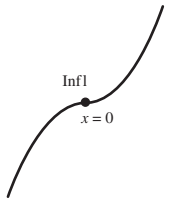
75. $y'' = -\frac{2}{3}(x + 1)^{-5/3}$



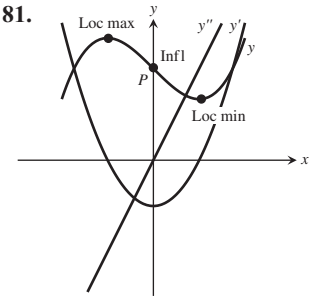
77. $y'' = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$



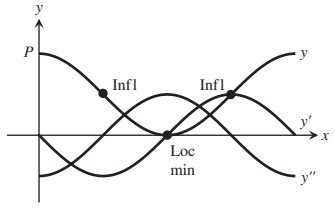
79. $y'' = \begin{cases} -2, & x < 0 \\ 2, & x > 0 \end{cases}$



81.



83.



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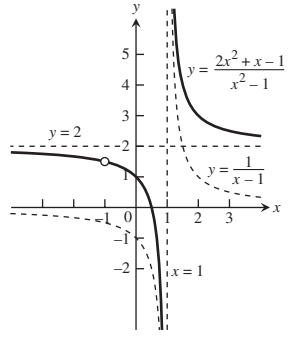

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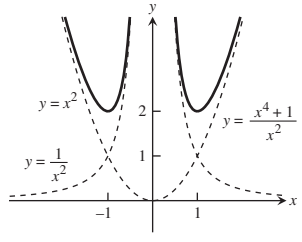
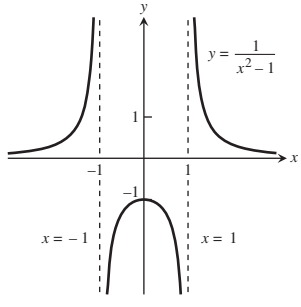


89.



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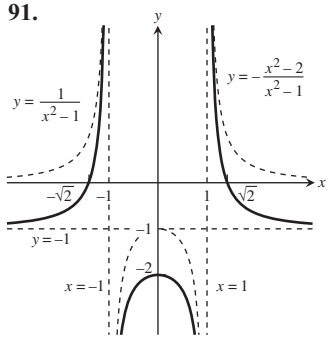
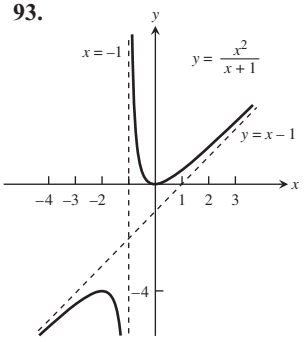
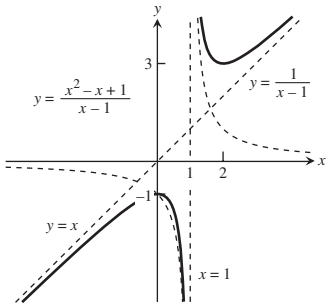


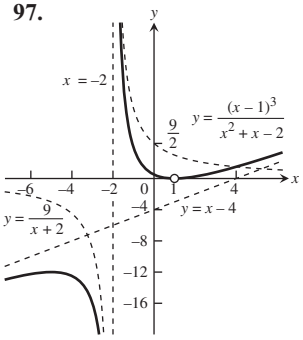
93.



95.



97.



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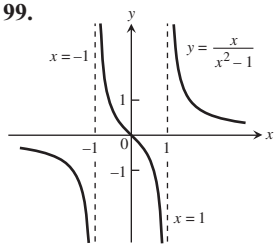
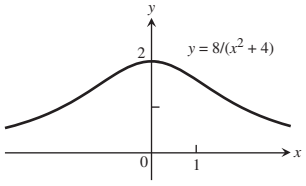


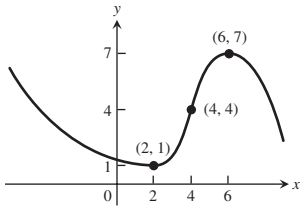
101.



103.

Point	y'	y''
P	$-$	$+$
Q	$+$	0
R	$+$	$-$
S	0	$-$
T	$-$	$-$

105.



107. **(a)** Towards origin: $0 \le t < 2$ and $6 \le t \le 10$; away from origin: $2 \le t \le 6$ and $10 \le t \le 15$

(b) $t = 2, t = 6, t = 10$ **(c)** $t = 5, t = 7, t = 13$

(d) Positive: $5 \le t \le 7, 13 \le t \le 15$; negative: $0 \le t \le 5, 7 \le t \le 13$

109. ≈ 60 thousand units

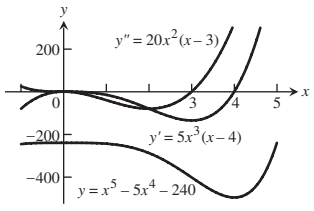
111. Local minimum at $x = 2$; inflection points at $x = 1$ and $x = 5/3$

115. $b = -3$

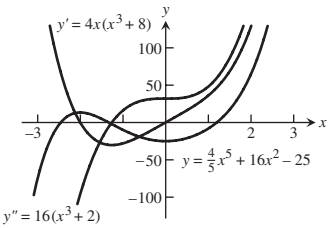
119. $-1, 2$

121. $a = 1, b = 3, c = 9$

123. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively. Inflection at $x = 3$, local maximum at $x = 0$, local minimum at $x = 4$.



125. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively. Inflection at $x = -\sqrt[3]{2}$; local maximum at $x = -2$; local minimum at $x = 0$.



Section 4.5, pp. 261–262

1. $-1/4$ **3.** $5/7$ **5.** $1/2$ **7.** $1/4$ **9.** $-23/7$ **11.** $5/7$ **13.** 0

15. -16 **17.** -2 **19.** $1/4$ **21.** 2 **23.** 3 **25.** -1

27. $\ln 3$ **29.** $\dfrac{1}{\ln 2}$ **31.** $\ln 2$ **33.** 1 **35.** $1/2$ **37.** $\ln 2$

39. $-\infty$ **41.** $-1/2$ **43.** -1 **45.** 1 **47.** 0 **49.** 2

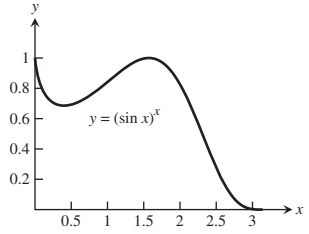
51. $1/e$ **53.** 1 **55.** $1/e$ **57.** $e^{1/2}$ **59.** 1 **61.** e^3

63. 0 **65.** 1 **67.** 3 **69.** 1 **71.** 0 **73.** ∞

75. (b) is correct. **77.** (d) is correct. **79.** $c = \dfrac{27}{10}$ **81.** **(b)** $\dfrac{-1}{2}$

83. -1 **87.** **(a)** $y = 1$ **(b)** $y = 0, y = \dfrac{3}{2}$

89. **(a)** We should assign the value 1 to $f(x) = (\sin x)^x$ to make it continuous at $x = 0$.



(c) The maximum value of $f(x)$ is close to 1 near the point $x \approx 1.55$ (see the graph in part (a)).

Section 4.6, pp. 268–274

1. 16 in., 4 in. by 4 in.

3. **(a)** $(x, 1 - x)$ **(b)** $A(x) = 2x(1 - x)$

(c) $\dfrac{1}{2}$ square units, 1 by $\dfrac{1}{2}$

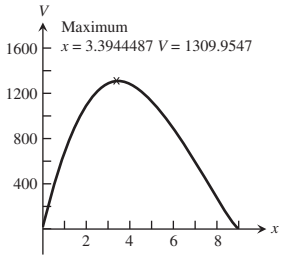
5. $\dfrac{14}{3} \times \dfrac{35}{3} \times \dfrac{5}{3}$ in., $\dfrac{2450}{27}$ in^3 **7.** 80,000 m^2; 400 m by 200 m

9. **(a)** The optimum dimensions of the tank are 10 ft on the base edges and 5 ft deep.

(b) Minimizing the surface area of the tank minimizes its weight for a given wall thickness. The thickness of the steel walls would likely be determined by other considerations such as structural requirements.

11. 9×18 in. **13.** $\dfrac{\pi}{2}$ **15.** $h : r = 8 : \pi$

17. **(a)** $V(x) = 2x(24 - 2x)(18 - 2x)$ **(b)** Domain: (0, 9)

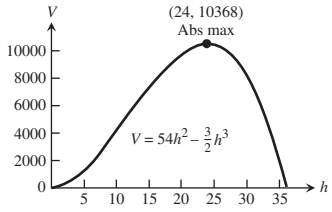


(c) Maximum volume ≈ 1309.95 in^3 when $x \approx 3.39$ in.
(d) $V'(x) = 24x^2 - 336x + 864$, so the critical point is at $x = 7 - \sqrt{13}$, which confirms the result in part (c).
(e) $x = 2$ in. or $x = 5$ in.

19. ≈ 2418.40 cm^3

21. **(a)** $h = 24$, $w = 18$

(b)



23. If r is the radius of the hemisphere, h the height of the cylinder, and V the volume, then $r = \left(\frac{3V}{8\pi}\right)^{1/3}$ and $h = \left(\frac{3V}{\pi}\right)^{1/3}$.

25. **(b)** $x = \frac{51}{8}$ **(c)** $L \approx 11$ in.

27. Radius $= \sqrt{2}$ m, height $= 1$ m, volume $= \frac{2\pi}{3}$ m^3

29. 1 **31.** $\frac{9b}{9 + \sqrt{3}\pi}$ m, triangle, $\frac{b\sqrt{3}\pi}{9 + \sqrt{3}\pi}$ m, circle

33. $\frac{3}{2} \times 2$ **35.** **(a)** 16 **(b)** -1

37. **(a)** $v(0) = 96$ ft/sec
(b) 256 ft at $t = 3$ sec
(c) Velocity when $s = 0$ is $v(7) = -128$ ft/sec.

39. ≈ 46.87 ft **41.** **(a)** $6 \times 6\sqrt{3}$ in.

43. **(a)** $4\sqrt{3} \times 4\sqrt{6}$ in.

45. **(a)** $10\pi \approx 31.42$ cm/sec; when $t = 0.5$ sec, 1.5 sec, 2.5 sec, 3.5 sec; $s = 0$, acceleration is 0.
(b) 10 cm from rest position; speed is 0.

47. **(a)** $s = ((12 - 12t)^2 + 64t^2)^{1/2}$
(b) -12 knots, 8 knots
(c) No
(e) $4\sqrt{13}$. This limit is the square root of the sums of the squares of the individual speeds.

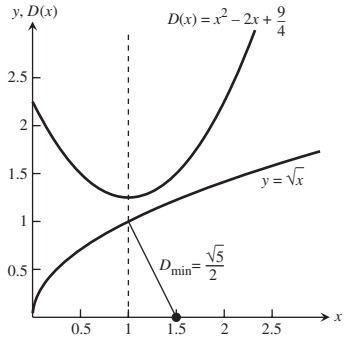
49. $x = \frac{a}{2}, v = \frac{ka^2}{4}$ **51.** $\frac{c}{2} + 50$

53. **(a)** $\sqrt{\frac{2km}{h}}$ **(b)** $\sqrt{\frac{2km}{h}}$

57. $4 \times 4 \times 3$ ft, \$288 **59.** $M = \frac{C}{2}$ **65.** **(a)** $y = -1$

67. **(a)** The minimum distance is $\frac{\sqrt{5}}{2}$.

(b) The minimum distance is from the point $(3/2, 0)$ to the point $(1, 1)$ on the graph of $y = \sqrt{x}$, and this occurs at the value $x = 1$, where $D(x)$, the distance squared, has its minimum value.

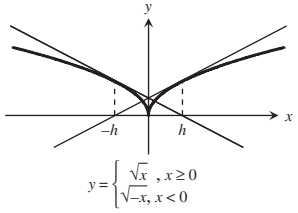


Section 4.7, pp. 277–279

1. $x_2 = -\frac{5}{3}, \frac{13}{21}$ **3.** $x_2 = -\frac{51}{31}, \frac{5763}{4945}$ **5.** $x_2 = \frac{2387}{2000}$

7. x_1, and all later approximations will equal x_0.

9.



11. The points of intersection of $y = x^3$ and $y = 3x + 1$ or $y = x^3 - 3x$ and $y = 1$ have the same x-values as the roots of part (i) or the solutions of part (iv). **13.** 1.165561185

15. **(a)** Two **(b)** 0.35003501505249 and -1.0261731615301

17. ± 1.3065629648764, ± 0.5411961001462 **19.** $x \approx 0.45$

21. 0.8192 **23.** 0, 0.53485 **25.** The root is 1.17951.

27. **(a)** For $x_0 = -2$ or $x_0 = -0.8$, $x_i \to -1$ as i gets large.
(b) For $x_0 = -0.5$ or $x_0 = 0.25$, $x_i \to 0$ as i gets large.
(c) For $x_0 = 0.8$ or $x_0 = 2$, $x_i \to 1$ as i gets large.
(d) For $x_0 = -\sqrt{21}/7$ or $x_0 = \sqrt{21}/7$, Newton's method does not converge. The values of x_i alternate between $-\sqrt{21}/7$ and $\sqrt{21}/7$ as i increases.

29. Answers will vary with machine speed.

Section 4.8, pp. 285–289

1. **(a)** x^2 **(b)** $\frac{x^3}{3}$ **(c)** $\frac{x^3}{3} - x^2 + x$

3. **(a)** x^{-3} **(b)** $-\frac{1}{3}x^{-3}$ **(c)** $-\frac{1}{3}x^{-3} + x^2 + 3x$

5. **(a)** $-\frac{1}{x}$ **(b)** $-\frac{5}{x}$ **(c)** $2x + \frac{5}{x}$

7. **(a)** $\sqrt{x^3}$ **(b)** $\sqrt{x}$ **(c)** $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x}$

9. **(a)** $x^{2/3}$ **(b)** $x^{1/3}$ **(c)** $x^{-1/3}$

11. **(a)** $\ln x$ **(b)** $7 \ln x$ **(c)** $x - 5 \ln x$

13. **(a)** $\cos(\pi x)$ **(b)** $-3 \cos x$ **(c)** $-\frac{1}{\pi}\cos(\pi x) + \cos(3x)$

15. **(a)** $\tan x$ **(b)** $2\tan\left(\frac{x}{3}\right)$ **(c)** $-\frac{2}{3}\tan\left(\frac{3x}{2}\right)$

17. **(a)** $-\csc x$ **(b)** $\frac{1}{5}\csc(5x)$ **(c)** $2\csc\left(\frac{\pi x}{2}\right)$

19. **(a)** $\frac{1}{3}e^{3x}$ **(b)** $-e^{-x}$ **(c)** $2e^{x/2}$

21. **(a)** $\frac{1}{\ln 3}3^x$ **(b)** $\frac{-1}{\ln 2}2^{-x}$ **(c)** $\frac{1}{\ln(5/3)}\left(\frac{5}{3}\right)^x$

23. **(a)** $2\sin^{-1}x$ **(b)** $\frac{1}{2}\tan^{-1}x$ **(c)** $\frac{1}{2}\tan^{-1}2x$

25. $\frac{x^2}{2}+x+C$ **27.** $t^3+\frac{t^2}{4}+C$ **29.** $\frac{x^4}{2}-\frac{5x^2}{2}+7x+C$

31. $-\frac{1}{x}-\frac{x^3}{3}-\frac{x}{3}+C$ **33.** $\frac{3}{2}x^{2/3}+C$

35. $\frac{2}{3}x^{3/2}+\frac{3}{4}x^{4/3}+C$ **37.** $4y^2-\frac{8}{3}y^{3/4}+C$

39. $x^2+\frac{2}{x}+C$ **41.** $2\sqrt{t}-\frac{2}{\sqrt{t}}+C$ **43.** $-2\sin t+C$

45. $-21\cos\frac{\theta}{3}+C$ **47.** $3\cot x+C$ **49.** $-\frac{1}{2}\csc\theta+C$

51. $\frac{1}{3}e^{3x}-5e^{-x}+C$ **53.** $-e^{-x}+\frac{4^x}{\ln 4}+C$

55. $4\sec x-2\tan x+C$ **57.** $-\frac{1}{2}\cos 2x+\cot x+C$

59. $\frac{t}{2}+\frac{\sin 4t}{8}+C$ **61.** $\ln|x|-5\tan^{-1}x+C$

63. $\frac{3x^{(\sqrt{3}+1)}}{\sqrt{3}+1}+C$ **65.** $\tan\theta+C$ **67.** $-\cot x-x+C$

69. $-\cos\theta+\theta+C$

83. **(a)** Wrong: $\frac{d}{dx}\left(\frac{x^2}{2}\sin x+C\right)=\frac{2x}{2}\sin x+\frac{x^2}{2}\cos x=x\sin x+\frac{x^2}{2}\cos x$

(b) Wrong: $\frac{d}{dx}(-x\cos x+C)=-\cos x+x\sin x$

(c) Right: $\frac{d}{dx}(-x\cos x+\sin x+C)=-\cos x+x\sin x+\cos x=x\sin x$

85. **(a)** Wrong: $\frac{d}{dx}\left(\frac{(2x+1)^3}{3}+C\right)=\frac{3(2x+1)^2(2)}{3}=2(2x+1)^2$

(b) Wrong: $\frac{d}{dx}((2x+1)^3+C)=3(2x+1)^2(2)=6(2x+1)^2$

(c) Right: $\frac{d}{dx}((2x+1)^3+C)=6(2x+1)^2$

87. Right **89.** (b) **91.** $y=x^2-7x+10$

93. $y=-\frac{1}{x}+\frac{x^2}{2}-\frac{1}{2}$ **95.** $y=9x^{1/3}+4$

97. $s=t+\sin t+4$ **99.** $r=\cos(\pi\theta)-1$

101. $v=\frac{1}{2}\sec t+\frac{1}{2}$ **103.** $v=3\sec^{-1}t-\pi$

105. $y=x^2-x^3+4x+1$ **107.** $r=\frac{1}{t}+2t-2$

109. $y=x^3-4x^2+5$ **111.** $y=-\sin t+\cos t+t^3-1$

113. $y=2x^{3/2}-50$ **115.** $y=x-x^{4/3}+\frac{1}{2}$

117. $y=-\sin x-\cos x-2$

119. **(a)** **(i)** 33.2 units, **(ii)** 33.2 units, **(iii)** 33.2 units **(b)** True

121. $t=88/k$, $k=16$

123. **(a)** $v=10t^{3/2}-6t^{1/2}$ **(b)** $s=4t^{5/2}-4t^{3/2}$

127. **(a)** $-\sqrt{x}+C$ **(b)** $x+C$ **(c)** $\sqrt{x}+C$ **(d)** $-x+C$ **(e)** $x-\sqrt{x}+C$ **(f)** $-x-\sqrt{x}+C$

Practice Exercises, pp. 289–293

1. No **3.** No minimum; absolute maximum: $f(1)=16$; critical points: $x=1$ and $11/3$

5. Absolute minimum: $g(0)=1$; no absolute maximum; critical point $x=0$

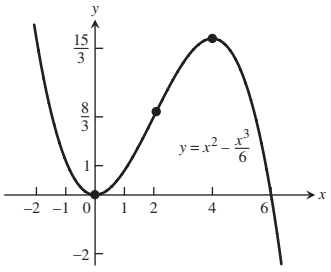
7. Absolute minimum: $2-2\ln 2$ at $x=2$; absolute maximum 1 at $x=1$

9. Yes, except at $x=0$ **11.** No **15.** **(b)** one

17. **(b)** 0.8555 99677 2 **23.** Global minimum value of $\frac{1}{2}$ at $x=2$

25. **(a)** $t=0,6,12$ **(b)** $t=3,9$ **(c)** $6<t<12$ **(d)** $0<t<6$, $12<t<14$

27.



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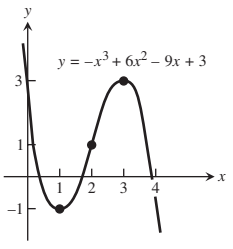

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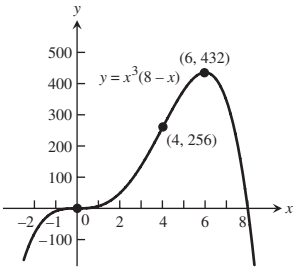

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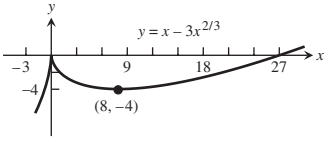

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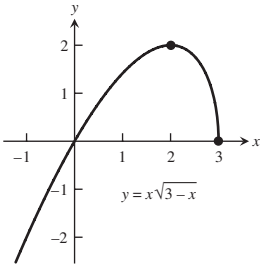

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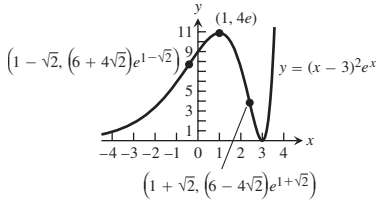
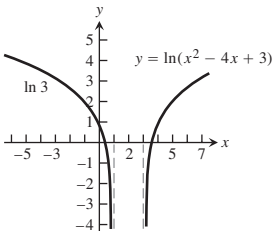


39.



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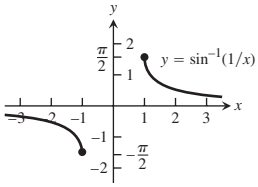
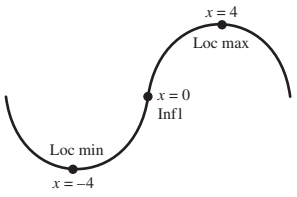


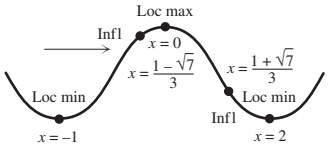
43. **(a)** Local maximum at $x = 4$, local minimum at $x = -4$, inflection point at $x = 0$

(b)



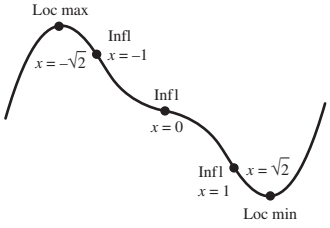
45. **(a)** Local maximum at $x = 0$, local minima at $x = -1$ and $x = 2$, inflection points at $x = (1 \pm \sqrt{7})/3$

(b)

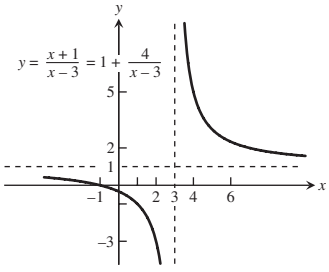


47. **(a)** Local maximum at $x = -\sqrt{2}$, local minimum at $x = \sqrt{2}$, inflection points at $x = \pm 1$ and 0

(b)



53.



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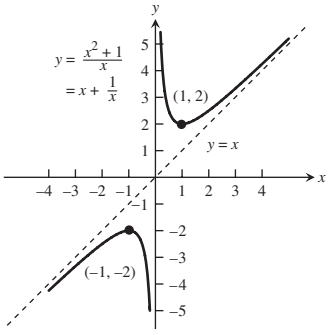

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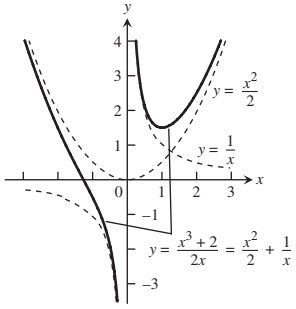
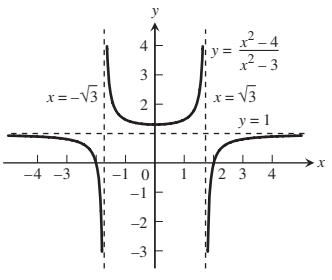


59.

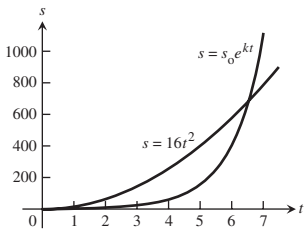


61. 5 **63.** 0 **65.** 1 **67.** 3/7 **69.** 0 **71.** 1
73. ln 10 **75.** ln 2 **77.** 5 **79.** $-\infty$ **81.** 1 **83.** e^{bk}
85. **(a)** 0, 36 **(b)** 18, 18 **87.** 54 square units
89. height = 2, radius = $\sqrt{2}$
91. $x = 5 - \sqrt{5}$ hundred $\approx$ 276 tires, $y = 2(5 - \sqrt{5})$ hundred $\approx$ 553 tires
93. Dimensions: base is 6 in. by 12 in., height = 2 in.; maximum volume = 144 in.^{3}
95. $x_5 = 2.1958\ 23345$ **97.** $\frac{x^4}{4} + \frac{5}{2}x^2 - 7x + C$
99. $2t^{3/2} - \frac{4}{t} + C$ **101.** $-\frac{1}{r+5} + C$ **103.** $(\theta^2 + 1)^{3/2} + C$
105. $\frac{1}{3}(1 + x^4)^{3/4} + C$ **107.** $10 \tan \frac{s}{10} + C$
109. $-\frac{1}{\sqrt{2}} \csc \sqrt{2}\,\theta + C$ **111.** $\frac{1}{2}x - \sin \frac{x}{2} + C$
113. $3 \ln x - \frac{x^2}{2} + C$ **115.** $\frac{1}{2}e^t + e^{-t} + C$
117. $\frac{\theta^{2-\pi}}{2 - \pi} + C$ **119.** $\frac{3}{2} \sec^{-1}|x| + C$
121. $y = x - \frac{1}{x} - 1$ **123.** $r = 4t^{5/2} + 4t^{3/2} - 8t$
125. Yes, $\sin^{-1}(x)$ and $-\cos^{-1}(x)$ differ by the constant $\pi/2$.
127. $1/\sqrt{2}$ units long by $1/\sqrt{e}$ units high, $A = 1/\sqrt{2e} \approx 0.43$ units^2
129. Absolute maximum = 0 at $x = e/2$, absolute minimum = -0.5 at $x = 0.5$
131. $x = \pm 1$ are the critical points; $y = 1$ is a horizontal asymptote in both directions; absolute minimum value of the function is $e^{-\sqrt{2}/2}$ at $x = -1$, and absolute maximum value is $e^{\sqrt{2}/2}$ at $x = 1$.
133. **(a)** Absolute maximum of $2/e$ at $x = e^2$, inflection point $(e^{8/3}, (8/3)e^{-4/3})$, concave up on $(e^{8/3}, \infty)$, concave down on $(0, e^{8/3})$
(b) Absolute maximum of 1 at $x = 0$, inflection points $(\pm 1/\sqrt{2}, 1/\sqrt{e})$, concave up on $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$, concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$
(c) Absolute maximum of 1 at $x = 0$, inflection point $(1, 2/e)$, concave up on $(1, \infty)$, concave down on $(-\infty, 1)$

Additional and Advanced Exercises, pp. 293–296

1. The function is constant on the interval.
3. The extreme points will not be at the end of an open interval.
5. **(a)** A local minimum at $x = -1$, points of inflection at $x = 0$ and $x = 2$ **(b)** A local maximum at $x = 0$ and local minima at $x = -1$ and $x = 2$, points of inflection at $x = \frac{1 \pm \sqrt{7}}{3}$
9. No **11.** $a = 1, b = 0, c = 1$ **13.** Yes
15. Drill the hole at $y = h/2$.
17. $r = \frac{RH}{2(H - R)}$ for $H > 2R$, $r = R$ if $H \le 2R$
19. **(a)** $\frac{10}{3}$ **(b)** $\frac{5}{3}$ **(c)** $\frac{1}{2}$ **(d)** 0 **(e)** $-\frac{1}{2}$ **(f)** 1 **(g)** $\frac{1}{2}$ **(h)** 3
21. **(a)** $\frac{c - b}{2e}$ **(b)** $\frac{c + b}{2}$ **(c)** $\frac{b^2 - 2bc + c^2 + 4ae}{4e}$ **(d)** $\frac{c + b + t}{2}$
23. $m_0 = 1 - \frac{1}{q}, m_1 = \frac{1}{q}$

25. $s = ce^{kt}$



27. **(a)** $k = -38.72$ **(b)** 25 ft

29. Yes, $y = x + C$ **31.** $v_0 = \dfrac{2\sqrt{2}}{3}b^{3/2}$

CHAPTER 5

Section 5.1, pp. 304–306

1. **(a)** 0.125 **(b)** 0.21875 **(c)** 0.625 **(d)** 0.46875

3. **(a)** 1.066667 **(b)** 1.283333 **(c)** 2.666667 **(d)** 2.083333

5. 0.3125, 0.328125 **7.** 1.5, 1.574603

9. **(a)** 87 in. **(b)** 87 in. **11.** **(a)** 3490 ft **(b)** 3840 ft

13. **(a)** 74.65 ft/sec **(b)** 45.28 ft/sec **(c)** 146.59 ft

15. $\dfrac{31}{16}$ **17.** 1

19. **(a)** Upper = 758 gal, lower = 543 gal
(b) Upper = 2363 gal, lower = 1693 gal
(c) ≈ 31.4 h, ≈ 32.4 h

21. **(a)** 2 **(b)** $2\sqrt{2} \approx 2.828$
(c) $8\sin\left(\dfrac{\pi}{8}\right) \approx 3.061$
(d) Each area is less than the area of the circle, π. As n increases, the polygon area approaches π.

Section 5.2, pp. 312–313

1. $\dfrac{6(1)}{1+1} + \dfrac{6(2)}{2+1} = 7$

3. $\cos(1)\pi + \cos(2)\pi + \cos(3)\pi + \cos(4)\pi = 0$

5. $\sin\pi - \sin\dfrac{\pi}{2} + \sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}-2}{2}$ **7.** All of them **9.** b

11. $\displaystyle\sum_{k=1}^{6} k$ **13.** $\displaystyle\sum_{k=1}^{4} \frac{1}{2^k}$ **15.** $\displaystyle\sum_{k=1}^{5} (-1)^{k+1}\frac{1}{k}$

17. **(a)** −15 **(b)** 1 **(c)** 1 **(d)** −11 **(e)** 16

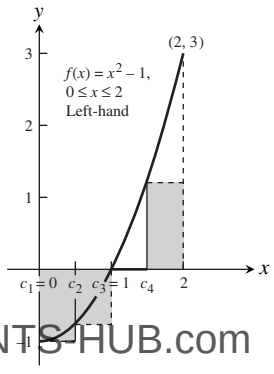
19. **(a)** 55 **(b)** 385 **(c)** 3025

21. −56 **23.** −73 **25.** 240 **27.** 3376

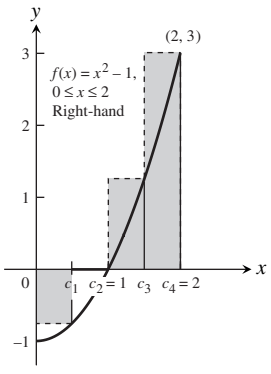
29. **(a)** 21 **(b)** 3500 **(c)** 2620

31. **(a)** $4n$ **(b)** cn **(c)** $(n^2 - n)/2$

33. **(a)**



(b)



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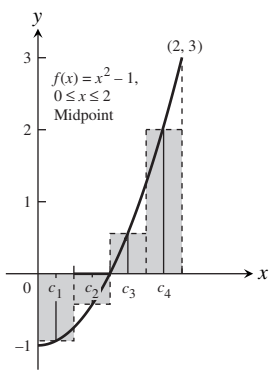
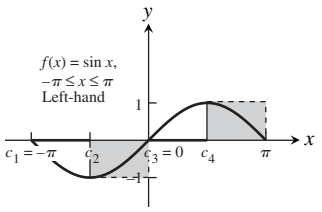
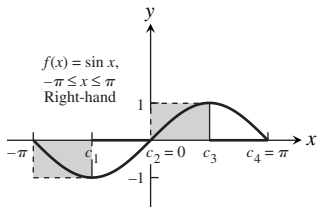


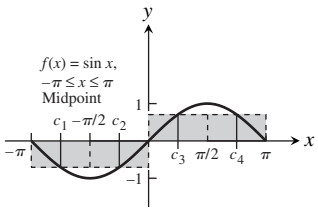
35. **(a)**



(b)



(c)



37. 1.2 **39.** $\dfrac{2}{3} - \dfrac{1}{2n} - \dfrac{1}{6n^2}$, $\dfrac{2}{3}$ **41.** $12 + \dfrac{27n+9}{2n^2}$, 12

43. $\dfrac{5}{6} + \dfrac{6n+1}{6n^2}$, $\dfrac{5}{6}$ **45.** $\dfrac{1}{2} + \dfrac{1}{n} + \dfrac{1}{2n^2}$, $\dfrac{1}{2}$

Section 5.3, pp. 321–325

1. $\displaystyle\int_0^2 x^2\,dx$ **3.** $\displaystyle\int_{-7}^{5} (x^2 - 3x)\,dx$ **5.** $\displaystyle\int_2^3 \frac{1}{1-x}\,dx$

7. $\displaystyle\int_{-\pi/4}^{0} \sec x\,dx$

9. **(a)** 0 **(b)** −8 **(c)** −12 **(d)** 10 **(e)** −2 **(f)** 16

11. **(a)** 5 **(b)** $5\sqrt{3}$ **(c)** −5 **(d)** −5

13. **(a)** 4 **(b)** −4 **15.** Area = 21 square units

17. Area = $9\pi/2$ square units **19.** Area = 2.5 square units

21. Area = 3 square units **23.** $b^2/4$ **25.** $b^2 - a^2$

27. **(a)** 2π **(b)** π **29.** 1/2 **31.** $3\pi^2/2$ **33.** 7/3

35. 1/24 **37.** $3a^2/2$ **39.** $b/3$ **41.** −14 **43.** −2

45. −7/4 **47.** 7 **49.** 0

51. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 3x^2\,dx = b^3$$

53. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 2x\,dx = b^2$$

55. $\text{av}(f) = 0$ **57.** $\text{av}(f) = -2$ **59.** $\text{av}(f) = 1$

61. **(a)** $\text{av}(g) = -1/2$ **(b)** $\text{av}(g) = 1$ **(c)** $\text{av}(g) = 1/4$

63. $c(b - a)$ **65.** $b^3/3 - a^3/3$ **67.** 9 **69.** $b^4/4 - a^4/4$

71. $a = 0$ and $b = 1$ maximize the integral.

73. Upper bound $= 1$, lower bound $= 1/2$

75. For example, $\displaystyle\int_0^1 \sin(x^2)\,dx \le \int_0^1 dx = 1$

77. $\displaystyle\int_a^b f(x)\,dx \ge \int_a^b 0\,dx = 0$ **79.** Upper bound $= 1/2$

Section 5.4, pp. 333–336

1. 6 **3.** $-10/3$ **5.** 8 **7.** 1 **9.** $2\sqrt{3}$ **11.** 0

13. $-\pi/4$ **15.** $1 - \dfrac{\pi}{4}$ **17.** $\dfrac{2 - \sqrt{2}}{4}$ **19.** $-8/3$

21. $-3/4$ **23.** $\sqrt{2} - \sqrt[4]{8} + 1$ **25.** -1 **27.** 16

29. $7/3$ **31.** $2\pi/3$ **33.** $\dfrac{1}{\pi}(4^\pi - 2^\pi)$ **35.** $\dfrac{1}{2}(e - 1)$

37. $\sqrt{26} - \sqrt{5}$ **39.** $(\cos\sqrt{x})\left(\dfrac{1}{2\sqrt{x}}\right)$ **41.** $4t^5$

43. $3x^2e^{-x^3}$ **45.** $\sqrt{1 + x^2}$ **47.** $-\dfrac{1}{2}x^{-1/2}\sin x$ **49.** 0

51. 1 **53.** $2xe^{(1/2)x^2}$ **55.** 1 **57.** 28/3 **59.** 1/2 **61.** π

63. $\dfrac{\sqrt{2}\pi}{2}$ **65.** d, since $y' = \dfrac{1}{x}$ and $y(\pi) = \displaystyle\int_\pi^\pi \frac{1}{t}\,dt - 3 = -3$

67. b, since $y' = \sec x$ and $y(0) = \displaystyle\int_0^0 \sec t\,dt + 4 = 4$

69. $y = \displaystyle\int_2^x \sec t\,dt + 3$ **71.** $\dfrac{2}{3}bh$ **73.** \$9.00

75. **a.** $T(0) = 70°\text{F}$, $T(16) = 76°\text{F}$
$T(25) = 85°\text{F}$
b. $\text{av}(T) = 75°\text{F}$

77. $2x - 2$ **79.** $-3x + 5$

81. **(a)** True. Since f is continuous, g is differentiable by Part 1 of the Fundamental Theorem of Calculus.
(b) True: g is continuous because it is differentiable.
(c) True, since $g'(1) = f(1) = 0$.
(d) False, since $g''(1) = f'(1) > 0$.
(e) True, since $g'(1) = 0$ and $g''(1) = f'(1) > 0$.
(f) False: $g''(x) = f'(x) > 0$, so g'' never changes sign.
(g) True, since $g'(1) = f(1) = 0$ and $g'(x) = f(x)$ is an increasing function of x (because $f'(x) > 0$).

83. **(a)** $v = \dfrac{ds}{dt} = \dfrac{d}{dt}\displaystyle\int_0^t f(x)\,dx = f(t) \Rightarrow v(5) = f(5) = 2$ m/sec
(b) $a = df/dt$ is negative since the slope of the tangent line at $t = 5$ is negative.
(c) $s = \displaystyle\int_0^3 f(x)\,dx = \frac{1}{2}(3)(3) = \frac{9}{2}$ m since the integral is the area of the triangle formed by $y = f(x)$, the x-axis, and $x = 3$.
(d) $t = 6$ since after $t = 6$ to $t = 9$, the region lies below the x-axis.
(e) At $t = 4$ and $t = 7$, since there are horizontal tangents there.
(f) Toward the origin between $t = 6$ and $t = 9$ since the velocity is negative on this interval. Away from the origin between $t = 0$ and $t = 6$ since the velocity is positive there.
(g) Right or positive side, because the integral of f from 0 to 9 is positive, there being more area above the x-axis than below.

Section 5.5, pp. 342–344

1. $\dfrac{1}{6}(2x + 4)^6 + C$ **3.** $-\dfrac{1}{3}(x^2 + 5)^{-3} + C$

5. $\dfrac{1}{10}(3x^2 + 4x)^5 + C$ **7.** $-\dfrac{1}{3}\cos 3x + C$

9. $\dfrac{1}{2}\sec 2t + C$ **11.** $-6(1 - r^3)^{1/2} + C$

13. $\dfrac{1}{3}(x^{3/2} - 1) - \dfrac{1}{6}\sin(2x^{3/2} - 2) + C$

15. **(a)** $-\dfrac{1}{4}(\cot^2 2\theta) + C$ **(b)** $-\dfrac{1}{4}(\csc^2 2\theta) + C$

17. $-\dfrac{1}{3}(3 - 2s)^{3/2} + C$ **19.** $-\dfrac{2}{5}(1 - \theta^2)^{5/4} + C$

21. $(-2/(1 + \sqrt{x})) + C$ **23.** $\dfrac{1}{3}\tan(3x + 2) + C$

25. $\dfrac{1}{2}\sin^6\left(\dfrac{x}{3}\right) + C$ **27.** $\left(\dfrac{r^3}{18} - 1\right)^6 + C$

29. $-\dfrac{2}{3}\cos(x^{3/2} + 1) + C$ **31.** $\dfrac{1}{2\cos(2t + 1)} + C$

33. $-\sin\left(\dfrac{1}{t} - 1\right) + C$ **35.** $-\dfrac{\sin^2(1/\theta)}{2} + C$

37. $\dfrac{1}{16}(1 + t^4)^4 + C$ **39.** $\dfrac{2}{3}\left(2 - \dfrac{1}{x}\right)^{3/2} + C$

41. $\dfrac{2}{27}\left(1 - \dfrac{3}{x^3}\right)^{3/2} + C$

43. $\dfrac{1}{12}(x - 1)^{12} + \dfrac{1}{11}(x - 1)^{11} + C$

45. $-\dfrac{1}{8}(1 - x)^8 + \dfrac{4}{7}(1 - x)^7 - \dfrac{2}{3}(1 - x)^6 + C$

47. $\dfrac{1}{5}(x^2 + 1)^{5/2} - \dfrac{1}{3}(x^2 + 1)^{3/2} + C$ **49.** $\dfrac{-1}{4(x^2 - 4)^2} + C$

51. $e^{\sin x} + C$ **53.** $2\tan\left(e^{\sqrt{x}} + 1\right) + C$ **55.** $\ln|\ln x| + C$

57. $z - \ln(1 + e^z) + C$ **59.** $\dfrac{5}{6}\tan^{-1}\left(\dfrac{2r}{3}\right) + C$

61. $e^{\sin^{-1}x} + C$ **63.** $\dfrac{1}{3}(\sin^{-1}x)^3 + C$ **65.** $\ln|\tan^{-1}y| + C$

67. **(a)** $-\dfrac{6}{2 + \tan^3 x} + C$ **(b)** $-\dfrac{6}{2 + \tan^3 x} + C$
(c) $-\dfrac{6}{2 + \tan^3 x} + C$

69. $\dfrac{1}{6}\sin\sqrt{3(2r - 1)^2 + 6} + C$ **71.** $s = \dfrac{1}{2}(3t^2 - 1)^4 - 5$

73. $s = 4t - 2\sin\left(2t + \dfrac{\pi}{6}\right) + 9$

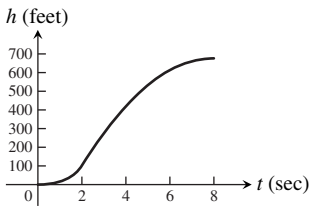
75. $s = \sin\left(2t - \dfrac{\pi}{2}\right) + 100t + 1$ **77.** 6 m

Section 5.6, pp. 350–354

1. **(a)** 14/3 **(b)** 2/3 **3.** **(a)** 1/2 **(b)** $-1/2$
5. **(a)** 15/16 **(b)** 0 **7.** **(a)** 0 **(b)** 1/8 **9.** **(a)** 4 **(b)** 0
11. **(a)** 1/6 **(b)** 1/2 **13.** **(a)** 0 **(b)** 0 **15.** $2\sqrt{3}$
17. 3/4 **19.** $3^{5/2} - 1$ **21.** 3 **23.** $\pi/3$ **25.** e
27. $\ln 3$ **29.** $(\ln 2)^2$ **31.** $\dfrac{1}{\ln 4}$ **33.** $\ln 2$ **35.** $\ln 27$ **37.** π
39. $\pi/12$ **41.** $2\pi/3$ **43.** $\sqrt{3} - 1$ **45.** $-\pi/12$
47. 16/3 **49.** $2^{5/2}$ **51.** $\pi/2$ **53.** 128/15 **55.** 4/3
57. 5/6 **59.** 38/3 **61.** 49/6 **63.** 32/3 **65.** 48/5
67. 8/3 **69.** 8 **71.** 5/3 (There are three intersection points.)
73. 18 **75.** 243/8 **77.** 8/3 **79.** 2 **81.** 104/15
83. 56/15 **85.** 4 **87.** $\dfrac{4}{3} - \dfrac{4}{\pi}$ **89.** $\pi/2$ **91.** 2 **93.** 1/2
95. 1 **97.** $\ln 16$ **99.** 2 **101.** $2 \ln 5$
103. **(a)** $(\pm\sqrt{c}, c)$ **(b)** $c = 4^{2/3}$ **(c)** $c = 4^{2/3}$
105. 11/3 **107.** 3/4 **109.** Neither **111.** $F(6) - F(2)$
113. **(a)** -3 **(b)** 3 **115.** $I = a/2$

Practice Exercises, pp. 354–357

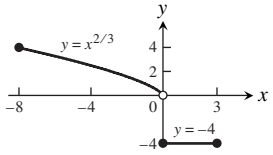
1. **(a)** about 680 ft **(b)**

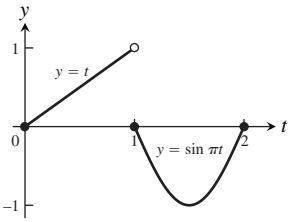


3. **(a)** $-1/2$ **(b)** 31 **(c)** 13 **(d)** 0
5. $\displaystyle\int_1^5 (2x-1)^{-1/2}\,dx = 2$ **7.** $\displaystyle\int_{-\pi}^{0} \cos\frac{x}{2}\,dx = 2$
9. **(a)** 4 **(b)** 2 **(c)** -2 **(d)** -2π **(e)** 8/5
11. 8/3 **13.** 62 **15.** 1 **17.** 1/6 **19.** 18 **21.** 9/8
23. $\dfrac{\pi^2}{32} + \dfrac{\sqrt{2}}{2} - 1$ **25.** 4 **27.** $\dfrac{8\sqrt{2} - 7}{6}$
29. Min: -4, max: 0, area: 27/4 **31.** 6/5 **33.** 1
37. $y = \displaystyle\int_5^x \left(\frac{\sin t}{t}\right) dt - 3$ **39.** $y = \sin^{-1} x$
41. $y = \sec^{-1} x + \dfrac{2\pi}{3}, x > 1$ **43.** $-4(\cos x)^{1/2} + C$
45. $\theta^2 + \theta + \sin(2\theta + 1) + C$ **47.** $\dfrac{t^3}{3} + \dfrac{4}{t} + C$
49. $-\dfrac{1}{3}\cos(2t^{3/2}) + C$ **51.** $\tan(e^x - 7) + C$ **53.** $e^{\tan x} + C$
55. $\dfrac{-\ln 7}{3}$ **57.** $\ln(9/25)$ **59.** $-\dfrac{1}{2}(\ln x)^{-2} + C$
61. $\dfrac{1}{2 \ln 3}\left(3^{x^2}\right) + C$ **63.** $\dfrac{3}{2}\sin^{-1} 2(r-1) + C$
65. $\dfrac{\sqrt{2}}{2}\tan^{-1}\left(\dfrac{x-1}{\sqrt{2}}\right) + C$ **67.** $\dfrac{1}{4}\sec^{-1}\left|\dfrac{2x-1}{2}\right| + C$
69. $e^{\sin^{-1}\sqrt{x}} + C$ **71.** $2\sqrt{\tan^{-1} y} + C$ **73.** 16
75. 2 **77.** 1 **79.** 8 **81.** $27\sqrt{3}/160$ **83.** $\pi/2$
85 $\sqrt{3}$ **87.** $6\sqrt{3} - 2\pi$ **89.** -1 **91.** 2 **93.** 1
95. $15/16 + \ln 2$ **97.** $e - 1$ **99.** 1/6 **101.** 9/14
103. $\dfrac{9 \ln 2}{4}$ **105.** π **107.** $\pi/\sqrt{3}$ **109.** $\sec^{-1}|2y| + C$
111. $\pi/12$ **113.** **(a)** b **(b)** b
117. **(a)** $\dfrac{d}{dx}(x \ln x - x + C) = x \cdot \dfrac{1}{x} + \ln x - 1 + 0 = \ln x$
(b) $\dfrac{1}{e-1}$
119. 25°F **121.** $\sqrt{2 + \cos^3 x}$ **123.** $\dfrac{-6}{3 + x^4}$
125. $\dfrac{dy}{dx} = \dfrac{-2}{x} e^{\cos(2 \ln x)}$ **127.** $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}\sqrt{1 - 2(\sin^{-1} x)^2}}$
129. Yes **131.** $-\sqrt{1 + x^2}$
133. Cost $\approx$ \$10,899 using a lower sum estimate

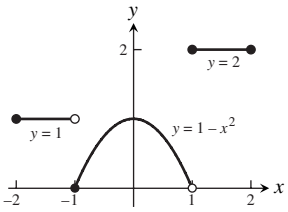
Additional and Advanced Exercises, pp. 358–361

1. **(a)** Yes **(b)** No **5.** **(a)** 1/4 **(b)** $\sqrt[3]{12}$
7. $f(x) = \dfrac{x}{\sqrt{x^2+1}}$ **9.** $y = x^3 + 2x - 4$
11. 36/5 **13.** $\dfrac{1}{2} - \dfrac{2}{\pi}$





15. 13/3



17. 1/2 **19.** $\pi/2$ **21.** $\ln 2$ **23.** 1/6 **25.** $\displaystyle\int_0^1 f(x)\,dx$
27. **(b)** πr^2
29. **(a)** 0 **(b)** -1 **(c)** $-\pi$ **(d)** $x = 1$
(e) $y = 2x + 2 - \pi$ **(f)** $x = -1, x = 2$ **(g)** $[-2\pi, 0]$
31. $2/x$ **33.** $\dfrac{\sin 4y}{\sqrt{y}} - \dfrac{\sin y}{2\sqrt{y}}$ **35.** $2x \ln|x| - x \ln\dfrac{|x|}{\sqrt{2}}$
37. $(\sin x)/x$ **39.** $x = 1$ **41.** $\dfrac{1}{\ln 2}, \dfrac{1}{2 \ln 2}, 2:1$ **43.** 2/17

CHAPTER 6

Section 6.1, pp. 371–374

1. 16 **3.** 16/3 **5.** **(a)** $2\sqrt{3}$ **(b)** 8 **7.** **(a)** 60 **(b)** 36
9. 8π **11.** 10 **13.** **(a)** $s^2 h$ **(b)** $s^2 h$ **15.** $\dfrac{2\pi}{3}$
17. $4 - \pi$ **19.** $\dfrac{32\pi}{5}$ **21.** 36π **23.** π **25.** $\dfrac{\pi}{2}\left(1 - \dfrac{1}{e^2}\right)$
27. $\dfrac{\pi}{2}\ln 4$ **29.** $\pi\left(\dfrac{\pi}{2} + 2\sqrt{2} - \dfrac{11}{3}\right)$ **31.** 2π **33.** 2π
35. $4\pi \ln 4$ **37.** $\pi^2 - 2\pi$ **39.** $\dfrac{2\pi}{3}$ **41.** $\dfrac{117\pi}{5}$
43. $\pi(\pi - 2)$ **45.** $\dfrac{4\pi}{3}$ **47.** 8π **49.** $\dfrac{7\pi}{6}$

51. **(a)** 8π **(b)** $\frac{32\pi}{5}$ **(c)** $\frac{8\pi}{3}$ **(d)** $\frac{224\pi}{15}$

53. **(a)** $\frac{16\pi}{15}$ **(b)** $\frac{56\pi}{15}$ **(c)** $\frac{64\pi}{15}$ **55.** $V = 2a^2b\pi^2$

57. **(a)** $V = \frac{\pi h^2(3a - h)}{3}$ **(b)** $\frac{1}{120\pi}$ m/sec

61. $V = 3308\ \text{cm}^3$ **63.** $\frac{4 - b + a}{2}$

Section 6.2, pp. 379–381

1. 6π **3.** 2π **5.** $14\pi/3$ **7.** 8π **9.** $5\pi/6$

11. $\frac{7\pi}{15}$ **13.** **(b)** 4π **15.** $\frac{16\pi}{15}(3\sqrt{2} + 5)$

17. $\frac{8\pi}{3}$ **19.** $\frac{4\pi}{3}$ **21.** $\frac{16\pi}{3}$

23. **(a)** 16π **(b)** 32π **(c)** 28π
(d) 24π **(e)** 60π **(f)** 48π

25. **(a)** $\frac{27\pi}{2}$ **(b)** $\frac{27\pi}{2}$ **(c)** $\frac{72\pi}{5}$ **(d)** $\frac{108\pi}{5}$

27. **(a)** $\frac{6\pi}{5}$ **(b)** $\frac{4\pi}{5}$ **(c)** 2π **(d)** 2π

29. **(a)** About the x-axis: $V = \frac{2\pi}{15}$; about the y-axis: $V = \frac{\pi}{6}$
(b) About the x-axis: $V = \frac{2\pi}{15}$; about the y-axis: $V = \frac{\pi}{6}$

31. **(a)** $\frac{5\pi}{3}$ **(b)** $\frac{4\pi}{3}$ **(c)** 2π **(d)** $\frac{2\pi}{3}$

33. **(a)** $\frac{4\pi}{15}$ **(b)** $\frac{7\pi}{30}$

35. **(a)** $\frac{24\pi}{5}$ **(b)** $\frac{48\pi}{5}$

37. **(a)** $\frac{9\pi}{16}$ **(b)** $\frac{9\pi}{16}$

39. Disk: 2 integrals; washer: 2 integrals; shell: 1 integral

41. **(a)** $\frac{256\pi}{3}$ **(b)** $\frac{244\pi}{3}$ **47.** $\pi\left(1 - \frac{1}{e}\right)$

Section 6.3, pp. 386–387

1. 12 **3.** $\frac{53}{6}$ **5.** $\frac{123}{32}$ **7.** $\frac{99}{8}$ **9.** 2

11. **(a)** $\int_{-1}^{2} \sqrt{1 + 4x^2}\, dx$ **(c)** ≈ 6.13

13. **(a)** $\int_{0}^{\pi} \sqrt{1 + \cos^2 y}\, dy$ **(c)** ≈ 3.82

15. **(a)** $\int_{-1}^{3} \sqrt{1 + (y + 1)^2}\, dy$ **(c)** ≈ 9.29

17. **(a)** $\int_{0}^{\pi/6} \sec x\, dx$ **(c)** ≈ 0.55

19. **(a)** $y = \sqrt{x}$ from (1, 1) to (4, 2)
(b) Only one. We know the derivative of the function and the value of the function at one value of x.

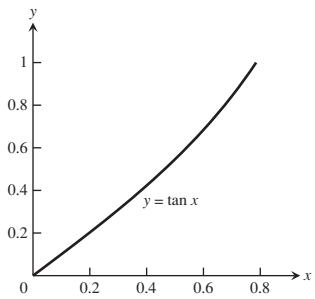
21. 1 **27.** Yes, $f(x) = \pm x + C$ where C is any real number.

31. $\frac{2}{27}(10^{3/2} - 1)$

Section 6.4, pp. 391–393

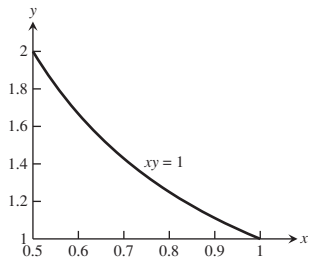
1. **(a)** $2\pi \int_{0}^{\pi/4} (\tan x)\sqrt{1 + \sec^4 x}\, dx$ **(c)** $S \approx 3.84$

(b)



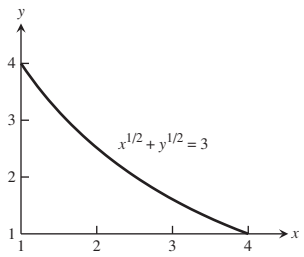
3. **(a)** $2\pi \int_{1}^{2} \frac{1}{y}\sqrt{1 + y^{-4}}\, dy$ **(c)** $S \approx 5.02$

(b)



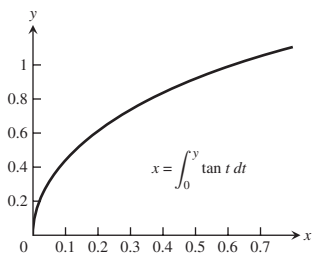
5. **(a)** $2\pi \int_{1}^{4} (3 - x^{1/2})^2\sqrt{1 + (1 - 3x^{-1/2})^2}\, dx$ **(c)** $S \approx 63.37$

(b)



7. **(a)** $2\pi \int_{0}^{\pi/3} \left(\int_{0}^{y} \tan t\, dt\right) \sec y\, dy$ **(c)** $S \approx 2.08$

(b)



9. $4\pi\sqrt{5}$ **11.** $3\pi\sqrt{5}$ **13.** $98\pi/81$ **15.** 2π

17. $\pi(\sqrt{8} - 1)/9$ **19.** $35\pi\sqrt{5}/3$ **21.** $\pi\left(\frac{15}{16} + \ln 2\right)$

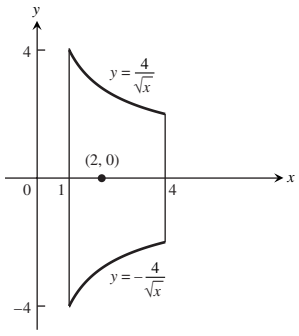
23. $253\pi/20$ **27.** Order 226.2 liters of each color.

Section 6.5, pp. 398–402

1. 400 N/m **3.** 4 cm, 0.08 J
5. **(a)** 7238 lb/in. **(b)** 905 in.-lb, 2714 in.-lb
7. 780 J **9.** 72,900 ft-lb **11.** 160 ft-lb
13. **(a)** 1,497,600 ft-lb **(b)** 1 hr, 40 min
(d) At 62.26 lb/ft^3: a) 1,494,240 ft-lb b) 1 hr, 40 min
At 62.59 lb/ft^3: a) 1,502,160 ft-lb b) 1 hr, 40.1 min
15. 37,306 ft-lb **17.** 7,238,299.47 ft-lb
19. 2446.25 ft-lb **21.** 15,073,099.75 J
25. 85.1 ft-lb **27.** 98.35 ft-lb **29.** 91.32 in.-oz
31. 5.144×10^{10} J **33.** 1684.8 lb
35. **(a)** 6364.8 lb **(b)** 5990.4 lb **37.** 1164.8 lb **39.** 1309 lb
41. **(a)** 12,480 lb **(b)** 8580 lb **(c)** 9722.3 lb
43. **(a)** 93.33 lb **(b)** 3 ft **45.** $\frac{wb}{2}$
47. No. The tank will overflow because the movable end will have moved only $3\frac{1}{3}$ ft by the time the tank is full.

Section 6.6, pp. 411–413

1. $\bar{x} = 0, \bar{y} = 12/5$ **3.** $\bar{x} = 1, \bar{y} = -3/5$
5. $\bar{x} = 16/105, \bar{y} = 8/15$ **7.** $\bar{x} = 0, \bar{y} = \pi/8$
9. $\bar{x} \approx 1.44, \bar{y} \approx 0.36$
11. $\bar{x} = \frac{\ln 4}{\pi}, \bar{y} = 0$ **13.** $\bar{x} = 7, \bar{y} = \frac{\ln 16}{12}$
15. $\bar{x} = 3/2, \bar{y} = 1/2$
17. **(a)** $\frac{224\pi}{3}$ **(b)** $\bar{x} = 2, \bar{y} = 0$
(c)



21. $\bar{x} = \bar{y} = 1/3$ **23.** $\bar{x} = a/3, \bar{y} = b/3$ **25.** $13\delta/6$
27. $\bar{x} = 0, \bar{y} = \frac{a\pi}{4}$
29. $\bar{x} = 1/2, \bar{y} = 4$ **31.** $\bar{x} = 6/5, \bar{y} = 8/7$
35. $V = 32\pi, S = 32\sqrt{2}\pi$
37. $4\pi^2$ **39.** $\bar{x} = 0, \bar{y} = \frac{2a}{\pi}$ **41.** $\bar{x} = 0, \bar{y} = \frac{4b}{3\pi}$
43. $\sqrt{2}\pi a^3(4 + 3\pi)/6$ **45.** $\bar{x} = \frac{a}{3}, \bar{y} = \frac{b}{3}$

Practice Exercises, pp. 413–415

1. $\frac{9\pi}{280}$ **3.** π^2 **5.** $\frac{72\pi}{35}$
7. **(a)** 2π **(b)** π **(c)** $12\pi/5$ **(d)** $26\pi/5$
9. **(a)** 8π **(b)** $1088\pi/15$ **(c)** $512\pi/15$
11. $\pi(3\sqrt{3} - \pi)/3$ **13.** π
15. $\frac{28\pi}{3}$ ft^3 **17.** $\frac{10}{3}$ **19.** $3 + \frac{1}{8}\ln 2$
21. $28\pi\sqrt{2}/3$ **23.** 4π **25.** 4640 J
27. 10 ft-lb, 30 ft-lb **29.** 418,208.81 ft-lb
31. 22,500π ft-lb, 257 sec **33.** $\bar{x} = 0, \bar{y} = 8/5$
35. $\bar{x} = 3/2, \bar{y} = 12/5$ **37.** $\bar{x} = 9/5, \bar{y} = 11/10$
39. 332.8 lb **41.** 2196.48 lb

Additional and Advanced Exercises, pp. 415–416

1. $f(x) = \sqrt{\frac{2x - a}{\pi}}$ **3.** $f(x) = \sqrt{C^2 - 1}\,x + a$, where $C \geq 1$
5. $\frac{\pi}{30\sqrt{2}}$ **7.** 28/3 **9.** $\frac{4h\sqrt{3mh}}{3}$
11. $\bar{x} = 0, \bar{y} = \frac{n}{2n + 1}$, $(0, 1/2)$
15. **(a)** $\bar{x} = \bar{y} = 4(a^2 + ab + b^2)/(3\pi(a + b))$
(b) $(2a/\pi, 2a/\pi)$
17. ≈2329.6 lb

CHAPTER 7

Section 7.1, pp. 425–427

1. $\ln\left(\frac{2}{3}\right)$ **3.** $\ln|y^2 - 25| + C$ **5.** $\ln|6 + 3\tan t| + C$
7. $\ln(1 + \sqrt{x}) + C$ **9.** 1 **11.** $2(\ln 2)^4$ **13.** 2
15. $2e^{\sqrt{r}} + C$ **17.** $-e^{-t^2} + C$ **19.** $-e^{1/x} + C$
21. $\frac{1}{\pi}e^{\sec \pi t} + C$ **23.** 1 **25.** $\ln(1 + e^r) + C$ **27.** $\frac{1}{2\ln 2}$
29. $\frac{1}{\ln 2}$ **31.** $\frac{6}{\ln 7}$ **33.** 32760 **35.** $3^{\sqrt{2}+1}$
37. $\frac{1}{\ln 10}\left(\frac{(\ln x)^2}{2}\right) + C$ **39.** $2(\ln 2)^2$ **41.** $\frac{3\ln 2}{2}$ **43.** $\ln 10$
45. $(\ln 10)\ln|\ln x| + C$ **47.** $y = 1 - \cos(e^t - 2)$
49. $y = 2(e^{-x} + x) - 1$ **51.** $y = x + \ln|x| + 2$ **53.** $\pi \ln 16$
55. $6 + \ln 2$ **57.** **(b)** 0.00469
69. **(a)** 1.89279 **(b)** −0.35621 **(c)** 0.94575 **(d)** −2.80735
(e) 5.29595 **(f)** 0.97041 **(g)** −1.03972 **(h)** −1.61181

Section 7.2, pp. 433–435

9. $\frac{2}{3}y^{3/2} - x^{1/2} = C$ **11.** $e^y - e^x = C$
13. $-x + 2\tan\sqrt{y} = C$ **15.** $e^{-y} + 2e^{\sqrt{x}} = C$
17. $y = \sin(x^2 + C)$ **19.** $\frac{1}{3}\ln|y^3 - 2| = x^3 + C$
21. $4\ln(\sqrt{y} + 2) = e^{x^2} + C$
23. **(a)** −0.00001 **(b)** 10,536 years **(c)** 82%
25. 54.88 g **27.** 59.8 ft **29.** 2.8147498×10^{14}
31. **(a)** 8 years **(b)** 32.02 years
33. 15.28 years **35.** 56,562 years
39. **(a)** 17.5 min **(b)** 13.26 min
41. −3°C **43.** About 6658 years **45.** 54.44%

Section 7.3, pp. 441–444

1. $\cosh x = 5/4, \tanh x = -3/5, \coth x = -5/3,$
$\operatorname{sech} x = 4/5, \operatorname{csch} x = -4/3$

3. $\sinh x = 8/15$, $\tanh x = 8/17$, $\coth x = 17/8$, $\operatorname{sech} x = 15/17$, $\operatorname{csch} x = 15/8$

5. $x + \frac{1}{x}$ **7.** e^{5x} **9.** e^{4x} **13.** $2\cosh\frac{x}{3}$

15. $\operatorname{sech}^2\sqrt{t} + \dfrac{\tanh\sqrt{t}}{\sqrt{t}}$ **17.** $\coth z$

19. $(\ln \operatorname{sech}\theta)(\operatorname{sech}\theta\tanh\theta)$ **21.** $\tanh^3 v$ **23.** 2

25. $\dfrac{1}{2\sqrt{x(1+x)}}$ **27.** $\dfrac{1}{1+\theta} - \tanh^{-1}\theta$

29. $\dfrac{1}{2\sqrt{t}} - \coth^{-1}\sqrt{t}$ **31.** $-\operatorname{sech}^{-1}x$ **33.** $\dfrac{\ln 2}{\sqrt{1+\left(\frac{1}{2}\right)^{2\theta}}}$

35. $|\sec x|$ **41.** $\dfrac{\cosh 2x}{2} + C$

43. $12\sinh\left(\dfrac{x}{2} - \ln 3\right) + C$ **45.** $7\ln|e^{x/7} + e^{-x/7}| + C$

47. $\tanh\left(x - \dfrac{1}{2}\right) + C$ **49.** $-2\operatorname{sech}\sqrt{t} + C$ **51.** $\ln\dfrac{5}{2}$

53. $\dfrac{3}{32} + \ln 2$ **55.** $e - e^{-1}$ **57.** $3/4$ **59.** $\dfrac{3}{8} + \ln\sqrt{2}$

61. $\ln(2/3)$ **63.** $\dfrac{-\ln 3}{2}$ **65.** $\ln 3$

67. **(a)** $\sinh^{-1}(\sqrt{3})$ **(b)** $\ln(\sqrt{3}+2)$

69. **(a)** $\coth^{-1}(2) - \coth^{-1}(5/4)$ **(b)** $\left(\dfrac{1}{2}\right)\ln\left(\dfrac{1}{3}\right)$

71. **(a)** $-\operatorname{sech}^{-1}\left(\dfrac{12}{13}\right) + \operatorname{sech}^{-1}\left(\dfrac{4}{5}\right)$

(b) $-\ln\left(\dfrac{1+\sqrt{1-(12/13)^2}}{(12/13)}\right) + \ln\left(\dfrac{1+\sqrt{1-(4/5)^2}}{(4/5)}\right)$

$= -\ln\left(\dfrac{3}{2}\right) + \ln(2) = \ln(4/3)$

73. **(a)** 0 **(b)** 0

77. **(b)** $\sqrt{\dfrac{mg}{k}}$ **(c)** $80\sqrt{5} \approx 178.89$ ft/sec **79.** 2π **81.** $\dfrac{6}{5}$

Section 7.4, pp. 448–449

1. **(a)** Slower **(b)** Slower **(c)** Slower **(d)** Faster **(e)** Slower **(f)** Slower **(g)** Same **(h)** Slower

3. **(a)** Same **(b)** Faster **(c)** Same **(d)** Same **(e)** Slower **(f)** Faster **(g)** Slower **(h)** Same

5. **(a)** Same **(b)** Same **(c)** Same **(d)** Faster **(e)** Faster **(f)** Same **(g)** Slower **(h)** Faster

7. d, a, c, b

9. **(a)** False **(b)** False **(c)** True **(d)** True **(e)** True **(f)** True **(g)** False **(h)** True

13. When the degree of f is less than or equal to the degree of g.

15. 1, 1

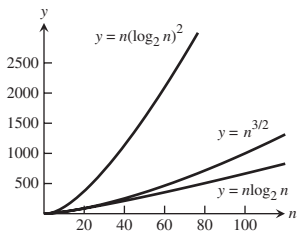
21. **(b)** $\ln(e^{17000000}) = 17{,}000{,}000 < (e^{17\times10^6})^{1/10^6}$
$= e^{17} \approx 24{,}154{,}952.75$

(c) $x \approx 3.4306311 \times 10^{15}$

(d) They cross at $x \approx 3.4306311 \times 10^{15}$.

23. **(a)** The algorithm that takes $O(n\log_2 n)$ steps

(b)



25. It could take one million for a sequential search; at most 20 steps for a binary search.

Practice Exercises, pp. 450–451

1. $-\cos e^x + C$ **3.** $\ln 8$ **5.** $2\ln 2$ **7.** $\frac{1}{2}(\ln(x-5))^2 + C$

9. $3\ln 7$ **11.** $2(\sqrt{2}-1)$ **13.** $y = \dfrac{\ln 2}{\ln(3/2)}$

15. $y = \ln x - \ln 3$ **17.** $y = \dfrac{1}{1-e^x}$

19. **(a)** Same rate **(b)** Same rate **(c)** Faster **(d)** Faster **(e)** Same rate **(f)** Same rate

21. **(a)** True **(b)** False **(c)** False **(d)** True **(e)** True **(f)** True

23. $1/3$ **25.** $1/e$ m/sec **27.** $\ln 5x - \ln 3x = \ln(5/3)$

29. $1/2$ **31.** $y = \left(\tan^{-1}\left(\dfrac{x+C}{2}\right)\right)^2$

33. $y^2 = \sin^{-1}(2\tan x + C)$

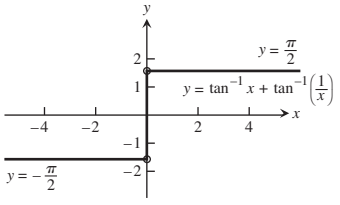
35. $y = -2 + \ln(2 - e^{-x})$ **37.** $y = 4x - 4\sqrt{x} + 1$

39. 18,935 years

Additional and Advanced Exercises, p.p 451–452

1. **(a)** 1 **(b)** $\pi/2$ **(c)** π

3. $\tan^{-1}x + \tan^{-1}\left(\dfrac{1}{x}\right)$ is a constant and the constant is $\dfrac{\pi}{2}$ for $x > 0$; it is $-\dfrac{\pi}{2}$ for $x < 0$.



7. $\bar{x} = \dfrac{\ln 4}{\pi}$, $\bar{y} = 0$

CHAPTER 8

Section 8.1, pp. 459–461

1. $-2x\cos(x/2) + 4\sin(x/2) + C$

3. $t^2\sin t + 2t\cos t - 2\sin t + C$ **5.** $\ln 4 - \dfrac{3}{4}$

7. $xe^x - e^x + C$ **9.** $-(x^2+2x+2)e^{-x} + C$

11. $y\tan^{-1}(y) - \ln\sqrt{1+y^2} + C$
13. $x\tan x + \ln|\cos x| + C$
15. $(x^3 - 3x^2 + 6x - 6)e^x + C$
17. $(x^2 - 7x + 7)e^x + C$
19. $(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120)e^x + C$
21. $\frac{1}{2}(-e^\theta\cos\theta + e^\theta\sin\theta) + C$
23. $\frac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + C$
25. $\frac{2}{3}\left(\sqrt{3s+9}\,e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}\right) + C$
27. $\frac{\pi\sqrt{3}}{3} - \ln(2) - \frac{\pi^2}{18}$
29. $\frac{1}{2}[-x\cos(\ln x) + x\sin(\ln x)] + C$
31. $\frac{1}{2}\ln|\sec x^2 + \tan x^2| + C$
33. $\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C$
35. $-\frac{1}{x}\ln x - \frac{1}{x} + C$ **37.** $\frac{1}{4}e^{x^4} + C$
39. $\frac{1}{3}x^2(x^2+1)^{3/2} - \frac{2}{15}(x^2+1)^{5/2} + C$
41. $-\frac{2}{5}\sin 3x\sin 2x - \frac{3}{5}\cos 3x\cos 2x + C$
43. $-\cos e^x + C$ **45.** $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$
47. $\frac{\pi^2 - 4}{8}$ **49.** $\frac{5\pi - 3\sqrt{3}}{9}$
51. **(a)** π **(b)** 3π **(c)** 5π **(d)** $(2n+1)\pi$
53. $2\pi(1 - \ln 2)$ **55.** **(a)** $\pi(\pi - 2)$ **(b)** 2π
57. **(a)** 1 **(b)** $(e-2)\pi$ **(c)** $\frac{\pi}{2}(e^2+9)$
(d) $\bar{x} = \frac{1}{4}(e^2+1), \bar{y} = \frac{1}{2}(e-2)$
59. $\frac{1}{2\pi}(1 - e^{-2\pi})$ **61.** $u = x^n, dv = \cos x\,dx$
63. $u = x^n, dv = e^{ax}\,dx$ **67.** $x\sin^{-1}x + \cos(\sin^{-1}x) + C$
69. $x\sec^{-1}x - \ln\left|x + \sqrt{x^2-1}\right| + C$ **71.** Yes
73. **(a)** $x\sinh^{-1}x - \cosh(\sinh^{-1}x) + C$
(b) $x\sinh^{-1}x - (1+x^2)^{1/2} + C$

Section 8.2, pp. 466–467

1. $\frac{1}{2}\sin 2x + C$ **3.** $-\frac{1}{4}\cos^4 x + C$ **5.** $\frac{1}{3}\cos^3 x - \cos x + C$
7. $-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$ **9.** $\sin x - \frac{1}{3}\sin^3 x + C$
11. $\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C$ **13.** $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$
15. 16/35 **17.** 3π
19. $-4\sin x\cos^3 x + 2\cos x\sin x + 2x + C$
21. $-\cos^4 2\theta + C$ **23.** 4 **25.** 2
27. $\sqrt{\frac{3}{2}} - \frac{2}{3}$ **29.** $\frac{4}{5}\left(\frac{3}{2}\right)^{5/2} - \frac{18}{35} - \frac{2}{7}\left(\frac{3}{2}\right)^{7/2}$ **31.** $\sqrt{2}$
33. $\frac{1}{2}\tan^2 x + C$ **35.** $\frac{1}{3}\sec^3 x + C$ **37.** $\frac{1}{3}\tan^3 x + C$
39. $2\sqrt{3} + \ln(2+\sqrt{3})$ **41.** $\frac{2}{3}\tan\theta + \frac{1}{3}\sec^2\theta\tan\theta + C$
43. 4/3 **45.** $2\tan^2 x - 2\ln(1+\tan^2 x) + C$
47. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \ln|\sec x| + C$
49. $\frac{4}{3} - \ln\sqrt{3}$ **51.** $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$ **53.** π
55. $\frac{1}{2}\sin x + \frac{1}{14}\sin 7x + C$
57. $\frac{1}{6}\sin 3\theta - \frac{1}{4}\sin\theta - \frac{1}{20}\sin 5\theta + C$
59. $-\frac{2}{5}\cos^5\theta + C$ **61.** $\frac{1}{4}\cos\theta - \frac{1}{20}\cos 5\theta + C$
63. $\sec x - \ln|\csc x + \cot x| + C$ **65.** $\cos x + \sec x + C$
67. $\frac{1}{4}x^2 - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + C$ **69.** $\ln(1+\sqrt{2})$
71. $\pi^2/2$ **73.** $\bar{x} = \frac{4\pi}{3}, \bar{y} = \frac{8\pi^2+3}{12\pi}$

Section 8.3, pp. 470–471

1. $\ln|\sqrt{9+x^2} + x| + C$ **3.** $\pi/4$ **5.** $\pi/6$
7. $\frac{25}{2}\sin^{-1}\left(\frac{t}{5}\right) + \frac{t\sqrt{25-t^2}}{2} + C$
9. $\frac{1}{2}\ln\left|\frac{2x}{7} + \frac{\sqrt{4x^2-49}}{7}\right| + C$
11. $7\left[\frac{\sqrt{y^2-49}}{7} - \sec^{-1}\left(\frac{y}{7}\right)\right] + C$ **13.** $\frac{\sqrt{x^2-1}}{x} + C$
15. $-\sqrt{9-x^2} + C$ **17.** $\frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C$
19. $\frac{-2\sqrt{4-w^2}}{w} + C$ **21.** $\frac{10}{3}\tan^{-1}\frac{5x}{6} + C$
23. $4\sqrt{3} - \frac{4\pi}{3}$ **25.** $-\frac{x}{\sqrt{x^2-1}} + C$
27. $-\frac{1}{5}\left(\frac{\sqrt{1-x^2}}{x}\right)^5 + C$ **29.** $2\tan^{-1}2x + \frac{4x}{(4x^2+1)} + C$
31. $\frac{1}{2}x^2 + \frac{1}{2}\ln|x^2-1| + C$ **33.** $\frac{1}{3}\left(\frac{v}{\sqrt{1-v^2}}\right)^3 + C$
35. $\ln 9 - \ln(1+\sqrt{10})$ **37.** $\pi/6$ **39.** $\sec^{-1}|x| + C$
41. $\sqrt{x^2-1} + C$ **43.** $\frac{1}{2}\ln|\sqrt{1+x^4} + x^2| + C$
45. $4\sin^{-1}\frac{\sqrt{x}}{2} + \sqrt{x}\sqrt{4-x} + C$
47. $\frac{1}{4}\sin^{-1}\sqrt{x} - \frac{1}{4}\sqrt{x}\sqrt{1-x}(1-2x) + C$
49. $y = 2\left[\frac{\sqrt{x^2-4}}{2} - \sec^{-1}\left(\frac{x}{2}\right)\right]$
51. $y = \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) - \frac{3\pi}{8}$ **53.** $3\pi/4$

55. (a) $\frac{1}{12}(\pi + 6\sqrt{3} - 12)$

(b) $\bar{x} = \dfrac{3\sqrt{3} - \pi}{4(\pi + 6\sqrt{3} - 12)}$, $\bar{y} = \dfrac{\pi^2 + 12\sqrt{3}\pi - 72}{12(\pi + 6\sqrt{3} - 12)}$

57. (a) $-\frac{1}{3}x^2(1 - x^2)^{3/2} - \frac{2}{15}(1 - x^2)^{5/2} + C$

(b) $-\frac{1}{3}(1 - x^2)^{3/2} + \frac{1}{5}(1 - x^2)^{5/2} + C$

(c) $\frac{1}{5}(1 - x^2)^{5/2} - \frac{1}{3}(1 - x^2)^{3/2} + C$

Section 8.4, pp. 479–480

1. $\dfrac{2}{x - 3} + \dfrac{3}{x - 2}$ **3.** $\dfrac{1}{x + 1} + \dfrac{3}{(x + 1)^2}$

5. $\dfrac{-2}{z} + \dfrac{-1}{z^2} + \dfrac{2}{z - 1}$ **7.** $1 + \dfrac{17}{t - 3} + \dfrac{-12}{t - 2}$

9. $\frac{1}{2}[\ln|1 + x| - \ln|1 - x|] + C$

11. $\frac{1}{7}\ln|(x + 6)^2(x - 1)^5| + C$ **13.** $(\ln 15)/2$

15. $-\frac{1}{2}\ln|t| + \frac{1}{6}\ln|t + 2| + \frac{1}{3}\ln|t - 1| + C$ **17.** $3\ln 2 - 2$

19. $\frac{1}{4}\ln\left|\dfrac{x + 1}{x - 1}\right| - \dfrac{x}{2(x^2 - 1)} + C$ **21.** $(\pi + 2\ln 2)/8$

23. $\tan^{-1} y - \dfrac{1}{y^2 + 1} + C$

25. $-(s - 1)^{-2} + (s - 1)^{-1} + \tan^{-1} s + C$

27. $\frac{2}{3}\ln|x - 1| + \frac{1}{6}\ln|x^2 + x + 1| - \sqrt{3}\tan^{-1}\left(\dfrac{2x + 1}{\sqrt{3}}\right) + C$

29. $\frac{1}{4}\ln\left|\dfrac{x - 1}{x + 1}\right| + \frac{1}{2}\tan^{-1} x + C$

31. $\dfrac{-1}{\theta^2 + 2\theta + 2} + \ln(\theta^2 + 2\theta + 2) - \tan^{-1}(\theta + 1) + C$

33. $x^2 + \ln\left|\dfrac{x - 1}{x}\right| + C$

35. $9x + 2\ln|x| + \dfrac{1}{x} + 7\ln|x - 1| + C$

37. $\dfrac{y^2}{2} - \ln|y| + \frac{1}{2}\ln(1 + y^2) + C$ **39.** $\ln\left(\dfrac{e^t + 1}{e^t + 2}\right) + C$

41. $\frac{1}{5}\ln\left|\dfrac{\sin y - 2}{\sin y + 3}\right| + C$

43. $\dfrac{(\tan^{-1} 2x)^2}{4} - 3\ln|x - 2| + \dfrac{6}{x - 2} + C$

45. $\ln\left|\dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}\right| + C$ **47.** $2\sqrt{1 + x} + \ln\left|\dfrac{\sqrt{x + 1} - 1}{\sqrt{x + 1} + 1}\right| + C$

49. $\frac{1}{4}\ln\left|\dfrac{x^4}{x^4 + 1}\right| + C$

51. $x = \ln|t - 2| - \ln|t - 1| + \ln 2$ **53.** $x = \dfrac{6t}{t + 2} - 1$

55. $3\pi \ln 25$ **57.** 1.10 **59.** (a) $x = \dfrac{1000e^{4t}}{499 + e^{4t}}$ (b) 1.55 days

Section 8.5, pp. 485–486

1. $\dfrac{2}{\sqrt{3}}\left(\tan^{-1}\sqrt{\dfrac{x - 3}{3}}\right) + C$

3. $\sqrt{x - 2}\left(\dfrac{2(x - 2)}{3} + 4\right) + C$

5. $\dfrac{(2x - 3)^{3/2}(x + 1)}{5} + C$

7. $\dfrac{-\sqrt{9 - 4x}}{x} - \dfrac{2}{3}\ln\left|\dfrac{\sqrt{9 - 4x} - 3}{\sqrt{9 - 4x} + 3}\right| + C$

9. $\dfrac{(x + 2)(2x - 6)\sqrt{4x - x^2}}{6} + 4\sin^{-1}\left(\dfrac{x - 2}{2}\right) + C$

11. $-\dfrac{1}{\sqrt{7}}\ln\left|\dfrac{\sqrt{7} + \sqrt{7 + x^2}}{x}\right| + C$

13. $\sqrt{4 - x^2} - 2\ln\left|\dfrac{2 + \sqrt{4 - x^2}}{x}\right| + C$

15. $\dfrac{e^{2t}}{13}(2\cos 3t + 3\sin 3t) + C$

17. $\dfrac{x^2}{2}\cos^{-1} x + \dfrac{1}{4}\sin^{-1} x - \dfrac{1}{4}x\sqrt{1 - x^2} + C$

19. $\dfrac{x^3}{3}\tan^{-1} x - \dfrac{x^2}{6} + \dfrac{1}{6}\ln(1 + x^2) + C$

21. $-\dfrac{\cos 5x}{10} - \dfrac{\cos x}{2} + C$ **23.** $8\left[\dfrac{\sin(7t/2)}{7} - \dfrac{\sin(9t/2)}{9}\right] + C$

25. $6\sin(\theta/12) + \dfrac{6}{7}\sin(7\theta/12) + C$

27. $\frac{1}{2}\ln(x^2 + 1) + \dfrac{x}{2(1 + x^2)} + \frac{1}{2}\tan^{-1} x + C$

29. $\left(x - \dfrac{1}{2}\right)\sin^{-1}\sqrt{x} + \dfrac{1}{2}\sqrt{x - x^2} + C$

31. $\sin^{-1}\sqrt{x} - \sqrt{x - x^2} + C$

33. $\sqrt{1 - \sin^2 t} - \ln\left|\dfrac{1 + \sqrt{1 - \sin^2 t}}{\sin t}\right| + C$

35. $\ln\left|\ln y + \sqrt{3 + (\ln y)^2}\right| + C$

37. $\ln|x + 1 + \sqrt{x^2 + 2x + 5}| + C$

39. $\dfrac{x + 2}{2}\sqrt{5 - 4x - x^2} + \dfrac{9}{2}\sin^{-1}\left(\dfrac{x + 2}{3}\right) + C$

41. $-\dfrac{\sin^4 2x\cos 2x}{10} - \dfrac{2\sin^2 2x\cos 2x}{15} - \dfrac{4\cos 2x}{15} + C$

43. $\dfrac{\sin^3 2\theta\cos^2 2\theta}{10} + \dfrac{\sin^3 2\theta}{15} + C$

45. $\tan^2 2x - 2\ln|\sec 2x| + C$

47. $\dfrac{(\sec \pi x)(\tan \pi x)}{\pi} + \dfrac{1}{\pi}\ln|\sec \pi x + \tan \pi x| + C$

49. $\dfrac{-\csc^3 x\cot x}{4} - \dfrac{3\csc x\cot x}{8} - \dfrac{3}{8}\ln|\csc x + \cot x| + C$

51. $\frac{1}{2}[\sec(e^t - 1)\tan(e^t - 1) + \ln|\sec(e^t - 1) + \tan(e^t - 1)|] + C$

53. $\sqrt{2} + \ln(\sqrt{2} + 1)$ **55.** $\pi/3$

57. $2\pi\sqrt{3} + \pi\sqrt{2}\ln(\sqrt{2} + \sqrt{3})$

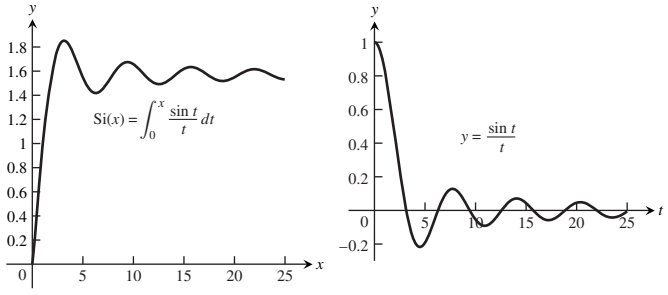
59. $\bar{x} = 4/3$, $\bar{y} = \ln\sqrt{2}$ **61.** 7.62 **63.** $\pi/8$ **67.** $\pi/4$

Section 8.6, pp. 493–495

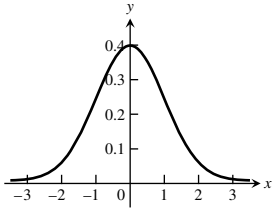
1. **I:** **(a)** 1.5, 0 **(b)** 1.5, 0 **(c)** 0%
II: **(a)** 1.5, 0 **(b)** 1.5, 0 **(c)** 0%
3. **I:** **(a)** 2.75, 0.08 **(b)** 2.67, 0.08 **(c)** 0.0312 ≈ 3%
II: **(a)** 2.67, 0 **(b)** 2.67, 0 **(c)** 0%
5. **I:** **(a)** 6.25, 0.5 **(b)** 6, 0.25 **(c)** 0.0417 ≈ 4%
II: **(a)** 6, 0 **(b)** 6, 0 **(c)** 0%
7. **I:** **(a)** 0.509, 0.03125 **(b)** 0.5, 0.009 **(c)** 0.018 ≈ 2%
II: **(a)** 0.5, 0.002604 **(b)** 0.5, 0.0004 **(c)** 0%
9. **I:** **(a)** 1.8961, 0.161 **(b)** 2, 0.1039 **(c)** 0.052 ≈ 5%
II: **(a)** 2.0045, 0.0066 **(b)** 2, 0.00454 **(c)** 0.2%
11. **(a)** 1 **(b)** 2 **13.** **(a)** 116 **(b)** 2
15. **(a)** 283 **(b)** 2 **17.** **(a)** 71 **(b)** 10
19. **(a)** 76 **(b)** 12 **21.** **(a)** 82 **(b)** 8
23. 15,990 ft^3 **25.** ≈10.63 ft
27. **(a)** ≈0.00021 **(b)** ≈1.37079 **(c)** ≈0.015%
31. **(a)** ≈5.870 **(b)** $|E_T| \le 0.0032$
33. 21.07 in. **35.** 14.4

Section 8.7, pp. 505–507

1. $\pi/2$ **3.** 2 **5.** 6 **7.** $\pi/2$ **9.** $\ln 3$ **11.** $\ln 4$ **13.** 0
15. $\sqrt{3}$ **17.** π **19.** $\ln\left(1 + \frac{\pi}{2}\right)$ **21.** -1 **23.** 1
25. $-1/4$ **27.** $\pi/2$ **29.** $\pi/3$ **31.** 6 **33.** $\ln 2$
35. Diverges **37.** Converges **39.** Converges **41.** Converges
43. Diverges **45.** Converges **47.** Converges **49.** Diverges
51. Converges **53.** Converges **55.** Diverges **57.** Converges
59. Diverges **61.** Converges **63.** Converges
65. **(a)** Converges when $p < 1$ **(b)** Converges when $p > 1$
67. 1 **69.** 2π **71.** $\ln 2$ **73.** **(b)** ≈0.88621
75. **(a)**



(b) $\pi/2$
77. **(a)**



(b) ≈0.683, ≈0.954, ≈0.997

Practice Exercises, pp. 507–509

1. $(x + 1)(\ln(x + 1)) - (x + 1) + C$
3. $x\tan^{-1}(3x) - \frac{1}{6}\ln(1 + 9x^2) + C$
5. $(x + 1)^2e^x - 2(x + 1)e^x + 2e^x + C$
7. $\frac{2e^x \sin 2x}{5} + \frac{e^x \cos 2x}{5} + C$
9. $2\ln|x - 2| - \ln|x - 1| + C$
11. $\ln|x| - \ln|x + 1| + \frac{1}{x + 1} + C$
13. $-\frac{1}{3}\ln\left|\frac{\cos\theta - 1}{\cos\theta + 2}\right| + C$
15. $4\ln|x| - \frac{1}{2}\ln(x^2 + 1) + 4\tan^{-1}x + C$
17. $\frac{1}{16}\ln\left|\frac{(v - 2)^5(v + 2)}{v^6}\right| + C$
19. $\frac{1}{2}\tan^{-1}t - \frac{\sqrt{3}}{6}\tan^{-1}\frac{t}{\sqrt{3}} + C$
21. $\frac{x^2}{2} + \frac{4}{3}\ln|x + 2| + \frac{2}{3}\ln|x - 1| + C$
23. $\frac{x^2}{2} - \frac{9}{2}\ln|x + 3| + \frac{3}{2}\ln|x + 1| + C$
25. $\frac{1}{3}\ln\left|\frac{\sqrt{x + 1} - 1}{\sqrt{x + 1} + 1}\right| + C$ **27.** $\ln|1 - e^{-s}| + C$
29. $-\sqrt{16 - y^2} + C$ **31.** $-\frac{1}{2}\ln|4 - x^2| + C$
33. $\ln\frac{1}{\sqrt{9 - x^2}} + C$ **35.** $\frac{1}{6}\ln\left|\frac{x + 3}{x - 3}\right| + C$
37. $-\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$ **39.** $\frac{\tan^5 x}{5} + C$
41. $\frac{\cos\theta}{2} - \frac{\cos 11\theta}{22} + C$ **43.** $4\sqrt{1 - \cos(t/2)} + C$
45. At least 16 **47.** $T = \pi, S = \pi$ **49.** 25°F
51. **(a)** ≈2.42 gal **(b)** ≈24.83 mi/gal
53. $\pi/2$ **55.** 6 **57.** $\ln 3$ **59.** 2 **61.** $\pi/6$
63. Diverges **65.** Diverges **67.** Converges
69. $\frac{2x^{3/2}}{3} - x + 2\sqrt{x} - 2\ln\left(\sqrt{x} + 1\right) + C$
71. $\ln\left|\frac{\sqrt{x}}{\sqrt{x^2 + 1}}\right| - \frac{1}{2}\left(\frac{x}{\sqrt{x^2 + 1}}\right)^2 + C$
73. $-2\cot x - \ln|\csc x + \cot x| + \csc x + C$
75. $\frac{1}{12}\ln\left|\frac{3 + v}{3 - v}\right| + \frac{1}{6}\tan^{-1}\frac{v}{3} + C$
77. $\frac{\theta\sin(2\theta + 1)}{2} + \frac{\cos(2\theta + 1)}{4} + C$ **79.** $\frac{1}{4}\sec^2\theta + C$
81. $2\left(\frac{\left(\sqrt{2 - x}\right)^3}{3} - 2\sqrt{2 - x}\right) + C$ **83.** $\tan^{-1}(y - 1) + C$
85. $\frac{1}{4}\ln|z| - \frac{1}{4z} - \frac{1}{4}\left[\frac{1}{2}\ln(z^2 + 4) + \frac{1}{2}\tan^{-1}\left(\frac{z}{2}\right)\right] + C$
87. $-\frac{1}{4}\sqrt{9 - 4t^2} + C$ **89.** $\ln\left(\frac{e^t + 1}{e^t + 2}\right) + C$ **91.** 1/4
93. $\frac{2}{3}x^{3/2} + C$ **95.** $-\frac{1}{5}\tan^{-1}(\cos 5t) + C$
97. $2\sqrt{r} - 2\ln(1 + \sqrt{r}) + C$
99. $\frac{1}{2}x^2 - \frac{1}{2}\ln(x^2 + 1) + C$

101. $\frac{2}{3}\ln|x+1| + \frac{1}{6}\ln|x^2-x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

103. $\frac{4}{7}\left(1+\sqrt{x}\right)^{7/2} - \frac{8}{5}\left(1+\sqrt{x}\right)^{5/2} + \frac{4}{3}\left(1+\sqrt{x}\right)^{3/2} + C$

105. $2\ln\left|\sqrt{x}+\sqrt{1+x}\right| + C$

107. $\ln x - \ln|1+\ln x| + C$

109. $\frac{1}{2}x^{\ln x} + C$ **111.** $\frac{1}{2}\ln\left|\frac{1-\sqrt{1-x^4}}{x^2}\right| + C$

113. **(b)** $\frac{\pi}{4}$ **115.** $x - \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right) + C$

Additional and Advanced Exercises, pp. 510–512

1. $x(\sin^{-1}x)^2 + 2(\sin^{-1}x)\sqrt{1-x^2} - 2x + C$

3. $\frac{x^2\sin^{-1}x}{2} + \frac{x\sqrt{1-x^2} - \sin^{-1}x}{4} + C$

5. $\frac{1}{2}\left(\ln\left(t-\sqrt{1-t^2}\right) - \sin^{-1}t\right) + C$

7. 0 **9.** $\ln(4) - 1$ **11.** 1 **13.** $32\pi/35$ **15.** 2π

17. **(a)** π **(b)** $\pi(2e-5)$

19. **(b)** $\pi\left(\frac{8(\ln 2)^2}{3} - \frac{16(\ln 2)}{9} + \frac{16}{27}\right)$ **21.** $\left(\frac{e^2+1}{4}, \frac{e-2}{2}\right)$

23. $\sqrt{1+e^2} - \ln\left(\frac{\sqrt{1+e^2}}{e} + \frac{1}{e}\right) - \sqrt{2} + \ln\left(1+\sqrt{2}\right)$

25. $\frac{12\pi}{5}$ **27.** $a = \frac{1}{2}, -\frac{\ln 2}{4}$ **29.** $\frac{1}{2} < p \le 1$

33. $\frac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + C$

35. $\frac{\cos x \sin 3x - 3\sin x\cos 3x}{8} + C$

37. $\frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$ **39.** $x\ln(ax) - x + C$

41. $\frac{2}{1-\tan(x/2)} + C$ **43.** 1 **45.** $\frac{\sqrt{3}\pi}{9}$

47. $\frac{1}{\sqrt{2}}\ln\left|\frac{\tan(t/2)+1-\sqrt{2}}{\tan(t/2)+1+\sqrt{2}}\right| + C$

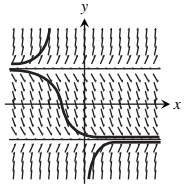
49. $\ln\left|\frac{1+\tan(\theta/2)}{1-\tan(\theta/2)}\right| + C$

CHAPTER 9

Section 9.1, pp. 520–522

1. (d) **3.** (a)

5.



7. $y' = x - y;\ y(1) = -1$ **9.** $y' = -(1+y)\sin x;\ y(0) = 2$

11. $y(\text{exact}) = \frac{x}{2} - \frac{4}{x},\ y_1 = -0.25,\ y_2 = 0.3,\ y_3 = 0.75$

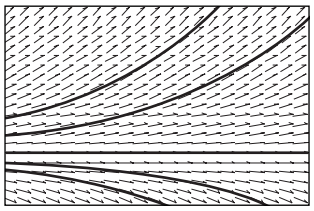
13. $y(\text{exact}) = 3e^{x(x+2)},\ y_1 = 4.2,\ y_2 = 6.216,\ y_3 = 9.697$

15. $y(\text{exact}) = e^{x^2} + 1,\ y_1 = 2.0,\ y_2 = 2.0202,\ y_3 = 2.0618$

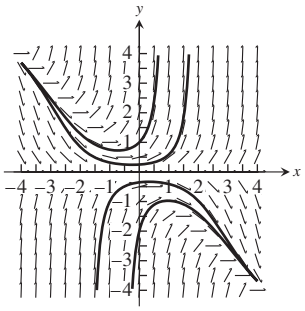
17. $y \approx 2.48832$, exact value is e.

19. $y \approx -0.2272$, exact value is $1/\left(1-2\sqrt{5}\right) \approx -0.2880$.

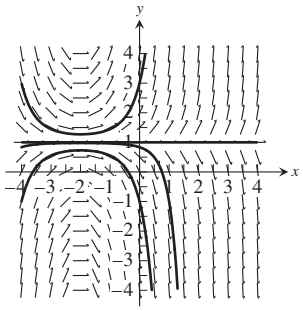
23.



25.



27.



35. Euler's method gives $y \approx 3.45835$; the exact solution is $y = 1 + e \approx 3.71828$.

37. $y \approx 1.5000$; exact value is 1.5275.

Section 9.2, pp. 526–528

1. $y = \frac{e^x + C}{x},\quad x > 0$ **3.** $y = \frac{C - \cos x}{x^3},\quad x > 0$

5. $y = \frac{1}{2} - \frac{1}{x} + \frac{C}{x^2},\quad x > 0$ **7.** $y = \frac{1}{2}xe^{x/2} + Ce^{x/2}$

9. $y = x(\ln x)^2 + Cx$

11. $s = \frac{t^3}{3(t-1)^4} - \frac{t}{(t-1)^4} + \frac{C}{(t-1)^4}$

13. $r = (\csc\theta)(\ln|\sec\theta| + C),\quad 0 < \theta < \pi/2$

15. $y = \frac{3}{2} - \frac{1}{2}e^{-2t}$ **17.** $y = -\frac{1}{\theta}\cos\theta + \frac{\pi}{2\theta}$

19. $y = 6e^{x^2} - \frac{e^{x^2}}{x+1}$ **21.** $y = y_0e^{kt}$

23. **(b)** is correct, but **(a)** is not. **25.** $t = \frac{L}{R}\ln 2$ sec

27. **(a)** $i = \frac{V}{R} - \frac{V}{R}e^{-3} = \frac{V}{R}(1-e^{-3}) \approx 0.95\frac{V}{R}$ amp **(b)** 86%

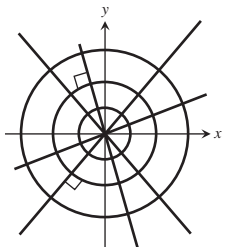
29. $y = \frac{1}{1+Ce^{-x}}$ **31.** $y^3 = 1 + Cx^{-3}$

Section 9.3, pp. 533–534

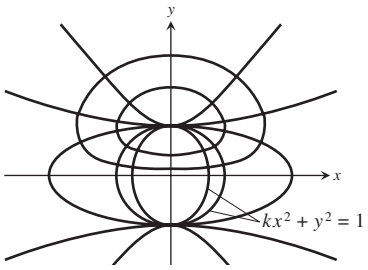
1. **(a)** 168.5 m **(b)** 41.13 sec

3. $s(t) = 4.91\left(1 - e^{-(22.36/39.92)t}\right)$

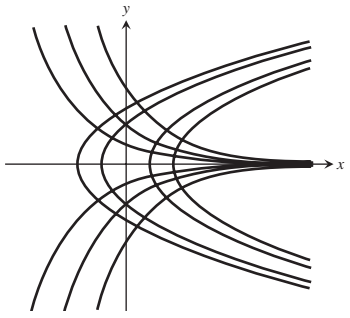
5. $x^2 + y^2 = C$



7. $\ln|y| - \frac{1}{2}y^2 = \frac{1}{2}x^2 + C$



9. $y = \pm\sqrt{2x + C}$



13. **(a)** 10 lb/min **(b)** $(100 + t)$ gal **(c)** $4\left(\dfrac{y}{100 + t}\right)$ lb/min

(d) $\dfrac{dy}{dt} = 10 - \dfrac{4y}{100 + t}$, $y(0) = 50$,

$$y = 2(100 + t) - \frac{150}{\left(1 + \dfrac{t}{100}\right)^4}$$

(e) Concentration $= \dfrac{y\,(25)}{\text{amt. brine in tank}} = \dfrac{188.6}{125} \approx 1.5$ lb/gal

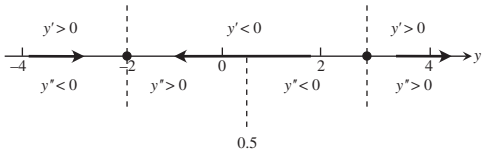
15. $y(27.8) \approx 14.8$ lb, $t \approx 27.8$ min

Section 9.4, pp. 540–541

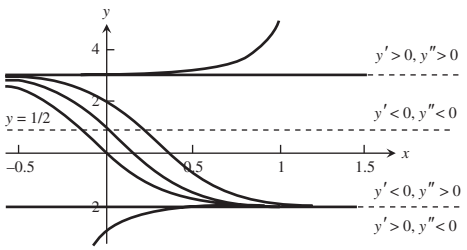
1. $y' = (y + 2)(y - 3)$

(a) $y = -2$ is a stable equilibrium value and $y = 3$ is an unstable equilibrium.

(b) $y'' = 2(y + 2)\left(y - \dfrac{1}{2}\right)(y - 3)$



(c)

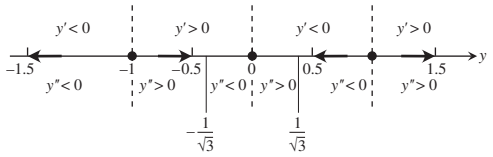


3. $y' = y^3 - y = (y + 1)y(y - 1)$

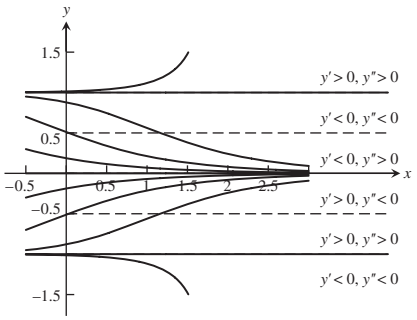
(a) $y = -1$ and $y = 1$ are unstable equilibria and $y = 0$ is a stable equilibrium.

(b) $y'' = (3y^2 - 1)y'$

$= 3(y + 1)\left(y + 1/\sqrt{3}\right)y\left(y - 1/\sqrt{3}\right)(y - 1)$



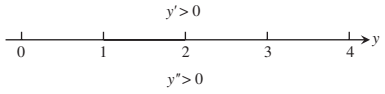
(c)



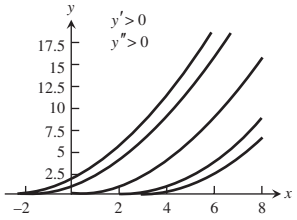
5. $y' = \sqrt{y}, y > 0$

(a) There are no equilibrium values.

(b) $y'' = \dfrac{1}{2}$



(c)

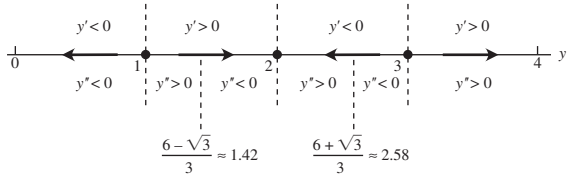


7. $y' = (y - 1)(y - 2)(y - 3)$

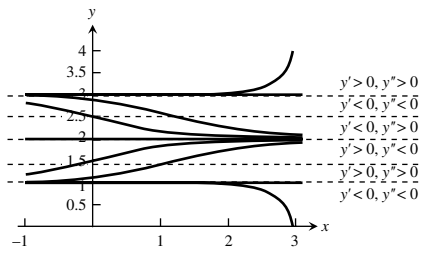
(a) $y = 1$ and $y = 3$ are unstable equilibria and $y = 2$ is a stable equilibrium.

(b) $y'' = (3y^2 - 12y + 11)(y - 1)(y - 2)(y - 3) =$

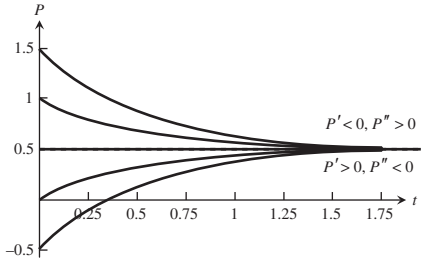
$$3(y - 1)\left(y - \frac{6 - \sqrt{3}}{3}\right)(y - 2)\left(y - \frac{6 + \sqrt{3}}{3}\right)(y - 3)$$



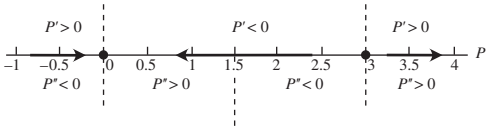
(c)

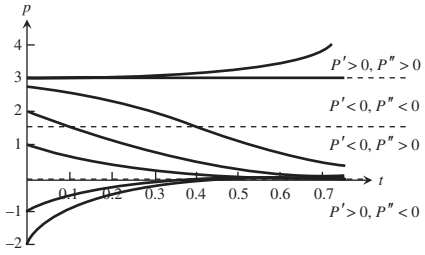


9. $\dfrac{dP}{dt} = 1 - 2P$ has a stable equilibrium at $P = \dfrac{1}{2}$;
$\dfrac{d^2P}{dt^2} = -2\dfrac{dP}{dt} = -2(1 - 2P)$.

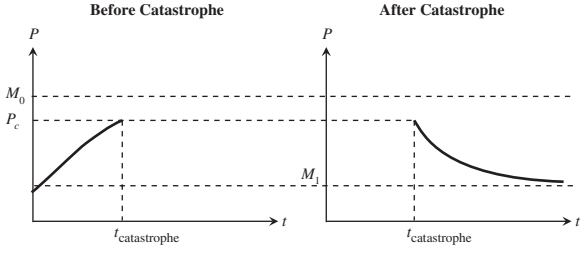


11. $\dfrac{dP}{dt} = 2P(P - 3)$ has a stable equilibrium at $P = 0$ and an unstable equilibrium at $P = 3$; $\dfrac{d^2P}{dt^2} = 2(2P - 3)\dfrac{dP}{dt} = 4P(2P - 3)(P - 3)$.





13. Before the catastrophe, the population exhibits logistic growth and $P(t)$ increases toward M_0, the stable equilibrium. After the catastrophe, the population declines logistically and $P(t)$ decreases toward M_1, the new stable equilibrium.

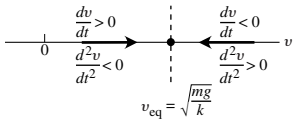


15. $\dfrac{dv}{dt} = g - \dfrac{k}{m}v^2$, $g, k, m > 0$ and $v(t) \geq 0$

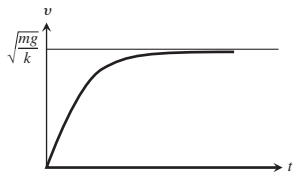
Equilibrium: $\dfrac{dv}{dt} = g - \dfrac{k}{m}v^2 = 0 \Rightarrow v = \sqrt{\dfrac{mg}{k}}$

Concavity: $\dfrac{d^2v}{dt^2} = -2\left(\dfrac{k}{m}v\right)\dfrac{dv}{dt} = -2\left(\dfrac{k}{m}v\right)\left(g - \dfrac{k}{m}v^2\right)$

(a)



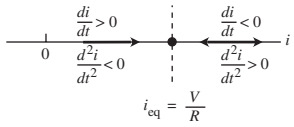
(b)



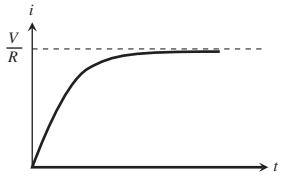
(c) $v_{\text{terminal}} = \sqrt{\dfrac{160}{0.005}} = 178.9 \text{ ft/sec} = 122 \text{ mph}$

17. $F = F_p - F_r$; $ma = 50 - 5|v|$; $\dfrac{dv}{dt} = \dfrac{1}{m}(50 - 5|v|)$. The maximum velocity occurs when $\dfrac{dv}{dt} = 0$ or $v = 10$ ft/sec.

19. Phase line:



If the switch is closed at $t = 0$, then $i(0) = 0$, and the graph of the solution looks like this:



As $t \to \infty$, $i(t) \to i_{\text{steady state}} = \dfrac{V}{R}$.

Section 9.5, pp. 545–547

1. Seasonal variations, nonconformity of the environments, effects of other interactions, unexpected disasters, etc.

3. This model assumes that the number of interactions is proportional to the product of x and y:

$\dfrac{dx}{dt} = (a - by)x, \quad a < 0,$

$\dfrac{dy}{dt} = m\left(1 - \dfrac{y}{M}\right)y - nxy = y\left(m - \dfrac{m}{M}y - nx\right).$

Rest points are (0, 0), unstable, and (0, M), stable.

5. **(a)** Logistic growth occurs in the absence of the competitor, and involves a simple interaction between the species: growth dominates the competition when either population is small, so it is difficult to drive either species to extinction.

(b) a: per capita growth rate for trout
m: per capita growth rate for bass
b: intensity of competition to the trout
n: intensity of competition to the bass
k_1: environmental carrying capacity for the trout
k_2: environmental carrying capacity for the bass
$\dfrac{a}{b}$: growth versus competition or net growth of trout
$\dfrac{m}{n}$: relative survival of bass

(c) $\dfrac{dx}{dt} = 0$ when $x = 0$ or $y = \dfrac{a}{b} - \dfrac{a}{bk_1}x$,

$\dfrac{dy}{dt} = 0$ when $y = 0$ or $y = k_2 - \dfrac{k_2 n}{m}x$.

By picking $a/b > k_2$ and $m/n > k_1$, we insure that an equilibrium point exists inside the first quadrant.

Practice Exercises, pp. 547–548

1. $y = -\ln\left(C - \dfrac{2}{5}(x-2)^{5/2} - \dfrac{4}{3}(x-2)^{3/2}\right)$

3. $\tan y = -x\sin x - \cos x + C$ **5.** $(y+1)e^{-y} = -\ln|x| + C$

7. $y = C\dfrac{x-1}{x}$ **9.** $y = \dfrac{x^2}{4}e^{x/2} + Ce^{x/2}$

11. $y = \dfrac{x^2 - 2x + C}{2x^2}$ **13.** $y = \dfrac{e^{-x} + C}{1 + e^{x}}$ **15.** $xy + y^3 = C$

17. $y = \dfrac{2x^3 + 3x^2 + 6}{6(x+1)^2}$ **19.** $y = \dfrac{1}{3}(1 - 4e^{-x^3})$

21. $y = e^{-x}(3x^3 - 3x^2)$

23.

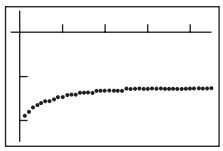
x	y
0	0
0.1	0.1000
0.2	0.2095
0.3	0.3285
0.4	0.4568
0.5	0.5946
0.6	0.7418
0.7	0.8986
0.8	1.0649
0.9	1.2411
1.0	1.4273

x	y
1.1	1.6241
1.2	1.8319
1.3	2.0513
1.4	2.2832
1.5	2.5285
1.6	2.7884
1.7	3.0643
1.8	3.3579
1.9	3.6709
2.0	4.0057

25. $y(3) \approx 0.8981$

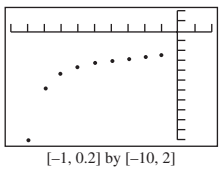
27.

(a)



[−0.2, 4.5] by [−2.5, 0.5]

(b) Note that we choose a small interval of x-values because the y-values decrease very rapidly and our calculator cannot handle the calculations for $x \le -1$. (This occurs because the analytic solution is $y = -2 + \ln(2 - e^{-x})$, which has an asymptote at $x = -\ln 2 \approx -0.69$. Obviously, the Euler approximations are misleading for $x \le -0.7$.)



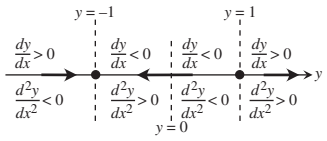
[−1, 0.2] by [−10, 2]

29. $y(\text{exact}) = \dfrac{1}{2}x^2 - \dfrac{3}{2}$; $y(2) \approx 0.4$; exact value is $\dfrac{1}{2}$.

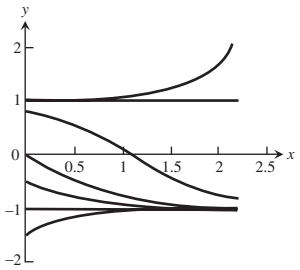
31. $y(\text{exact}) = -e^{(x^2-1)/2}$; $y(2) \approx -3.4192$; exact value is $-e^{3/2} \approx -4.4817$.

33. **(a)** $y = -1$ is stable and $y = 1$ is unstable.

(b) $\dfrac{d^2y}{dx^2} = 2y\dfrac{dy}{dx} = 2y(y^2 - 1)$



(c)



Additional and Advanced Exercises, pp. 548–549

1. **(a)** $y = c + (y_0 - c)e^{-k(A/V)t}$

(b) Steady-state solution: $y_\infty = c$

5. $x^2(x^2 + 2y^2) = C$

7. $\ln|x| + e^{-y/x} = C$

9. $\ln|x| - \ln|\sec(y/x - 1) + \tan(y/x - 1)| = C$

CHAPTER 10

Section 10.1, pp. 559–562

1. $a_1 = 0, a_2 = -1/4, a_3 = -2/9, a_4 = -3/16$

3. $a_1 = 1, a_2 = -1/3, a_3 = 1/5, a_4 = -1/7$

5. $a_1 = 1/2, a_2 = 1/2, a_3 = 1/2, a_4 = 1/2$

7. $1, \dfrac{3}{2}, \dfrac{7}{4}, \dfrac{15}{8}, \dfrac{31}{16}, \dfrac{63}{32}, \dfrac{127}{64}, \dfrac{255}{128}, \dfrac{511}{256}, \dfrac{1023}{512}$

9. $2, 1, -\dfrac{1}{2}, -\dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, -\dfrac{1}{32}, -\dfrac{1}{64}, \dfrac{1}{128}, \dfrac{1}{256}$

11. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 **13.** $a_n = (-1)^{n+1}, n \ge 1$

15. $a_n = (-1)^{n+1}(n)^2, n \ge 1$ **17.** $a_n = \dfrac{2^{n-1}}{3(n+2)}, n \ge 1$

19. $a_n = n^2 - 1, n \ge 1$ **21.** $a_n = 4n - 3, n \ge 1$

23. $a_n = \dfrac{3n+2}{n!}, n \ge 1$ **25.** $a_n = \dfrac{1 + (-1)^{n+1}}{2}, n \ge 1$

27. Converges, 2 **29.** Converges, −1 **31.** Converges, −5

33. Diverges **35.** Diverges **37.** Converges, 1/2

39. Converges, 0 **41.** Converges, $\sqrt{2}$ **43.** Converges, 1

45. Converges, 0 **47.** Converges, 0 **49.** Converges, 0

51. Converges, 1 **53.** Converges, e^7 **55.** Converges, 1

57. Converges, 1 **59.** Diverges **61.** Converges, 4

63. Converges, 0 **65.** Diverges **67.** Converges, e^{-1}

69. Converges, $e^{2/3}$ **71.** Converges, x $(x > 0)$

73. Converges, 0 **75.** Converges, 1 **77.** Converges, 1/2

79. Converges, 1 **81.** Converges, $\pi/2$ **83.** Converges, 0

85. Converges, 0 **87.** Converges, 1/2 **89.** Converges, 0

91. 8 **93.** 4 **95.** 5 **97.** $1 + \sqrt{2}$ **99.** $x_n = 2^{n-2}$