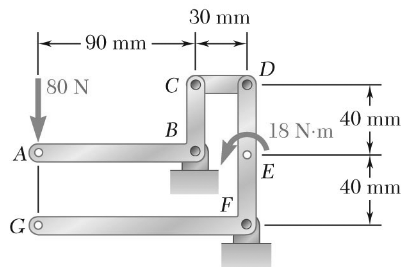
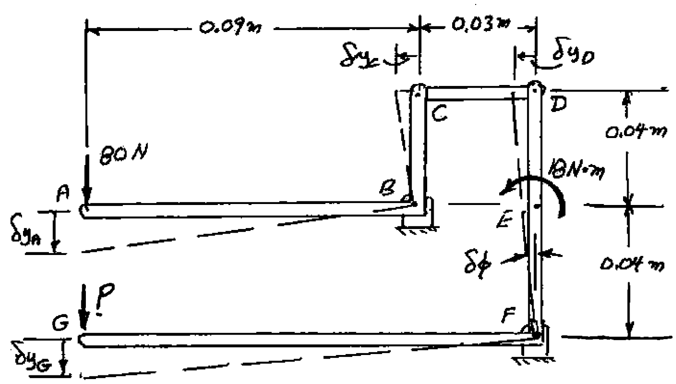
CHAPTER 10



PROBLEM 10.1

Determine the vertical force **P** that must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{0.04}{0.09}\delta y_A = \frac{4}{9} y_A \leftarrow, \qquad \delta y_D = \delta y_C = \frac{4}{9}\delta y_A \leftarrow$$

$$\delta y_G = \frac{0.12 \text{ m}}{0.08 \text{ m}}\delta y_D = 1.5\left(\frac{4}{9}\delta y_A\right) = \frac{2}{3}\delta y_A \downarrow$$

$$\delta\phi = \frac{\delta y_D}{0.08} = \frac{4}{9}\delta y_A / 0.08 = \frac{4}{0.72}\delta y_A = \frac{50}{9}\delta y_A \circlearrowright$$

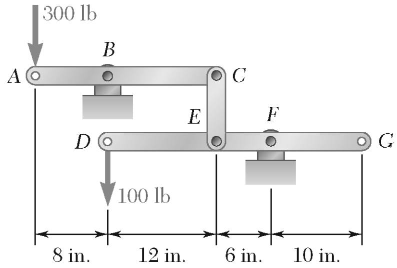
Virtual Work:

$$\delta U = 0: \quad (80 \text{ N})\delta y_A + (18 \text{ N}\cdot\text{m})\delta\phi + P\delta y_G = 0$$

$$80\delta y_A + 18\left(\frac{50}{9}\delta y_A\right) + P\left(\frac{2}{3}\delta y_A\right) = 0$$

$$80 + 100 + \frac{2}{3}P = 0$$

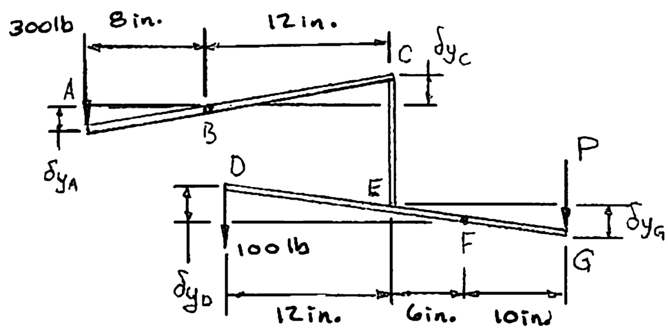
$$P = -270 \text{ N} \qquad \mathbf{P} = 270 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 10.2

Determine the vertical force **P** that must be applied at *G* to maintain the equilibrium of the linkage.

SOLUTION



Assuming $\delta y_A\downarrow$

it follows

$$\delta y_C = \frac{12}{8}\delta y_A = 1.5\delta y_A\uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A\uparrow$$

$$\delta y_D = \frac{18}{6}\delta y_A = 3(1.5\delta y_A) = 4.5\delta y_A\uparrow$$

$$\delta y_G = \frac{10}{6}\delta y_A = \frac{10}{6}(1.5\delta y_A) = 2.5\delta y_A\downarrow$$

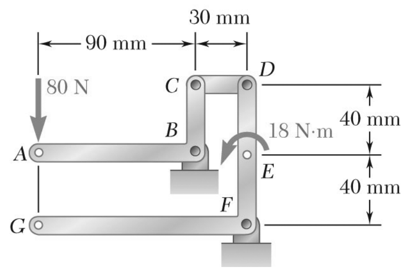
Then, by virtual work

$$\delta U = 0: \quad (300\text{ lb})\delta y_A - (100\text{ lb})\delta y_D + P\delta y_G = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + P(2.5\delta y_A) = 0$$

$$300 - 450 + 2.5P = 0$$

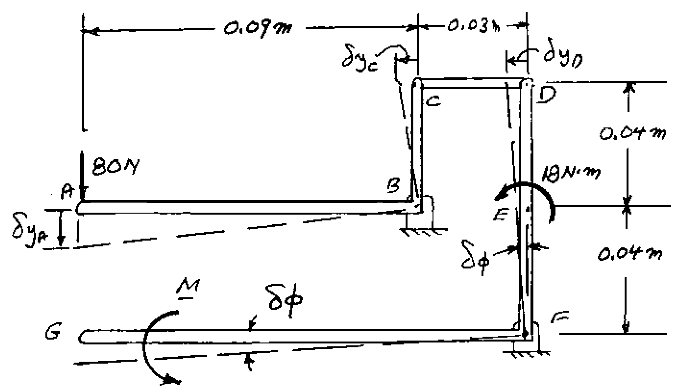
$P = 60.0$ lb $\qquad$ $\mathbf{P} = 60.0\text{ lb}\downarrow$ ◀



PROBLEM 10.3

Determine the couple **M** that must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{0.04}{0.09}\delta y_A = \frac{4}{9} y_A \leftarrow, \qquad \delta y_D = \delta y_C = \frac{4}{9}\delta y_A \leftarrow$$

$$\delta\phi = \frac{\delta y_C}{0.08} = \frac{4}{9}\delta y_A / 0.08 = \frac{4}{0.72}\delta y_A = \frac{50}{9}\delta y_A \circlearrowright$$

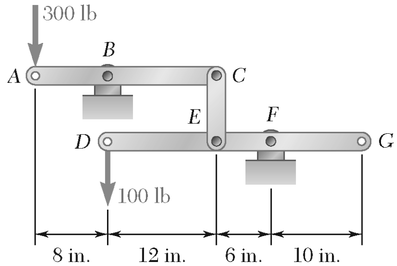
Virtual Work:

$$\delta U = 0: \qquad (80\text{ N})\delta y_A + (18\text{ N}\cdot\text{m})\delta\phi + M\,\delta\phi = 0$$

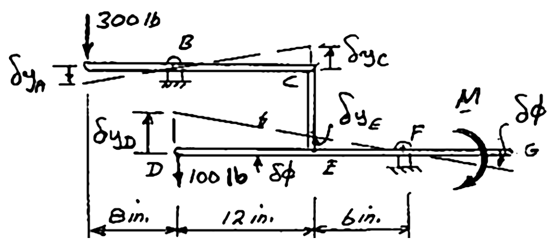
$$80\delta y_A + 18\left(\frac{50}{9}\delta y_A\right) + M\left(\frac{50}{9}\delta y_A\right) = 0$$

$$80 + 100 + \frac{50}{9}M = 0$$

$$M = -32.4\text{ N}\cdot\text{m} \qquad \mathbf{M} = 32.4\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

	**PROBLEM 10.4** Determine the couple **M** that must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{12}{8}\delta y_A = 1.5\delta y_A \uparrow, \qquad \delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{18}{6}\delta y_E = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta\phi = \frac{\delta y_E}{6} = \frac{1.5\delta y_A}{6} = \frac{1}{4}\delta y_A \circlearrowright$$

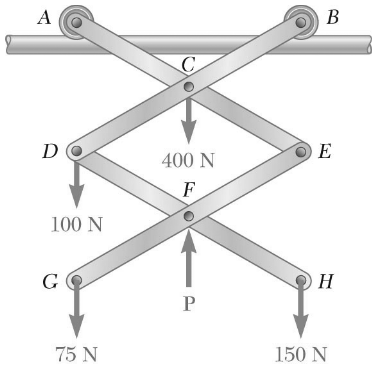
Virtual Work:

$$\delta U = 0: \quad (300 \text{ lb})\delta y_A - (100 \text{ lb})\delta y_D + M\,\delta\phi = 0$$

$$300\delta y_A - 100(1.5\delta y_A) + M\left(\frac{1}{4}\delta y_A\right) = 0$$

$$300 - 450 + \frac{1}{4}M = 0$$

$$M = +600 \text{ lb}\cdot\text{in.} \qquad \mathbf{M} = 600 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$



PROBLEM 10.5

Determine the force **P** required to maintain the equilibrium of the linkage shown. All members are of the same length and the wheels at *A* and *B* roll freely on the horizontal rod.

SOLUTION

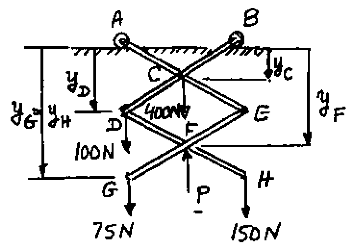
Using y_C as independent variable:

$$y_D = 2y_C \qquad \delta y_D = 2\delta y_C$$

$$y_F = 3y_C \qquad \delta y_F = 3\delta y_C$$

$$y_G = y_H = 4y_C$$

$$\delta y_G = \delta y_H = 4\delta y_C$$



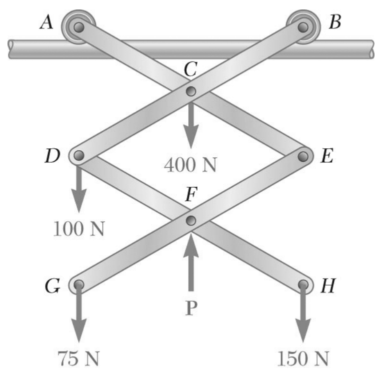
Virtual Work:

$$\delta U = (400\text{ N})\delta y_C + (100\text{ N})\delta y_D - P\delta y_F + (75\text{ N})\delta y_G + (150\text{ N})\delta y_H = 0$$

$$400\delta y_C + 100(2\delta y_C) - P(3\delta y_C) + (75 + 150)(4\delta y_C) = 0$$

$$3P = 400 + 200 + 900 \qquad P = +500\text{ N}$$

$\mathbf{P} = 500\text{ N}\uparrow$ ◀



PROBLEM 10.6

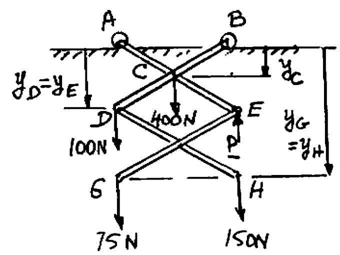
Solve Problem 10.5 assuming that the vertical force **P** is applied at Point *E*.

PROBLEM 10.5 Determine the force **P** required to maintain the equilibrium of the linkage shown. All members are of the same length and the wheels at *A* and *B* roll freely on the horizontal rod.

SOLUTION

Using y_C as independent variable:

$$y_D = y_E = 2y_C$$
$$\delta y_D = \delta y_E = 2\delta y_C$$
$$y_G = y_H = 4y_C$$
$$\delta y_G = \delta y_A = 4\delta y_C$$

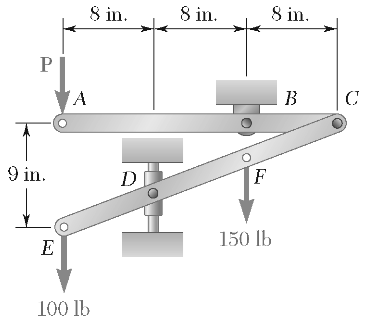


Virtual Work:

$$\delta U = (400 \text{ N})\delta y_C + (100 \text{ N})\delta y_D - P\delta y_E + (75 \text{ N})\delta y_G + (150 \text{ N})\delta y_H = 0$$
$$400\delta y_C + 100(2\delta y_C) - P(2\delta y_C) + (75+150)(4\delta y_C) = 0$$

$$2P = 400 + 200 + 900 \qquad P = +750 \text{ N}$$

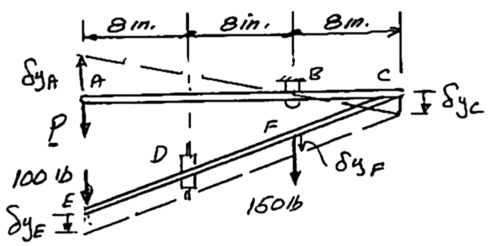
$$\mathbf{P} = 750 \text{ N}\uparrow \blacktriangleleft$$



PROBLEM 10.7

The two-bar linkage shown is supported by a pin and bracket at *B* and a collar at *D* that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \uparrow$:

$$\delta y_C = \frac{8\text{ in.}}{16\text{ in.}}\delta y_A; \quad \delta y_C = \frac{1}{2}\delta y_A \downarrow$$

Since bar *CD* move in translation

$$\delta y_E = \delta y_F = \delta y_C$$

or

$$\delta y_E = \delta y_F = \frac{1}{2} y_A \downarrow$$

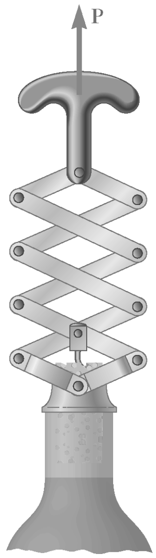
<u>Virtual Work</u>:

$$\delta U = 0: \quad -P\delta y_A + (100\text{ lb})\delta y_E + (150\text{ lb})\delta y_F = 0$$

$$-P\delta y_A + 100\left(\frac{1}{2}\delta y_A\right) + 150\left(\frac{1}{2}\delta y_A\right) = 0$$

$$P = 125\text{ lb}$$

$$\mathbf{P} = 125.0\text{ lb} \downarrow \blacktriangleleft$$



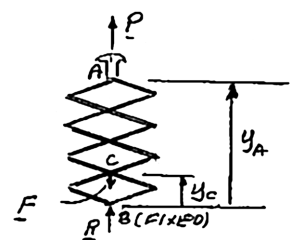
PROBLEM 10.8

Knowing that the maximum friction force exerted by the bottle on the cork is 60 lb, determine (*a*) the force **P** that must be applied to the corkscrew to open the bottle, (*b*) the maximum force exerted by the base of the corkscrew on the top of the bottle.

SOLUTION

From sketch $$y_A = 4y_C$$

Thus $$\delta y_A = 4\delta y_C$$



(*a*) Virtual Work: $$\delta U = 0: \quad P\delta y_A - F\delta y_C = 0$$

$$P(4\delta y_C) - F\delta y_C = 0$$

$$P = \frac{1}{4}F$$

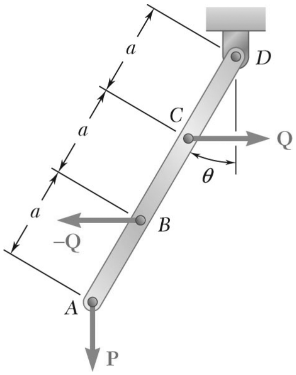
$$F = 60\text{ lb}: \quad P = \frac{1}{4}(60\text{ lb}) = 15\text{ lb}$$

$\mathbf{P} = 15.00\text{ lb}\uparrow$ ◀

(*b*) Free body: Corkscrew

$$+\uparrow \Sigma F_y = 0 \quad R + P - F = 0; \quad R + 15\text{ lb} - 60\text{ lb} = 0$$

On corkscrew: $\mathbf{R} = 45\text{ lb}\uparrow$. On bottle: $\mathbf{R} = 45.0\text{ lb}\downarrow$ ◀



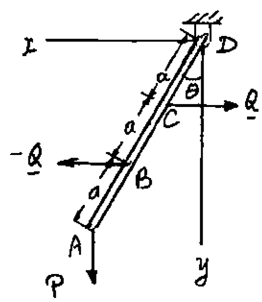
PROBLEM 10.9

Rod AD is acted upon by a vertical force **P** at end A, and by two equal and opposite horizontal forces of magnitude Q at points B and C. Derive an expression for the magnitude Q of the horizontal forces required for equilibrium.

SOLUTION

We have

$$x_C = a\sin\theta$$
$$\delta x_C = a\cos\theta\delta\theta$$
$$x_B = 2a\sin\theta$$
$$\delta x_B = 2a\cos\theta\delta\theta$$
$$y_A = 3a\cos\theta$$
$$\delta y_A = -3a\sin\theta\delta\theta$$

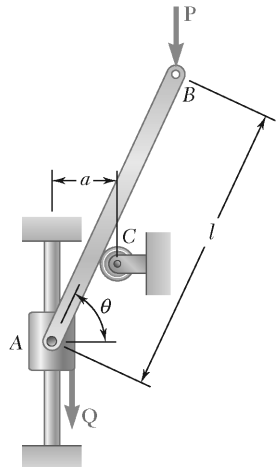


Virtual Work: We note that **P** tends to increase y_A and **−Q** tends to increase x_B, while **Q** tends to decrease x_C. Therefore

$$\delta U = P\delta y_A + Q\delta x_B - Q\delta x_C = 0$$
$$= P(-3a\sin\theta\delta\theta) + Q(2a\cos\theta\delta\theta) - Q(a\cos\theta\delta\theta) = 0$$

$$Q\cos\theta = 3P\sin\theta'$$

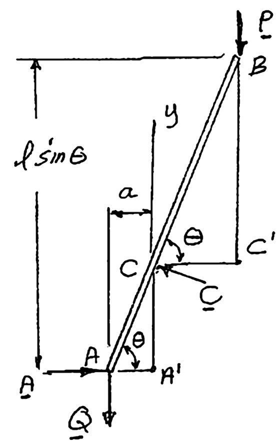
$$Q = 3P\tan\theta \blacktriangleleft$$



PROBLEM 10.10

The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

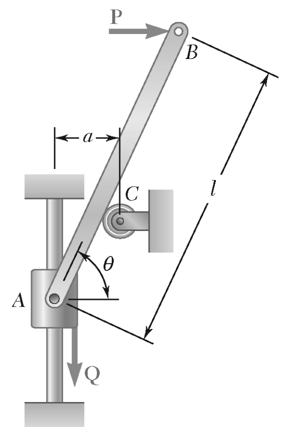
$$\delta y_B = l\cos\theta\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Virtual Work: $\delta U = 0$: $Q\delta y_A - P\delta y_B = 0$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

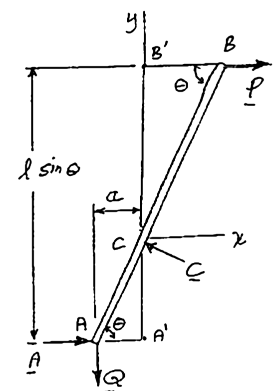
$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right) \blacktriangleleft$$



PROBLEM 10.11

The slender rod AB is attached to a collar A and rests on a small wheel at C. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\delta\theta$$

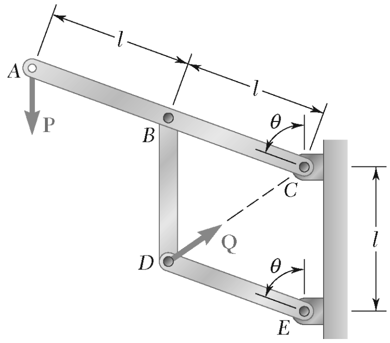
<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

or $$Pl\sin\theta\cos^2\theta = Qa$$

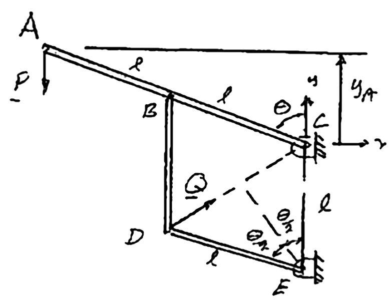
$$Q = P\frac{l}{a}\sin\theta\cos^2\theta \blacktriangleleft$$



PROBLEM 10.12

Knowing that the line of action of the force **Q** passes through Point *C*, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION



We have

$$y_A = 2l\cos\theta; \quad \delta y_A = -2l\sin\theta\delta\theta$$

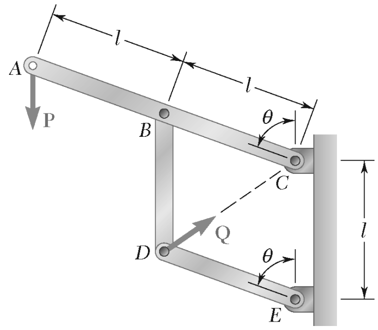
$$CD = 2l\sin\frac{\theta}{2}; \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$Q = 2P\frac{\sin\theta}{\cos(\theta/2)} \blacktriangleleft$$

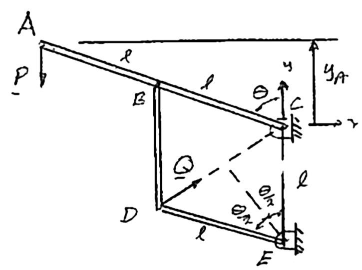


PROBLEM 10.13

Solve Problem 10.12 assuming that the force **P** applied at Point A acts horizontally to the left.

PROBLEM 10.12 Knowing that the line of action of the force **Q** passes through Point C, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION



We have

$$x_A = 2l\cos\theta; \quad \delta x_A = 2l\cos\theta\delta\theta$$

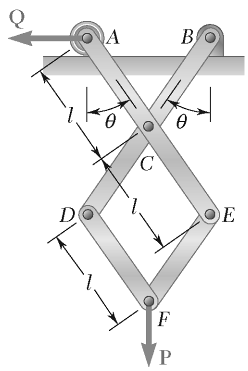
$$CD = 2l\sin\frac{\theta}{2}; \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta x_A - Q\delta(CD) = 0$$

$$P(2l\cos\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

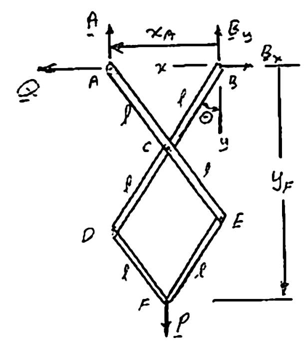
$$Q = 2P\frac{\cos\theta}{\cos(\theta/2)} \blacktriangleleft$$



PROBLEM 10.14

The mechanism shown is acted upon by the force **P**; derive an expression for the magnitude of the force **Q** required to maintain equilibrium.

SOLUTION



Virtual Work:

We have $$x_A = 2l\sin\theta$$
$$\delta x_A = 2l\cos\theta\delta\theta$$

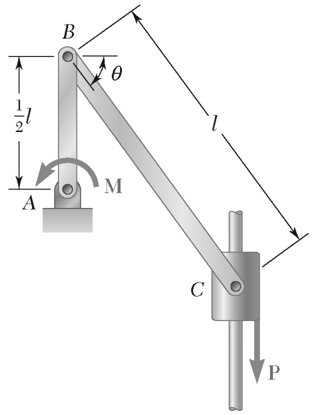
and $$y_F = 3l\cos\theta$$
$$\delta y_F = -3l\sin\theta\delta\theta$$

Virtual Work: $$\delta U = 0: \quad Q\delta x_A + P\delta y_F = 0$$

$$Q(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

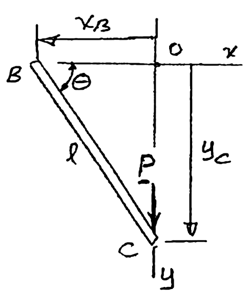
$$Q = \frac{3}{2}P\tan\theta \blacktriangleleft$$

PROBLEM 10.15



Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



We have $\quad x_B = l\cos\theta$

$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$

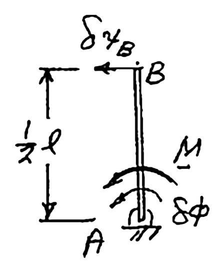
$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta$$

Now $\quad \delta x_B = \frac{1}{2}l\delta\theta$

Substituting from Equation (1)

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

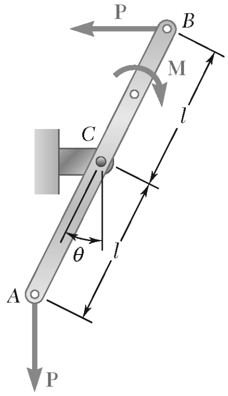


or $\quad \delta\phi = -2\sin\theta\delta\theta$

Virtual Work: $\quad \delta U = 0: \quad M\,\delta\varphi + P\delta y_C = 0$

$$M(-2\sin\theta\delta\theta) + P(l\cos\theta\delta\theta) = 0$$

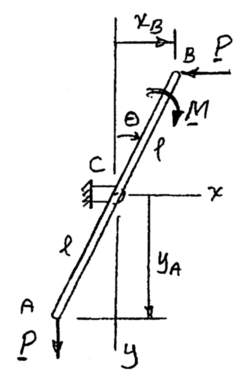
or $\quad M = \frac{1}{2}Pl\frac{\cos\theta}{\sin\theta} \qquad M = \frac{Pl}{2\tan\theta}$ ◀



PROBLEM 10.16

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



We have

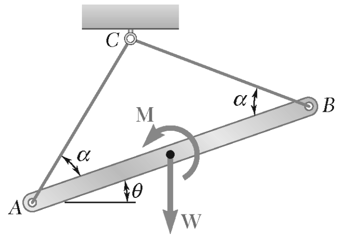
$$x_B = l\sin\theta$$
$$\delta x_B = l\cos\theta\delta\theta$$
$$y_A = l\cos\theta$$
$$\delta y_A = -l\sin\theta\delta\theta$$

Virtual Work:

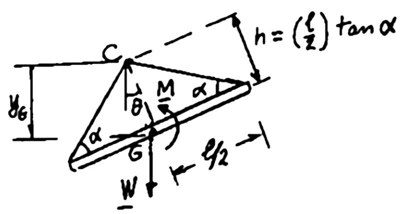
$$\delta U = 0: \quad M\delta\theta - P\delta x_B + P\delta y_A = 0$$

$$M\delta\theta - P(l\cos\theta\delta\theta) + P(-l\sin\theta\delta\theta) = 0$$

$$M = Pl(\sin\theta + \cos\theta) \blacktriangleleft$$

	**PROBLEM 10.17** A uniform rod *AB* of length *l* and weight *W* is suspended from two cords *AC* and *BC* of equal length. Derive an expression for the magnitude of the couple **M** required to maintain equilibrium of the rod in the position shown.

SOLUTION



$$y_G = h\cos\theta = \frac{1}{2}l\tan\alpha\cos\theta$$

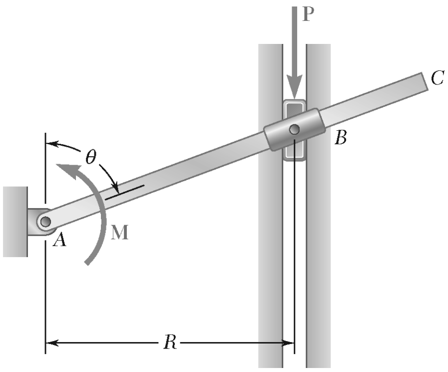
$$\delta y_G = -\frac{1}{2}l\tan\alpha\sin\theta\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = W\delta y_G + M\delta\theta = 0$$

$$W\left(-\frac{1}{2}l\tan\alpha\sin\theta\delta\theta\right) + M\delta\theta = 0$$

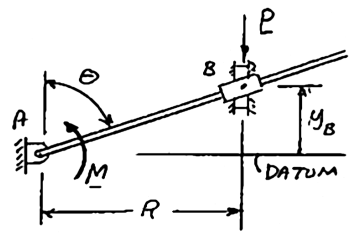
$$M = \frac{1}{2}Wl\tan\alpha\sin\theta \quad \blacktriangleleft$$



PROBLEM 10.18

Collar B can slide along rod AC and is attached by a pin to a block that can slide in the vertical slot shown. Derive an expression for the magnitude of the couple **M** required to maintain equilibrium.

SOLUTION



$$y_B = \frac{R}{\tan(90° - \theta)}$$

$$\delta y_B = \frac{-R\delta\theta}{\cos^2(90° - \theta)}$$

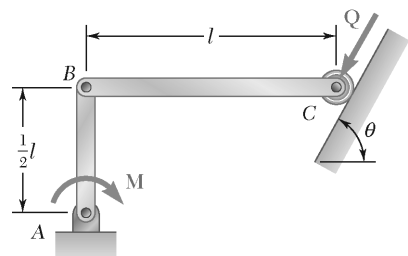
$$\delta y_B = \frac{-R\delta\theta}{\sin^2\theta}$$

Virtual Work: $\delta U = 0$: $\delta U = -M\,\delta\theta - P\delta y_B = 0$

$$-M\,\delta\theta + PR\frac{1}{\sin^2\theta}\delta\theta = 0$$

$$M = \frac{PR}{\sin^2\theta}$$

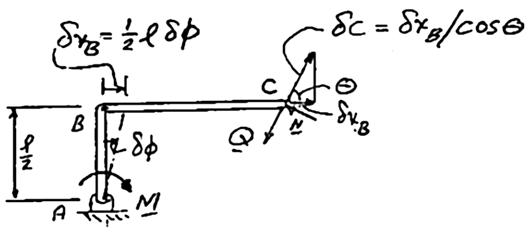
$M = PR\csc^2\theta$ ◀



PROBLEM 10.19

For the linkage shown, determine the couple **M** required for equilibrium when $l = 1.8$ ft, $Q = 40$ lb, and $\theta = 65°$.

SOLUTION



$$\delta C = \frac{\frac{1}{2} l \delta\phi}{\cos\theta}$$

Virtual Work:

$$\delta U = 0: \quad M\,\delta\phi - Q\delta C = 0$$

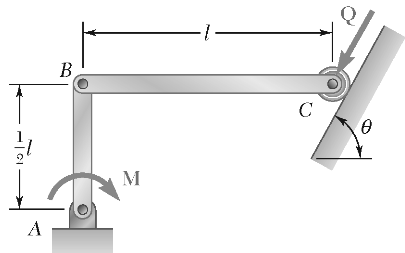
$$M\,\delta\phi - Q\left(\frac{1}{2}\frac{l}{\cos\theta}\right)\delta\phi = 0$$

$$M = \frac{1}{2}\frac{Ql}{\cos\theta}$$

Data:

$$M = \frac{1}{2}\frac{(40\text{ lb})(1.8\text{ ft})}{\cos 65°} = 85.18\text{ lb}\cdot\text{ft}$$

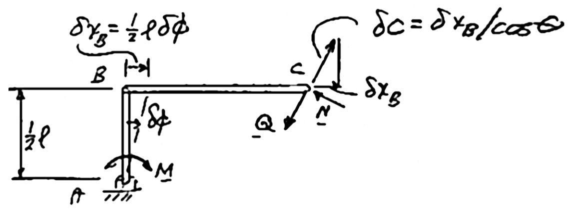
$\mathbf{M} = 85.2$ lb · ft ↻ ◀



PROBLEM 10.20

For the linkage shown, determine the force **Q** required for equilibrium when $l = 18$ in., $M = 600$ lb · in., and $\theta = 70°$.

SOLUTION



$$\delta C = \frac{1}{2}\frac{l\delta\phi}{\cos\theta}$$

Virtual Work:

$$\delta U = 0: \qquad M\,\delta\phi - Q\delta C = 0$$

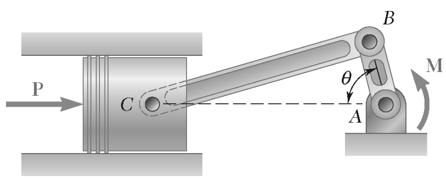
$$M\,\delta\phi - Q\left(\frac{1}{2}\frac{l}{\cos\theta}\right)\delta\phi = 0$$

$$Q = \frac{2M\cos\theta}{l}$$

Data:

$$Q = \frac{2(600 \text{ lb}\cdot\text{in.})\cos 70°}{18 \text{ in.}} = 22.801 \text{ lb}$$

Q = 22.8 lb ↗ 70.0° ◀

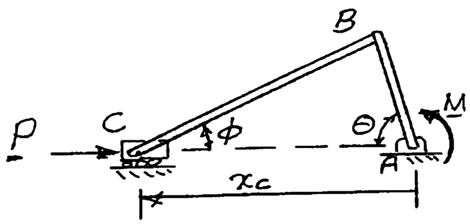


PROBLEM 10.21

A 4-kN force **P** is applied as shown to the piston of the engine system. Knowing that $AB = 50$ mm and $BC = 200$ mm, determine the couple **M** required to maintain the equilibrium of the system when (*a*) $\theta = 30°$, (*b*) $\theta = 150°$.

SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \qquad (1)$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \qquad (2)$$

Now, from Equation (1)

$$\cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$$

or

$$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \qquad (3)$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or

$$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

Then

$$\delta x_C = -\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$$

<u>Virtual Work</u>: $\delta U = 0$: $-P\delta x_C - M\,\delta\theta = 0$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\,\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \qquad (4)$$

(*a*) $P = 4$ kN, $\theta = 30°$

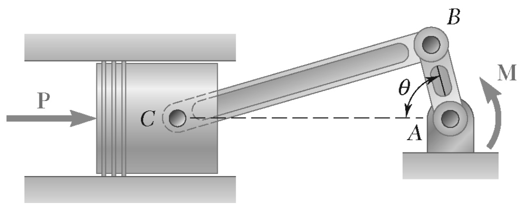
Eq. (1): $\sin\phi = \dfrac{50 \text{ mm}}{200 \text{ mm}}\sin 30° \qquad \phi = 7.181°$

Eq. (4): $M = (0.05 \text{ m})\dfrac{\sin(30°+7.181°)}{\cos 7.818°}(4 \text{ kN})$ $\qquad \mathbf{M} = 121.8 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) $P = 4$ kN, $\theta = 150°$

Eq. (1): $\sin\phi = \dfrac{50 \text{ mm}}{200 \text{ mm}}\sin 160° \qquad \phi = 7.181°$

Eq. (4): $M = (0.05 \text{ m})\dfrac{\sin(150°+7.181°)}{\cos 7.181°}(4 \text{ kN})$ $\qquad \mathbf{M} = 78.2 \text{ N}\cdot\text{m}$ ↻ ◀

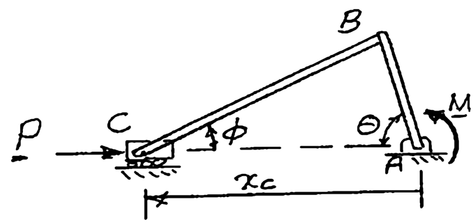


PROBLEM 10.22

A couple **M** of magnitude 100 N · m is applied as shown to the crank of the engine system. Knowing that $AB = 50$ mm and $BC = 200$ mm, determine the force **P** required to maintain the equilibrium of the system when (*a*) $\theta = 60°$, (*b*) $\theta = 120°$.

SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \qquad (1)$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \qquad (2)$$

Now, from Equation (1) $\quad \cos\phi\delta\phi = \dfrac{AB}{BC}\cos\theta\delta\theta$

or

$$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \qquad (3)$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or

$$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

Then $\quad \delta x_C = -\dfrac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$

Virtual Work: $\delta U = 0: \quad -P\delta x_C - M\,\delta\theta = 0$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\,\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \qquad (4)$$

(*a*) $M = 100\ \text{N}\cdot\text{m}, \quad \theta = 60°$

Eq. (1): $\sin\phi = \dfrac{50\ \text{mm}}{200\ \text{mm}}\sin 60° \qquad \phi = 12.504°$

Eq. (4): $100\ \text{N}\cdot\text{m} = (0.05\ \text{m})\dfrac{\sin\,(60° + 12.504°)}{\cos 12.504°}P$

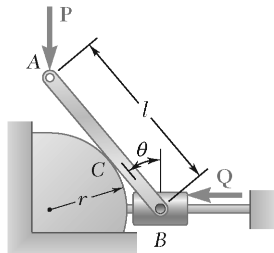
$P = 2047\ \text{N}$ $\qquad$ $\mathbf{P} = 2.05\ \text{kN} \longrightarrow$ ◀

(*b*) $M = 100\ \text{N}\cdot\text{m}, \quad \theta = 120°$

Eq. (1): $\sin\phi = \dfrac{50\ \text{mm}}{200\ \text{mm}}\sin 120° \quad \phi = 12.504°$

Eq. (4): $100\ \text{N}\cdot\text{m} = (0.05\ \text{m})\dfrac{\sin\,(120° + 12.504°)}{\cos 12.504°}P$

$P = 2649\ \text{N}$ $\qquad$ $\mathbf{P} = 2.65\ \text{kN} \longrightarrow$ ◀

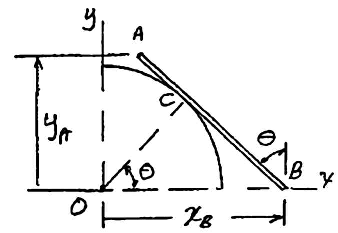


PROBLEM 10.23

A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N.

SOLUTION

Geometry



$$OC = r$$

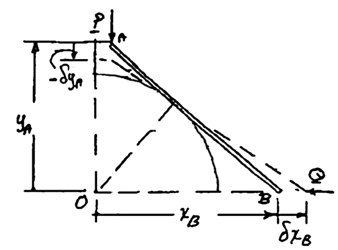
$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta$$

$$\delta y_A = -l\sin\theta\delta\theta$$



Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

$$Pl\sin\theta\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

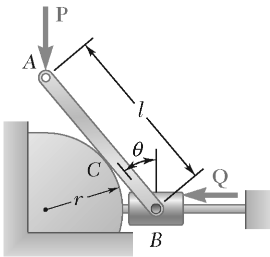
$$\cos^2\theta = \frac{Qr}{Pl} \qquad (1)$$

Then, with $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N

$$\cos^2\theta = \frac{(80\text{ N})(60\text{ mm})}{(40\text{ N})(200\text{ mm})} = 0.6$$

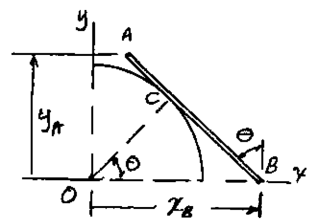
or $\theta = 39.231°$ $\qquad \theta = 39.2°$ ◀

PROBLEM 10.24



A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb.

SOLUTION



Geometry

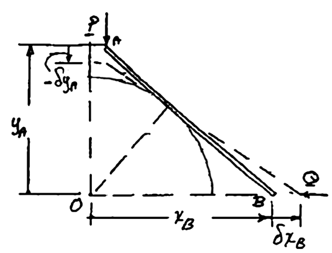
$$OC = r$$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta; \quad \delta y_A = -l\sin\theta\delta\theta$$



Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

$$Pl\sin\theta\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \tag{1}$$

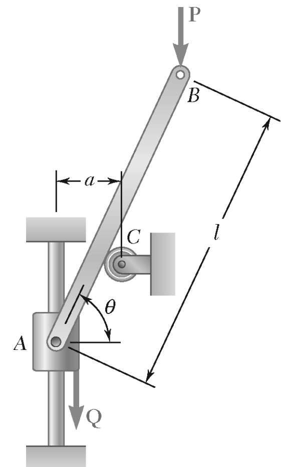
Then, with $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb

Eq. (1) becomes:

$$\cos^2\theta = \frac{(150\text{ lb})(5\text{ in.})}{(75\text{ lb})(14\text{ in.})} = 0.71429$$

$\theta = 32.3°$ ◀

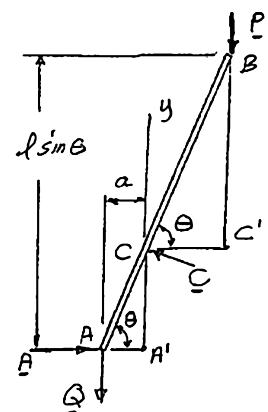
PROBLEM 10.25



Determine the value of θ corresponding to the equilibrium position of the rod of Problem 10.10 when $l = 30$ in., $a = 5$ in., $P = 25$ lb, and $Q = 40$ lb.

PROBLEM 10.10 The slender rod AB is attached to a collar A and rests on a small wheel at C. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad -Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

or

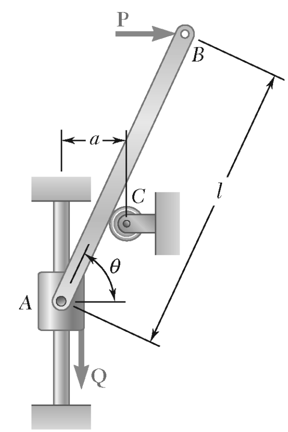
$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right)$$

PROBLEM 10.25 (Continued)

with $l = 30 \text{ in.}, a = 5 \text{ in.}, P = 25 \text{ lb}, \text{ and } Q = 40 \text{ lb}$

$$(40 \text{ lb}) = (25 \text{ lb})\left(\frac{30 \text{ in.}}{5 \text{ in.}}\cos^3\theta - 1\right)$$

or $\cos^3\theta = 0.4333$ $\theta = 40.8°$ ◀

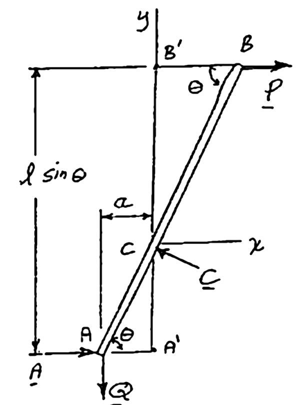


PROBLEM 10.26

Determine the values of θ corresponding to the equilibrium position of the rod of Problem 10.11 when $l = 600$ mm, $a = 100$ mm, $P = 50$ N, and $Q = 90$ N.

PROBLEM 10.11 The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

$$Pl\sin\theta\cos^2\theta = Qa$$

or

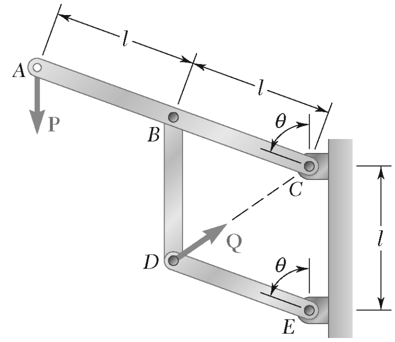
$$Q = P\frac{l}{a}\sin\theta\cos^2\theta$$

with $$l = 600 \text{ mm}, a = 100 \text{ mm}, P = 50 \text{ N, and } Q = 90 \text{ N}$$

$$90 \text{ N} = (50 \text{ N})\frac{600 \text{ mm}}{100 \text{ mm}}\sin\theta\cos^2\theta$$

or $$\sin\theta\cos^2\theta = 0.300$$

Solving numerically $$\theta = 19.81° \text{ and } 51.9° \blacktriangleleft$$



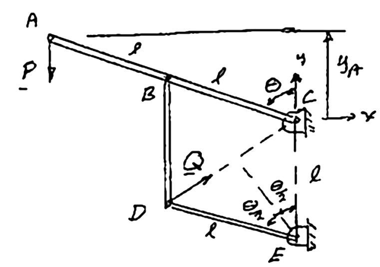
PROBLEM 10.27

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.12 when $P = 80$ N and $Q = 100$ N.

PROBLEM 10.12 Knowing that the line of action of the force **Q** passes through Point C, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION

From geometry



$$y_A = 2l\cos\theta, \quad \delta y_A = -2l\sin\theta\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}, \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or

$$Q = 2P\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

with

$$P = 80 \text{ N}, \quad Q = 100 \text{ N}$$

$$(100 \text{ N}) = 2(80 \text{ N})\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

$$\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

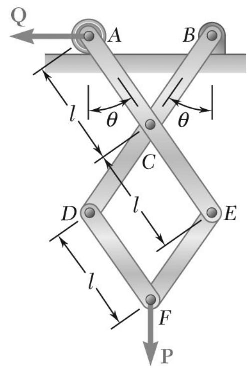
or

$$\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

$$\theta = 36.42°$$

$\theta = 36.4°$ ◀

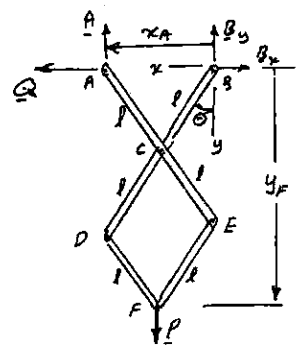
(Additional solutions discarded as not applicable are $\theta = \pm 180°$)



PROBLEM 10.28

Determine the value of θ corresponding to the equilibrium position of the mechanism of Prob. 10.14 when $P = 270$ N and $Q = 960$ N.

SOLUTION



<u>Virtual Work</u>:

$$x_A = 2l\sin\theta, \quad \delta x_A = 2l\cos\theta\delta\theta$$
$$y_F = 3l\cos\theta, \quad \delta y_F = -3l\sin\theta\delta\theta$$

$$\delta U = 0: \qquad Q\delta x_A + P\delta y_F = 0$$
$$\delta U = 0: \qquad Q\delta\theta x_A + P\delta y_F = 0$$

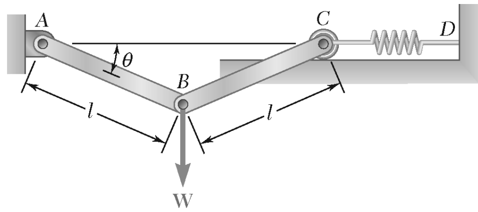
$$Q(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

$$Q = \frac{3}{2}P\tan\theta$$

<u>Data</u>: $P = 270 \text{ N}, \quad Q = 960 \text{ N}$

$$(960 \text{ N}) = \frac{3}{2}(270 \text{ N})\tan\theta$$

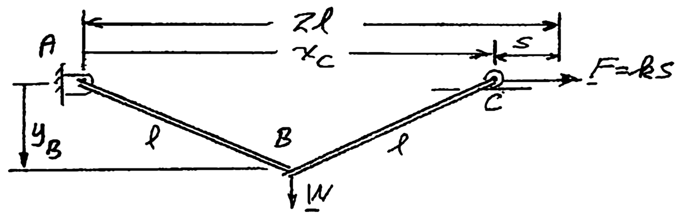
$\theta = 67.1°$ ◀



PROBLEM 10.29

A load **W** of magnitude 600 N is applied to the linkage at B. The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \quad \delta x_C = -2l\sin\theta\delta\theta$$
$$y_B = l\sin\theta \quad \delta y_B = l\cos\theta\delta\theta$$
$$F = ks = k(2l - x_C) = 2kl(1-\cos\theta)$$

Virtual Work: $\delta U = 0$: $F\delta x_C + W\delta y_B = 0$

$$2kl(1-\cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1-\cos\theta)\sin\theta = Wl\cos\theta$$

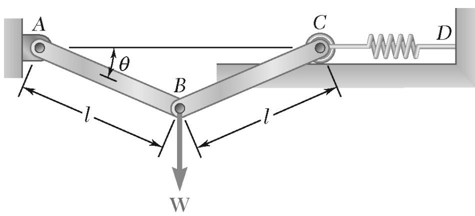
or $$(1-\cos\theta)\tan\theta = \frac{W}{4kl}$$

Given: $l = 0.3$ m, $W = 600$ N, $k = 2500$ N/m

Then $$(1-\cos\theta)\tan\theta = \frac{600\text{ N}}{4(2500\text{ N/m})(0.3\text{ m})}$$

or $$(1-\cos\theta)\tan\theta = 0.2$$

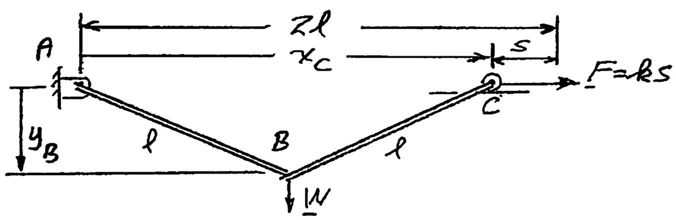
Solving numerically $\theta = 40.22°$ $\theta = 40.2°$ ◀



PROBLEM 10.30

A vertical load **W** is applied to the linkage at B. The constant of the spring is k, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage, derive an equation in θ, W, l, and k that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \quad \delta x_C = -2l\sin\theta\delta\theta$$

$$y_B = l\sin\theta \quad \delta y_B = l\cos\theta\delta\theta$$

$$F = ks = k(2l - x_C) = 2kl(1-\cos\theta)$$

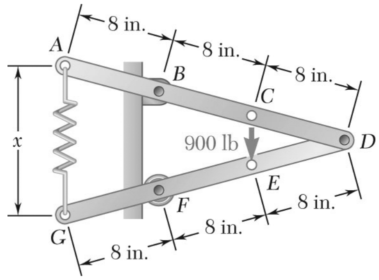
Virtual Work: $\delta U = 0$: $\quad F\delta x_C + W\delta y_B = 0$

$$2kl(1-\cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1-\cos\theta)\sin\theta = Wl\cos\theta$$

or $$(1-\cos\theta)\tan\theta = \frac{W}{4kl}$$

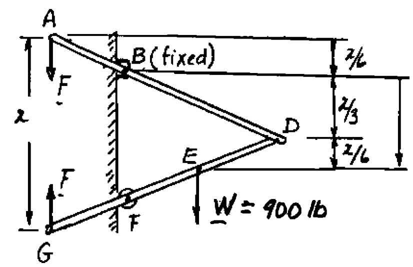
From above $$(1-\cos\theta)\tan\theta = \frac{W}{4kl} \blacktriangleleft$$



PROBLEM 10.31

Two bars *AD* and *DG* are connected by a pin at *D* and by a spring *AG*. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of x corresponding to equilibrium when a 900-lb load is applied at *E* as shown.

SOLUTION



$$y_E = \frac{x}{3} + \frac{x}{6} = \frac{x}{2} \qquad \delta y_E = \frac{1}{2}\delta x$$

$$F = ks = (125 \text{ lb/in.})(x - 12 \text{ in.})$$

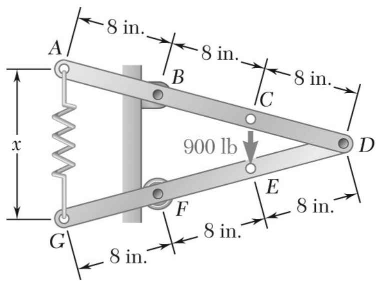
Virtual Work: $\delta U = 0$: $\quad F\delta x + W\delta y_E = 0$

$$-(125)(x-12)\delta x + (900)\left(\frac{1}{2}\delta x\right) = 0$$

$$-125x + 1500 + 450 = 0$$

$x = 15.60$ in. ◀

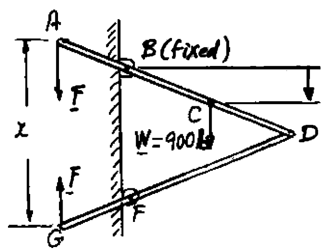
PROBLEM 10.32



Solve Problem 10.31 assuming that the 900-lb vertical force is applied at C instead of E.

PROBLEM 10.31 Two bars AD and DG are connected by a pin at D and by a spring AG. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of x corresponding to equilibrium when a 900-lb load is applied at E as shown.

SOLUTION



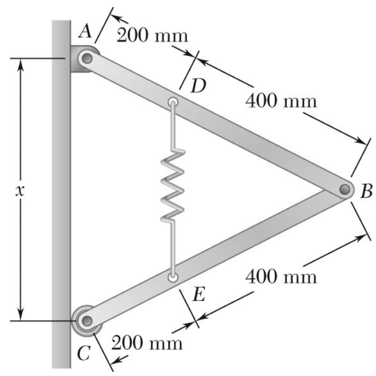
$$y_C = \frac{1}{6}x \quad \delta y_C = \frac{1}{6}\delta x$$

$$F = ks = (125 \text{ lb/in.})(x - 12 \text{ in.})$$

Virtual Work: $$\delta U = -F\delta x + W\delta y_C = 125(x-12)\delta x + 900\left(\frac{1}{6}\delta x\right) = 0$$

$$-125x + 1500 + 150 = 0$$

$x = 13.20$ in. ◀

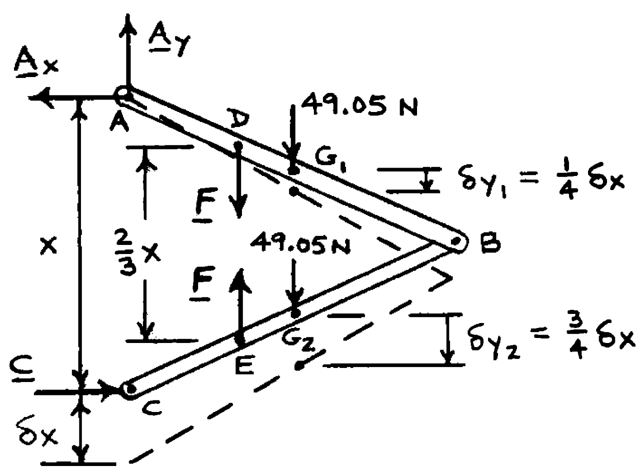


PROBLEM 10.33

Two 5-kg bars AB and BC are connected by a pin at B and by a spring DE. Knowing that the spring is 150 mm long when unstretched and that the constant of the spring is 1 kN/m, determine the value of x corresponding to equilibrium.

SOLUTION

First note: $$W_{\text{bar}} = (5\text{ kg})(9.81\text{ m/s}^2) = 49.05\text{ N}$$



During the virtual displacement, Points D and E move apart a distance $\delta(DE) = \frac{2}{3}\delta x$ and the total work done by the forces exerted at D and E is $-F\left(\frac{2}{3}\delta x\right)$

$$\delta U = 0: \quad -F\left(\frac{2}{3}\delta x\right) + 49.05\text{ N}\left(\frac{1}{4}\delta x\right) + 49.05\text{ N}\left(\frac{3}{4}\delta x\right) = 0$$

$$F = 73.575\text{ N}$$

For $F = 73.575$ N, elongation of spring is

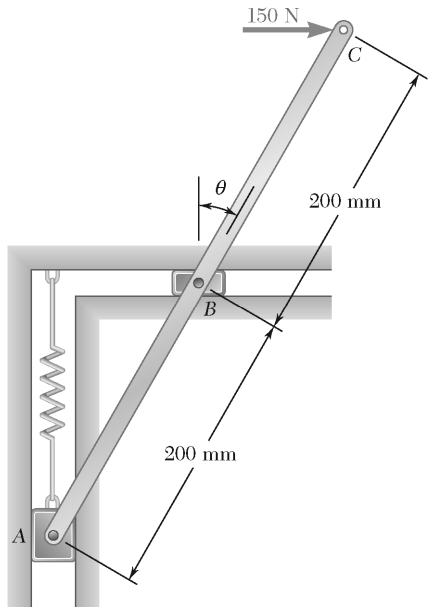
$$\frac{F}{k} = \frac{73.575\text{ N}}{1000\text{ N/m}} = 73.575\times 10^{-3}\text{ m}$$

$$= 73.575\text{ mm}$$

Since undeformed length of spring is 150 mm, total length is

$$DE = \frac{2}{3}x = 150\text{ mm} + 73.575\text{ mm}$$

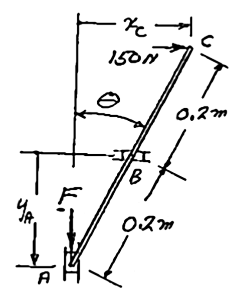
$x = 355$ mm ◀



PROBLEM 10.34

Rod ABC is attached to blocks A and B that can move freely in the guides shown. The constant of the spring attached at A is $k = 3$ kN/m, and the spring is unstretched when the rod is vertical. For the loading shown, determine the value of θ corresponding to equilibrium.

SOLUTION



$$x_C = (0.4 \text{ m})\sin\theta$$
$$\delta x_C = 0.4\cos\theta\delta\theta$$
$$y_A = (0.2 \text{ m})\cos\theta$$
$$\delta y_A = -0.2\sin\theta\delta\theta$$

Spring:

Unstretched length $= 0.2$ m

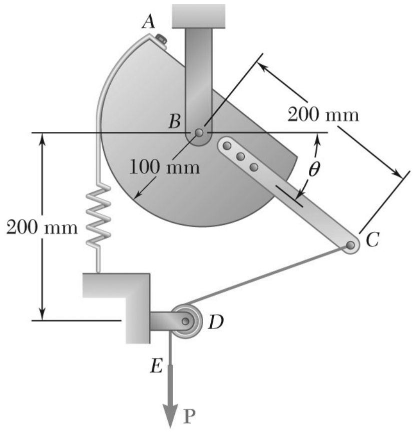
$$F = k(0.2 \text{ m} - y_A) = k(0.2 - 0.2\cos\theta)$$
$$= (300 \text{ N/m})(0.2)(1-\cos\theta)$$
$$F = 600(1-\cos\theta)$$

Virtual Work: $\delta U = 0$: $(150 \text{ N})\delta x_C + F\delta y_A = 0$

$$150(0.4\cos\theta\delta\theta) + 600(1-\cos\theta)(-0.2\sin\theta\delta\theta) = 0$$

$$\frac{150(0.4)}{600(0.2)} = \frac{1}{2}: \quad \frac{1}{2} = (1-\cos\theta)\tan\theta$$

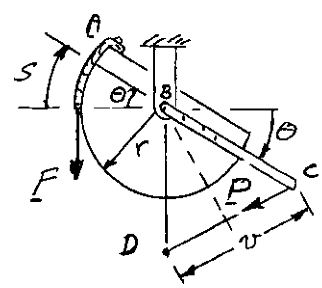
Solve by trial and error: $\theta = 57.2°$ ◀



PROBLEM 10.35

A vertical force **P** of magnitude 150 N is applied to end E of cable CDE, which passes over a small pulley D and is attached to the mechanism at C. The constant of the spring is k = 4 kN/m, and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium.

SOLUTION



$$l = BC = 0.2 \text{ m}$$
$$r = 0.1 \text{ m}$$

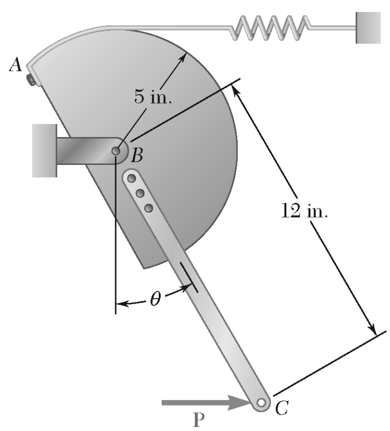
$\angle CBD = 90° - \theta$

$$v = 2l \sin\left(45° - \frac{\theta}{2}\right)$$
$$\delta v = -l\cos\left(45° - \frac{\theta}{2}\right)\delta\theta$$
$$s = r\theta \qquad \delta s = r\delta\theta$$
$$F = ks = kr\theta$$

Virtual Work: $\delta U = 0: \quad -P\delta v - F\delta s = 0$

$$-P\left[-l\cos\left(45° - \frac{\theta}{2}\right)\right]\delta\theta - kr\theta(r\delta\theta) = 0$$
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$
$$\frac{Pl}{kr^2} = \frac{(150 \text{ N})(0.2 \text{ m})}{(4000 \text{ N/m})(0.1 \text{ m})^2} = 0.75$$
$$0.75 = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$
$$\theta = 0.67623 \text{ rad} = 38.745°$$

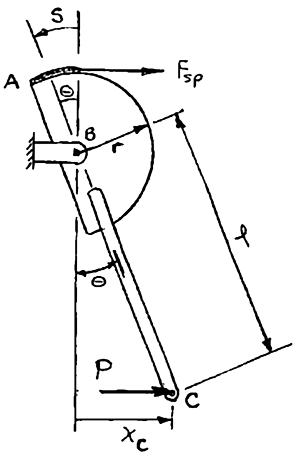
$\theta = 38.7°$ ◀



PROBLEM 10.36

A horizontal force **P** of magnitude 40 lb is applied to the mechanism at *C*. The constant of the spring is $k = 9$ lb/in., and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism, determine the value of θ corresponding to equilibrium.

SOLUTION



$$s = r\theta$$
$$\delta s = r\delta\theta$$

Spring is unstretched at $\theta = 0°$

$$F_{SP} = ks = kr\theta$$
$$x_C = l\sin\theta$$
$$\delta x_C = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_C - F_{SP}\delta s = 0$$

$$P(l\cos\theta\delta\theta) - kr\theta(r\delta\theta) = 0$$

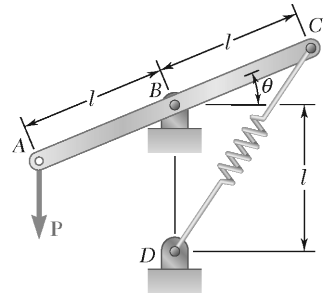
or
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\theta}$$

Thus
$$\frac{(40 \text{ lb})(12 \text{ in.})}{(9 \text{ lb/in.})(5 \text{ in.})^2} = \frac{\theta}{\cos\theta}$$

or
$$\frac{\theta}{\cos\theta} = 2.1333$$

$$\theta = 1.054 \text{ rad} = 60.39°$$

$\theta = 60.4°$ ◀

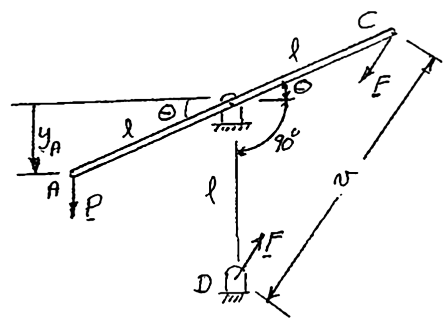


PROBLEM 10.37

Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 300 \text{ N}, \quad l = 400 \text{ mm}, \quad k = 5 \text{ kN/m}.$$

SOLUTION



$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Spring: $v = CD$

Unstretched when $\theta = 0$

so that $v_0 = \sqrt{2}l$

For θ: $v = 2l\sin\left(\frac{90^\circ + \theta}{2}\right)$

$$\delta v = l\cos\left(45^\circ + \frac{\theta}{2}\right)\delta\theta$$

Stretched length: $s = v - v_0 = 2l\sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}l$

Then $F = ks = kl\left[2\sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}\right]$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta y_A - F\delta v = 0$$

$$Pl\cos\theta\delta\theta - kl\left[2\sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}\right]l\cos\left(45^\circ + \frac{\theta}{2}\right)\delta\theta = 0$$

or

$$\frac{P}{kl} = \frac{1}{\cos\theta}\left[2\sin\left(45^\circ + \frac{\theta}{2}\right)\cos\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45^\circ + \frac{\theta}{2}\right)\right]$$

$$= \frac{1}{\cos\theta}\left[2\sin\left(45^\circ + \frac{\theta}{2}\right)\cos\left(45^\circ + \frac{\theta}{2}\right)\cos\theta - \sqrt{2}\cos\left(45^\circ + \frac{\theta}{2}\right)\right]$$

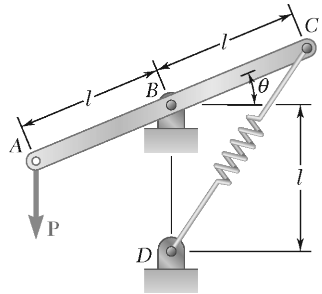
$$= 1 - \sqrt{2}\frac{\cos\left(45^\circ \frac{\theta}{2}\right)}{\cos\theta}$$

Now, with $P = 300\text{ N}, \quad l = 400\text{ mm}, \quad \text{and} \quad k = 5\text{ kN/m}$

$$\frac{(300\text{ N})}{(5000\text{ N/m})(0.4\text{ m})} = 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

or
$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.60104$$

Solving numerically $\theta = 22.6°$ ◀



PROBLEM 10.38

Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 75 \text{ lb}, \quad l = 15 \text{ in.}, \quad k = 20 \text{ lb/in.}$$

SOLUTION

From the analysis of Problem 10.37, we have

$$\frac{P}{kl} = 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

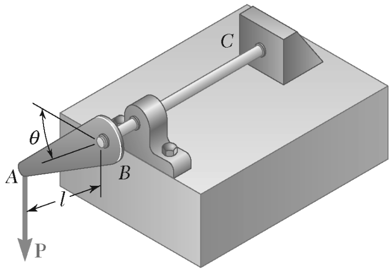
with $\quad P = 75 \text{ lb}, \quad l = 15 \text{ in.} \quad \text{and} \quad k = 20 \text{ lb/in.}$

$$\frac{(75 \text{ lb})}{(20 \text{ lb/in.})(15 \text{ in.})} = 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.53033$$

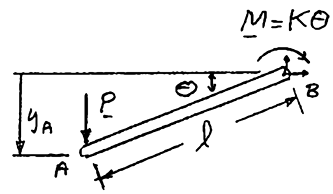
Solving numerically $\quad \theta = 51.1° \blacktriangleleft$



PROBLEM 10.39

The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C. The torsional spring constant of the shaft BC is K; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100$ N, $l = 250$ mm, and $K = 12.5$ N · m/rad.

SOLUTION



We have $$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - M\delta\theta = 0$$

$$Pl\cos\theta\delta\theta - K\theta\delta\theta = 0$$

or $$\frac{\theta}{\cos\theta} = \frac{Pl}{K} \qquad (1)$$

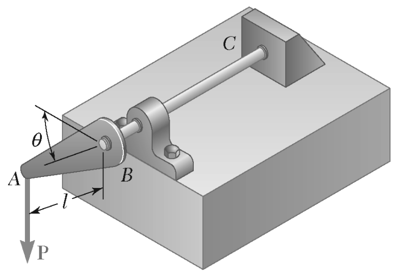
with $P = 100$ N, $l = 250$ mm and $K = 12.5$ N · m/rad

$$\frac{\theta}{\cos\theta} = \frac{(100 \text{ N})(0.250 \text{ m})}{12.5 \text{ N}\cdot\text{m/rad}}$$

or $$\frac{\theta}{\cos\theta} = 2.0000$$

Solving numerically $\theta = 59.0°$ ◀

PROBLEM 10.40



Solve Problem 10.39 assuming that $P = 350\text{ N}$, $l = 250\text{ mm}$, and $K = 12.5\text{ N}\cdot\text{m/rad}$. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, $360° < \theta < 450°$.

PROBLEM 10.39 The lever *AB* is attached to the horizontal shaft *BC* that passes through a bearing and is welded to a fixed support at *C*. The torsional spring constant of the shaft *BC* is *K*; that is, a couple of magnitude *K* is required to rotate end *B* through 1 rad. Knowing that the shaft is untwisted when *AB* is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100\text{ N}$, $l = 250\text{ mm}$, and $K = 12.5\text{ N}\cdot\text{m/rad}$.

SOLUTION

Using Equation (1) of Problem 10.39 and

$$P = 350\text{ N},\quad l = 250\text{ mm}\quad\text{and}\quad K = 12.5\text{ N}\cdot\text{m/rad}$$

We have

$$\frac{\theta}{\cos\theta} = \frac{(350\text{ N})(0.250\text{ m})}{12.5\text{ N}\cdot\text{m/rad}}$$

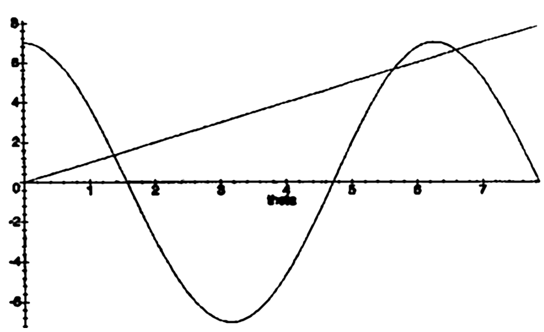
or

$$\frac{\theta}{\cos\theta} = 7 \quad\text{or}\quad \theta = 7\cos\theta \qquad (1)$$

The solutions to this equation can be shown graphically using any appropriate graphing tool, such as Maple, with the command: plot ({theta, 7 * cos(theta)}, $t = 0.5$ * Pi/2);

Thus, we plot $y = \theta$ and $y = 7\cos\theta$ in the range

$$0 \le \theta \le \frac{5\pi}{2}$$



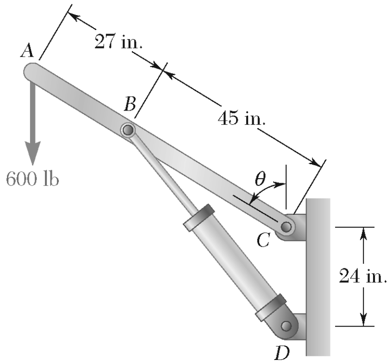
PROBLEM 10.40 (Continued)

We observe that there are three points of intersection, which implies that Equation (1) has three roots in the specified range of θ.

$$0 \le \theta \le 90°\left(\frac{\pi}{2}\right); \quad \theta = 1.37333 \text{ rad}, \quad \theta = 78.69° \qquad \theta = 78.7° \blacktriangleleft$$

$$270 \le \theta \le 360°\left(\frac{3\pi}{2} \le \theta \le 2\pi\right); \quad \theta = 5.65222 \text{ rad}, \quad \theta = 323.85° \qquad \theta = 324° \blacktriangleleft$$

$$360 \le \theta \le 450°\left(2\pi \le \theta \le \frac{5\pi}{2}\right); \quad \theta = 6.61597 \text{ rad}, \quad \theta = 379.07 \qquad \theta = 379° \blacktriangleleft$$

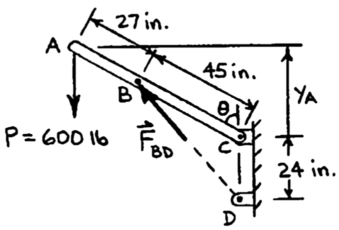


PROBLEM 10.41

The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *B* when $\theta = 65°$.

SOLUTION

We have



$$y_A = (72\text{ in.})\cos\theta \qquad \delta y_A = -72\sin\theta\delta\theta$$

$$(BD)^2 = (BC)^2 + (CD)^2 - 2(BC)(CD)\cos(180° - \theta)$$
$$= (45)^2 + (24)^2 + 2(45)(24)\cos\theta$$

$$(BD)^2 = 2601 + 2160\cos\theta \qquad (1)$$

Differentiating: $$2(BD)\delta(BD) = -2160\sin\theta\delta\theta$$

$$\delta(BD) = -\frac{1080}{BD}\sin\theta\delta\theta \qquad (2)$$

Virtual Work: Noting that **P** tends to decrease y_A and $\mathbf{F}_{BD}$ tends to increase *BD*, write

$$\delta U = -P\delta y_A + F_{BD}\delta(BD) = 0$$

$$-P(-72\sin\theta\delta\theta) + F_{BD}\left(-\frac{1080}{BD}\sin\theta\delta\theta\right) = 0$$

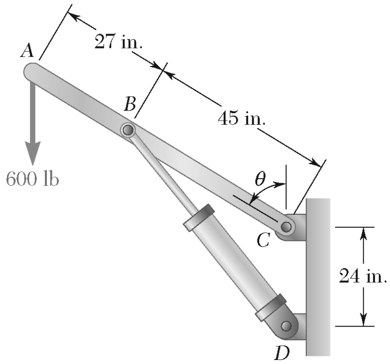
$$F_{BD} = \frac{1}{15}(BD)P$$

or, since $$P = 600\text{ lb:} \quad F_{BD} = \frac{600}{15}(BD) = (40\text{ lb})(BD) \qquad (3)$$

Making $\theta = 65°$ in Eq. (1), we have

$$(BD)^2 = 2601 + 2160\cos 65° = 3513.9$$
$$BD = 59.278$$

Carrying into Eq. (3): $$F_{BD} = (40\text{ lb})(59.278) = 2371.1\text{ lb}$$ $F_{BD} = 2370\text{ lb}$ ◀



PROBLEM 10.42

The position of boom ABC is controlled by the hydraulic cylinder BD. For the loading shown, (*a*) express the force exerted by the hydraulic cylinder on pin B as a function of the length BD, (*b*) determine the smallest possible value of the angle θ if the maximum force that the cylinder can exert on pin B is 2.5 kips.

SOLUTION

(*a*) See solution of Problem 10.41 for the derivation of Eq. (3):

$$F_{BD} = (40 \text{ lb})(BD) \blacktriangleleft$$

(*b*) For $(F_{BD})_{\max} = 2.5 \text{ kips} = 2500 \text{ lb}$, we have

$$2500 \text{ lb} = (40 \text{ lb})(BD)$$

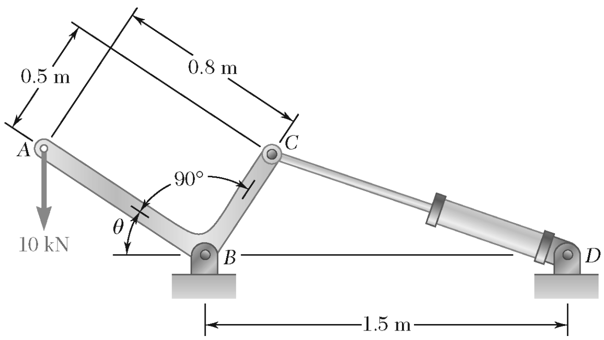
$$BD = 62.5$$

Carrying this value into Eq. (1) of Problem 10.41, write

$$(BD)^2 = 2601 + 2160\cos\theta$$

$$(62.5)^2 = 2601 + 2160\cos\theta$$

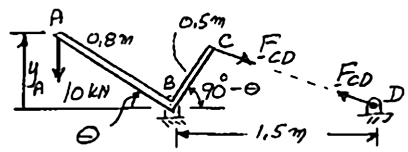
$$\cos\theta = 0.60428 \qquad \theta = 52.8° \blacktriangleleft$$



PROBLEM 10.43

The position of member ABC is controlled by the hydraulic cylinder CD. For the loading shown, determine the force exerted by the hydraulic cylinder on pin C when $\theta = 55°$.

SOLUTION



$$y_A = (0.8 \text{ m})\sin\theta$$

$$\delta y_A = 0.8\cos\theta\delta\theta$$

$$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos(90° - \theta)$$

$$CD^2 = 0.5^2 + 1.5^2 - 2(0.5)(1.5)\sin\theta$$

$$CD^2 = 2.5 - 1.5\sin\theta \qquad (1)$$

$$2(CD)(\delta_{CD}) = -1.5\cos\theta\delta\theta \quad \delta_{CD} = -\frac{3\cos\theta}{4CD}\delta\theta$$

Virtual Work: $\delta U = 0: \quad -(10\text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$

$$-10(0.8\cos\theta\delta\theta) - F_{CD}\left(-\frac{3\cos\theta}{4CD}\delta\theta\right) = 0$$

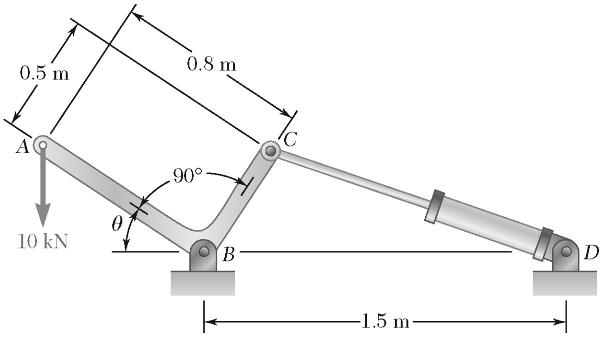
$$F_{CD} = \frac{32}{3}CD \qquad (2)$$

For $\theta = 55°$:

Eq. (1): $CD^2 = 2.5 - 1.5\sin 55° = 1.2713; \quad CD = 1.1275 \text{ m}$

Eq. (2): $F_{CD} = \frac{32}{3}CD = \frac{32}{3}(1.1275) = 12.027 \text{ kN}$

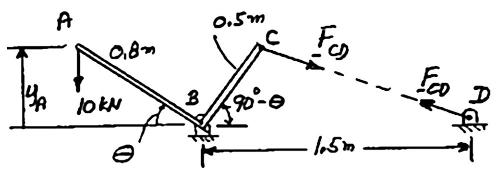
$\mathbf{F}_{CD} = 12.03 \text{ kN} \searrow$ ◀



PROBLEM 10.44

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. Determine the angle θ knowing that the hydraulic cylinder exerts a 15-kN force on pin *C*.

SOLUTION



$$y_A = (0.8\text{ m})\sin\theta$$

$$\delta y_A = 0.8\cos\theta\delta\theta$$

$$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos(90° - \theta)$$

$$CD^2 = 0.5^2 + 1.5^2 - 2(0.5)(1.5)\sin\theta$$

$$CD^2 = 2.5 - 1.5\sin\theta \quad (1)$$

$$2(CD)(\delta_{CD}) = -1.5\cos\theta\delta\theta; \qquad \delta_{CD} = -\frac{3\cos\theta}{4CD}\delta\theta$$

Virtual Work: $\delta U = 0$: $\quad -(10\text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$

$$-10(0.8\cos\theta\delta\theta) - F_{CD}\left(-\frac{3\cos\theta}{4CD}\delta\theta\right) = 0$$

$$F_{CD} = \frac{32}{3}CD \quad (2)$$

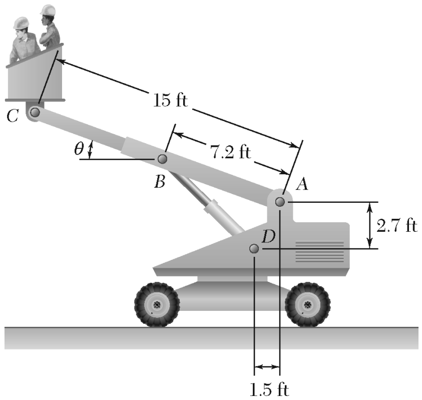
For $F_{CD} = 15\text{ kN}$:

Eq. (2): $\quad 15\text{ kN} = \frac{32}{3}CD$: $\qquad CD = \frac{45}{32} = 1.40625\text{ m}$

Eq. (1): $\quad (1.40625)^2 = 2.5 - 1.5\sin\theta$: $\qquad \sin\theta = 0.34831$

$$\theta = 20.38°$$

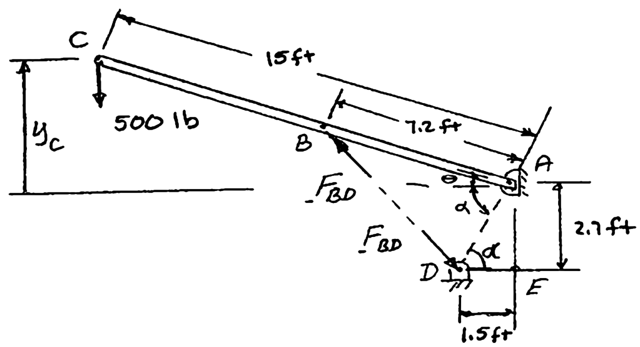
$\theta = 20.4°$ ◀



PROBLEM 10.45

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION



In ΔADE: $\tan\alpha = \dfrac{AE}{DE} = \dfrac{2.7 \text{ ft}}{1.5 \text{ ft}}$

$$\alpha = 60.945°$$

$$AD = \frac{2.7 \text{ ft}}{\sin 60.945°} = 3.0887 \text{ m}$$

From the geometry:

$$y_C = (15 \text{ ft})\sin\theta$$

$$\delta y_C = (15 \text{ ft})\cos\theta\delta\theta$$

Then, in triangle *BAD*: Angle $BAD = \alpha + \theta$

Law of cosines:

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(\alpha + \theta)$$

or

$$BD^2 = (7.2 \text{ ft})^2 + (3.0887 \text{ ft})^2 - 2(7.2 \text{ ft})(3.0887 \text{ ft})\cos(\alpha + \theta)$$

$$BD^2 = 61.380 \text{ ft}^2 - (44.477\cos(\alpha + \theta)) \text{ ft}^2 \quad (1)$$

And then

$$2(BD)(\delta BD) = (44.477\sin(\alpha+\theta))\delta\theta$$

$$\delta BD = \frac{44.477\sin(\alpha+\theta)}{2(BD)}\delta\theta$$

<u>Virtual Work</u>: $\delta U = 0{:}\quad -P\delta y_C + F_{BD}\delta BD = 0$

Substituting $$-(500\text{ lb})(15\text{ ft})\cos\theta\delta\theta + F_{BD}\left[\frac{(44.477\text{ ft}^2)\sin(\alpha+\theta)}{2(BD)}\delta\theta\right] = 0$$

or $$F_{BD} = \left[337.25\frac{\cos\theta}{\sin(\alpha+\theta)}BD\right]\text{ lb/ft} \qquad (2)$$

Now, with $\theta = 20°$ and $\alpha = 60.945°$

Equation (1):

$$BD^2 = 61.380 - 44.477\cos(60.945° + 20°)$$

$$BD^2 = 54.380$$

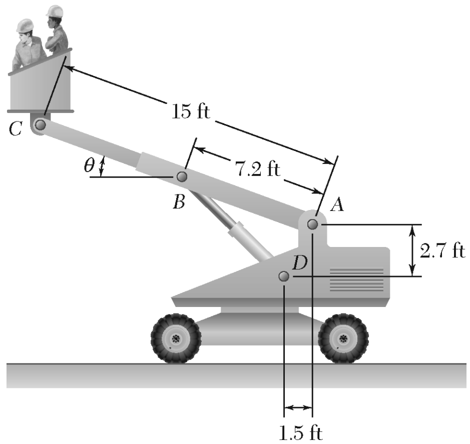
$$BD = 7.3743\text{ ft}$$

Equation (2):

$$F_{BD} = \left[337.25\frac{\cos 20°}{\sin(60.945° + 20°)}(7.3743\text{ ft})\right]\text{ lb/ft}$$

or $F_{BD} = 2366\text{ lb}$ $\quad\quad \mathbf{F}_{BD} = 2370\text{ lb}$ ↖ ◀

PROBLEM 10.46



Solve Problem 10.45 assuming that the workers are lowered to a point near the ground so that $\theta = -20°$.

PROBLEM 10.45 The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION

Using the figure and analysis of Problem 10.45, including Equations (1) and (2), and with $\theta = -20°$, we have

Equation (1):

$$BD^2 = 61.380 - 44.477\cos(60.945° - 20°)$$

$$BD^2 = 27.785$$

$$BD = 5.2711 \text{ ft}$$

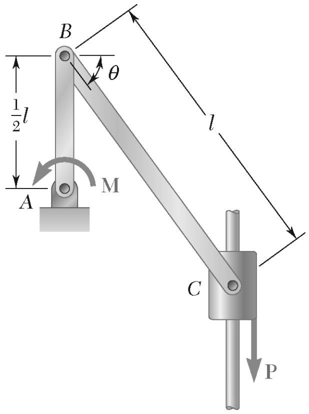
Equation (2):

$$F_{BD} = 337.25\frac{\cos(-20°)}{\sin(60.945° - 20°)}(5.2711)$$

$$F_{BD} = 2549 \text{ lb}$$

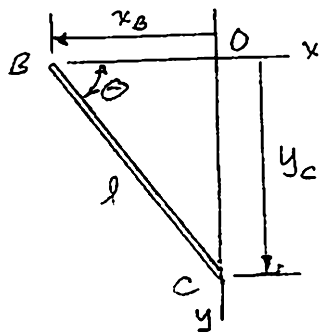
or $\mathbf{F}_{BD} = 2550 \text{ lb} \nwarrow$ ◀

PROBLEM 10.47



Denoting by μ_s the coefficient of static friction between collar C and the vertical rod, derive an expression for the magnitude of the largest couple **M** for which equilibrium is maintained in the position shown. Explain what happens if $\mu_s \geq \tan\theta$.

SOLUTION



Member BC: We have

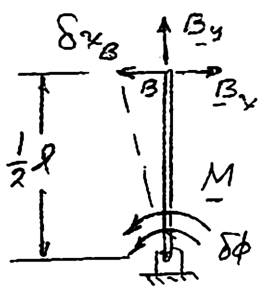
$$x_B = l\cos\theta$$

$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$

and

$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta \qquad (2)$$



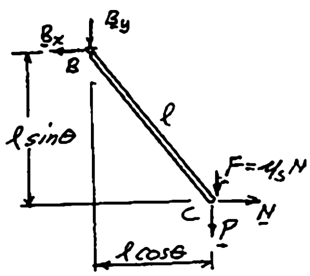
Member AB: We have

$$\delta x_B = \frac{1}{2}l\delta\phi$$

Substituting from Equation (1),

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or
$$\delta\phi = -2\sin\theta\delta\theta \qquad (3)$$



Free body of rod BC

For $M_{\max}$, motion of collar C impends upward

$$+\circlearrowleft \Sigma M_B = 0: \quad Nl\sin\theta - (P + \mu_s N)(l\cos\theta) = 0$$

$$N\tan\theta - \mu_s N = P$$

$$N = \frac{P}{\tan\theta - \mu_s}$$

PROBLEM 10.47 (Continued)

Virtual Work:

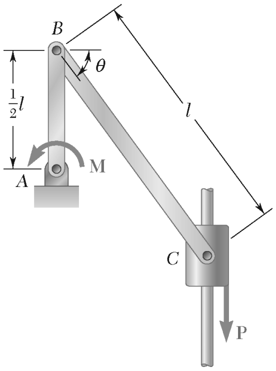
$$\delta U = 0: \quad M\,\delta\phi + (P + \mu_s N)\delta y_C = 0$$

$$M(-2\sin\theta\delta\theta) + (P + \mu_s N)l\cos\theta\delta\theta = 0$$

$$M_{\max} = \frac{(P + \mu_s N)}{2\tan\theta}l = \frac{P + \mu_s \frac{P}{\tan\theta - \mu_s}}{2\tan\theta}l$$

$$\text{or} \quad M_{\max} = \frac{Pl}{2(\tan\theta - \mu_s)} \blacktriangleleft$$

If $\mu_s = \tan\theta$, $M = \infty$, system becomes *self-locking*.



PROBLEM 10.48

Knowing that the coefficient of static friction between collar C and the vertical rod is 0.40, determine the magnitude of the largest and smallest couple **M** for which equilibrium is maintained in the position shown, when $\theta = 35°$, $l = 600$ mm, and $P = 300$ N.

SOLUTION

From the analysis of Problem 10.50, we have

$$M_{\max} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

With $\qquad \theta = 35°, \quad l = 0.6 \text{ m}, \quad P = 300 \text{ N}$

$$M_{\max} = \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35° - 0.4)}$$

$$= 299.80 \text{ N}\cdot\text{m} \qquad M_{\max} = 300 \text{ N}\cdot\text{m} ◀$$

For $M_{\min}$, motion of C impends downward and F acts upward. The equations of Problem 10.50 can still be used if we replace μ_s by $-\mu_s$. Then

$$M_{\min} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

Substituting,

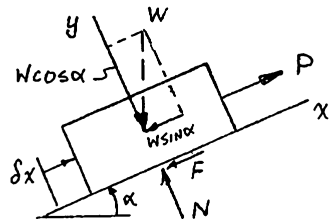
$$M_{\min} = \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35° + 0.4)}$$

$$= 81.803 \text{ N}\cdot\text{m} \qquad M_{\min} = 81.8 \text{ N}\cdot\text{m} ◀$$

PROBLEM 10.49

A block of weight W is pulled up a plane forming an angle α with the horizontal by a force **P** directed along the plane. If μ is the coefficient of friction between the block and the plane, derive an expression for the mechanical efficiency of the system. Show that the mechanical efficiency cannot exceed $\frac{1}{2}$ if the block is to remain in place when the force **P** is removed.

SOLUTION



$$\text{Input work} = P\delta x$$

$$\text{Output work} = (W\sin\alpha)\delta x$$

Efficiency:

$$\eta = \frac{W\sin\alpha\delta x}{P\delta x} \quad \text{or} \quad \eta = \frac{W\sin\alpha}{P} \qquad (1)$$

$$+\nearrow \Sigma F_x = 0: \quad P - F - W\sin\alpha = 0 \quad \text{or} \quad P = W\sin\alpha + F \qquad (2)$$

$$+\nwarrow \Sigma F_y = 0: \quad N - W\cos\alpha = 0 \quad \text{or} \quad N = W\cos\alpha$$

$$F = \mu N = \mu W\cos\alpha$$

Equation (2): $$P = W\sin\alpha + \mu W\cos\alpha = W(\sin\alpha + \mu\cos\alpha)$$

Equation (1): $$\eta = \frac{W\sin\alpha}{W(\sin\alpha + \mu\cos\alpha)} \quad \text{or} \quad \eta = \frac{1}{1 + \mu\cot\alpha}$$ ◀

If block is to remain in place when $P = 0$, we know (see Chapter 8) that $\phi_s \geq \alpha$ or, since

$$\mu = \tan\phi_s, \quad \mu \geq \tan\alpha$$

Multiply by $\cot\alpha$: $$\mu\cot\alpha \geq \tan\alpha\cot\alpha = 1$$

Add 1 to each side: $$1 + \mu\cot\alpha \geq 2$$

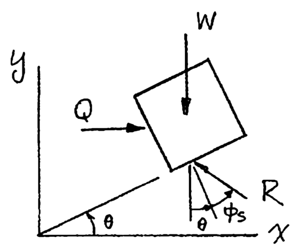
Recalling the expression for η, we find $$\eta \geq \frac{1}{2}$$ ◀

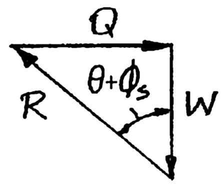
PROBLEM 10.50

Derive an expression for the mechanical efficiency of the jack discussed in Section 8.6. Show that if the jack is to be self-locking, the mechanical efficiency cannot exceed $\frac{1}{2}$.

SOLUTION

Recall Figure 8.9*a*. Draw force triangle





$$Q = W\tan(\theta+\phi_s)$$

$$y = x\tan\theta \text{ so that } \delta y = \delta x\tan\theta$$

$$\text{Input work} = Q\delta x = W\tan(\theta+\phi_s)\delta x$$

$$\text{Output work} = W\delta y = W(\delta x)\tan\theta$$

Efficiency: $\eta = \dfrac{W\tan\theta\delta x}{W\tan(\theta+\phi_s)\delta x}$; $\qquad \eta = \dfrac{\tan\theta}{\tan(\theta+\phi_s)}$ ◀

From Page 432, we know the jack is self-locking if

$$\phi \geq \theta$$

Then $\qquad \theta+\phi_s \geq 2\theta$

so that $\qquad \tan(\theta+\phi_s) \geq \tan 2\theta$

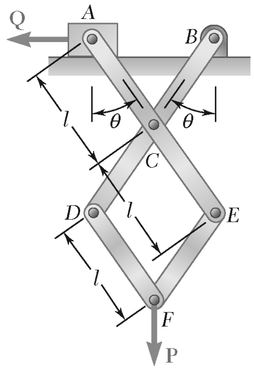
From above $\qquad \eta = \dfrac{\tan\theta}{\tan(\theta+\phi_s)}$

It then follows that $\qquad \eta \leq \dfrac{\tan\theta}{\tan 2\theta}$

But $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Then $\qquad \eta \leq \dfrac{\tan\theta(1-\tan^2\theta)}{2\tan\theta} = \dfrac{1-\tan^2\theta}{2} \qquad \eta \leq \dfrac{1}{2}$ ◀

PROBLEM 10.51

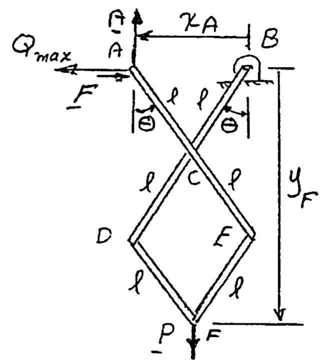


Denoting by μ_s the coefficient of static friction between the block attached to rod ACE and the horizontal surface, derive expressions in terms of P, μ_s, and θ for the largest and smallest magnitude of the force **Q** for which equilibrium is maintained.

SOLUTION

For the linkage:

$$+\circlearrowleft \Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \quad \text{or} \quad \mathbf{A} = \frac{P}{2} \uparrow$$



Then: $$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2}\mu_s P$$

Now $$x_A = 2l\sin\theta$$
$$\delta x_A = 2l\cos\theta\delta\theta$$

and $$y_F = 3l\cos\theta$$
$$\delta y_F = -3l\sin\theta\delta\theta$$

Virtual Work: $$\delta U = 0: \quad (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

$$\left(Q_{\max} - \frac{1}{2}\mu_s P\right)(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

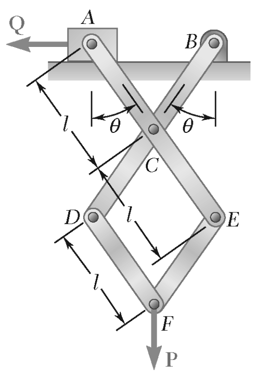
or $$Q_{\max} = \frac{3}{2}P\tan\theta + \frac{1}{2}\mu_s P$$

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s) \blacktriangleleft$$

For $Q_{\min}$, motion of A impends to the right and **F** acts to the left. We change μ_s to $-\mu_s$ and find

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s) \blacktriangleleft$$

PROBLEM 10.52



Knowing that the coefficient of static friction between the block attached to rod ACE and the horizontal surface is 0.15, determine the magnitude of the largest and smallest force **Q** for which equilibrium is maintained when $\theta = 30°$, $l = 0.2$ m, and $P = 40$ N.

SOLUTION

Using the results of Problem 10.48 with

$$\theta = 30°$$
$$l = 0.2 \text{ m}$$
$$P = 40 \text{ N}, \quad \text{and} \quad \mu_s = 0.15$$

We have

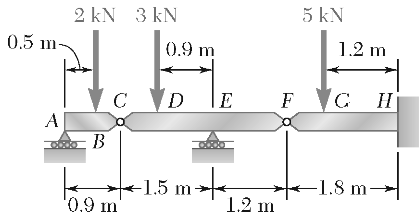
$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s)$$
$$= \frac{(40 \text{ N})}{2}(3\tan 30° + 0.15)$$
$$= 37.64 \text{ N}$$

$Q_{\max} = 37.6$ N ◀

and

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s)$$
$$= \frac{(40 \text{ N})}{2}(3\tan 30° - 0.15)$$
$$= 31.64 \text{ N}$$

$Q_{\min} = 31.6$ N ◀

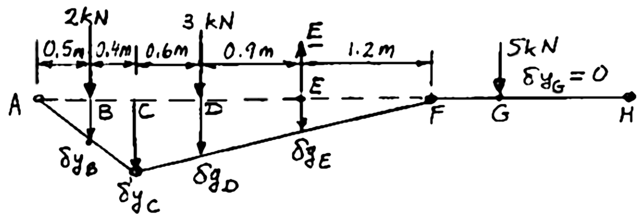


PROBLEM 10.53

Using the method of virtual work, determine the reaction at E.

SOLUTION

We release the support at E and assume a virtual displacement δy_E for Point E.



<u>From similar triangles</u>:

$$\delta y_D = \frac{2.1}{1.2}\delta y_E = 1.75\delta y_E$$

$$\delta y_C = \frac{2.7}{1.2}\delta y_E = 2.25\delta y_E$$

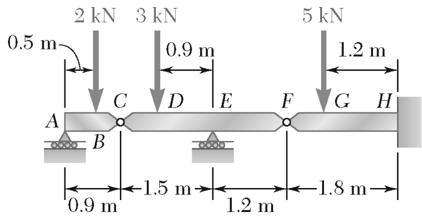
$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(2.25\delta y_E) = 1.25\delta y_E$$

<u>Virtual Work</u>:

$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - E\delta y_E = 0$$

$$2(1.25\delta y_E) + 3(1.75\delta y_E) - E\delta y_E = 0$$

$E = +7.75$ kN $\qquad$ $\mathbf{E} = 7.75\text{ kN}\uparrow$ ◀

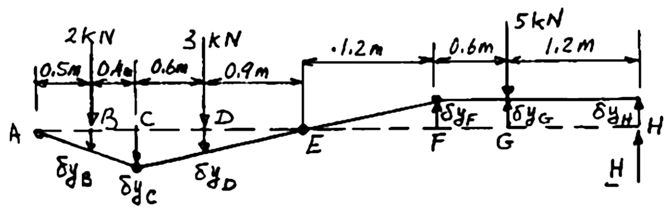


PROBLEM 10.54

Using the method of virtual work, determine separately the force and couple representing the reaction at H.

SOLUTION

Force at H. We give a vertical virtual displacement δy_H to Point H, keeping member FH horizontal.



From the geometry of the diagram:

$$\delta y_F = \delta y_G = \delta y_H$$

$$\delta y_D = \frac{0.9}{1.2}\delta y_F = \frac{0.9}{1.2}\delta y_H = 0.75\delta y_H$$

$$\delta y_C = \frac{1.5}{0.9}\delta y_D = \frac{1.5}{0.9}(0.75\delta y_H) = 1.25\delta y_H$$

$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(1.25\delta y_H) = 0.69444\delta y_H$$

Virtual Work:

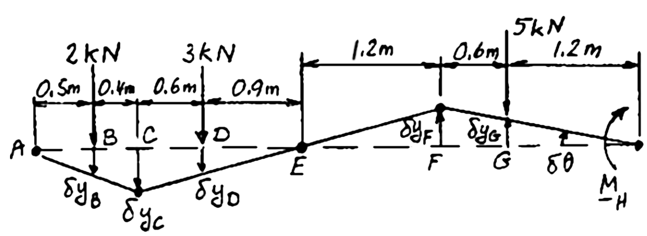
$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - (5\text{ kN})\delta y_G + H\delta y_H = 0$$

$$2(0.69444\delta y_H) + 3(0.75\delta y_H) - 5\delta y_H + H\delta y_H = 0$$

$$H = +1.3611\text{ kN}$$

$$\mathbf{H} = 1.361\text{ kN}\uparrow ◀$$

Couple at H. We rotate beam FH through $\delta\theta$ about Point H.



PROBLEM 10.54 (Continued)

From the geometry of the diagram:

$$\delta y_G = 1.2\delta\theta \qquad \delta y_F = 1.8\delta\theta$$

$$\delta y_D = \frac{0.9}{1.2}\delta y_F = \frac{0.9}{1.2}(1.8\delta\theta) = 1.35\delta\theta$$

$$\delta y_C = \frac{1.5}{0.9}\delta y_D = \frac{1.5}{0.9}(1.35\delta\theta) = 2.25\delta\theta$$

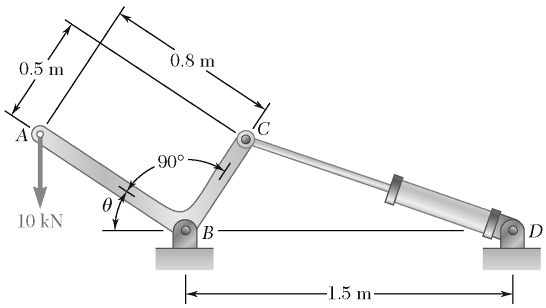
$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(2.25\delta\theta) = 1.25\delta\theta$$

Virtual Work:

$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - (5\text{ kN})\delta y_G + M_H\delta\theta = 0$$

$$2(1.25\delta\theta) + 3(1.35\delta\theta) - 5(1.2\delta\theta) + M_H\delta\theta = 0$$

$$M_H = -0.550\text{ kN}\cdot\text{m} \qquad \mathbf{M}_H = 550\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

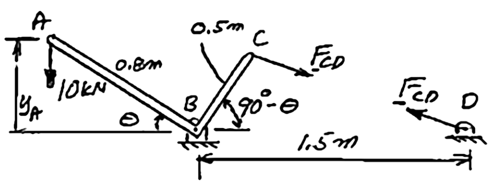


PROBLEM 10.55

Referring to Problem 10.43 and using the value found for the force exerted by the hydraulic cylinder *CD*, determine the change in the length of *CD* required to raise the 10-kN load by 15 mm.

PROBLEM 10.43 The position of member *ABC* is controlled by the hydraulic cylinder *CD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *C* when $\theta = 55°$.

SOLUTION



Virtual Work: Assume both δy_A and δ_{CD} increase

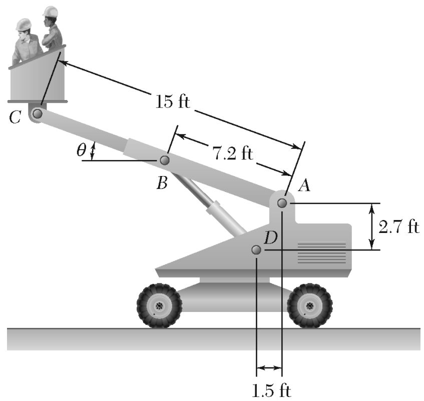
$$\delta U = 0: \quad -(10\text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$$

Substitute: $\delta y_A = 15\text{ mm}$ and $F_{CD} = 12.03\text{ kN}$

$$-(10\text{ kN})(15\text{ mm}) - (12.03\text{ kN})\delta_{CD} = 0$$

$$\delta_{CD} = -12.47\text{ mm}$$

The negative sign indicates that *CD* shortened $\qquad \delta_{CD} = 12.47\text{ mm shorter}$ ◀

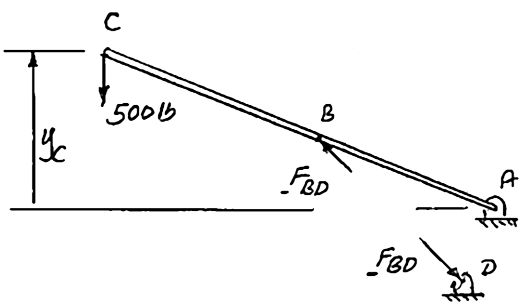


PROBLEM 10.56

Referring to Problem 10.45 and using the value found for the force exerted by the hydraulic cylinder *BD*, determine the change in the length of *BD* required to raise the platform attached at *C* by 2.5 in.

PROBLEM 10.45 The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION



Virtual Work: Assume both δy_C and δ_{BD} increase

$$\delta U = 0: \quad -(500\text{ lb})\delta y_C + F_{BD}\delta_{BD} = 0$$

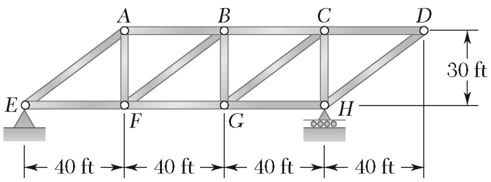
Substitute: $\delta y_C = 2.5$ in. and $F_{BD} = 2370$ lb

$$-(500\text{ lb})(2.5\text{ in.}) + (2370\text{ lb})\delta_{BD} = 0$$

$$\delta_{BD} = +0.527\text{ in.}$$

The positive sign indicates that cylinder *BD* increases in length $\delta_{BD} = 0.527$ in. longer ◀

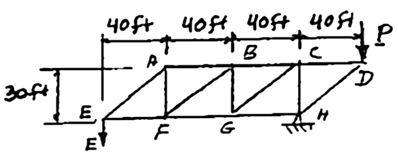
PROBLEM 10.57



Determine the vertical movement of joint D if the length of member BF is increased by 1.5 in. (*Hint:* Apply a vertical load at joint D, and, using the methods of Chapter 6, compute the force exerted by member BF on joints B and F. Then apply the method of virtual work for a virtual displacement resulting in the specified increase in length of member BF. This method should be used only for small changes in the lengths of members.)

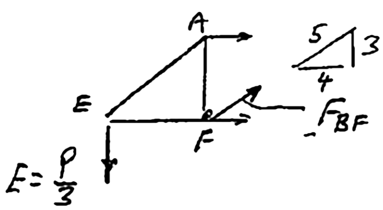
SOLUTION

Apply vertical load P at D.



$$+\circlearrowleft \Sigma M_H = 0: \quad -P(40\text{ ft}) + E(120\text{ ft}) = 0$$

$$\mathbf{E} = \frac{P}{3}\downarrow$$

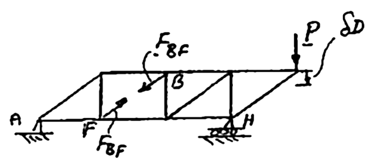


$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{3} = 0$$

$$F_{BF} = \frac{5}{9}P$$

Virtual Work:

We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of Points B and F as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

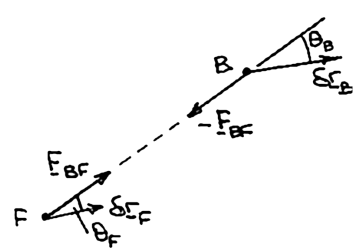


Virtual Work:

$$\delta U = 0: \qquad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$$

$$P\delta D + F_{BF}\delta r_F \cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = 0$$



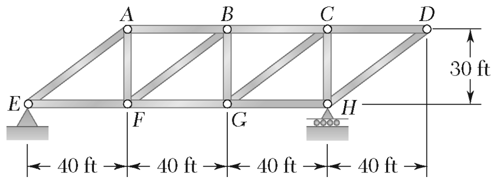
where $(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(1.5\text{ in.}) = 0$$

$$\delta D = 0.833\text{ in.}$$

$$\delta D = 0.833\text{ in.}\downarrow \blacktriangleleft$$

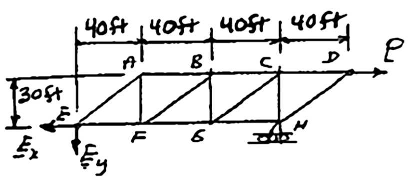


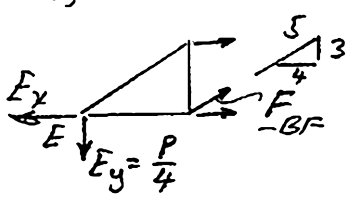
PROBLEM 10.58

Determine the horizontal movement of joint D if the length of member BF is increased by 1.5 in. (See the hint for Problem 10.57.)

SOLUTION

Apply horizontal load P at D.





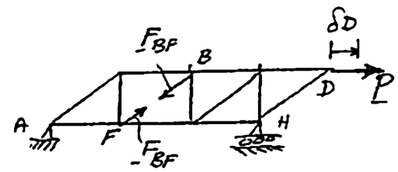
$$+\circlearrowleft \Sigma M_H = 0: \quad P(30\text{ ft}) - E_y(120\text{ ft}) = 0$$

$$\mathbf{E}_y = \frac{P}{4}\downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{4} = 0$$

$$F_{BF} = \frac{5}{12}P$$

We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of Points B and F as $\delta \mathbf{r}_B$ and $\delta \mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

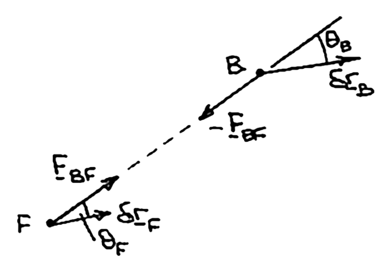


Virtual Work:

$$\delta U = 0: \qquad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$$

$$P\delta D + F_{BF}\delta r_F\cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = 0$$



where $(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

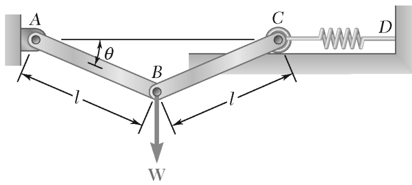
$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(1.5\text{ in.}) = 0$$

$$\delta D = 0.625\text{ in.}$$

$\delta D = 0.625$ in. $\longrightarrow$ ◀

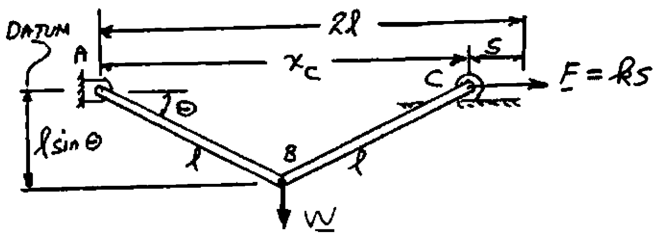
PROBLEM 10.59



Using the method of Section 10.8, solve Problem 10.29.

PROBLEM 10.29 A load **W** of magnitude 600 N is applied to the linkage at *B*. The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



$$W = 600 \text{ N}$$

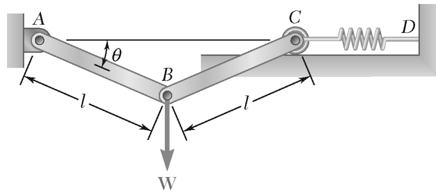
$$l = 0.3 \text{ m}, \quad \text{and} \quad k = 2500 \text{ N} \cdot \text{m}$$

We have

$$(l - \cos\theta)\tan\theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

$$= 0.2$$

Solving numerically $\theta = 40.2° \blacktriangleleft$

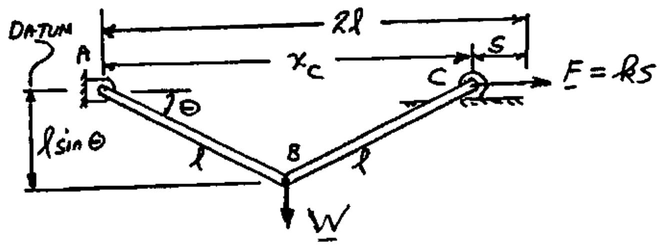


PROBLEM 10.60

Using the method of Section 10.8, solve Problem 10.30.

PROBLEM 10.30 A vertical load **W** is applied to the linkage at B. The constant of the spring is k, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage, derive an equation in θ, W, l, and k that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$V = \frac{1}{2}ks^2 + Wy_B$$

$$V = \frac{1}{2}k(2i - x_C)^2 + Wy_B$$

$$x_C = 2l\cos\theta \quad \text{and} \quad y_B = -l\sin\theta$$

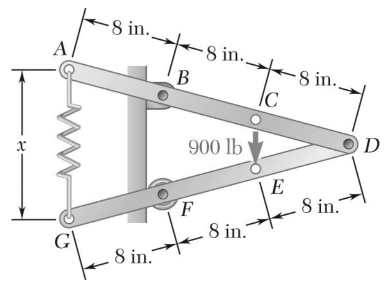
Thus

$$V = \frac{1}{2}k(2l - 2l\cos\theta)^2 - Wl\sin\theta$$

$$= 2kl^2(1-\cos\theta)^2 - Wl\sin\theta$$

$$\frac{dV}{d\theta} = 2kl^2 2(1-\cos\theta)\sin\theta - Wl\cos\theta = 0$$

$$(1-\cos\theta)\tan\theta = \frac{W}{4kl} \blacktriangleleft$$



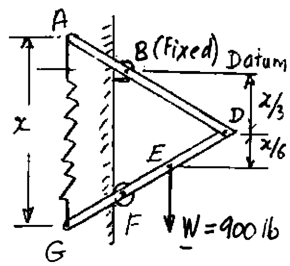
PROBLEM 10.61

Using the method of Section 10.8, solve Problem 10.31.

PROBLEM 10.31 Two bars *AD* and *DG* are connected by a pin at *D* and by a spring *AG*. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of *x* corresponding to equilibrium when a 900-lb load is applied at *E* as shown.

SOLUTION

$$V = \frac{1}{2}ks^2 + Wy_E$$



But $$s = x - 12 \text{ in.}$$

and $$y_E = -\frac{x}{3} - \frac{x}{6}$$

$$= -\frac{x}{2}$$

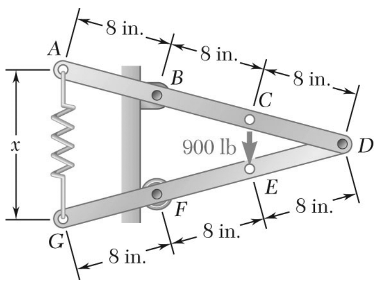
Thus $$V = \frac{1}{2}k(x-12)^2 - W\left(\frac{x}{2}\right)$$

$$\frac{dV}{dx} = k(x-12) - \frac{1}{2}W = 0$$

$$x = 12 + \frac{W}{2k} = 12 \text{ in.} + \frac{900 \text{ lb}}{2(125 \text{ lb/in.})}$$

$x = 15.60$ in. ◀

PROBLEM 10.62

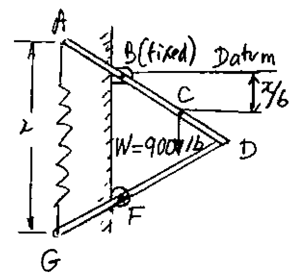


Using the method of Section 10.8, solve Problem 10.32.

PROBLEM 10.32 Solve Problem 10.31 assuming that the 900-lb vertical force is applied at *C* instead of *E*.

PROBLEM 10.31 Two bars *AD* and *DG* are connected by a pin at *D* and by a spring *AG*. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of *x* corresponding to equilibrium when a 900-lb load is applied at *E* as shown.

SOLUTION



$$V = \frac{1}{2}ks^2 + Wy_C$$

But $$s = x - 12 \text{ in.}$$

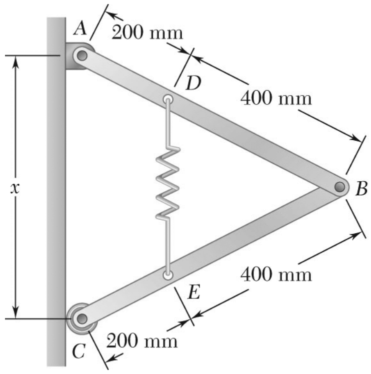
and $$y_E = -\frac{x}{6}$$

Thus $$V = \frac{1}{2}k(x-12)^2 - \frac{1}{6}Wx$$

$$\frac{dV}{dx} = k(x-12) - \frac{1}{6}W = 0$$

$$x = 12 + \frac{W}{6k} = 12 \text{ in.} + \frac{900 \text{ lb}}{6(125 \text{ lb/in.})}$$

$x = 13.20$ in. ◀



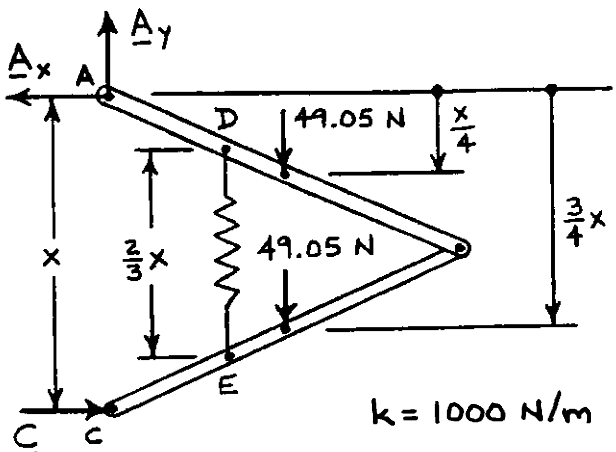
PROBLEM 10.63

Using the method of Section 10.8, solve Problem 10.33.

PROBLEM 10.33 Two 5-kg bars *AB* and *BC* are connected by a pin at *B* and by a spring *DE*. Knowing that the spring is 150 mm long when unstretched and that the constant of the spring is 1 kN/m, determine the value of *x* corresponding to equilibrium.

SOLUTION

First note: $W_{\text{bar}} = (5\text{ kg})(9.81\text{ m/s}^2) = 49.05\text{ N}$



Since unstretched length of spring is 150 mm, or 0.15 m, we have

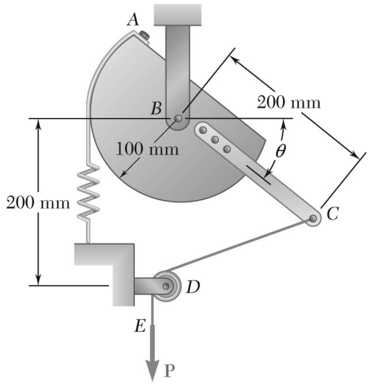
$$\Delta_S = \frac{2}{3}x - 0.15$$

$$\mathbf{V} = \frac{1}{2}k\Delta_S^2 - (49.05\text{ N})\frac{x}{4} - (49.05\text{ N})\frac{3x}{4}$$

$$V = \frac{1}{2}(1000\text{ N/m})\left(\frac{2}{3}x - 0.15\right)^2 - 12.2625x - 36.7875x$$

$$\frac{dV}{dx} = 1000\left(\frac{2}{3}x - 0.15\right)\frac{2}{3} - 12.2625 - 36.7875 = 0$$

$x = 0.335$ m $\qquad\qquad$ $x = 335$ mm ◀

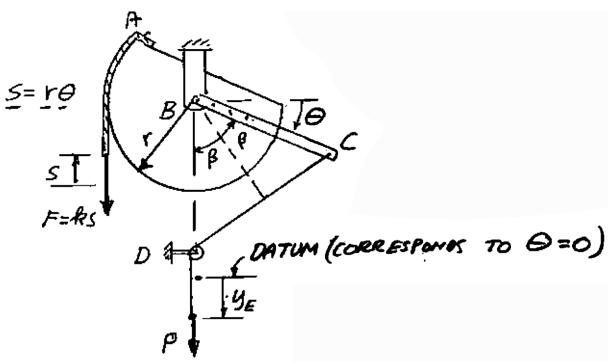


PROBLEM 10.64

Using the method of Section 10.8, solve Problem 10.35.

PROBLEM 10.35 A vertical force **P** of magnitude 150 N is applied to end *E* of cable *CDE*, which passes over a small pulley *D* and is attached to the mechanism at *C*. The constant of the spring is k = 4 kN/m, and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium.

SOLUTION



$$\beta = \frac{1}{2}(90° - \theta) = 45° - \frac{\theta}{2}$$

$$BC = BD = l$$

$$CD = 2l\sin\beta = 2l\sin\left(45 - \frac{\theta}{2}\right)$$

For $\theta = 0$: $(CD)_0 = 2l\sin 45 = \sqrt{2}\,l$

$$y_E = (CD)_0 - CD$$

$$= \sqrt{2}\,l - 2l\sin\left(45° - \frac{\theta}{2}\right)$$

<u>Potential energy</u>:

$$V = \frac{1}{2}ks^2 - Py_E$$

$$V = \frac{1}{2}k(r\theta)^2 - P\left[\sqrt{2}l - 2l\sin\left(45° - \frac{\theta}{2}\right)\right]$$

$$\frac{dV}{d\theta} = kr^2\theta + 2Pl\cos\left(45° - \frac{\theta}{2}\right)\left(-\frac{1}{2}\right) = 0$$

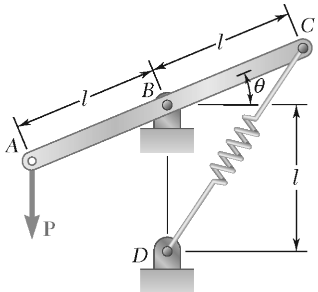
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$

$$\frac{Pl}{kr^2} = \frac{(150\text{ N})(0.2\text{ m})}{(4000\text{ N/m})(0.1\text{ m})^2} = 0.75$$

$$0.75 = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$

Solve by trial and error: $\theta = 0.67623$ rad

$\theta = 38.745°$ $\theta = 38.7°$ ◀



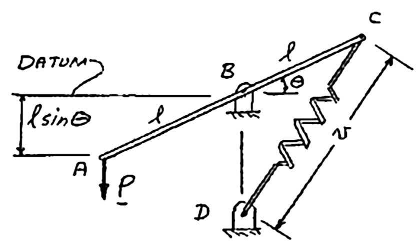
PROBLEM 10.65

Using the method of Section 10.8, solve Problem 10.37.

PROBLEM 10.37 and 10.38 Knowing that the constant of spring *CD* is k and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

PROBLEM 10.37 $P = 300$ N, $l = 400$ mm, $k = 5$ kN/m.

SOLUTION



Spring
$$v = 2l\sin\left(\frac{90° + \theta}{2}\right)$$

$$v = 2l\sin\left(45° + \frac{\theta}{2}\right)$$

Unstretched $(\theta = 0)$
$$v_0 = 2l\sin 45° = \sqrt{2}l$$

Deflection of spring
$$s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]^2 + P(-l\sin\theta)$$

$$\frac{dV}{d\theta} = kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]\cos\left(45° + \frac{\theta}{2}\right) - Pl\cos\theta = 0$$

$$\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right] = \frac{P}{kl}\cos\theta$$

$$\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right) = \frac{P}{kl}\cos\theta$$

Divide each member by cos θ
$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

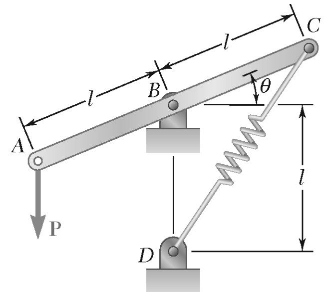
Then with $P = 300$ N, $l = 0.4$ m and $k = 5000$ N/m

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{300 \text{ N}}{(5000 \text{ N/m})(0.4 \text{ m})}$$

$$= 0.15$$

or
$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.60104$$

Solving numerically $\theta = 22.6°$ ◀



PROBLEM 10.66

Using the method of Section 10.8, solve Problem 10.38.

PROBLEM 10.37 and 10.38 Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

PROBLEM 10.38 $P = 75$ lb, $l = 15$ in., $k = 20$ lb/in.

SOLUTION

Using the results of Problem 10.65 with $P = 75$ lb, $l = 15$ in. and $k = 20$ lb/in., we have

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

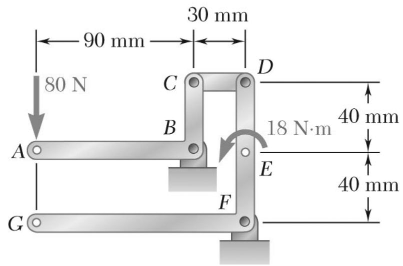
$$= \frac{75 \text{ lb}}{(20 \text{ lb/in.})(15 \text{ in.})}$$

$$= 0.25$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.53033$$

Solving numerically $\theta = 51.058°$ $\theta = 51.1°$ ◀

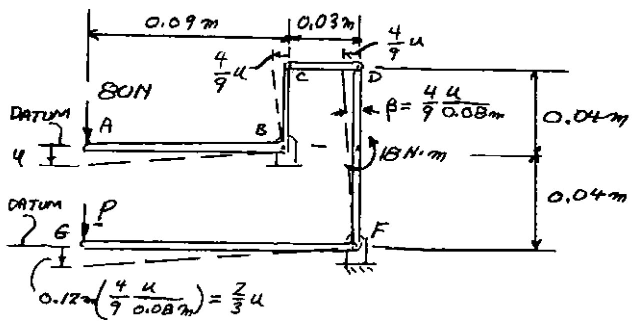


PROBLEM 10.67

Show that equilibrium is neutral in Problem 10.1.

PROBLEM 10.1 Determine the vertical force **P** that must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



We have $y_A = -u, \quad y_G = -\frac{2}{3}u, \quad \beta = \frac{4u}{0.72}$

$$V = (80\text{ N})y_A + P(y_G) - (18\text{ N}\cdot\text{m})\beta$$

$$V = 80(-u) + P\left(-\frac{2}{3}u\right) - (18)\frac{4u}{0.72}$$

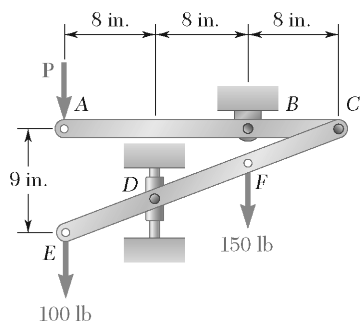
$$\frac{dV}{du} = -80 - \frac{2}{3}P - 100 = 0$$

$$P = 270\text{ N}$$

$\mathbf{P} = 270\text{ N}\uparrow$ ◀

Substituting $P = 270$ N in the expression for V, we have $V = 0$. Thus V is constant

and equilibrium is neutral ◀

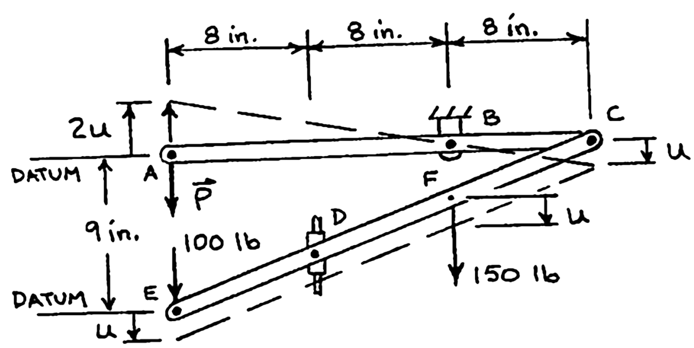


PROBLEM 10.68

Show that equilibrium is neutral in Problem 10.7.

PROBLEM 10.7 The two-bar linkage shown is supported by a pin and bracket at *B* and a collar at *D* that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

SOLUTION



$$y_A = 2u$$

$$y_E = -u$$

$$y_F = -u$$

$$V = Py_A + (100 \text{ lb})y_E + (150 \text{ lb})y_F$$

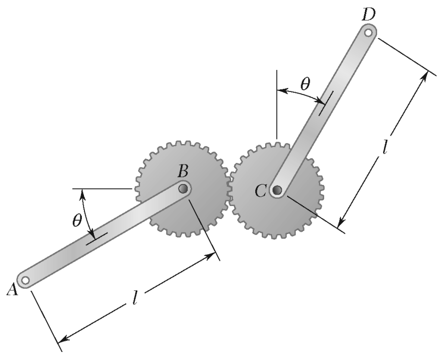
$$V = P(2u) + (100 \text{ lb})(-u) + (150 \text{ lb})(-u)$$

$$\frac{dV}{du} = 2P - 100 - 150 = 0$$

$$P = 125 \text{ lb}$$

Now, substitute $P = 125$ lb in expression for V, making $V = 0$. Thus, V is constant and

<u>equilibrium is neutral.</u> ◀



PROBLEM 10.69

Two uniform rods, each of mass m, are attached to gears of equal radii as shown. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

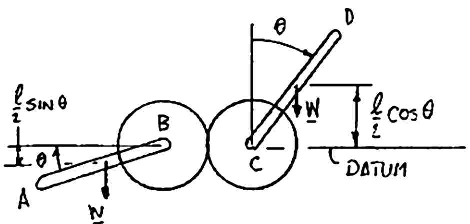
Potential energy

$$V = W\left(-\frac{l}{2}\sin\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \quad W = mg$$

$$= W\frac{l}{2}(\cos\theta - \sin\theta)$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-\sin\theta - \cos\theta)$$

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin\theta - \cos\theta)$$



For equilibrium: $\frac{dV}{d\theta} = 0$: $\sin\theta = -\cos\theta$

or $\tan\theta = -1$

Thus $\theta = -45.0°$ and $\theta = 135.0°$

Stability:

At $\theta = -45.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}[\sin(-45°) - \cos 45°]$$

$$= \frac{Wl}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) < 0$$

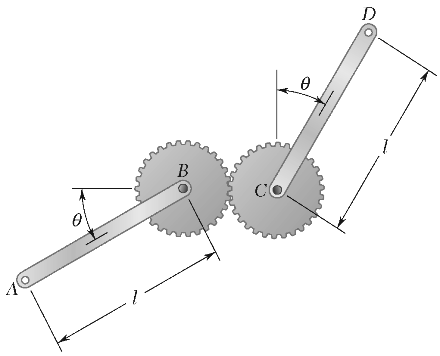
$\theta = -45.0°$, Unstable ◀

At $\theta = 135.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin 135° - \cos 135°)$$

$$= \frac{Wl}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) > 0$$

$\theta = 135.0°$, Stable ◀



PROBLEM 10.70

Two uniform rods, AB and CD, are attached to gears of equal radii as shown. Knowing that $W_{AB} = 8$ lb and $W_{CD} = 4$ lb, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

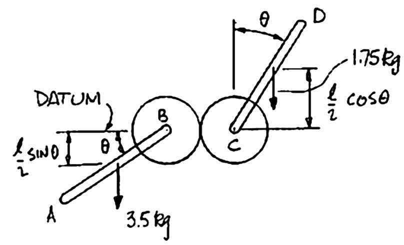
Potential energy

$$V = (3.5 \text{ kg} \times 9.81 \text{ m/s}^2)\left(-\frac{l}{2}\sin\theta\right) + (1.75 \text{ kg} \times 9.81 \text{ m/s}^2)\left(\frac{l}{2}\cos\theta\right)$$

$$= (8.5838 \text{ N})l(-2\sin\theta + \cos\theta)$$

$$\frac{dV}{d\theta} = (8.5838 \text{ N})l(-2\cos\theta - \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = (8.5838 \text{ N})l(2\sin\theta - \cos\theta)$$



Equilibrium: $\frac{dV}{d\theta} = 0$: $\quad -2\cos\theta - \sin\theta = 0$

or $\tan\theta = -2$

Thus $\theta = -63.4°$ and $116.6°$

Stability:

At $\theta = -63.4°$:

$$\frac{d^2V}{d\theta^2} = (8.5838 \text{ N})l[2\sin(-63.4°) - \cos(-63.4°)]$$

$$= (8.5838 \text{ N})l(-1.788 - 0.448) < 0$$

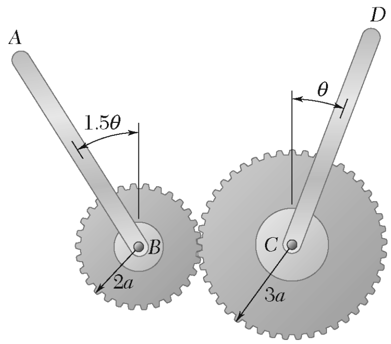
$\theta = -63.4°$, Unstable ◀

At $\theta = 116.6°$:

$$\frac{d^2V}{d\theta^2} = (8.5838 \text{ N})l[2\sin(116.6°) - \cos(116.6°)]$$

$$= (8.5838 \text{ N})l(1.788 + 0.447) > 0$$

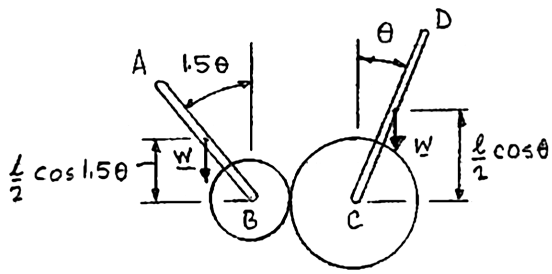
$\theta = 116.6°$, Stable ◀



PROBLEM 10.71

Two uniform rods, each of mass m and length l, are attached to gears as shown. For the range $0 \le \theta \le 180°$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential energy $$V = W\left(\frac{l}{2}\cos 1.5\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \quad W = mg$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-1.5\sin 1.5\theta) + \frac{Wl}{2}(-\sin\theta)$$

$$= -\frac{Wl}{2}(1.5\sin 1.5\theta + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

For equilibrium $$\frac{dV}{d\theta} = 0: \quad 1.5\sin 1.5\theta + \sin\theta = 0$$

Solutions: One solution, by inspection, is $\theta = 0$, and a second angle less than 180° can be found numerically:

$$\theta = 2.4042 \text{ rad} = 137.8°$$

Now $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

At $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 0° + \cos 0°)$$

$$= -\frac{Wl}{2}(3.25)(< 0)$$

$\theta = 0$, Unstable ◀

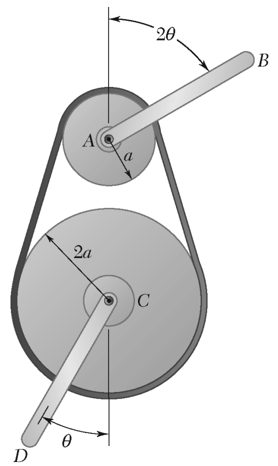
At $\theta = 137.8°$:

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}[2.25\cos(1.5\times 137.8°) + \cos 137.8°]$$

$$= \frac{Wl}{2}(2.75)(>0)$$

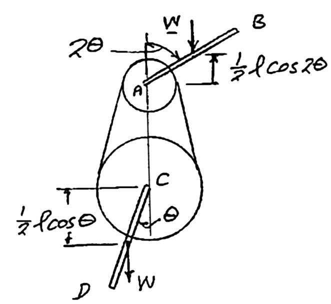
$\theta = 137.8°$, Stable ◀

PROBLEM 10.72



Two uniform rods, each of mass m and length l, are attached to drums that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the drums, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



$$W = mg$$

$$V = W\left(\frac{l}{2}\cos 2\theta\right) - W\left(\frac{l}{2}\cos\theta\right)$$

$$\frac{dV}{d\theta} = W\frac{l}{2}(-2\sin 2 + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = W\frac{1}{2}(-4\cos 2\theta - \cos\theta)$$

Equilibrium: $\frac{dV}{d\theta} = 0$: $\frac{Wl}{2}(-2\sin 2\theta + \sin\theta) = 0$

or $\sin\theta(-4\cos\theta + 1) = 0$

Solving $\theta = 0,\ 75.5°,\ 180°,\ \text{and}\ 284°$

Stability: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4\cos 2\theta - \cos\theta)$

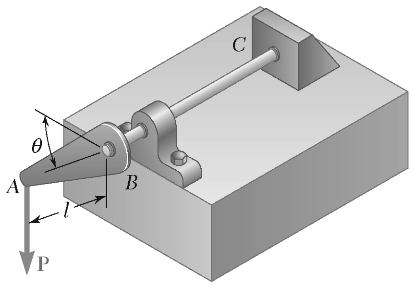
At $\theta = 0$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 - 1) < 0$ $\theta = 0$, Unstable ◀

At $\theta = 75.5°$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$ $\theta = 75.5°$, Stable ◀

At $\theta = 180°$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 + 1) < 0$ $\theta = 180.0°$, Unstable ◀

At $\theta = 284°$: $\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$ $\theta = 284°$, Stable ◀

PROBLEM 10.73



Using the method of Section 10.8, solve Problem 10.39. Determine whether the equilibrium is stable, unstable, or neutral. (*Hint:* The potential energy corresponding to the couple exerted by a torsion spring is $\frac{1}{2}K\theta^2$, where K is the torsional spring constant and θ is the angle of twist.)

PROBLEM 10.39 The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C. The torsional spring constant of the shaft BC is K; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100$ N, $l = 250$ mm, and $K = 12.5$ N · m/rad.

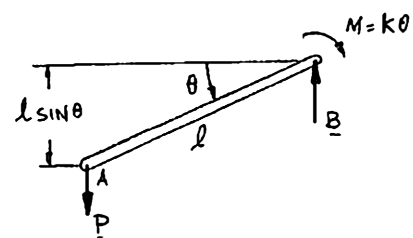
SOLUTION

Potential energy

$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$



Equilibrium:

$$\frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$$

For

$$P = 100 \text{ N}, \quad l = 0.25 \text{ m.}, \quad K = 12.5 \text{ N} \cdot \text{m/rad}$$

$$\cos\theta = \frac{12.5 \text{ N} \cdot \text{m/rad}}{(100)(0.25 \text{ m})}\theta$$

$$= 0.500\theta$$

Solving numerically, we obtain

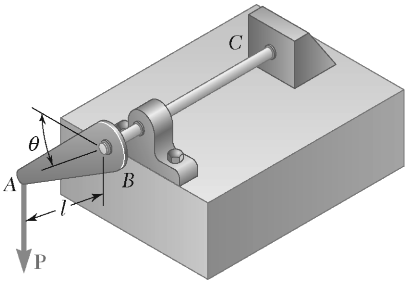
$$\theta = 1.02967 \text{ rad} = 59.000°$$

$\theta = 59.0°$ ◀

Stability

$$\frac{d^2V}{d\theta^2} = (12.5 \text{ N} \cdot \text{m/rad}) + (100 \text{ N})(0.25 \text{ m})\sin 59.0° > 0$$

Stable ◀

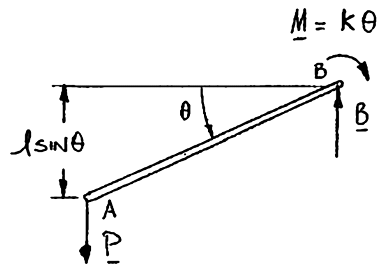


PROBLEM 10.74

In Problem 10.40, determine whether each of the positions of equilibrium is stable, unstable, or neutral. (See hint for Problem 10.73.)

PROBLEM 10.40 Solve Problem 10.39 assuming that $P = 350$ N, $l = 250$ mm, and $K = 12.5\text{ N}\cdot\text{m/rad}$. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, $360° < \theta < 450°$.

SOLUTION



Potential energy $$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium $$\frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$$

For $$P = 350\text{ N}, \quad l = 0.250\text{ m} \quad \text{and} \quad K = 12.5\text{ N}\cdot\text{m/rad}$$

$$\cos\theta = \frac{12.5\text{ N}\cdot\text{m/rad}}{(350\text{ N})(0.250\text{ m})}\theta$$

or $$\cos\theta = \frac{\theta}{7}$$

Solving numerically $$\theta = 1.37333\text{ rad}, \quad 5.652\text{ rad}, \quad \text{and} \quad 6.616\text{ rad}$$

or $$\theta = 78.7°, \ 323.8°, \ 379.1°$$

Stability at $\theta = 78.7°$: $$\frac{d^2V}{d\theta^2} = (12.5\text{ N}\cdot\text{m/rad}) + (350\text{ N})(0.250\text{ m})\sin 78.7°$$

$$= 98.304 > 0$$

$\theta = 78.7°$, Stable ◀

At $\theta = 323.8°$: $$\frac{d^2V}{d\theta^2} = (12.5\text{ N}\cdot\text{m/rad}) + (350\text{ N})(0.250\text{ m})\sin 323.8°$$

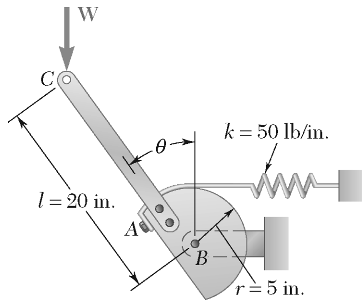
$$= -39.178\text{ N}\cdot\text{m} < 0$$

$\theta = 324°$, Unstable ◀

At At $\theta = 379.1°$: $$\frac{d^2V}{d\theta^2} = (12.5\text{ N}\cdot\text{m/rad}) + (350\text{ N})(0.250\text{ m})\sin 379.1°$$

$$= 44.132\text{ N}\cdot\text{m} > 0$$

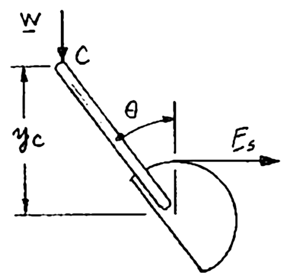
$\theta = 379°$, Stable ◀



PROBLEM 10.75

A load **W** of magnitude 100 lb is applied to the mechanism at *C*. Knowing that the spring is unstretched when $\theta = 15°$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION



We have

$$y_C = l\cos\theta$$

$$V = \frac{1}{2}k[r(\theta-\theta_0)]^2 + Wy_C \quad \theta_0 = 15° = \frac{\pi}{12}\text{rad}$$

$$= \frac{1}{2}kr^2(\theta-\theta_0)^2 + Wl\cos\theta$$

$$\frac{dV}{d\theta} = kr^2(\theta-\theta_0) - Wl\sin\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad kr^2(\theta-\theta_0) - wl\sin\theta = 0 \qquad (1)$$

with $W = 100$ lb, $R = 50$ lb./in., $l = 20$ in., and $r = 5$ in.

$$(50\text{ lb./in.})(25\text{ in.}^2)\left(\theta - \frac{\pi}{12}\right) - (100\text{ lb})(20\text{ in.})\sin\,\theta = 0$$

or

$$0.625\theta - \sin\theta = 0.16362$$

Solving numerically

$$\theta = 1.8145\text{ rad} = 103.97°$$

$$\theta = 104.0° \blacktriangleleft$$

Stability

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta \qquad (2)$$

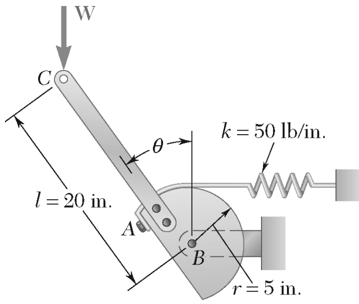
or

$$= 1250 - 2000\cos\theta$$

For $\theta = 104.0°$:

$$= 1734\text{ in.}\cdot\text{lb} > 0$$

Stable ◀



PROBLEM 10.76

A load **W** of magnitude 100 lb is applied to the mechanism at *C*. Knowing that the spring is unstretched when $\theta = 30°$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION

Using the solution of Problem 10.75, particularly Equation (1), with 15° replaced by $30°\left(\frac{\pi}{6}\text{ rad}\right)$:

For equilibrium $$kr^2\left(\theta - \frac{\pi}{6}\right) - Wl\sin\theta = 0$$

With $k = 50$ lb/in., $W = 100$ lb, $r = 5$ in., and $l = 20$ in.

$$(50\text{ lb/in.})(25\text{ in.}^2)\left(\theta - \frac{\pi}{6}\right) - (100\text{ lb})(20\text{ in.})\sin\theta = 0$$

or $$1250\theta - 654.5 - 200\sin\theta = 0$$

Solving numerically, $$\theta = 1.9870\text{ rad } = 113.8°$$

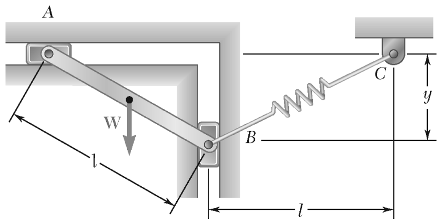
$\theta = 113.8°$ ◀

Stability: Equation (2), Problem 75:

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta$$

or $$= 1250 - 2000\cos\theta$$

For $\theta = 113.8°$: $$= 2057\text{ in.}\cdot\text{lb} > 0$$ Stable ◀

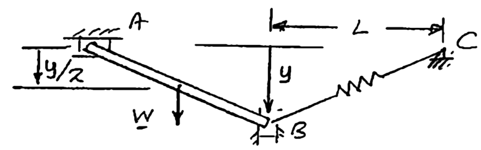


PROBLEM 10.77

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. Knowing that the spring is unstretched when $y = 0$, determine the value of *y* corresponding to equilibrium when $W = 80$ N, $l = 500$ mm, and $k = 600$ N/m.

SOLUTION

Deflection of spring = *s*, where $s = \sqrt{l^2 + y^2} - l$



$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 - y^2}}$$

Potential energy: $V = \frac{1}{2}ks^2 - W\frac{y}{2}$

$$\frac{dV}{dy} = ks\frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k\left(\sqrt{l^2 + y^2} - l\right)\frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k\left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y - \frac{1}{2}W$$

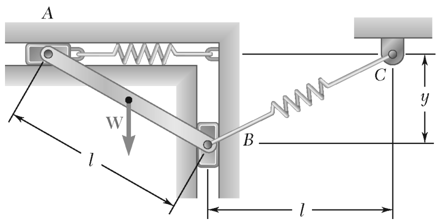
Equilibrium $\frac{dV}{dy} = 0: \quad \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y = \frac{1}{2}\frac{W}{k}$

Now $W = 80$ N, $l = 0.500$ m, and $k = 600$ N/m

Then $\left(1 - \frac{0.500 \text{ m}}{\sqrt{(0.500)^2 + y^2}}\right)y = \frac{1}{2}\frac{(80 \text{ N})}{(600 \text{ N/m})}$

or $\left(1 - \frac{0.500}{\sqrt{0.25 + y^2}}\right)y = 0.066667$

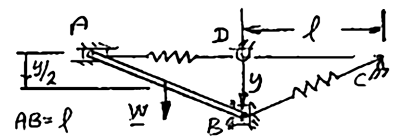
Solving numerically, $y = 0.357$ m $\qquad y = 357$ mm ◀



PROBLEM 10.78

A slender rod AB, of weight W, is attached to two blocks A and B that can move freely in the guides shown. Knowing that both springs are unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $W = 80$ N, $l = 550$ mm, and $k = 600$ N/m.

SOLUTION



Spring deflections

$$S_{AD} = l - \sqrt{l^2 - y^2}$$

$$S_{BC} = \sqrt{l^2 + y^2} - l$$

$$V = \frac{1}{2}kS_{AD}^2 + \frac{1}{2}kS_{BC}^2 - W\frac{y}{2}$$

$$V = \frac{1}{2}k\left(l - \sqrt{l^2 - y^2}\right)^2 + \frac{1}{2}k\left(\sqrt{l^2 + y^2} - l\right)^2 - W\frac{y}{2}$$

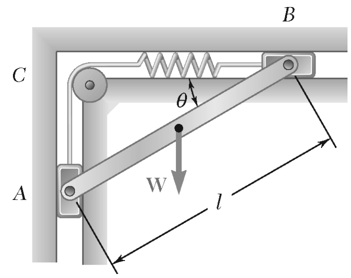
$$\frac{dV}{dy} = k\left(l - \sqrt{l^2 - y^2}\right)\left(\frac{y}{\sqrt{l^2 - y^2}}\right) + k\left(\sqrt{l^2 + y^2} - l\right)\left(\frac{y}{\sqrt{l^2 + y^2}}\right) - \frac{W}{2}$$

$$\frac{dV}{dy} = 0: \quad \left[\left(\frac{l}{\sqrt{l^2 - y^2}} - 1\right) + \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)\right] y = \frac{W}{2k}$$

Data: $W = 80$ N, $l = 0.5$ m, $k = 600$ N/m

$$\left[\frac{0.5}{\sqrt{(0.5)^2 - y^2}} - \frac{0.5}{\sqrt{(0.5)^2 + y^2}}\right] y = \frac{80}{2(1200)} = 0.066667$$

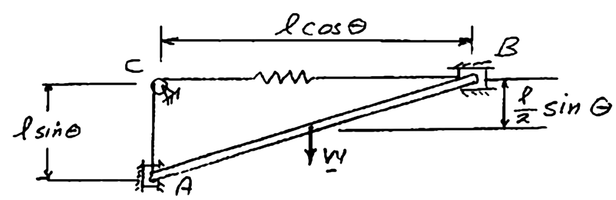
Solve by trial and error: $y = 0.252$ m $\qquad y = 252$ mm ◀



PROBLEM 10.79

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when *AB* is horizontal. Neglecting the weight of the blocks, derive an equation in θ, *W*, *l*, and *k* that must be satisfied when the rod is in equilibrium.

SOLUTION



Elongation of spring: $s = l\sin\theta + l\cos\theta - l$

$s = l(\sin\theta + \cos\theta - 1)$

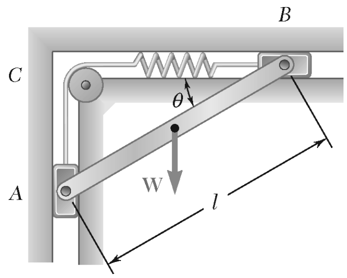
Potential energy: $V = \frac{1}{2}ks^2 - W\frac{1}{2}\sin\theta \quad W = mg$

$= \frac{1}{2}kl^2(\sin\theta + \cos\theta - 1)^2 - mg\frac{l}{2}\sin\theta$

$$\frac{dV}{d\theta} = kl^2(\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{1}{2}mgl\cos\theta \qquad (1)$$

Equilibrium: $\frac{dV}{d\theta} = 0: \quad (\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{mg}{2kl}\cos\theta = 0$

or $\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0$ ◀



PROBLEM 10.80

A slender rod AB, of weight W, is attached to two blocks A and B that can move freely in the guides shown. Knowing that the spring is unstretched when AB is horizontal, determine three values of θ corresponding to equilibrium when $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the results of Problem 10.79, particularly the condition of equilibrium

$$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0$$

Now, with $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in.

$$\frac{W}{2kl} = \frac{300 \text{ lb}}{(16 \text{ in.})(75 \text{ lb/in.})} = 0.25$$

Thus: $\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - 0.25\right] = 0$

$$\cos\theta = 0 \quad \text{and} \quad (\sin\theta + \cos\theta - 1)(1 - \tan\theta) = 0.25$$

First equation yields $\theta = 90°$. Solving the second equation by trial, we find $\theta = 9.39°$ and $34.16°$

Values of θ for equilibrium are

$$\theta = 9.39°,\ 34.2°, \text{ and } 90.0°$$

Stability: we differentiate Eq. (1).

$$\frac{d^2y}{ds^2} = kl^2[(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta - 1)(-\sin\theta - \cos\theta)] + \frac{1}{2}wl\sin\theta$$

$$= kl^2\left[\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta - \sin^2\theta - \cos^2\theta - 2\cos\theta\sin\theta + \sin\theta + \cos\theta + \frac{W}{2kl}\sin\theta\right]$$

$$= kl^2\left[\left(1 + \frac{W}{2kl}\right)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

$$\frac{d^2V}{d\theta^2} = kl^2(1.25\sin\theta + \cos\theta - 2\sin 2\theta)$$

$\theta = 9.39°$:

$$\frac{d^2V}{d\theta^2} = kl^2(1.25\sin 9.4 + \cos 9.4 - 2\sin 18.8)$$

$$= kl^2(+0.55) < 0$$

Stable ◀

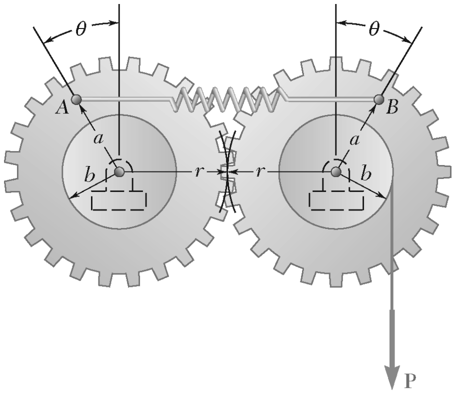
PROBLEM 10.80 (Continued)

$\theta = 34.2°$: $$\frac{d^2v}{d\theta^2} = kl^2(1.25\sin 34.2° + \cos 34.3° - 2\sin 68.4°)$$

$$= kl^2(-0.33) < 0$$ Unstable ◀

$\theta = 90.0°$: $$\frac{d^2V}{d\theta^2} = kl^2(1.25\sin 90° + \cos 90° - 2\sin 180°)$$

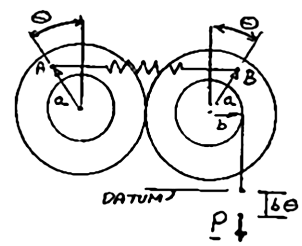
$$= kl^2(1.25) > 0$$ Stable ◀



PROBLEM 10.81

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, determine two values of the angle θ corresponding to equilibrium when $P = 30$ lb, $a = 4$ in., $b = 3$ in., $r = 6$ in., and $k = 5$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Elongation of spring

$$s = 2(a\sin\theta) = 2a\sin\theta$$

$$V = \frac{1}{2}ks^2 - Pb\theta$$

$$= \frac{1}{2}k(2a\sin\theta)^2 - Pb\theta$$

$$\frac{dV}{d\theta} = 4ka^2\sin\theta\cos\theta - Pb$$

$$= 2ka^2\sin 2\theta - Pb \qquad (1)$$

Equilibrium $\quad \dfrac{dV}{d\theta} = 0: \quad \sin 2\theta = \dfrac{Pb}{2ka^2}$

$$\sin 2\theta = \frac{(30\text{ lb})(3\text{ in.})}{2(5\text{ lb/in.})(4\text{ in.})^2}; \quad \sin 2\theta = 0.5625$$

$$2\theta = 34.229° \quad \text{and} \quad 145.771°$$

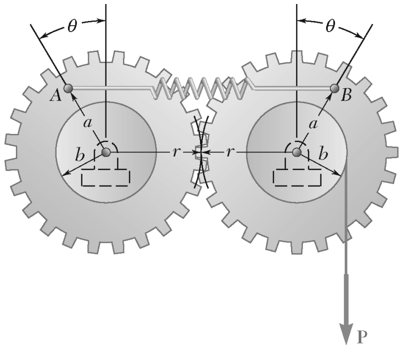
$$\theta = 17.11° \quad \text{and} \quad 72.9° \blacktriangleleft$$

Stability: We differentiate Eq. (1)

$$\frac{d^2V}{d\theta^2} = 4ka^2\cos 2\theta$$

$\theta = 17.11°: \quad \dfrac{d^2v}{d\theta^2} = 4ka^2\cos 34.2° = 4ka^2(0.83) > 0$ Stable ◀

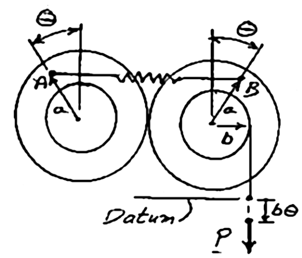
$\theta = 72.9°: \quad \dfrac{d^2v}{d\theta^2} = 4ka^2\cos 145.8° = 4ka^2(-0.83) < 0$ Unstable ◀



PROBLEM 10.82

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, and given that $a = 60$ mm, $b = 45$ mm, $r = 90$ mm, and $k = 6$ kN/m, determine (a) the range of values of P for which a position of equilibrium exists, (b) two values of θ corresponding to equilibrium if the value of P is equal to half the upper limit of the range found in part a.

SOLUTION



Elongation of spring

$$s = 2(a\sin\theta) = 2a\sin\theta$$

Potential energy

$$V = \frac{1}{2}ks^2 - Pb\theta = \frac{1}{2}k(2a\sin\theta)^2 - Pb\theta$$

$$V = 2ka^2\sin^2\theta - Pb\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad \frac{dV}{d\theta} = 4ka^2\sin\theta\cos\theta - Pb \tag{1}$$

$$= 2ka^2\sin 2\theta - Pb = 0$$

$$\sin 2\theta = \frac{Pb}{2ka^2}; \quad \text{For} \quad P_{max}; \ \frac{P_{max}b}{2ka^2} = 1$$

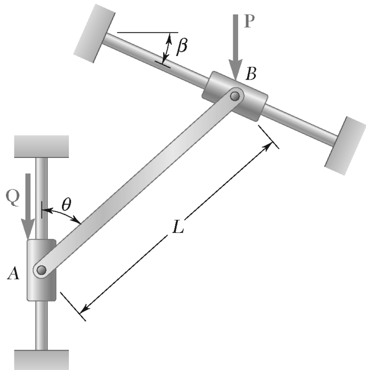
(a)

$$\frac{P_{max}(0.045\text{ m})}{2(6000\text{ N/m})(0.06\text{ m})^2} = 1$$

$P_{max} = 960$ N ◀

(b) For $P = \frac{1}{2}P_{max}$, $\quad \sin 2\theta = \frac{1}{2}; 2\theta = 30°$ and $150°$

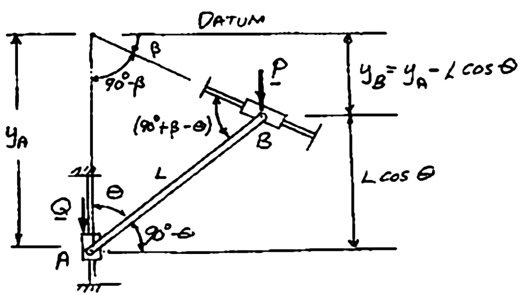
$\theta = 15.00°$ and $75.0°$ ◀



PROBLEM 10.83

A slender rod AB is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30°$ and $P = Q = 400$ N, determine the value of the angle θ corresponding to equilibrium.

SOLUTION



Law of Sines

$$\frac{y_A}{\sin(90° + \beta - \theta)} = \frac{L}{\sin(90 - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{L}{\cos\beta}$$

or

$$y_A = L\frac{\cos(\theta - \beta)}{\cos\beta}$$

From the figure:

$$y_B = L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta$$

Potential Energy:

$$V = -Py_B - Qy_A = -P\left[L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta\right] - QL\frac{\cos(\theta - \beta)}{\cos\beta}$$

$$\frac{dV}{d\theta} = -PL\left[-\frac{\sin(\theta - \beta)}{\cos\beta} + \sin\theta\right] + QL\frac{\sin(\theta - \beta)}{\cos\beta}$$

$$= L(P + Q)\frac{\sin(\theta - \beta)}{\cos\beta} - PL\sin\theta$$

PROBLEM 10.83 (Continued)

Equilibrium $$\frac{dV}{d\theta} = 0: \quad L(P+Q)\frac{\sin(\theta-\beta)}{\cos\beta} - PL\sin\theta = 0$$

or $$(P+Q)\sin(\theta-\beta) = P\sin\theta\cos\beta$$

$$(P+Q)(\sin\theta\cos\beta - \cos\theta\sin\beta) = P\sin\theta\cos\beta$$

or $$-(P+Q)\cos\theta\sin\beta + Q\sin\theta\cos\beta = 0$$

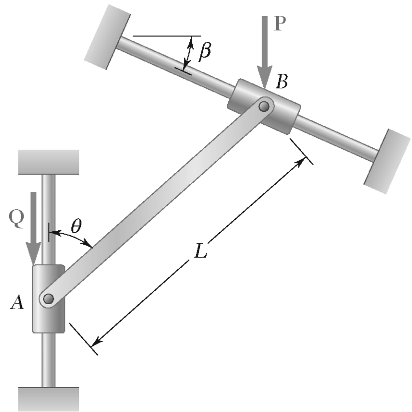
$$-\frac{P+Q}{Q}\frac{\sin\beta}{\cos\beta} + \frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = \frac{P+Q}{Q}\tan\beta \tag{2}$$

With $$P = Q = 400 \text{ N}, \quad \beta = 30°$$

$$\tan\theta = \frac{800 \text{ N}}{400 \text{ N}}\tan 30° = 1.1547$$

$\theta = 49.1°$ ◀



PROBLEM 10.84

A slender rod AB is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30°$, $P = 100$ N, and $Q = 25$ N, determine the value of the angle θ corresponding to equilibrium.

SOLUTION

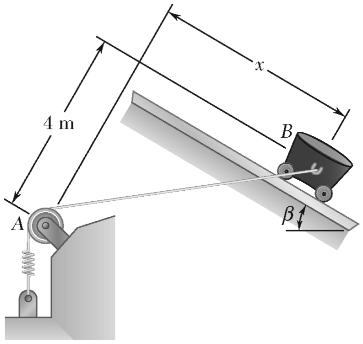
Using Equation (2) of Problem 10.83, with $P = 100$ N, $Q = 25$ N, and $\beta = 30°$, we have

$$\tan\theta = \frac{(100\text{ N})(25\text{ N})}{(25\text{ N})}\tan 30°$$

$$= 57.735$$

$$\theta = 89.007°$$

$\theta = 89.0°$ ◀

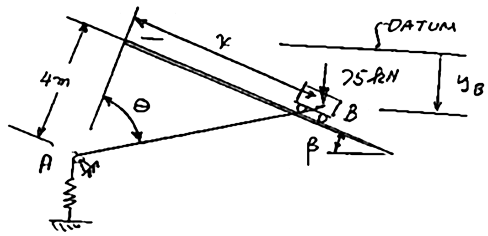


PROBLEM 10.85

Cart B, which weighs 75 kN, rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m, and the spring is unstretched when $x = 0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 30°$.

SOLUTION



$$x = (4\text{ m})\tan\theta \qquad (1)$$

$$y_B = x\sin\beta = 4\tan\theta\sin\beta$$

$$AC = (4\text{ m})\cos\theta$$

For $x = 0$, $\quad (AC)_0 = 4\text{ m}$

Stretch of spring. $\quad s = AC - (AC)_0 = \dfrac{4}{\cos\theta} - 4 = 4\left(\dfrac{1}{\cos\theta} - 1\right)$

$$V = \frac{1}{2}ks^2 - (75\text{ kN})y_B$$

$$= \frac{1}{2}(5\text{ kN/m})\,16\left(\frac{1}{\cos\theta} - 1\right)^2 - (75\text{ kN})4\tan\theta\sin\beta$$

$$\frac{dV}{d\theta} = 80\left(\frac{1}{\cos\theta} - 1\right)\frac{\sin\theta}{\cos^2\theta} - 300\frac{\sin\beta}{\cos^2\theta}$$

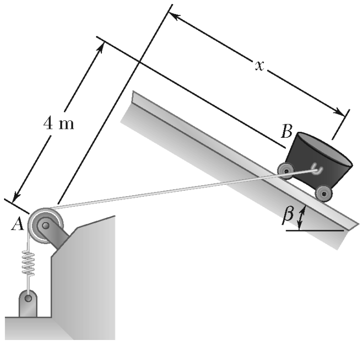
Equilibrium $\quad \dfrac{dV}{d\theta} = 0: \quad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75\sin\beta \qquad (2)$

Given: $\quad \beta = 30°, \quad \sin\theta = 0.5$

Eq. (2): $\quad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75(0.5) = 1.875$

Solve by trial and error: $\quad \theta = 70.46°$

Eq. (1): $\quad x = (4\text{ m})\tan 70.46° \qquad x = 11.27\text{ m}$ ◀

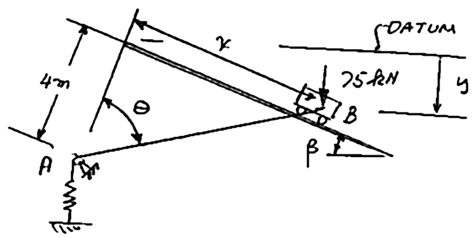


PROBLEM 10.86

Cart B, which weighs 75 kN, rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m, and the spring is unstretched when $x = 0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 60°$.

SOLUTION



$$x = (4 \text{ m})\tan\theta \qquad (1)$$

$$y_B = x\sin\beta = 4\tan\theta\sin\beta$$

$$AC = (4 \text{ m})\cos\theta$$

For $x = 0$, $\quad (AC)_0 = 4 \text{ m}$

Stretch of spring: $\quad s = AC - (AC)_0 = \dfrac{4}{\cos\theta} - 4 = 4\left(\dfrac{1}{\cos\theta} - 1\right)$

$$V = \frac{1}{2}ks^2 - (75 \text{ kN})y_B$$

$$= \frac{1}{2}(5 \text{ kN/m})\,16\left(\frac{1}{\cos\theta} - 1\right)^2 - (75 \text{ kN})4\tan\theta\sin\beta$$

$$\frac{dV}{d\theta} = 80\left(\frac{1}{\cos\theta} - 1\right)\frac{\sin\theta}{\cos^2\theta} - 300\frac{\sin\beta}{\cos^2\theta}$$

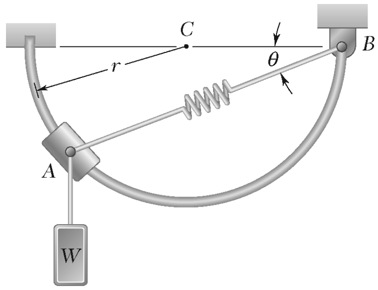
<u>Equilibrium</u> $\quad \dfrac{dV}{d\theta} = 0: \quad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75\sin\beta \qquad (2)$

<u>Given</u>: $\quad \beta = 60°, \sin\theta = 0.86603$

Eq. (2): $\quad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75(0.86603) = 3.2476$

Solve by trial and error: $\quad \theta = 76.67°$

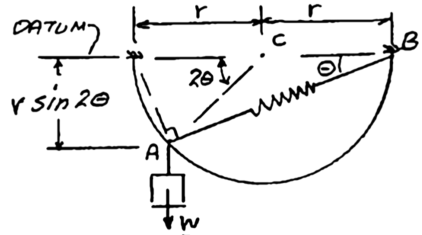
Eq. (1): $\quad x = (4 \text{ m})\tan 26.67° \qquad x = 16.88 \text{ m}$ ◀



PROBLEM 10.87

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $W = 50$ lb, $r = 9$ in., and $k = 15$ lb/in.

SOLUTION



Stretch of spring

$$s = AB - r$$

$$s = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

Potential energy:

$$V = \frac{1}{2}ks^2 - Wr\sin 2\theta \qquad W = mg$$

$$V = \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\sin 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta - 2Wr\cos 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta - Wr\cos 2\theta = 0$$

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -\frac{W}{kr}$$

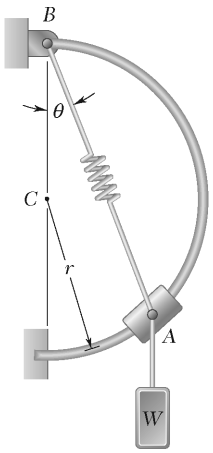
Now

$$\frac{W}{kr} = \frac{(50 \text{ lb})}{(15 \text{ lb/in.})(9 \text{ in.})} = 0.37037$$

Then

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -0.37037$$

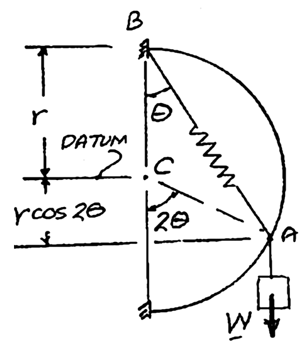
Solving numerically, $\theta = 0.95637 \text{ rad} = 54.8°$ $\qquad \theta = 54.8°$ ◀



PROBLEM 10.88

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $W = 50$ lb, $r = 9$ in., and $k = 15$ lb/in.

SOLUTION



Stretch of spring

$$s = AB - r = 2(r\cos\theta) - r$$
$$s = r(2\cos\theta - 1)$$
$$V = \frac{1}{2}ks^2 - Wr\cos 2\theta$$
$$= \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\cos 2\theta$$
$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta + 2Wr\sin 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta + Wr\sin 2\theta = 0$$
$$-kr^2(2\cos\theta - 1)\sin\theta + Wr(2\sin\theta\cos\theta) = 0$$

or

$$\frac{(2\cos\theta - 1)\sin\theta}{2\cos\theta} = \frac{W}{kr}$$

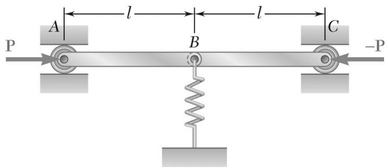
Now

$$\frac{W}{kr} = \frac{(50\text{ lb})}{(15\text{ lb/in.})(9\text{ in.})} = 0.37037$$

Then

$$\frac{2\cos\theta - 1}{2\cos\theta} = 0.37037$$

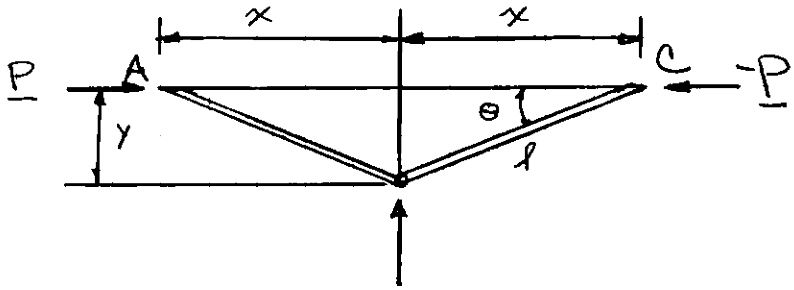
Solving $\theta = 37.4°$ ◀



PROBLEM 10.89

Two bars *AB* and *BC* of negligible weight are attached to a single spring of constant *k* that is unstretched when the bars are horizontal. Determine the range of values of the magnitude *P* of two equal and opposite forces **P** and **–P** for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$x = l\cos\theta$$
$$y = l\sin\theta$$

$$V = 2Px + \frac{1}{2}ky^2$$

$$v = 2Pl\cos\theta + \frac{1}{2}kl^2\sin^2\theta$$

$$\frac{dV}{d\theta} = -2Pl\sin\theta + kl^2\sin\theta\cos\theta$$

$$= -2Pl\sin\theta + \frac{1}{2}kl^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -2Pl\cos\theta + kl^2\cos 2\theta \qquad (1)$$

For equilibrium position $\theta = 0$ to be stable

$$\frac{d^2V}{d\theta^2} = -2Pl + kl^2 > 0 \qquad (2)$$

$$P < \frac{1}{2}kl \blacktriangleleft$$

PROBLEM 10.89 (Continued)

Note: For $P = \frac{1}{2}kl$, we have $\frac{d^2V}{d\theta^2} = 0$ and we must determine which is the first derivative to be $\neq 0$.

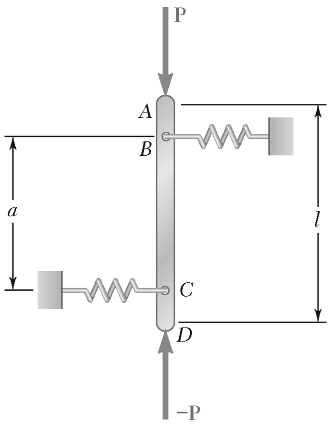
Differentiating Eq. (1):

$$\frac{d^3V}{d\theta^3} = +2Pl\sin\theta - 2kl^2\sin 2\theta = 0 \text{ for } \theta = 0$$

$$\frac{d^4V}{d\theta^4} = 2Pl\cos\theta - 4kl^2\cos 2\theta = 2Pl - 4kl^2 \text{ for } \theta = 0$$

But $P = \frac{1}{2}kl$. Thus $\frac{d^4V}{d\theta^4} = kl^2 - 4kl^2 < 0$ and we conclude that the equilibrium is unstable for $P = \frac{1}{2}kl$.

The sign < in Eq. (2) is thus correct.



PROBLEM 10.90

A vertical bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Determine the range of values of the magnitude P of two equal and opposite vertical forces **P** and **−P** for which the equilibrium position is stable if (*a*) $AB = CD$, (*b*) $AB = 2CD$.

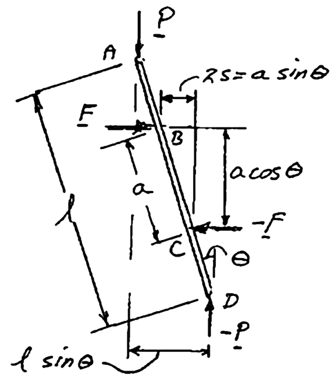
SOLUTION

For both (*a*) and (*b*): Since **P** and **−P** are vertical, they form a couple of moment

$$M_P = +Pl\sin\theta$$

The forces **F** and **−F** exerted by springs must, therefore, also form a couple, with moment

$$M_F = -Fa\cos\theta$$



We have

$$dU = M_P d\theta + M_F d\theta$$
$$= (Pl\sin\theta - Fa\cos\theta)d\theta$$

but

$$F = ks = k\left(\frac{1}{2}a\sin\theta\right)$$

Thus,

$$dU = \left(Pl\sin\theta - \frac{1}{2}ka^2\sin\theta\cos\theta\right)d\theta$$

From Equation (10.19), page 580, we have

$$dV = -dU = -Pl\sin\theta d\theta + \frac{1}{4}ka^2\sin 2\theta d\theta$$

or

$$\frac{dV}{d\theta} = -Pl\sin\theta + \frac{1}{4}ka^2\sin 2\theta$$

and

$$\frac{d^2V}{d\theta^2} = -Pl\cos\theta + \frac{1}{2}ka^2\cos 2\theta \qquad (1)$$

For $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = -Pl + \frac{1}{2}ka^2$$

For Stability: $\dfrac{d^2V}{d\theta^2} > 0, \quad -Pl + \dfrac{1}{2}ka^2 > 0$

or (for Parts *a* and *b*) $P < \dfrac{ka^2}{2l}$ ◀

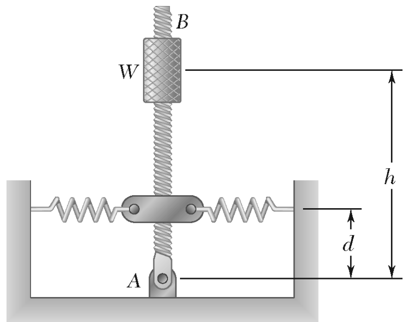
Note: To check that equilibrium is unstable for $P = \frac{ka^2}{2l}$, we differentiate (1) twice:

$$\frac{d^3V}{d\theta^3} = +Pl\sin\theta - ka^2\sin 2\theta = 0, \quad \text{for} \quad \theta = 0,$$

$$\frac{d^4V}{d\theta^4} = Pl\cos\theta - 2ka^2\cos 2\theta$$

For $\theta = 0$ $\dfrac{d^4V}{d\theta^4} = Pl - 2ka^2 = \dfrac{ka^2}{2} - 2ka^2 < 0$

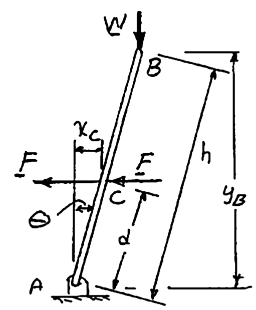
Thus, equilibrium is unstable when $P = \dfrac{ka^2}{2l}$



PROBLEM 10.91

Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 25$ in., $d = 12$ in., and $W = 80$ lb, determine the range of values of k for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION



We have $\qquad x_C = d\sin\theta \qquad y_B = h\cos\theta$

Potential Energy:

$$V = 2\left(\frac{1}{2}kx_C^2 + Wy_B\right)$$

$$= kd^2\sin^2\theta + Wh\cos\theta$$

Then

$$\frac{dV}{d\theta} = 2kd^2\sin\theta\cos\theta - Wh\sin\theta$$

$$= kd^2\sin 2\theta - Wh\sin\theta$$

and

$$\frac{d^2V}{d\theta^2} = 2kd^2\cos 2\theta - Wh\cos\theta \qquad (1)$$

For equilibrium position $\theta = 0$ to be stable, we must have

$$\frac{d^2V}{d\theta^2} = 2kd^2 - Wh > 0$$

or

$$kd^2 > \frac{1}{2}Wh \qquad (2)$$

Note: For $kd^2 = \frac{1}{2}Wh$, we have $\frac{d^2V}{d\theta^2} = 0$, so that we must determine which is the first derivative that is not equal to zero. Differentiating Equation (1), we write

$$\frac{d^3V}{d\theta^3} = -4kd^2\sin 2\theta + Wh\sin\theta = 0 \qquad \text{for } \theta = 0$$

$$\frac{d^4V}{d\theta^2} = -8kd^2\cos 2\theta + Wh\cos\theta$$

For $\theta = 0$:

$$\frac{d^4V}{d\theta^4} = -8kd^2 + Wh$$

PROBLEM 10.91 (Continued)

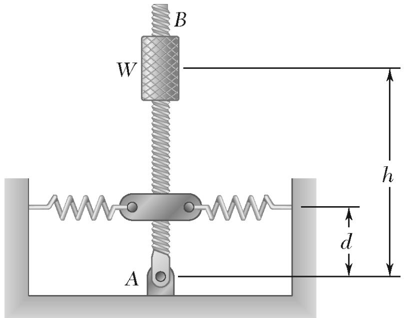
Since $kd^2 = \frac{1}{2}Wh$, $\frac{d^4V}{d\theta^4} = -4Wh + Wh < 0$, we conclude that the equilibrium is unstable for $kd^2 = \frac{1}{2}Wh$ and the $>$ sign in Equation (2) is correct.

With $\quad W = 80 \text{ lb}, \quad h = 25 \text{ in.}, \quad \text{and} \quad d = 12 \text{ in.}$

Equation (2) gives $\quad k(12 \text{ in.})^2 > \dfrac{1}{2}(80 \text{ lb})(25 \text{ in.})$

or $\quad k > 6.944 \text{ lb/in.} \qquad k > 6.94 \text{ lb/in.}$ ◀

PROBLEM 10.92



Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 45$ in., $k = 6$ lb/in., and $W = 60$ lb, determine the smallest distance d for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

Using Equation (2) of Problem 10.91 with

$$h = 45 \text{ in.}, \quad k = 6 \text{ lb/in.}, \quad \text{and} \quad W = 60 \text{ lb}$$

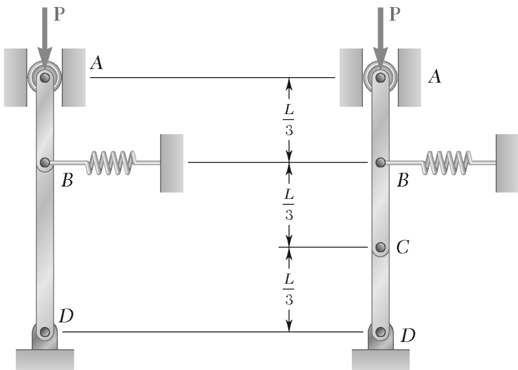
$$(6 \text{ lb/in.})d^2 > \frac{1}{2}(60 \text{ lb})(45 \text{ in.})$$

or

$$d^2 > 225 \text{ in.}^2$$

$$d > 15.0000 \text{ in.}$$

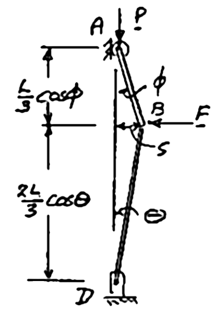
smallest $d = 15.00$ in. ◀



PROBLEM 10.93

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$s = \frac{L}{3}\sin\phi = \frac{2L}{3}\sin\theta$$

For small values of ϕ and θ

$$\phi = 2\theta$$

$$V = P\left(\frac{L}{3}\cos\phi + \frac{2L}{3}\cos\theta\right) + \frac{1}{2}ks^2$$

$$V = \frac{PL}{3}(\cos 2\theta + 2\cos\theta) + \frac{1}{2}k\left(\frac{2L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}(-2\sin 2\theta - 2\sin\theta) + \frac{2}{9}kL^2\sin\theta\cos\theta$$

$$= -\frac{PL}{3}(2\sin 2\theta + 2\sin\theta) + \frac{2}{9}kL^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{PL}{3}(4\cos 2\theta + 2\cos\theta) + \frac{4}{9}kL^2\cos 2\theta$$

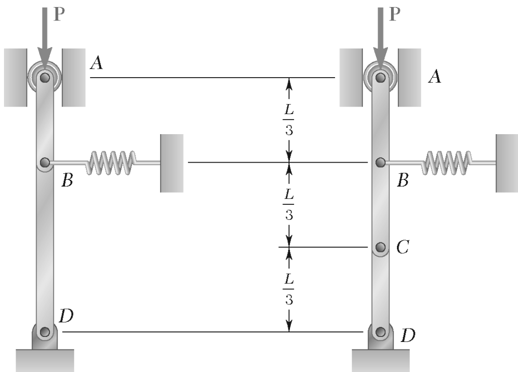
when $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = -\frac{6PL}{3} + \frac{4}{9}kL^2$$

<u>For stability</u>:

$$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{4}{9}kL^2 > 0$$

$$P < \frac{2}{9}kL \blacktriangleleft$$

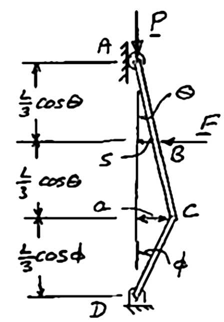


PROBLEM 10.94

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION

$$a = \frac{2L}{3}\sin\theta = \frac{L}{3}\sin\phi$$



For small values of ϕ and θ

$$\phi = 2\theta$$

$$s = \frac{L}{3}\sin\theta$$

$$V = P\left(\frac{2L}{3}\cos\theta + \frac{L}{3}\cos\phi\right) + \frac{1}{2}ks^2$$

$$= \frac{PL}{3}(2\cos\theta + \cos 2\theta) + \frac{1}{2}k\left(\frac{L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}(-2\sin\theta - 2\sin 2\theta) + \frac{kL^2}{9}\sin\theta\cos\theta$$

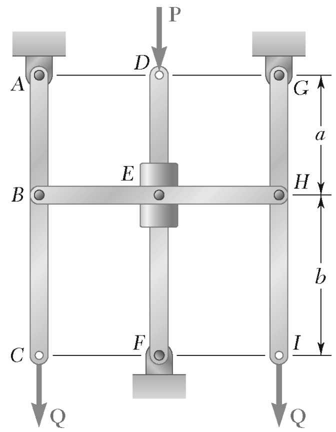
$$\frac{dV}{d\theta} = -\frac{2PL}{3}(\sin\theta + \sin 2\theta) + \frac{kL^2}{18}\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{2PL}{3}(\cos\theta + 2\cos 2\theta) + \frac{kL^2}{9}\cos 2\theta$$

when $\theta = 0$:
$$\frac{dV^2}{d\theta^2} = -2PL + \frac{kL^2}{9}$$

For stability:
$$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{kL^2}{9} > 0$$

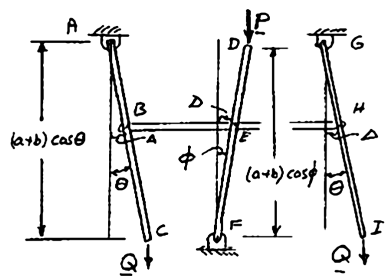
$$P < \frac{1}{18}kL \blacktriangleleft$$



PROBLEM 10.95

The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of *Q* for which the equilibrium of the system is stable in the position shown when $a = 24$ in., $b = 20$ in., and $P = 150$ lb.

SOLUTION



First note $\qquad A = a\sin\theta = b\sin\phi$

For small values of θ and ϕ: $\qquad a\theta = b\phi$

or $\qquad \phi = \dfrac{a}{b}\theta$

$$V = P(a+b)\cos\phi - 2Q(a+b)\cos\theta$$

$$= (a+b)\left[P\cos\left(\frac{a}{b}\theta\right) - 2Q\cos\theta\right]$$

$$\frac{dV}{d\theta} = (a+b)\left[-\frac{a}{b}P\sin\left(\frac{a}{b}\theta\right) + 2Q\sin\theta\right]$$

$$\frac{d^2V}{d\theta^2} = (a+b)\left[-\frac{a^2}{b^2}P\cos\left(\frac{a}{b}\theta\right) + 2Q\cos\theta\right]$$

when $\theta = 0$: $\qquad \dfrac{d^2V}{d\theta^2} = (a+b)\left(-\dfrac{a^2}{b^2}P + 2Q\right)$

PROBLEM 10.95 (Continued)

Stability: $$\frac{d^2V}{d\theta^2} > 0: \quad -\frac{a^2}{b^2}P + 2Q > 0$$

$$P < 2\frac{b^2}{a^2}Q \qquad (1)$$

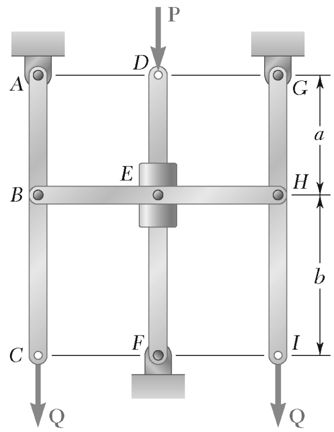
or

$$Q > \frac{a^2}{2b^2}P \qquad (2)$$

with $P = 150$ lb, $a = 24$ in., and $b = 20$ in.

Equation (1): $$Q > \frac{(24 \text{ in.})^2}{2(20 \text{ in.})^2}(150 \text{ lb}) = 108.000 \text{ lb}$$

For stability $Q > 108.0$ lb ◀



PROBLEM 10.96

The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown when $a = 150$ mm, $b = 200$ mm, and $Q = 45$ N.

SOLUTION

Using Equation (2) of Problem 10.95 with

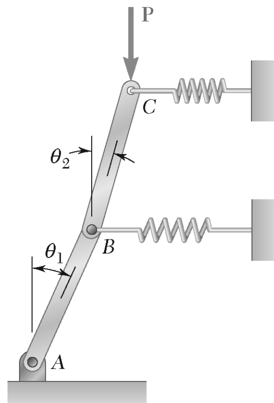
$$Q = 45 \text{ N}, \quad a = 150 \text{ mm}, \quad \text{and} \quad b = 200 \text{ mm}$$

Equation (2)

$$P < 2\frac{(200 \text{ mm})^2}{(150 \text{ mm})^2}(45 \text{ N})$$

$$= 160.000 \text{ N}$$

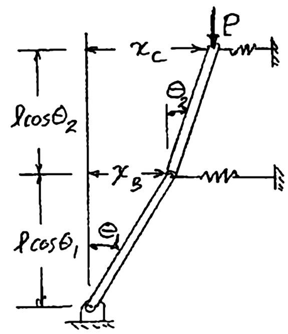
For stability $P < 160.0$ N ◀



PROBLEM 10.97*

Bars AB and BC, each of length l and of negligible weight, are attached to two springs, each of constant k. The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of P for which the equilibrium position is stable.

SOLUTION



We have
$$x_B = l\sin\theta$$
$$x_C = l\sin\theta_1 + l\sin\theta_2$$
$$y_C = l\cos\theta_1 + l\cos\theta_2$$
$$V = Py_C + \frac{1}{2}kx_B^2 + \frac{1}{2}kx_C^2$$

or
$$V = Pl(\cos\theta_1 + \cos\theta_2) + \frac{1}{2}kl^2\left[\sin^2\theta_1 + (\sin\theta_1 + \sin\theta_2)^2\right]$$

For small values of θ_1 and θ_2:

$$\sin\theta_1 \approx \theta_1, \quad \sin\theta_2 \approx \theta_2, \quad \cos\theta_1 \approx 1 - \frac{1}{2}\theta_1^2, \quad \cos\theta_2 \approx 1 - \frac{1}{2}\theta_2^2$$

Then
$$V = Pl\left(1 - \frac{\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2}\right) + \frac{1}{2}kl^2\left[\theta_1^2 + (\theta_1 + \theta_2)^2\right]$$

and
$$\frac{\partial V}{\partial\theta_1} = -Pl\theta_1 + kl^2[\theta_1 + (\theta_1 + \theta_2)]$$
$$\frac{\partial V}{\partial\theta_2} = -Pl\theta_2 + kl^2(\theta_1 + \theta_2)$$
$$\frac{\partial^2 V}{\partial\theta_1^2} = -Pl + 2kl^2 \qquad \frac{\partial^2 V}{\partial\theta_2^2} = -Pl + kl^2$$
$$\frac{\partial^2 V}{\partial\theta_1\partial\theta_2} = kl^2$$

Stability: Conditions for stability (see Page 583).

PROBLEM 10.97* (Continued)

For $\theta_1 = \theta_2 = 0$: $\dfrac{\partial V}{\partial \theta_1} = \dfrac{\partial V}{\partial \theta_2} = 0$ (condition satisfied)

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0$$

Substituting $(kl^2)^2 - (-Pl + 2kl^2)(-Pl + kl) < 0$

$$k^2 l^4 - P^2 l^2 + 3Pkl^3 - 2k^2 l^4 < 0$$

$$P^2 - 3klP + k^2 l^2 > 0$$

Solving $P < \dfrac{3-\sqrt{5}}{2}kl$ or $P > \dfrac{3+\sqrt{5}}{2}kl$

or $P < 0.382kl$ or $P > 2.62kl$

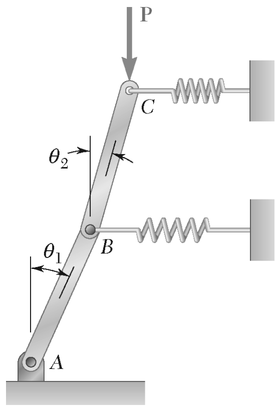
$$\frac{\partial^2 V}{\partial \theta_1^2} > 0: \quad -Pl + 2kl^2 > 0$$

or $$P < \frac{1}{2}kl$$

$$\frac{\partial^2 V}{\partial \theta_2^2} > 0: \quad -Pl + kl^2 > 0$$

or $$P < kl$$

Therefore, all conditions for stable equilibrium are satisfied when $0 \le P < 0.382kl$ ◀



PROBLEM 10.98*

Solve Problem 10.97 knowing that $l = 800$ mm and $k = 2.5$ kN/m.

PROBLEM 10.97* Bars AB and BC, each of length l and of negligible weight, are attached to two springs, each of constant k. The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of P for which the equilibrium position is stable.

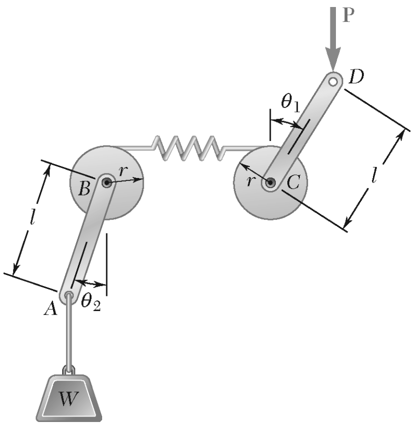
SOLUTION

From the analysis of Problem 10.97 with

$$l = 800 \text{ mm} \quad \text{and} \quad k = 2.5 \text{ kN/m}$$

$$P < 0.382kl = 0.382(2500 \text{ N/m})(0.8 \text{ m}) = 764 \text{ N}$$

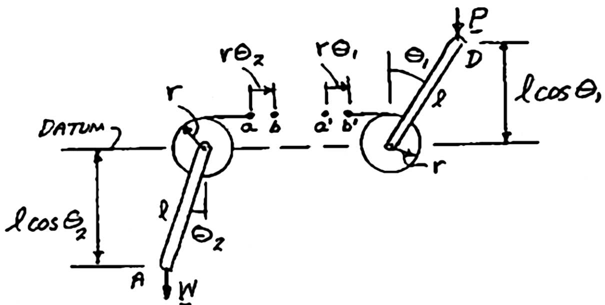
$P < 764$ N ◀



PROBLEM 10.99*

Two rods of negligible weight are attached to drums of radius r that are connected by a belt and spring of constant k. Knowing that the spring is undeformed when the rods are vertical, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION



Left end of spring moves from a to b. Right end of spring moves from a' to b'. Elongation of spring

$$s = a'b' - ab = r\theta_1 - r\theta_2 = r(\theta_1 - \theta_2)$$

$$V = \frac{1}{2}ks^2 + pl\cos\theta_1 - wl\cos\theta_2$$

$$= \frac{1}{2}kr^2(\theta_1 - \theta_2)^2 + pl\cos\theta_1 - wl\cos\theta_2$$

$$\frac{\partial v}{\partial \theta_1} = kr^2(\theta_1 - \theta_2) - pl\sin\theta_1$$

$$\frac{\partial v}{\partial \theta_2} = -kr^2(\theta_1 - \theta_2) + wl\sin\theta_2$$

$$\frac{\partial^2 v}{\partial \theta_1^2} = kr^2 - pl\cos\theta_1$$

$$\frac{\partial^2 v}{\partial \theta_2^2} = kr^2 + wl\cos\theta_2$$

$$\frac{\partial^2 v}{\partial \theta_1 \partial \theta_2} = -kr^2$$

PROBLEM 10.99* (Continued)

For $\theta_1 = \theta_2 = 0$: $\dfrac{\partial^2 v}{\partial\theta_1^2} = kr^2 - pl, \quad \dfrac{\partial^2 v}{\partial\theta_2^2} = +kr^2 + wl, \quad \dfrac{r^2 v}{\partial\theta, \partial\theta_2} = -kr^2$

Conditions for stability (see Page 583)

$$\left(\frac{\partial^2 v}{\partial\theta_1\partial\theta_2}\right)^2 - \frac{\partial^2 v}{\partial\theta_1^2}\cdot\frac{\partial^2 v}{\partial\theta_2^2} < 0$$

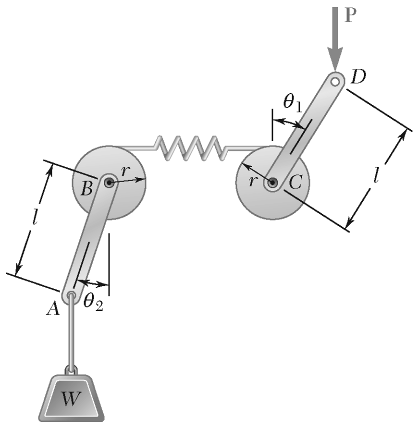
$$(kr^2)^2 - (kr^2 - pl)(kr^2 + wl) < 0$$

$$pl(kr^2 + wl) - kr^2 wl < 0$$

$$P < \frac{wkr^2}{kr^2 + wl}; \quad P < \frac{kr^2}{l}\left(\frac{W}{\frac{kr^2}{l} + w}\right) \quad \triangleleft$$

$$\frac{\partial^2 v}{\partial\theta_1^2} >: \quad kr^2 - pl > 0; \quad P < \frac{kr^2}{l} \quad \triangleleft$$

We choose: $P < \dfrac{kr^2}{l}\left(\dfrac{W}{\frac{kr^2}{l} + W}\right)$ ◀



PROBLEM 10.100*

Solve Problem 10.99 knowing that k = 20 lb/in., r = 3 in., l = 6 in., and (a) W = 15 lb, (b) W = 60 lb.

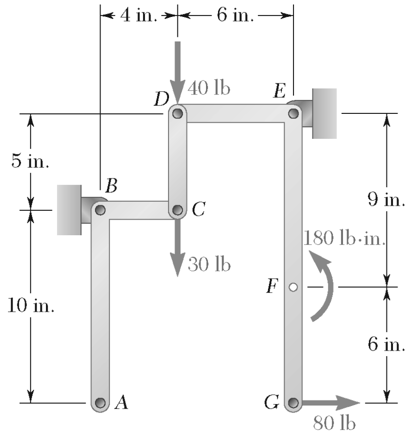
PROBLEM 10.99* Two rods of negligible weight are attached to drums of radius r that are connected by a belt and spring of constant k. Knowing that the spring is undeformed when the rods are vertical, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION

$$k = 20 \text{ lb/in.}$$
$$r = 3 \text{ in.}$$
$$l = 6 \text{ in.}$$
$$\frac{kr^2}{l} = \frac{(20 \text{ lb/in.})(3 \text{ in.})^2}{6 \text{ in.}} = 30 \text{ lb}$$

(a) $W = 15 \text{ lb}: \quad P < (30 \text{ lb})\dfrac{15 \text{ lb}}{(30 \text{ lb}) + (15 \text{ lb})}$ $\qquad P < 10.00 \text{ lb}$ ◀

(b) $W = 60 \text{ lb}: \quad P < (30 \text{ lb})\dfrac{60 \text{ lb}}{(30 \text{ lb}) + (60 \text{ lb})}$ $\qquad P < 20.0 \text{ lb}$ ◀

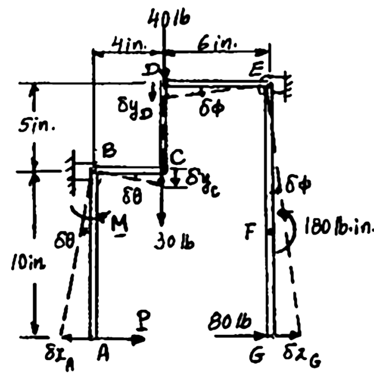


PROBLEM 10.101

Determine the horizontal force **P** that must be applied at A to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta$ ↻



$$\delta x_A = 10\delta\theta \leftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \circlearrowright$$

$$\delta x_G = 15\delta\phi = 15\left(\frac{2}{3}\delta\theta\right) = 10\delta\theta \rightarrow$$

<u>Virtual Work</u>: We shall assume that a force **P** and a couple **M** are applied to member ABC as shown.

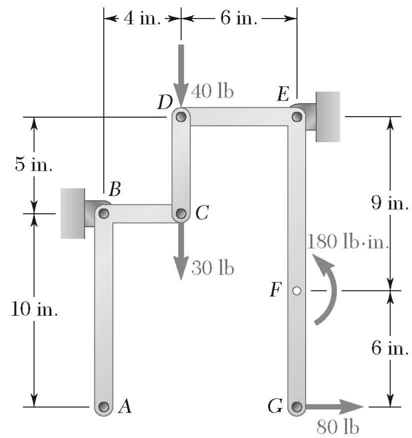
$$\delta U = -P\delta x_A - M\,\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

$$-P(10\delta\theta) - M\,\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

$$-10P - M + 120 + 160 + 120 + 800 = 0$$

$$(10\text{ in.})P + M = 1200\text{ lb}\cdot\text{in.} \qquad (1)$$

Making $M = 0$ in Eq. (1): $P = +120.0\text{ lb}$ $\qquad$ $\mathbf{P} = 120.0\text{ lb} \rightarrow$ ◀



PROBLEM 10.102

Determine the couple **M** that must be applied to member *ABC* to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta$ ↻

$$\delta x_A = 10\delta\theta \leftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

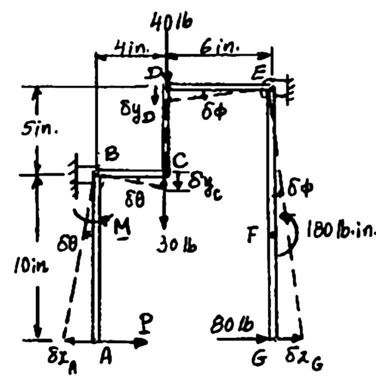
$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \circlearrowright$$

$$\delta x_G = 15\delta\phi$$

$$= 15\left(\frac{2}{3}\delta\theta\right)$$

$$= 10\delta\theta \rightarrow$$



Virtual Work: We shall assume that a force **P** and a couple **M** are applied to member *ABC* as shown.

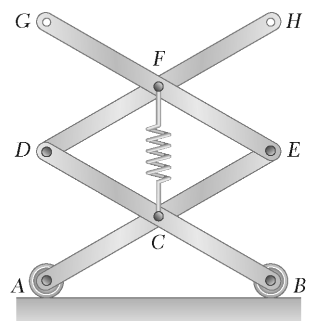
$$\delta U = -P\delta x_A - M\,\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

$$-P(10\delta\theta) - M\,\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

$$-10P - M + 120 + 160 + 120 + 800 = 0$$

$$(10\text{ in.})P + M = 1200\text{ lb}\cdot\text{in.} \qquad (1)$$

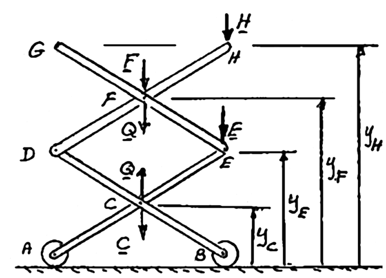
Now from Eq. (1) for $P = 0$ $\qquad$ $\mathbf{M} = 1200\text{ lb}\cdot\text{in.}$ ◀



PROBLEM 10.103

A spring of constant 15 kN/m connects Points C and F of the linkage shown. Neglecting the weight of the spring and linkage, determine the force in the spring and the vertical motion of Point G when a vertical downward 120-N force is applied (*a*) at Point C, (*b*) at Points C and H.

SOLUTION



$$y_G = 4y_C$$
$$y_H = 4y_C \quad \delta y_H = 4\delta y_C$$
$$y_F = 3y_C \quad \delta y_F = 3\delta y_C$$
$$y_E = 2y_C \quad \delta y_E = 2\delta y_C$$

For spring:

$$\Delta = y_F - y_C$$

Q = Force in spring (assumed in tension)

$$Q = +k\Delta = k(y_F - y_C) = k(3y_C - y_C) = 2ky_C \qquad (1)$$

(*a*) $\mathbf{C} = 120 \text{ N}, \quad \mathbf{E} = \mathbf{F} = \mathbf{H} = 0$

Virtual Work:

$$\delta U = 0: \quad -(120 \text{ N})\delta y_C + Q\delta y_C - Q\delta y_F = 0$$
$$-120\delta y_C + Q\delta y_C - Q(3\delta y_C) = 0$$
$$Q = -60 \text{ N} \qquad Q = 60.0 \text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C, \quad -60 \text{ N} = 2(15 \text{ kN/m})y_C, \quad y_C = -2 \text{ mm}$

At Point G: $y_G = 4y_C = 4(-2 \text{ mm}) = -8 \text{ mm} \qquad \mathbf{y}_G = 8.00 \text{ mm}\downarrow \blacktriangleleft$

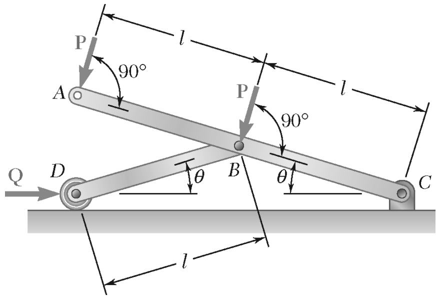
(*b*) $\mathbf{C} = \mathbf{H} = 120 \text{ N}, \mathbf{E} = \mathbf{F} = 0$

Virtual Work:

$$\delta U = 0: \quad -(120 \text{ N})\delta y_C - (120 \text{ N})y_H + Q\delta y_C - Q\delta y_F = 0$$
$$-120\delta y_C - 120(4\delta y_C) + Q\delta y_C - Q(3\delta y_C) = 0$$
$$Q = -300 \text{ N} \qquad Q = 300 \text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C \quad -300 \text{ N} = 2(15 \text{ kN/m})y_C, \quad y_C = -10 \text{ mm}$

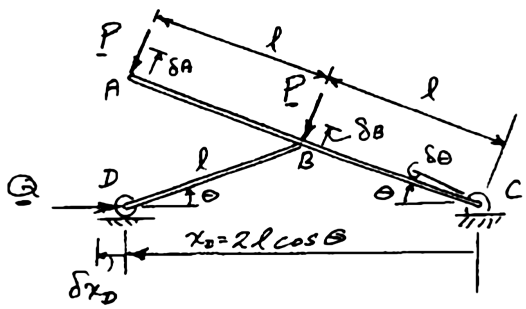
At Point G: $y_G = 4y_C = 4(-10 \text{ mm}) = -40 \text{ mm} \qquad \mathbf{y}_G = 40.0 \text{ mm}\downarrow \blacktriangleleft$



PROBLEM 10.104

Derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the mechanism shown.

SOLUTION



We have $x_D = 2l\cos\theta$ so that $\delta x_D = -2l\sin\theta\delta\theta$

$\delta A = 2l\delta\theta$

$\delta B = l\delta\theta$

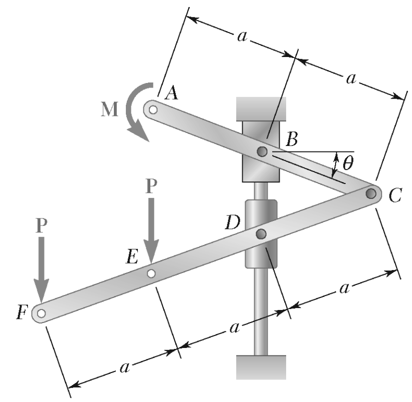
Virtual Work:

$$\delta U = 0: \quad -Q\delta x_D - P\delta A - P\delta B = 0$$

$$-Q(-2l\sin\theta\delta\theta) - P(2l\delta\theta) - P(l\delta\theta) = 0$$

$$2Ql\sin\theta - 3Pl = 0$$

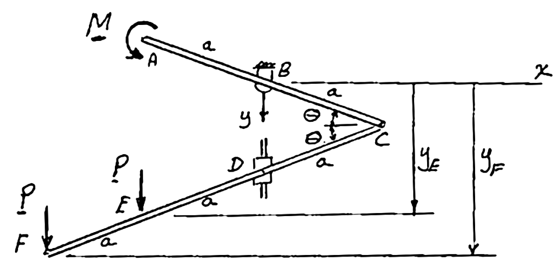
$$Q = \frac{3}{2}\frac{P}{\sin\theta} \blacktriangleleft$$



PROBLEM 10.105

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



$$y_E = 3a\sin\theta \quad \delta y_E = 3a\cos\theta\delta\theta$$

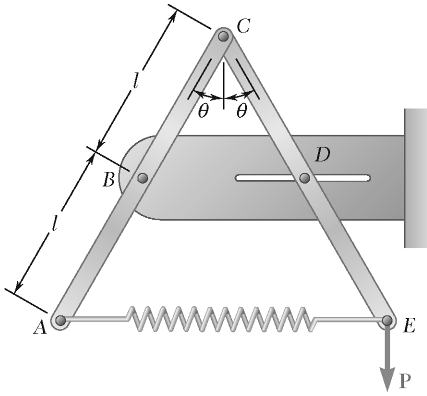
$$y_F = 4a\sin\theta \quad \delta y_F = 4a\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -M\,\delta\theta + P\delta y_E + P\delta y_F = 0$$

$$-M\,\delta\theta + P(3a\cos\theta\delta\theta) + P(4a\cos\theta\delta\theta) = 0$$

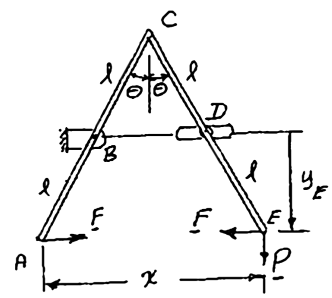
$$M = 7Pa\cos\theta \;◀$$



PROBLEM 10.106

Two rods AC and CE are connected by a pin at C and by a spring AE. The constant of the spring is k, and the spring is unstretched when $\theta = 30°$. For the loading shown, derive an equation in P, θ, l, and k that must be satisfied when the system is in equilibrium.

SOLUTION



$$y_E = l\cos\theta$$

$$\delta y_E = -l\sin\theta\,\delta\theta$$

Spring:

Unstretched length $= 2l$

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\delta\theta$$

$$F = k(x - 2l)$$

$$F = k(4l\sin\theta - 2l)$$

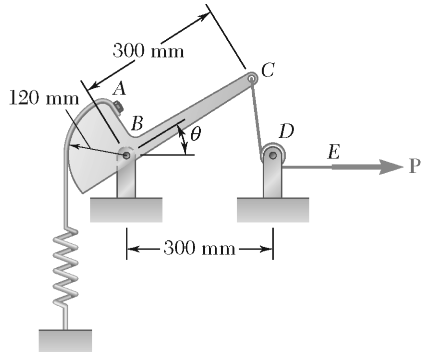
Virtual Work:

$$\delta U = 0: \quad P\delta y_E - F\delta x = 0$$

$$P(-l\sin\theta\,\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\,\delta\theta) = 0$$

$$-P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

or $$\frac{P}{8kl} = (1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta} \qquad \frac{P}{8kl} = \frac{1-2\sin\theta}{\tan\theta} \blacktriangleleft$$



PROBLEM 10.107

A force **P** of magnitude 240 N is applied to end *E* of cable *CDE*, which passes under pulley *D* and is attached to the mechanism at *C*. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium. The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 90°$.

SOLUTION

$$s = r\left(\frac{\pi}{2} - \theta\right)$$

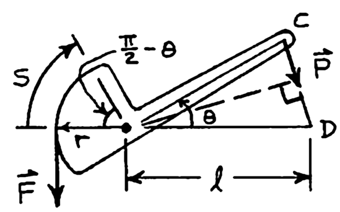
$$\delta s = -r\delta\theta$$

$$F = ks = kr\left(\frac{\pi}{2} - \theta\right)$$

$$CD = 2l\sin\frac{\theta}{2}$$

$$\delta(CD) = 2l\cos\frac{\theta}{2}\left(\frac{1}{2}\delta\theta\right)$$

$$= l\cos\frac{\theta}{2}\delta\theta$$



Virtual Work:

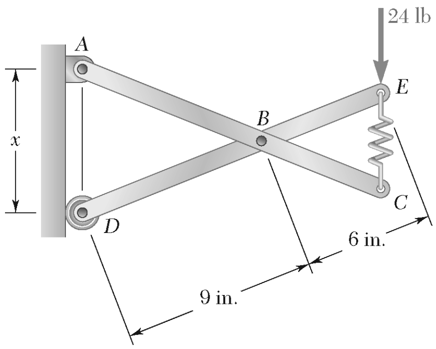
Since **F** tends to decrease s and **P** tends to decrease CD, we have

$$\delta U = -F\delta s - P\delta(CD) = 0$$

$$-kr\left(\frac{\pi}{2} - \theta\right)(-r\delta\theta) - P\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$\frac{\frac{\pi}{2} - \theta}{\cos\frac{\theta}{2}} = \frac{pl}{kr^2} = \frac{(240\text{ N})(0.3\text{ m})}{(4000\text{ N/m})(0.12\text{ m})^2} = 1.25$$

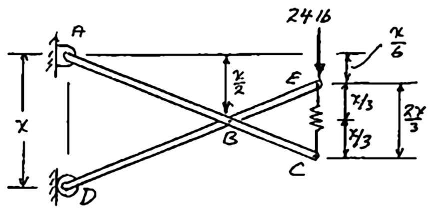
Solving by trial and error: $\theta = 0.33868$ rad $\qquad \theta = 19.40°$ ◀



PROBLEM 10.108

Two identical rods *ABC* and *DBE* are connected by a pin at *B* and by a spring *CE*. Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance *x* corresponding to equilibrium when a 24-lb load is applied at *E* as shown.

SOLUTION



Deformation of spring

$$s = EC - 4 \text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24 \text{ lb})\frac{x}{6} = \frac{1}{2}(8 \text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 4x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 4$$

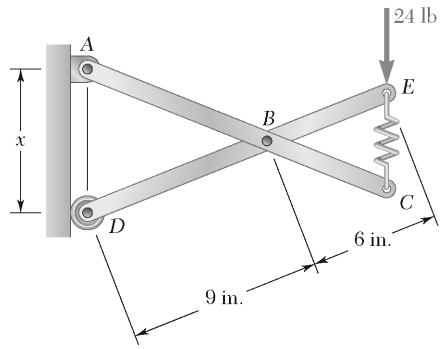
Equilibrium:

$$\frac{dV}{dx} = 0 \qquad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 4 = 0$$

$$\frac{2x}{3} - 4 = 4\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + \frac{3}{4}$$

$x = 7.13$ in. ◀

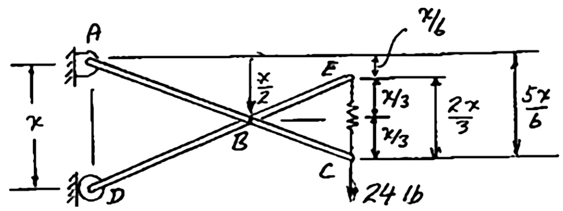


PROBLEM 10.109

Solve Problem 10.108 assuming that the 24-lb load is applied at *C* instead of *E*.

PROBLEM 10.108 Two identical rods *ABC* and *DBE* are connected by a pin at *B* and by a spring *CE*. Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance *x* corresponding to equilibrium when a 24-lb load is applied at *E* as shown.

SOLUTION



Deformation of spring

$$s = EC - 4 \text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24 \text{ lb})\frac{5x}{6} = \frac{1}{2}(8 \text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 20x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 20$$

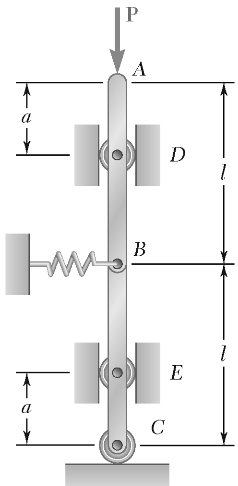
Equilibrium: $\frac{dV}{dx} = 0 \qquad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 20 = 0$

$$\frac{2x}{3} - 4 = 20\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + 3.75$$

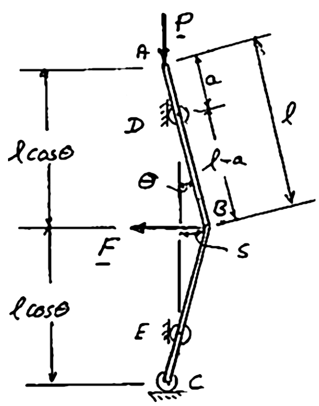
$x = 11.63$ in. ◀

PROBLEM 10.110



Two bars *AB* and *BC* are attached to a single spring of constant *k* that is unstretched when the bars are vertical. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$s = (l-a)\sin\theta$$

$$V = P(2l\cos\theta) + \frac{1}{2}ks^2$$

$$= 2Pl\cos\theta + \frac{1}{2}k(l-a)^2\sin^2\theta$$

$$\frac{dV}{d\theta} = -2Pl\sin\theta + k(l-a)^2\sin\theta\cos\theta$$

$$= -2Pl\sin\theta + \frac{1}{2}k(l-a)^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -2Pl\cos\theta + k(l-a)^2\cos 2\theta \qquad (1)$$

when $\theta = 0$: $\dfrac{d^2V}{d\theta^2} = -2Pl + k(l-a)^2$

Stability: $\dfrac{d^2V}{d\theta^2} > 0$: $\quad -2Pl + k(l-a)^2 > 0$ $\qquad P < \dfrac{k(l-a)^2}{2l}$ ◀

To check whether equilibrium is unstable for $P = \frac{k(l-a)^2}{2l}$, we differentiate

Eq. (1) twice:

$$\frac{d^3V}{d\theta^3} = 2Pl\sin\theta - 2k(l-a)^2\sin 2\theta = 0, \quad \text{For} \quad \theta = 0$$

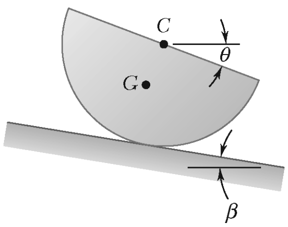
$$\frac{d^4V}{d\theta^4} = 2Pl\cos\theta - 4k(l-a)^2\cos 2\theta$$

PROBLEM 10.110 (Continued)

For $G = 0$ and $$P = \frac{k(l-a)^2}{2l}$$

$$\frac{d^4V}{d\theta^4} = 2Pl - 4k(l-a)^2$$
$$= k(l-a)^3 - 4k(l-a)^2 < 0$$

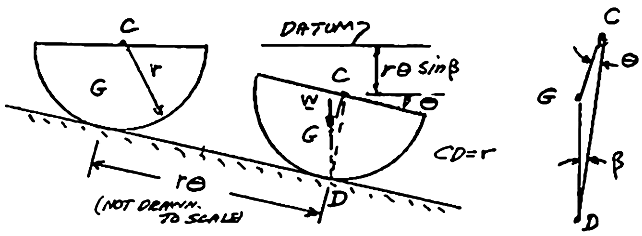
Thus equilibrium is unstable for $$P = \frac{k(l-a)^2}{2l}$$



PROBLEM 10.111

A homogeneous hemisphere of radius r is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine the angle θ corresponding to equilibrium when $\beta = 10°$.

SOLUTION



Detail

$$CG = \frac{3}{8}r$$

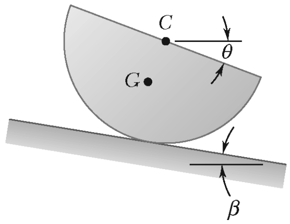
$$V = W(-r\theta \sin\beta - (CG)\cos\theta)$$

$$\frac{dV}{d\theta} = W\left(-r\sin\beta + \frac{3}{8}r\sin\theta\right)$$

Equilibrium: $\frac{dV}{d\theta} = 0, \quad -\sin\beta + \frac{3}{8}\sin\theta = 0$

$$\sin\beta = \frac{3}{8}\sin\theta \qquad (1)$$

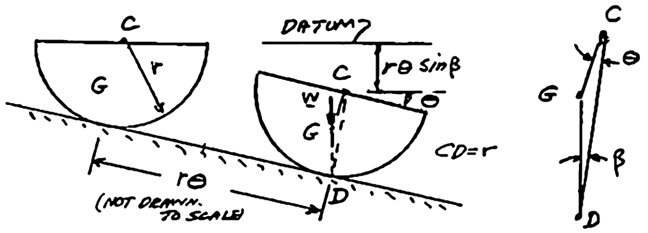
For $\beta = 10°$ $\quad \sin 10° = \frac{3}{8}\sin\theta \qquad \sin\theta = 0.46306, \quad \theta = 27.6°$ ◀



PROBLEM 10.112

A homogeneous hemisphere of radius r is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine (*a*) the largest angle β for which a position of equilibrium exists, (*b*) the angle θ corresponding to equilibrium when the angle β is equal to half the value found in part *a*.

SOLUTION



$$CG = \frac{3}{8}r$$

$$V = W(-r\theta\sin\beta - (CG)\cos\theta)$$

$$\frac{dV}{d\theta} = W\left(-r\sin\beta + \frac{3}{8}r\sin\theta\right)$$

Equilibrium: $\dfrac{dV}{d\theta} = 0, \quad -\sin\beta + \dfrac{3}{8}\sin\theta = 0$

$$\sin\beta = \frac{3}{8}\sin\theta \qquad (1)$$

(*a*) For β_{max}, $\theta = 90°$

Eq. (1) $\quad \sin\beta_{max} = \dfrac{3}{8}\sin 90°, \quad \sin\beta_{max} = \dfrac{3}{8} = 22.02° \qquad \beta_{max} = 22.0°$ ◀

(*b*) When $\beta = \frac{1}{2}\beta_{max} = 11.01°$

Eq. (1) $\quad \sin 11.01° = \dfrac{3}{8}\sin\theta; \quad \sin\theta = 0.5093 \qquad \theta = 30.6°$ ◀

Note: We can also use ΔCGD and law of sines to derive Eq. (1).

$$\frac{\sin\beta}{CG} = \frac{\sin\theta}{CD}; \quad \sin\beta = \frac{CG}{CD}\sin\theta; \quad \sin\beta = \frac{3}{8}\sin\theta$$